Abstract: This article reexamines sequential entry of firms in a Hotelling model of spatial product differentiation as analyzed by Neven (1987). Contrary to Neven, I show that the pattern of locations is generally asymmetric in the case of a duopoly. Profits are non-monotonic in market size, even in the range where the number of firms does not change. The firm that bears the “burden” of entry deterrence gains from lower barriers to entry as long as entry deterrence is possible. Equilibrium profits of all firms may be larger in situations in which more firms are active.

Keywords: Hotelling model, entry deterrence, strategic location choice

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1. Introduction

This note reexamines sequential entry of firms in a Hotelling model of spatial product differentiation as analyzed by Neven (1987). The setup consists of a standard Hotelling model with quadratic transportation costs (see, e.g., Tirole, 1988). There are no barriers to entry other than fixed entry costs and early entrants, in general, locate strategically in order to deter further entry. Following recent empirical work (see, e.g. Bresnahan and Reiss, 1991), the main question of the paper is how the equilibrium industry structure varies with changes in market size. Considering changes in market size for given fixed costs implies a slight deviation from Neven who analyzes changes in fixed costs for a given market size. The two approaches are equivalent in terms of equilibrium locations, equilibrium prices and the number of active firms. However, they exhibit different patterns of profits. The slightly different approach allows the derivation of new results as well as the correction of some of the conclusions drawn in Neven (1987).

Contrary to Neven, I show that the pattern of locations is in general asymmetric in the case of a duopoly. Profits are non-monotonic in market size, even in the range in which entry must be deterred but where the number of firms does not change. The firm which bears the “burden” of entry deterrence gains from lower barriers to entry as long as entry deterrence is possible. Finally, the simulations show that equilibrium profits of all firms may be larger in situations in which more firms are active. Larger market size more than compensates for the intensified competition, which results from lower barriers to entry.

After a first version of this paper was written, I discovered that Economides, Howell, and Meza (2002, henceforth EHM) also examine sequential entry in the Hotelling model with quadratic transportation costs. Such as Neven, they analyze market structure as a function of fixed costs and derive equilibrium locations, prices and profits. Additionally, they analyze consumer welfare and calculate various measures of the degree of asymmetry among active firms such as the Herfindahl-Hirschman index of market concentration. The simulation results
of EHM largely coincide with my results for the duopoly case. As Neven (and contrary to the first version of my paper), EHM also calculate the equilibrium for the case in which three firms must deter entry, as well as the case in which four firms are active. The results EHM report differ in two important and interesting respects from Neven. First, they claim that a range exists in the three firm case in which the profit of the second entrant is lower than that of the third, contradicting Neven’s claim that early entrants systematically earn higher profits than later firms. Second, they report what they call a discontinuity in the locations of the three firms, meaning that for certain parameter values the first entrant leaves the center location and switches positions with the second entrant. Again, this is a result not reported by Neven. Since EHM allow for locations on a rather coarse grid only (steps of $1/100^{th}$ of the unit interval) in their simulations, I checked whether their results are an artifact of this constraint. My simulations confirm the first result regarding profits. Concerning the discontinuity in locations, I also find a switch of the first entrant towards the edge of the market, but for parameter values which are completely different from the ones EHM report. In both cases, my calculations allow for rather intuitive explanations of the respective phenomena.

My simulations consider parameter constellations for which at the most three firms are active. For more firms, the problem seems to become too complex to derive reliable results. This is indicated by the fact that in the four firm case the claimed equilibrium locations of both Neven and EHM do not seem to constitute an equilibrium at least for some parameter values. I find that a fifth entrant could profitably enter in the cases I checked. Given Neven’s and EHM’s ‘equilibrium’ locations for the case of four firms, a fifth entrant could earn up to 10% and 2.5%, respectively, of fixed costs as pure profits.

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1 The results of the simulation have been derived using Mathematica. They are available as both a Mathematica notebook and as a PDF file at my homepage on http://mailbox.univie.ac.at/Georg.Goetz.
The remainder of the article is organized as follows: Section 2 describes the model, Section 3 examines the equilibrium of the model as a function of market size, Section 4 concludes.

2. The model

In order to facilitate comparability, I use Neven's (1987) setup. There is a continuum of consumers distributed uniformly over the unit segment [0,1]. The density and the total population is \( N \). As each consumer has a unit demand, \( N \) is a measure of market size. For a consumer whose location and most preferred variety is \( \hat{x} \), the (indirect) utility from consuming a good which is sold at a price \( p_i \) at location \( x_i \) is

\[
U_i = a - t (x_i - \hat{x})^2 - p_i.
\]

With respect to the (common) reservation price \( a \) I assume: \( a \geq 5/4 \). As I show below, this assumption guarantees that all consumers will always buy a product. I choose the indices of the firms in a way that firm \( i \) is located to the left of firm \( j \), i.e. \( x_i < x_j \), if \( i < j \). With respect to the transport costs parameter \( t \) I assume: \( t = 1 \). Nevertheless, the model yields conclusions about the effects of changes in transport costs. Operating profits are proportional to \( t \). Thus, a change in \( t \) has the same effect on operating profits as a change in market size \( N \).

The consumer \( \alpha \) who is indifferent between firm \( i \) and firm \( j \) is defined by the condition

\[
\alpha_{i,j} = \frac{p_j - p_i}{2(x_j - x_i)} + \frac{x_j + x_i}{2}.
\]

Suppose \( n \) firms are active. Analogously to Neven (1987) I define the sets

\[
L = \{0, \alpha_{i_1,j_1}, \ldots, \alpha_{i_{n-1},j_n}\} \quad \text{and} \quad R = \{\alpha_{i_{n-1},j_{n+1}}, \ldots, \alpha_{i_n,n+1}\}
\]

The aggregate demand faced by firm \( i \) is given by

\[
D_i = \max\{0, N(\min R - \max L)\}.
\]

The profit function of firm \( i \) reads
\[ \pi_i = (p_i - c_i)D_i - f_i, \]
where \( c_i \) and \( f_i \) denote firm \( i \)'s marginal and fixed costs, respectively. I assume \( c_i = 0 \) and \( f_i = 25 \). This does not restrict generality as the equilibrium depends only on the ratio of fixed costs to market size. The value of \( f_i \) is chosen because it yields nice numbers.

The structure of the game is as follows: In the first stage, firms enter sequentially and choose the specification of their product, i.e. their location, upon entry. After the entry stage is completed, price competition takes place in a second stage. The game is solved by backward induction. Neven (1987) shows that for any vector of products (i.e. locations) chosen by the firms a unique equilibrium of the second-stage price game exists. The arguments that ensure existence and uniqueness are concavity of the demand and the profit function as well as the linearity of the reaction functions in the rivals' prices. Contrary to the second stage of the game, no general proof of existence of a unique equilibrium is available for the location stage. In the simulations I calculate the equilibria of the whole game directly. In order to ensure that the configurations derived from first order conditions are globally optimal, I check the global behavior of the profit function for representative cases.

3. The equilibrium of the model

3.1 One and two active firms

The equilibrium of the game is a function of market size \( N \). The results are depicted in Figures 1, 2 and 3.

[Insert Figure 1 about here]

[Insert Figure 2 about here]

[Insert Figure 3 about here]

The diagrams show the equilibrium locations, the equilibrium profits and the equilibrium prices as a function of market size. These relationships are not continuous. In what follows I
depict the threshold values of market size for which changes in the type of competition takes place.

i) For $N < 144$, a second firm cannot profitably enter. The monopolist locates in the center of the market and thus deters entry. The equilibrium price is equal to $a - 1/4$, which guarantees that the whole market is covered. With the above assumption that $a \geq 5/4$, a market size $N \geq 25$ is a sufficient condition to support one firm.\(^2\)

ii) For $144 \leq N < 200$, we find a duopoly with maximum product differentiation.\(^3\) Entry of a third firm is blockaded.

iii) For $200 \leq N < 468.9$, entry is no longer blockaded. The first entrant deters further entry by unilaterally moving closer to the center as $N$ grows.

The above statement follows from figure 1. A formal proof can be found in the Appendix. The result is in contrast to Neven’s claim that both firms will move inside and choose symmetric locations (Neven 1987, p. 429). Contrary to Neven, the first entrant bears the "burden" of entry deterrence alone in this range. The burden is put in quotation marks as the profit of the first entrant increases while that of the second is constant in spite of the increase in market size.

iv) For $468.9 \leq N < 967.6$, the first entrant can no longer deter entry alone. The second entrant moves closer to the center in order to deter entry. The first entrant will move closer to the edge of the market.

In this range the condition for entry deterrence at both the center and at location zero are binding. The calculation of the subgame perfect is complicated by the fact that the optimal location for entry at the left edge is not zero but a positive value. That is, the optimal location must be calculated.

\(^2\) Note that the monopolist chooses to cover the whole market as long as $a \geq 3/4$.

\(^3\) Taking the fraction $f_i/N$ one gets the values Neven derives, i.e. .1736 and .125, resp.
At \( N = 967.6 \) the third entry deterrence constraint becomes binding. The two firms are located symmetrically at 0.330 and 0.670.\(^4\)

v) For \( 967.6 < N < 1136.9 \), we find an oligopoly with three firms. Further entry is blockaded. The equilibrium locations are: First entrant: 0.426, second entrant: 0.889, third entrant: 0.074.\(^5\)

Two general points arise from Figures 1-3.

First, market structure and profits are asymmetric, in general, once entry deterrence is taken into account. This result is in line with that of Gupta (1992) who analyses entry deterrence in a Hotelling model with spatial price discrimination. Gupta finds that in duopoly the first entrant usually locates closer to the center than the second. Tabuchi and Thisse (1995) also stress asymmetry. They allow for locations outside the range \([0,1]\) and show that the first entrant will locate in the center of the market while the second locates outside the market space. My calculations clearly show that quite asymmetric locations (consider for instance the equilibria for \( N \) close to 470) can arise even in the standard model. As regards profits, the initial asymmetry induced by the order of entry leads to a pattern of profits which is perfectly correlated with the order of entry: Early entrants earn higher profits. As will be shown in the next section, this result does not extend to the case in which three firms need to deter entry of a forth entrant.

\(^4\) This result again differs from Neven. His locations are .31 and .69. He claims that entry of a third firm cannot be prevented for values of \( f_l/N < 0.0255 \). My calculations show that the respective value is 0.0258.

\(^5\) The results for the equilibrium locations differ from Neven's in the order of 0.005. The third entrant earns higher profits in the locations I calculated, the derivative (w.r.t. location) of her profit function is positive at Neven’s location. Surprisingly, the prices, which are reported by Neven, differ from the ones I obtain if I use Neven’s locations. Again, the order of magnitude is about 0.005. See the documentation of the simulation for details.
Second, increases in market size have different effects on prices and profits. Prices decrease monotonically in market size, while profits may also be greater in larger markets. Greater market size implies lower barriers to entry (the ratio of fixed costs to market size falls). As a result potential competition is more intensive, requiring the incumbents to locate closer to each other. The resulting increase in actual competition adversely affects profits. However, by increasing demand, the increase in $N$ also exhibits a positive effect on profits. Figure 2 and the above discussion of the ranges iii) and iv)$^6$ show that the latter effect benefits only the firm that gains a larger market share due to the required entry deterring ‘relocation’. $^7$ The fact that multiple entrants must be deterred at the same time in the Hotelling model (in the relevant region) explains why it is not always the first entrant who gains from an increase in market size. The next section will show that (some) consumers might also loose if market size increases. The monotonicity of prices in market size experienced in the two firms case does not extend to the three firm case.

### 3.2 Three active firms

The results are depicted in Figures 4, 5 and 6.

[Insert Figure 4 about here]

[Insert Figure 5 about here]

[Insert Figure 6 about here]

vi) For $1136.9 \leq N < 1359.6$, further entry is no longer blockaded, a forth firm could profitably enter by locating between the first and the second entrant. As in the above cases (see iii), the first entrant deters entry unilaterally by moving closer to the center! To see that the “burden” of entry deterrence is solely borne by the first entrant, note that the second and the third entrant choose locations which are optimal in the absence of further entry.  

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$^6$ Profits of the first entrant fall in the latter range, whereas profits of the second rise.

$^7$ Of course, an increase in $N$ affects all incumbents positively in regions where entry is blockaded.
of the threat of entry. That is, they are not constrained by potential entry. For $N > 1329.7$, a third mover advantage arises in the sense that the third entrant earns a higher profit than the second. At the threshold value, the first entrant locates as close to the center as .491. The simulation shows that the second entrant earns lower profits than the third in a game with a fixed number of firms if the first entrant locates in the interval $[.491,.509]$. To understand this result, consider the case in which the first entrant locates at .5. As long as the second entrant locates at values greater than $7/8$, the third entrant locates closer to the center than the second obtaining a higher market share and higher profits. The third firm is more “aggressive” in this range than the second. The second entrant could keep the profits of the third lower than her own by locating at values smaller than $7/8$, but this would reduce her own profits. The existence of a third mover advantage, if the first entrant locates in a small and symmetric interval around the center, suggests the following explanation as to why being the second rather than the third entrant is beneficial in general. Taken as given the location of the first entrant, the second entrant can choose at which side of the market to locate. However, this possibility does not confer an advantage if the first entrant locates (almost exactly) at the center of the market, i.e. at .5.

vii) For $1359.6 \leq N < 1556.2$, the first entrant locates at .5.\textsuperscript{8} In this range, the first entrant can no longer prevent the second entrant from sharing the “burden” of entry deterrence. If the first entrant were to move closer towards the second, i.e., to locations greater than .5, the second entrant would switch to the other side of the market and obtain an even larger market share than if the first entrant locates at .5. Furthermore, note that – taking the necessity to deter entry into account – the distance

\textsuperscript{8} Neven does not seem to account for this pattern of locations. For the relevant range, the interval $[.009,.0245]$, he claims that the first entrant locates at values strictly smaller than .5 (see Neven, p. 429, Table 2).
between the second and third is maximized when the first entrant locates at .5. For the threshold value 1556.2, we obtain the equilibrium locations of the simultaneous entry game (1/8, 1/2, 7/8). The profits of the second and the third entrant are equal.

viii) For $1556.2 \leq N < 1840.9$, entry both to the left and to the right of the first entrant must be deterred. The first entrant moves towards the third, and the first and second entrant deter entry, while the third entrant chooses her unconstrained optimum location. In this range, the profit of the first entrant is falling in market size, while that of the second is increasing. The profit of the second entrant is higher than that of the third. The distance between the three firms decreases, the second entrant gets closer and closer to the center. Would the first entrant stay in the center, the second would eventually earn higher profits (for $N$ greater than about 2150), as entry deterrence requires an ever closer move to the center. As becomes clear next, the first entrant prevents that sort of second mover advantage by switching to the edge of the market.

ix) For $1840.9 \leq N < 2185.0$, the first entrant no longer chooses the central position. At the threshold value (1840.9) the profits from (optimally) locating at the center and at (optimally) locating at the edge are identical. The first entrant switches positions with the second. However, the position of the first entrant is closer to the center than that of the second entrant in the previous range. The first entrant locates in a way that just deters entry at 1. Additionally, the second entrant prevents entry between her and the first entrant. Note that the locations (third entrant: .101, second entrant: .406, first entrant: .713, $N = 1840.9$) differ also from the unconstrained locations with the first locating at the edge (third entrant: .057, second entrant: .340, first entrant: .702). The need to deter entry prevents the first entrant from locating as close to the center as she would like to. The switch in locations leads to jumps in the profits of the second and the third entrant. The second gains and the third looses. As market size increases, the first and the second entrant move closer towards one, and eventually (at 2185.4) entry
deterrence becomes binding between the second and the third entrant. All three incumbents must move in order to deter entry.

x) For $2185.0 \leq N < 2804.0$, all three incumbents move to deter entry until finally entry of a fourth entrant can no longer be deterred. The positions at the threshold value 2804.0 are: third entrant: .249, first entrant: .5, second entrant: .751. For small values of $N$ in this range the firm at the location close to 1, earns the highest profit. Therefore the first entrant takes this position. As soon as $N > 2420.9$, the center firm is the most profitable one, and the first entrant switches to the center position.

Concerning the pattern of prices as a function of market size in the case of three active firms, three results are worth mentioning: First, the firm at the center position always charges the lowest prices, except for small values of market size. In the respective range, the third entrant charges the lowest price due to the fact that the location of the center firm is biased towards 0. The identity of the firm located at the center changes, of course. Second, the prices of firms with adjacent positions may differ by up to 50% (for $N=1840.9$). This happens when the first entrant switches to the edge. The consumer indifferent between the second entrant (located at the center) and the first entrant is $\alpha_{2,3} = .671$. The comparison with the equilibrium location of the first entrant (.713) reveals that the first entrant serves hardly anyone other than consumers from her backyard. Third, contrary to the two active firm case, prices of all firms are no longer monotonically decreasing in market size. Although increases in market size decrease barriers to entry and therefore require that firms locate closer to each other, some firms may raise their prices if market size increases.

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9 The equilibrium locations derived by EHM and Neven are (.25, .5, .75) and (.245, .500, .755), respectively. My value threshold value of $N$ implies that for a ratio of fixed costs to market size ($F/N$) smaller than $0.00892$, entry can no longer be deterred. EHM and Neven both derive $F/N = .009$. The results are therefore rather similar.
Next I want to compare my results with the results of Neven and EHM. As mentioned in the introduction, the main motivation for analyzing the case of three active firms in the range of deterred entry was that EHM claim that the third entrant might earn higher profits than the second and that the pattern of locations might be discontinuous in the sense that the first entrant switches positions with the second. As described above, my numerical analysis confirms that both of these features arise; Neven does not account for either of these features! Despite the existence of a third mover advantage and a discontinuity in locations in both EHM and in my paper, the results are rather different. This becomes most obvious in the case of the discontinuity in locations. EHM claim rather loosely that a discontinuity in locations occurs “around $F = 0.018$” (p.17). From their Figure 8, one might conclude that “around” means something like the interval $[.016, .020]$. My analysis shows that the switch of the first entrant to the edge of the market occurs in a completely different range, namely in the interval $[.0103 = 25/2420.9, .0136 = 25/1840.9]$. If the ratio of fixed costs to market size is .018, the value for which EHM find a discontinuity, the profit of the first entrant is greater if he locates at the center location rather than at the edge (see the simulation). The differences in profits are so large that the result of EHM can hardly be explained by their restriction of the possible locations to ticks of $1/100^{th}$ of the unit interval.

The situation is somewhat different with respect to the third mover advantage result. I find that the third entrant earns higher profits than the second in the interval $[.0161 = 25/1556.2, .0188 = 25/1329.7]$. Unfortunately, Figure 10 of EHM, which depicts the pattern of profits is not very clear. It does not report the profits for $F/N = .017$ and $F/N = .019$; for $F/N = .018$ EHM report the profits of the case in which the first entrant locates at the edge. For $F/N = .020$, the profit of the second entrant is clearly lower than that of the third, while for both .022 and .016 the profit of the second entrant seems to be higher than that of the third.

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10 EHM seem to calculate the equilibrium values only for a very limited number of fixed costs values.
Apart from the fact that EHM obviously cannot find the third mover advantage around .018 due to the asserted discontinuity, the difference to my results could be due to the coarseness of their grid for the locations. Employing a grid with ticks of $1/100^{th}$ of the unit interval seems to imply a disadvantage beyond the fact that it naturally provides less exact results. While the small interval $[.491,.509]$ I derive makes the reason for the third mover advantage immediately obvious, the results seem to be much less clear when using EHM’s approach. At least, EHM do not give an economic interpretation as to why the second mover does not gain from moving earlier than the third if the first entrant locates at about .5.

4. Conclusions

Reexamination of the approach by Neven (1987) reveals a number of interesting results.

First, it clarifies how the ‘sharing of the burden of entry deterrence’ looks when location is the strategic variable of firms. Contrary to cases in which capital investment is the instrument to deter entry (see, e.g., McLean and Riordan, 1989), firms do not under-invest in entry deterrence. With location as the strategic variable, firms profit from participating in entry deterring activity. Early entrants provide as much of the entry deterring ‘investment’ in terms of strategic location as is compatible with potential entrants actually being deterred. Late entrants gain in terms of increasing profits as soon as early entrants are no longer able to deter entry alone; late entrants then share the entry deterring activity. The strategic moves required in this case typically increase the market share of the respective firm.

Second, it puts results regarding choice of locations in a two-firm framework into perspective. Once one allows firms to locate outside the market area, a standard result of duopoly models with sequential entry and location choice is that the first entrant locates at the center while the second entrant locates outside the unit interval (see Tabuchi and Thisse, 1995 and Tyagi, 2000). The extension to the three firm case shows that the first entrant does not necessarily select the central position even if firms exhibit identical costs. Under the threat of
entry, locating at the center, i.e. at a position which Tyagi (2000) calls the “most attractive location”, might not be optimal from the viewpoint of the first mover. The results for the three firm case also demonstrate that locating outside the range where the consumers are is an artifact of the duopoly case. All three incumbents locate strictly inside the unit interval if entry is blockaded. The edges of the market are not a binding constraint in location choice. This becomes even more obvious when considering the choices of the potential entrants in the three firms case which are actually deterred. The potential entrants (call them firm number 4 and 5) at the edges of the market (i.e. at 0 and at 1) would choose a location strictly inside the market area even though the incumbents locate at a distance to the edge as small as .249.

Finally, equilibrium profits of all active firms may be larger in situations in which more firms are active. To see this, compare the situation with two firms and a market size of about 250 with the case with three firms and a market size of about 2800. The reason is again the double effect of an increase in market size mentioned in the above paragraph. The result in terms of market size differs from the pattern of profits that Neven derives for c.p. changes in fixed costs. In the latter case, profits tend to fall as a result of an increasing number of firms (Neven, 1987, p. 427). Anderson et al. conclude from Neven’s work that “equilibrium profits decrease monotonically with the number of firms” (Anderson, de Palma, Thisse, 1992, p. 302). My result shows that such a general conclusion is hardly justified.

5. Appendix

Proof of the claim that the first entrant will deter a third entrant unilaterally in the range, in which the potential entrant would locate in the center and no other entry deterrence constraint is binding (200 ≤ N < 468.9). To prove the result, I first show that the third entrant will always locate exactly halfway between the two incumbents.

The reduced profit function of the potential entrant reads
\[ \pi^2(x_1,x_2,x_3) = \frac{(x_2 - x_1)(d + x_1 - x_3)(2 + d)^2 N}{18d} - f_L, \]  

(4)

where \( d \equiv x_3 - x_1 \) is the distance between the incumbents. It is straightforward to derive the optimal location of the second entrant. One obtains: \( x_2 = x_1 + d/2 \). Given this choice, the profit of the potential entrant depends only on the distance between the incumbents.

Given the (entry deterring) distance between the incumbents, one can determine the profit of the first entrant for two cases. First, both incumbents move to the center by the distance \( i/2 \). (The locations are \( i/2 \) and \( 1-i/2 \)) The profit of the first (and the second) entrant is \( (1-i)N/2 - f_L \) in this case. If only the first entrant would move to the center by the distance \( i \), her profit reads \( (1-i)(3+i)^2 N/18 - f_L \). (The profit of the second entrant equals \( (1-i)(3-i)^2 N/18 - f_L \)). The first entrant’s profit is greater if she deters entry unilaterally. As always with quadratic transportation costs and two firms (see Tirole, 1987, p. 281), the second entrant has an incentive to locate at maximum distance from the first entrant, i.e., at the edge of the market. The fact that the second entrant is not constrained by a threat of entry proves the claim.

6. References


Figure 1: Equilibrium locations in small markets (small $N$). First (solid line), second and third entrant (dashed lines).

Figure 2: Equilibrium profits in small markets. First (solid line), second and third entrant (dashed lines).
Figure 3: Equilibrium prices in small markets. First (solid line), second and third entrant (dashed lines).

Figure 4: Equilibrium locations in large markets (large $N$). First (solid line), second (dashed line) and third entrant (dotted line).
Figure 5: Equilibrium profits in large markets (large $N$). First (solid line), second (dashed line) and third entrant (dotted line).

Figure 6: Equilibrium prices in large markets (large $N$). First (solid line), second (dashed line) and third entrant (dotted line).