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Partial cross ownership and collusion

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Partial cross ownership and collusion

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Abstract

This article finds that non-controlling minority shareholdings among competitors lower the sustainability of collusion. This is the case under an even greater variety of situations than was indicated by earlier literature. The collusion destabilizing effect of minority shareholdings is mainly caused by their unilateral effects, and it is particularly prevalent in the presence of an effective antitrust authority.

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1 Introduction

This article shows in a formal model that non-controlling minority shareholdings (NCMS) among competitors *lower* the sustainability of collusive agreements under a great variety of circumstances and especially in the presence of an effective antitrust authority.

One speaks of non-controlling minority shareholdings if firm i buys a stake in a rival j that is lower than 50% and does not grant control rights, i.e., the buyer acquires a silent interest. The acquisition of non-controlling minority shareholdings is subject to merger control in some jurisdictions such as Austria, Germany, UK, US, and Japan but not in others like the European Union (EU). Therefore, in July 2014 the EU issued a White Paper discussing an amendment of the current EU Merger Regulation towards assessing non-controlling minority shareholdings without, however, reaching a conclusion. This topic had been on the agenda for some time, as it had already been addressed by the EU's Green Paper published in 2001.[1]

The present article contributes to this discussion by assessing the effects of NCMS on the sustainability of collusion. Paralleling the policy discussion, this topic has been studied in academia at least since the seminal contribution of Reynolds and Snapp (1986) without, however, having been fully resolved because the related literature (that is reviewed in Section 2) uses different assumptions, for example, on firms' profit function. This can be an impediment to the comparison of different articles' results.

Our article integrates established models on NCMS into a more comprehensive one. This fills a gap in the literature because we analyze several combinations of assumptions on, e.g., profit functions and models of competition that have not been studied jointly by prior literature. We also provide analytic proofs for effects that have only been established numerically by prior literature. Moreover, our model is the first to study analytically how a competition authority impacts the effect of minority shareholdings on the sustainability of

[1] Salop and O'Brien (2000) review some cases of minority shareholdings in the US and related literature, providing an introduction to the topic from the viewpoint of law and economics. They however concentrate on the unilateral effects of minority shareholdings rather than discussing their impact on collusion. Tzanaki (2015) presents more recent cases of minority shareholdings and discusses their legal treatment in the EU.

collusion.

The model indicates that NCMS have an ambiguous effect on collusion. On the one hand, a deviation from a collusive agreement is less profitable for a firm holding shares of another cartel firm, which contributes to stabilizing collusion. This is because the deviator would receive lower dividends from the cheated firm. On the other hand, minority shareholdings cause unilateral effects by raising profits in competition. This softens the punishment that a deviator expects after deviating from collusion, and it contributes to destabilizing collusion. Minority shareholdings destabilize collusion if the second effect prevails over the first as has been shown by prior literature (Malueg 1992).

We show that minority shareholdings destabilize collusion under a wider set of assumption than was suggested by earlier literature. This is especially the case in the presence of an antitrust authority, because a cartel firm must not only expect to be fined itself but – by receiving lower dividends – it also participates in the fines imposed on its co-conspirator. More importantly, we believe to be the first who show that although the shareholdings of firm i in firm j have opposing effects on its own incentives to sustain collusion, they destabilize collusion by making it harder for firm j to collude.

The article is structured as follows. Section 2 details how the present study contributes to the existing literature. Section 3 provides the model and studies the unilateral effects of NCMS in the stage game. Section 4 analyzes the effects of NCMS on the sustainability of collusion. The robustness of our results is established in Section 5. We present some case evidence in support of our theoretical predictions in Section 6. Section 7 concludes the article. Proofs are stated in the Appendix.

2 Literature Review

Reynolds and Snapp (1986) established that in a static Cournot model with symmetric firms and homogeneous goods NCMS cause unilateral effects. With NCMS it would be a best response for the firms to restrict aggregate output and raise aggregate profits. Empirical evidence of such unilateral effects on the fares of US airlines has been provided by Azar *et al.* (2016) for the related issue of common ownership. Trivieri (2007) shows that cross-ownership also reduced the degree of competition among Italian banks.

Reynolds and Snapp's (1986) analysis was extended to an infinitely repeated game by Malueg (1992) who shows that NCMS have an ambiguous impact on the stability of collusion. On the one hand, a colluding firm holding shares of its co-conspirator receives a lower short-run gain when cheating. This is because the deviator receives lower dividends when deviating because being cheated depresses the profits of its former co-conspirator. On the other hand, by softening competition the unilateral effects of NCMS also soften the long-run punishment that is imposed on the deviating firm. Hence, by lowering both the short-run gain from a deviation and the long-run punishment for such conduct NCMS have an ambiguous effect on the sustainability of collusion. Concerning the net effect, Malueg (1992) shows that NCMS lower the sustainability of collusion only when demand is convex.

We extend Malueg's (1992) seminal contribution in several directions. While he studies symmetric shareholdings only, we also allow for asymmetric NCMS. Besides Cournot competition with homogeneous goods, we also study Bertrand competition with homogeneous and with differentiated products. We present analytic proofs for effects that have been shown by Malueg (1992) only numerically. Finally, we introduce a competition authority and show that NCMS may de-stabilize collusion even for non-convex demand.

Some of our extensions were inspired by Flath (1991) and Flath (1992) who presents a static model to study the unilateral effects of NCMS not only in Cournot competition with homogeneous goods, but also in Bertrand competition with either differentiated or homogeneous goods.[2] Unfortunately, the results of Flath (1991) and Flath (1992) cannot be compared to those of Reynolds and Snapp (1986) or Malueg (1992). This is because Flath

[2]A related analysis was conducted by Reitman (1994) in a conjectural variations model. Reitman (1994) builds on the analysis of Kwoka (1992) who had studied joint ventures, where parent companies hold shares in a newly formed entity.

(1991) assumes the firms to maximize product-market profits plus dividends received before subtracting dividends paid. This models a situation where firm i holds shares in firm j and maximizes operating profits plus dividend income. Reynolds and Snapp (1986) and Malueg (1992) however assume that the sum of product-market profits plus dividends received minus dividends paid is to be maximized. This models a situation of common ownership where the majority shareholder of firm i also holds a minority share in firm j. We compare the effects caused by these different viewpoints by implementing our model under both assumptions.

In doing so, we also contribute to Gilo *et al.* (2006) who use the profit function assumed by Flath (1991) and extend his analysis to a dynamic game, which allows them to study the effect of minority shareholding on collusion. However, they only analyze a Bertrand model with homogeneous goods where minority shareholdings do not cause any unilateral effects such that the long-run punishment following a deviation is not softened by NCMS. They find minority shareholdings to stabilize collusion because the deviator only takes into account that a deviation lowers the dividends it receives from its former co-conspirators. We extend their model by assuming also Cournot competition with homogeneous goods and Bertrand competition with differentiated goods. Under these assumptions minority shareholdings may also stabilize collusion because of the second effect described above: By raising competitive profits minority shareholdings soften the long-run punishment following a deviation. Hence and contrary to the conclusion of Gilo *et al.* (2006), it is quite likely that NCMS hinder collusion.

We add a new element to this discussion by assuming an antitrust authority along the lines of Aubert *et al.* (2006), i.e., collusion may be detected with a certain probability and sanctioned thereafter. We show that in the presence of an effective antitrust authority NCMS are quite likely to lower the sustainability of collusion under a variety of conditions where the literature cited above suggested a stabilizing effect of NCMS on collusion. For example, under the assumption of a competition authority NCMS are found to destabilize collusion also for non-convex demand, which contributes to Malueg (1992), and even for Bertrand competition with homogeneous products, which extends the results of Gilo *et al.* (2006).

The literature cited above studies how firm i's shares of firm j affect the sustainability of collusion through the shareholdings' effect on firm i's critical discount factor. We also study how firm j's shares of firm i affect firm i's critical discount factor. As a central result, we find that minority shareholdings destabilize collusion in Cournot competition with homogeneous products and in Bertrand competition with differentiated products, i.e., firm i's discount factor rises when firm j raises its shareholdings in firm i. This is because under the assumption of imperfect competition firm j reduces the intensity of competition and makes higher profits after buying shares α_j of firm i even when competing. Hence, firm i reasons that it will only be punished softly after deviating from a collusive agreement, which lowers the sustainability of collusion.

The present article along with Reynolds and Snapp (1986), Flath (1991), Malueg (1992), and Gilo *et al.* (2006) concentrates on the most anticompetitive acquisition decisions, i.e., decisions being purely driven by the rationale of receiving a dividend and raising the acquirer's expected profits. Other authors have pointed out that minority shareholdings may be made, for example, to provide a financially constrained target firm with additional funds, to solve hold-up problems when engaging in joint investment projects (Ouimet 2013), or they may be driven by efficiency considerations such as the generation of economies of scope in the production process (Karle *et al.* 2011). Our model indicates that even inherently anti-competitive NCMS, which were only acquired to raise the profits of the acquirer, often have pro-competitive effects by disrupting collusion. To show this point most clearly we abstract from further efficiency considerations.

To summarize, we (i) compare Reynolds and Snapp (1986), Flath (1991), Malueg (1992), and Gilo *et al.* (2006) pointing out differences and similarities of their approaches, (ii) complete the analysis of assumptions that have not been studied jointly before, (iii) provide formal proofs for effects that have only been established numerically by prior literature, and (iv) add a competition authority. This establishes a coherent framework for the analysis of the effects of NCMS on the stability of collusion.

3 The Model

Subsection 3.1 presents the setup of the model. Subsection 3.2 establishes the unilateral effects of minority shareholdings in the stage game.

3.1 Setup

The timing of our game is based on the assumptions of Aubert *et al.* (2006). Two symmetric, risk-neutral firms i and j play an infinitely repeated game where, in each period, they have the opportunity to communicate before interacting on the product market. In the first stage, communication takes place if both firms agree to communicate. In the second stage, firms can always choose the strategy *compete*. If communication took place, they can choose either the strategy *compete* or the strategy *collude* instead.

Along with Aubert *et al.* (2006) communication is not treated in a game theoretic context as a device to overcome problems arising from, e.g., incomplete information about each firm's type. Those considerations are absent from our model. Communication is rather treated in an antitrust context as a prerequisite for explicit collusion. It leaves traces that can be discovered by an antitrust authority, which would ultimately prosecute the firms. Hence, communication is merely needed to establish that firms violate antitrust laws.

Firm i maximizes the expected discounted sum of its profits. Its discount factor is denoted $\delta_i \in (0, 1)$. An antitrust authority detects a collusive agreement with probability ρ in every period and imposes a fine F on every firm. In each period, the product market profit of firm i net of the expected fine is

$$
\begin{array}{ll}
\pi_{i,c} & \text{if both firms compete,} \\
\pi_{i,k} - \rho F & \text{if both firms collude,} \\
\pi_{i,d} - \rho F & \text{if firm } i \text{ competes and firm } j \text{ colludes,}
\end{array}
$$

$\pi_{i,-d} - \rho F \qquad$ if firm i colludes and firm j competes.

Further below we analyze three types of models, these are, Cournot competition with homogeneous products, Bertrand competition with homogeneous products, and Bertrand competition with differentiated products. In these models the conditions shown in (1) apply that impose a prisoner's dilemma structure on the game.

$$\pi_{i,d} > \pi_{i,k} > \pi_{i,c} \geq \pi_{i,-d} \text{ and } \pi_{i,d} + \pi_{i,-d} < 2\pi_{i,k} \tag{1}$$

In addition to Aubert *et al.* (2006), we assume that firm i may hold a stake α_i in firm j while firm j may hold a stake α_j in firm i. The stakes may be asymmetric (i.e., $\alpha_i \neq \alpha_j$). Equation (2) shows that firm i is assumed to maximize its total payoff, i.e., the profit earned in the product market plus its share α_i in the profits of the other firm j (Flath 1991, Gilo *et al.* 2006, Shelegia and Spiegel 2012).

$$max\ \hat{\pi}_i = \pi_i + \alpha_i \hat{\pi}_j \tag{2}$$

To be specific about notation, π_i denotes the reduced product-market profit / operating profit of firm i. It depends on the value of the shareholdings α_i and α_j, i.e., $\pi_i(\alpha_i, \alpha_j)$, as well as exogenous variables such as demand parameters. For reasons of conciseness, we often write π_i instead of $\pi_i(\alpha_i, \alpha_j)$. In line with earlier literature (Reynolds and Snapp 1986, Malueg 1992, Gilo *et al.* 2006) the values of the shareholdings α_i and α_j are assumed to have been chosen prior to the game analyzed here. Ouimet (2013) explores the reasons why firms invest in minority shareholdings. The firms are assumed to be cost-symmetric. The competitive, collusive, and deviant profits ($\hat{\pi}_{i,c}$, $\hat{\pi}_{i,k}$, $\hat{\pi}_{i,d}$) after dividends received can be expressed as in (3)-(5).

$$\hat{\pi}_{i,c} = \frac{\pi_{i,c} + \alpha_i \pi_{j,c}}{1 - \alpha_i \alpha_j} \tag{3}$$

$$\hat{\pi}_{i,k} = \frac{(\pi_{i,k} - \rho F) + \alpha_i(\pi_{j,k} - \rho F)}{1 - \alpha_i \alpha_j} \tag{4}$$

$$\hat{\pi}_{i,d} = \frac{(\pi_{i,d} - \rho F) + \alpha_i(\pi_{j,-d} - \rho F)}{1 - \alpha_i \alpha_j} \tag{5}$$

3.2 The Stage Game: Unilateral Effects

This subsection establishes the unilateral effects of minority shareholdings. It analyzes how changes in α_i and α_j affect the competitive, collusive, and deviant profits of firm i.

Cournot competition with homogeneous products

In Cournot competition with homogeneous products, firm i's profit $\hat{\pi}(q_i, q_j)$ is a function of the outputs q_i and q_j of the two firms. The best-response output $\hat{q}_i^R(q_j, \alpha_i)$ of firm i depends on q_j and α_i. The firms are said to compete if both play their best responses, making profits $\hat{\pi}_{i,c}(\alpha_i, \alpha_j) = \hat{\pi}_i \left(\hat{q}_i^R(\alpha_i), \hat{q}_j^R(\alpha_j)\right)$. The discussion below relies on a result that was established by Flath (1991) and Farrell and Shapiro (1990), and that is summarized in Lemma 1.

Lemma 1. $\frac{\partial \pi_{i,c}}{\partial \alpha_i} < 0$ *and* $\frac{\partial \pi_{i,c}}{\partial \alpha_j} > 0$

Proof. See Flath (1991), Farrell and Shapiro (1990), and Appendix A $\square$

Lemma 1 implies that the competitive product-market profits $\pi_{i,c}$ of firm i (as opposed to the total payoff $\hat{\pi}_{i,c}$) rise if firm j holds a higher share α_j in firm i. The product-market profits $\pi_{i,c}$ however fall if firm i holds a higher share α_i in firm j. This is because quantities are strategic substitutes, and firm i finds it optimal to reduce both its own output ($\partial \hat{q}_i^R / \partial \alpha_i < 0$) and its product-market profits $\pi_{i,c}$ in order to raise its total payoff $\hat{\pi}_{i,c}$ by raising the other firm's profit $\pi_{j,c}$ and receiving a higher dividend from firm j. Conversely, a higher value of α_j also raises $\pi_{i,c}$.

In line with the related literature (Malueg 1992, Aubert *et al.* 2006, Gilo *et al.* 2006), the firms are assumed to collude in the product market by setting a 50%-share of the monopoly output (i.e., $q_{i,k} = q_{j,k} = Q_k/2$). As the collusive output is independent of α_i and α_j, the collusive profits are also independent of the value of shareholdings by assumption (i.e., $\partial\pi_{i,k}/\partial\alpha_i = 0$, $\partial\pi_{i,k}/\partial\alpha_j = 0$).

Deviation profits are defined as $\hat{\pi}_{i,d}(\alpha_i) = \hat{\pi}_i\left(\hat{q}_i^R(\alpha_i), q_{j,k}\right)$ and $\hat{\pi}_{j,-d}(\alpha_i) = \hat{\pi}_j\left(\hat{q}_i^R(\alpha_i), q_{j,k}\right)$, i.e., firm i plays its best response while firm j sets the agreed-upon output. Lemma 2 establishes the effect of α_i on the product market profits in a deviation period.

Lemma 2. $\frac{\partial\pi_{i,d}}{\partial\alpha_i} < 0$ *and* $\frac{\partial\pi_{j,-d}}{\partial\alpha_i} > 0$

Proof. See Appendix A □

If firm i deviates from collusion, it receives a lower dividend from firm j as compared to continued collusion ($\alpha_i\pi_{j,-d} < \alpha_i\pi_{j,k}$). The higher the value of α_i the stronger is this effect and the lower is the profit $\hat{\pi}_{i,d}$ that firm i earns after the payment of dividends. Accordingly, cross-shareholdings $\alpha_i > 0$ induce the deviating firm i to set a lower deviation quantity than with $\alpha_i = 0$ and, thus, earn lower deviation profits. This leaves higher profits for firm j ($\partial\pi_{j,-d}/\partial\alpha_i > 0$).

Bertrand competition with differentiated products

Similar effects are found when assuming Bertrand competition with differentiated products. There is however one important difference to the Cournot model with homogeneous products: Because prices in the Bertrand model are strategic complements higher shareholdings α_i may raise the competitive product market profit $\pi_{i,c}$ of firm i. The profit only falls in α_i if product differentiation is low and if the cross-shareholdings α_i and α_j are sufficiently asymmetric.

To see this consider that firm i's profit $\hat{\pi}(p_i, p_j)$ is a function of the prices p_i and p_j of the two firms. Let $\hat{p}_i^R(p_j, \alpha_i)$ denote the best-response function of firm i. The firms compete if both play their best responses, making profits $\hat{\pi}_{i,c}(\alpha_i, \alpha_j) = \hat{\pi}_i\left(\hat{p}_i^R(\alpha_i), \hat{p}_j^R(\alpha_j)\right)$. Flath (1991) establishes Lemma 3.

Lemma 3. $\frac{\partial \pi_{i,c}}{\partial \alpha_j} > 0$ *and* $\frac{\partial \pi_{i,c}}{\partial \alpha_i} \begin{cases} > 0 & \text{if } (\alpha_i - \alpha_j) < \Delta\alpha^* \\ \leq 0 & \text{if } (\alpha_i - \alpha_j) \geq \Delta\alpha^* \end{cases}$

Proof. See Flath (1991) and Appendix A □

Unlike in Cournot competition with homogeneous products, firm i's competitive profit rises even for unilateral increases of its share α_i in firm j as long as α_i and α_j are not too asymmetric. This is because in a Bertrand model with differentiated products prices are strategic complements. Shareholdings α_i induce firm i to raise its price, and firm j follows suit. Therefore, even a somewhat asymmetric increase in α_i may cause unilateral effects.

The firms are assumed to collude in the product market by setting the same prices $p_{i,k}$ and $p_{j,k}$ that a jointly profit-maximizing monopolist would set. These prices are independent of α_i and α_j, which implies $\partial\pi_{i,k}/\partial\alpha_i = 0$ and $\partial\pi_{i,k}/\partial\alpha_j = 0$. The deviation profits are defined as $\hat{\pi}_{i,d}(\alpha_i) = \hat{\pi}_i\left(\hat{p}_i^R(\alpha_i), p_{j,k}\right)$ and $\hat{\pi}_{j,-d}(\alpha_i) = \hat{\pi}_j\left(\hat{p}_i^R(\alpha_i), p_{j,k}\right)$. Appendix A shows that Lemma 2 (i.e., $\partial\pi_{i,d}/\partial\alpha_i < 0$ and $\partial\pi_{j,-d}/\partial\alpha_i > 0$) applies also in Bertrand competition with differentiated goods.

Bertrand competition with homogeneous products

In Bertrand competition with homogeneous products "both firms set prices equal to marginal cost regardless of the state of any partial cross shareholding" (Flath 1991), i.e., the firms make zero profits ($\pi_{i,c} = 0, \partial\pi_{i,c}/\partial\alpha_i = 0, \text{and } \partial\pi_{j,c}/\partial\alpha_i = 0$), and minority shareholdings do not cause unilateral effects. Similarly, the collusive and the deviation profits are also independent of the value of minority shareholdings (i.e., $\partial\pi_{i,k}/\partial\alpha_i = 0$, $\partial\pi_{j,k}/\partial\alpha_i = 0$, $\partial\pi_{i,d}/\partial\alpha_i = 0$, $\partial\pi_{j,-d}/\partial\alpha_i = 0$). A deviating firm would cut the collusive price marginally and earn $\pi_{i,d} = 2\pi_{i,k}$ while the betrayed firm would earn $\pi_{j,-d} = 0$.

4 The Dynamic Game: Coordinated Effects

Using the framework introduced in Section 3, we study the effects of NCMS on the sustainability of collusion. Subsection 4.1 points out the forces that determine the effect of α_i on the critical discount factor δ_i^* in a general model. Subsection 4.2 applies this analysis to specific models of competition. Subsection 4.3 analyzes the effect of firm j's shareholdings α_j on firm i's critical discount factor δ_i^*. We find that an increase in α_j destabilizes collusion by raising δ_i^*. This effect is new as it has not been analyzed by any of the related articles. Section 4.4 shows that collusion is destabilized further if a higher level of shareholdings α_i raises the probability ρ of collusion being detected by a competition authority.

4.1 The Critical Discount Factor

Collusion is profitable for the firms if inequality (6) is satisfied. Collusion is sustainable if inequality (7) applies.

$$\pi_{i,k} - \pi_{i,c} > \rho F \quad \forall \quad i \in \{1, 2\} \tag{6}$$

$$\frac{\hat{\pi}_{i,k}}{1-\delta_i} > \hat{\pi}_{i,d} + \frac{\delta_i}{1-\delta_i}\hat{\pi}_{i,c} \quad \forall \quad i \in \{1, 2\} \tag{7}$$

The present value of deviation payoffs (i.e., the right-hand side of (7)) assumes a grim trigger strategy (Friedman 1971). This assumption is made to keep the model consistent with prior literature (Malueg 1992, Aubert *et al.* 2006, Gilo *et al.* 2006). It is relaxed in Section 5.2. Using equation (2) (i.e., $\hat{\pi}_i = \pi_i + \alpha_i \hat{\pi}_j$), the sustainability constraint (7) can be solved for the critical value δ_i^* of the discount factor as is shown in equation (8). For individual discount factors above this threshold, collusion is a stable outcome.

$$\delta_i > \frac{\hat{\pi}_{i,d} - \hat{\pi}_{i,k}}{\hat{\pi}_{i,d} - \hat{\pi}_{i,c}} = \frac{(\pi_{i,d} - \pi_{i,k}) + \alpha_i(\pi_{j,-d} - \pi_{j,k})}{(\pi_{i,d} - \rho F - \pi_{i,c}) + \alpha_i(\pi_{j,-d} - \rho F - \pi_{j,c})} \quad \equiv \delta_i^* \tag{8}$$

Proposition 1 establishes under which condition the critical discount factor rises in the value of minority shareholdings α_i.

Proposition 1. *The inequality* $\partial\delta_i^*/\partial\alpha_i > 0$ *applies if inequality* (9) *is satisfied.*

$$\frac{(\pi_{i,k} - \rho F - \pi_{i,c})(\pi_{i,d} - \pi_{j,-d})}{(\pi_{i,d} + \alpha_i \pi_{j,-d}) - (\pi_{i,k} + \alpha_i \pi_{j,k})} < \frac{\partial \pi_{i,c}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{j,c}}{\partial \alpha_i} \tag{9}$$

Proof. See Appendix B. □

The terms in equation (9) take the following signs: Collusion must be profitable to be established ($\pi_{i,k} - \rho F - \pi_{i,c} > 0$, see equation (6)). The deviating firm earns a higher profit than the betrayed firm ($\pi_{i,d} - \pi_{j,-d} > 0$). Given the prisoner dilemma structure of the game a firm earns a higher profit (including dividends) by deviating from a collusive agreement as compared to adhering to it ($(\pi_{i,d} + \alpha_i \pi_{j,-d}) - (\pi_{i,k} + \alpha_i \pi_{j,k}) > 0$). The term $(\partial\pi_{i,c}/\partial\alpha_i) + \alpha_i(\partial\pi_{j,c}/\partial\alpha_i)$ captures the unilateral effects of the minority shareholdings. Lemma 4 establishes that this sum is non-negative for any of the models of competition introduced in Section 3.2.

Lemma 4. $\frac{\partial \pi_{i,c}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{j,c}}{\partial \alpha_i} \geq 0$

Proof. See Appendix B. □

Higher shareholdings α_i have two effects on the sustainability of collusion. Effect 1: Malueg (1992) argues that higher shareholdings have a destabilizing effect on collusion by causing unilateral effects and softening the punishment following a deviation. This can also be seen in our model where one finds $\partial\delta_i^*/\partial\alpha_i > 0$ when the right-hand side of inequality (9) is sufficiently high. Effect 2: NCMS have a stabilizing effect on collusion because a higher value of α_i causes a greater loss of dividend income if firm i deviates from the collusive agreement. This makes a firm more reluctant to deviate as can be seen from inequality (9) because $\alpha_i(\pi_{j,k} - \pi_{j,-d})$ measures the loss of dividends received from firm j if firm i deviates. A higher value of α_i raises the left-hand side of (9) and, thus, contributes to situations with $\partial\delta_i^*/\partial\alpha_i < 0$.

In addition to prior literature, our framework also allows to study the effect of an antitrust authority on the stability of collusion. This is done by solving inequality (9) for ρF, which yields (10).

$$\rho F > \pi_{i,k} - \pi_{i,c} - \frac{\left[\frac{\partial \pi_{i,c}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{j,c}}{\partial \alpha_i}\right] \left[(\pi_{i,d} + \alpha_i \pi_{j,-d}) - (\pi_{i,k} + \alpha_i \pi_{j,k})\right]}{\pi_{i,d} - \pi_{j,-d}} \equiv (\rho F)^* \qquad (10)$$

One finds $\partial \delta_i^* / \partial \alpha_i > 0$ if $\rho F > (\rho F)^*$ applies, i.e., a high value of the expected sanction makes it more likely that minority shareholdings destabilize collusion. This is because of a *double deterrence* effect. In a situation without minority shareholdings firm i only fears the sanctions being imposed on itself. However, in a situation with minority shareholdings the competition authority also imposes a fine on firm j, which lowers the expected dividend income of firm i. This has an effect on the sustainability of collusion. A high value of ρF causes the expected dividend income of firm i to be low whether the firm adheres to the collusive agreement or not. This makes a deviation relatively more profitable for firm i.

In other words, a high value of ρF weakens the stabilizing Effect 2 explained above, which stated that minority shareholdings ($\alpha_i > 0$) make firm i more hesitant to deviate as the deviation would lower its dividend income. Mathematically, a higher value of ρF lowers the left-hand side of (9), which makes it more likely to find $\partial \delta_i^* / \partial \alpha_i > 0$. As Effect 2 is weakened for higher values of ρF, Effect 1 gains importance, which stated that higher shareholdings α_i destabilize collusion especially when they soften punishments by causing unilateral effects. To summarize, our model shows that minority shareholdings raise δ_i^* even for $\rho F = 0$ if their unilateral effects are strong enough. This destabilizing effect is enhanced by $\rho F > 0$.

Our results extend those obtained by Malueg (1992) who, however, concentrated on the study of Cournot competition with homogeneous goods, restricted his analysis to symmetric shareholdings, did not allow for a competition authority, and obtained his central results only by numerical simulation. This section advanced his analysis by providing formal proofs in a more general model, i.e., without assuming a specific model of competition, and while allowing both for asymmetric shareholdings and a competition authority.

4.2 Coordinated Effects in Different Models of Competition

This section explores the effect of minority shareholdings on the stability of collusion in different models of competition. This extends and reconciles the results of Gilo *et al.* (2006), who find that minority shareholdings never hinder tacit collusion in Bertrand competition with homogeneous goods, and those of Malueg (1992), who suggests just the opposite in Cournot competition. We find that in the presence of a competition authority minority shareholdings destabilize collusion even under a wider set of assumptions than was predicted by Malueg (1992).

Bertrand competition with homogeneous goods

Assuming price competition with homogeneous goods creates a relatively simplistic version of our model because this model assumes the absence of unilateral effects ($\partial\pi_{i,c}/\partial\alpha_i = 0, \partial\pi_{j,c}/\partial\alpha_j = 0$). Therefore, the right-hand side of inequality (9) takes a value of zero, which implies $\partial\delta_i^*/\partial\alpha_i < 0 \quad \forall \quad \alpha_i$. In the absence of unilateral effects the collusion-destabilizing Effect 1, which was discussed in the context of Proposition 1, vanishes because NCMS cannot destabilize collusion by softening punishments. This only leaves Effect 2: Higher shareholdings α_i stabilize collusion because firm i would receive a lower dividend income otherwise, i.e., when deviating from a collusive agreement. This result is in line with the finding of Gilo *et al.* (2006) who show that under the assumption of Bertrand competition with homogeneous goods "an increase in firm [i]'s stake in firm [j] never hinders tacit collusion". However, this result cannot be supported for imperfectly competitive industries as is explored in the following.

Cournot competition with homogeneous goods

Malueg (1992) found by means of numerical simulations that in a Cournot model with homogeneous goods a symmetric increase in shareholdings may very well hinder tacit collusion. This was formally proven in our general model in Subsection 4.1. Malueg (1992) shows further that a symmetric increase of shareholdings α_i raises δ_i^* for convex demand but not for linear or concave demand. Lemma 5 however suggests that under the assumption of a sufficiently effective competition authority higher minority shareholdings destabilize collusion even for non-convex demand.

Lemma 5. *For $\rho F > (\rho F)^*$ one finds $\partial \delta_i^* / \partial \alpha_i > 0$ even for non-convex demand.*

Proof. See Appendix B. □

Lemma 5 shows that in the presence of an antitrust authority pro-competitive (i.e., collusion-destabilizing) effects of minority shareholdings are even more prevalent than was predicted by prior literature.[3]

Bertrand competition with differentiated goods

This subsection compares the results obtained for Cournot competition with homogeneous products to Bertrand competition with differentiated products. We argue that the pro-competitive effects of NCMS (i.e., $\partial \delta_i^* / \partial \alpha_i > 0$) are more likely in Bertrand competition with differentiated goods than in Cournot competition with homogeneous goods. This is because in Bertrand competition with differentiated goods asymmetric increases of α_i raise the right-hand side of inequality (9) more strongly than in Cournot competition with homogeneous goods.

Note that a formal proof of this point would necessarily remain imperfect. For a meaningful comparison, one would have to make assumptions about the functional forms of demand

[3] Note that Malueg (1992) assumed the firms to maximize the profit function $\tilde{\pi}_i = (1 - \alpha_j)\pi_i + \alpha_i \pi_j$ rather than $\hat{\pi}_i = \pi_i + \alpha_i \hat{\pi}_j$. Although these functions model different aspects of minority shareholdings (i.e., common ownership vs. cross shareholdings) their effect on our results is minor and has no impact on our qualitative conclusions as will be discussed in Section 5.

and costs, while imposing constraints on, e.g., market size, product differentiation, and demand elasticities to ensure that at least for $\alpha_i = \alpha_j = 0$ the profits and their first derivatives are of about the same size in both models, such that left-hand side and the right-hand side of inequality (9) would be comparable in Cournot and in Bertrand competition. Therefore, we followed Malueg (1992) and resorted to a numerical simulation whose results we present in the following.

Lemma 1 established that in Cournot competition with homogeneous products a unilateral increase of the shareholdings α_i of firm i in firm j raise the product market profits of firm j ($\partial\pi_{j,c}/\partial\alpha_i > 0$) but lowers the profits of firm i ($\partial\pi_{i,c}/\partial\alpha_i < 0$). In contrast, Lemma 3 showed for Bertrand competition with differentiated products that both partial derivatives are positive as long as the shareholdings of the two firms are not too asymmetric ($\partial\pi_{j,c}/\partial\alpha_i > 0$, and $\partial\pi_{i,c}/\partial\alpha_i > 0$ if $(\alpha_i - \alpha_j) < \Delta\alpha^*$). This suggests that in Cournot competition especially symmetric increases of α_i and α_j have the capability of lowering the sustainability of collusion ($\partial\delta_i^*/\partial\alpha_i > 0$).

In Bertrand competition with differentiated goods this is even the case for asymmetric increases of α_i, as is confirmed by our numerical evaluations of the model. This finding supports our suggestion that pro-competitive effects of NCMS are even more prevalent than was suggested by Malueg's (1992) seminal contribution.

4.3 The Effect of α_j on δ_i^*

Our previous analysis was concerned with the effects of firm i's stake in firm j on its own critical discount factor δ_i^*. Proposition 2 extends both our own analysis and prior literature by establishing that under the assumptions of our model an asymmetric increase of firm j's stake α_j in firm i never facilitates collusion, i.e., it never lowers δ_i^* of firm i.

Proposition 2. $\frac{\partial\delta_i^*}{\partial\alpha_j} \geq 0 \quad \forall \quad \alpha_j$

Proof. See Appendix B. □

Proposition 2 suggests an important result because it challenges the findings of Gilo *et al.* (2006). Under their assumption of Bertrand competition with homogeneous products (causing $\partial\pi_{i,c}/\partial\alpha_j = 0$ and $\partial\pi_{j,c}/\partial\alpha_j = 0$) one finds $\partial\delta_i^*/\partial\alpha_i < 0$ (see Subsection 4.2) and $\partial\delta_i^*/\partial\alpha_j = 0$ (see Appendix B). They thus conclude that in the absence of unilateral effects minority shareholdings never hinder collusion.

However, once one allows for the existence of unilateral effects by assuming Cournot competition or Bertrand competition with differentiated products, one finds a remarkably different result. Although an asymmetric expansion of the minority shareholdings α_i of firm i in firm j does not necessarily make it harder for firm i to sustain collusion (see Subsection 4.2), Proposition 2 shows that an asymmetric expansion of α_j always raises the temptation of firm i to deviate ($\partial\delta_i^*/\partial\alpha_j > 0$). This is because of the incentive of firm j to soften competition after acquiring shares of firm i. This raises the competitive profits of both firms and softens the grim trigger punishments. We conclude that under the assumption of profit function (2) an asymmetric increase of minority shareholdings never facilitates collusion in our model.

4.4 Endogenous Detection Probability

It is straightforward to extend our analysis to the case of an endogenous detection probability, where an increase in minority shareholdings leads to a greater chance that collusion is detected by an antitrust authority. This confirms the hypothesis of Reynolds and Snapp (1986, p. 149) who suggested that partial "ownership [...] could actually [destabilize collusion] if such an involvement (however small) drew the attention of antitrust agencies." Our model is the first to formally investigate this hypothesis because it is the first that explicitly models a competition authority in the context of minority shareholdings. We extend our prior analysis by assuming the detection probability of collusion to rise in the value of the shareholdings, i.e., $\partial\rho/\partial\alpha_i > 0$. Proposition 3 shows that an increase of α_i can raise δ_i^* even in Bertrand competition with homogeneous goods if ρ rises strongly enough in α_i.

Proposition 3. *If* $\partial\rho/\partial\alpha_i > [2(\pi_{i,k} - \rho F)]/[(1-\alpha_i^2)F]$ *then* $\partial\delta_i^*/\partial\alpha_i > 0$ *in Bertrand competition with homogeneous goods.*

Proof. See Appendix B □

If the value of the detection probability rises strongly enough in α_i, minority shareholdings α_i make collusion harder to sustain by raising the critical discount factor δ_i^*. This is even the case under conditions, where minority shareholdings would otherwise have the capability to stabilize collusion (i.e., Bertrand competition with homogeneous products). As a tentative policy conclusion, we infer that competition authorities need not be concerned much about the coordinated effects of minority shareholdings as long as they keep an eye on industries where such shareholdings are prevalent.

5 Robustness Checks

Subsection 5.1 shows that our main results continue to apply if one assumes a different profit function than (2). Subsection 5.2 illustrates that minority shareholdings need not stabilize collusion if firm j can credibly commit to engage in off-equilibrium behavior by sanctioning a deviation of firm i using harsher than grim trigger punishments.

5.1 Profit Function

Up to this point, the article relied on profit function (2) (i.e., $\hat{\pi}_i = \pi_i + \alpha_i\hat{\pi}_j$) that modeled minority shareholdings of firm i in firm j. A different profit function was used by Reynolds and Snapp (1986) and Malueg (1992), which is shown in (11) and models, for example, a situation where the majority shareholder of firm i also holds minority shares in firm j.

$$max\ \tilde{\pi}_i = (1-\alpha_j)\pi_i + \alpha_i\pi_j \tag{11}$$

This section demonstrates that – despite the different interpretations of the profit functions – only the functional form but not the economic interpretation of our results changes if one assumes profit function (11). To see this, consider that $\tilde{\delta}_i^*$ denotes the critical discount factor under this assumption.

$$\delta_i > \frac{(1-\alpha_j)(\pi_{i,d}-\pi_{i,k})+\alpha_i(\pi_{j,-d}-\pi_{j,k})}{(1-\alpha_j)(\pi_{i,d}-\rho F-\pi_{i,c})+\alpha_i(\pi_{j,-d}-\rho F-\pi_{j,c})} \equiv \tilde{\delta}_i^* \tag{12}$$

Proposition 4 establishes that an increase in α_i raises $\tilde{\delta}_i^*$ under the same conditions that also raised δ_i^* when assuming profit function (2).

Proposition 4. *The inequality* $\partial\tilde{\delta}_i^*/\partial\alpha_i > 0$ *applies if inequality* (13) *is satisfied.*

$$\begin{aligned} &\frac{(1-\alpha_j)\left[(\pi_{i,k}-\rho F-\pi_{i,c})\pi_{j,-d}-(\pi_{j,k}-\rho F-\pi_{j,c})\pi_{i,d}-(\pi_{i,c}-\pi_{j,c})\pi_{i,k}\right]}{(1-\alpha_j)(\pi_{i,d}-\pi_{i,k})+\alpha_i(\pi_{j,-d}-\pi_{j,k})} \\ &\qquad < (1-\alpha_j)\frac{\partial\pi_{i,c}}{\partial\alpha_i}+\alpha_i\frac{\partial\pi_{j,c}}{\partial\alpha_i} \end{aligned} \tag{13}$$

Proof. See Appendix C. □

Proposition 4 can be interpreted in the same way as Proposition 1. NCMS contribute to raising the critical discount factor $\tilde{\delta}_i^*$ if they soften competition, i.e., if they cause unilateral effects that are sufficiently strong. NCMS have a depressing effect on $\tilde{\delta}_i^*$ because a deviation by firm i results in lower dividends received from firm j. A high value of ρF facilitates situations with $\partial\tilde{\delta}_i^*/\partial\alpha_i > 0$. These are the same effects that were derived above under Gilo *et al.*'s (2006) profit function (2).

Proposition 5 is the equivalent to Proposition 2.

Proposition 5. *The inequality* $\partial\tilde{\delta}_i^*/\partial\alpha_j > 0$ *applies if inequality* (14) *is satisfied.*

$$\begin{aligned} &\frac{\alpha_i\left[\pi_{i,k}(\pi_{j,c}-\pi_{j,-d})+\pi_{i,d}(\pi_{j,k}-\pi_{j,c})+\pi_{i,c}(\pi_{j,-d}-\pi_{j,k})\right]}{(1-\alpha_j)(\pi_{i,d}-\pi_{i,k})+\alpha_i(\pi_{j,-d}-\pi_{j,k})} \\ &\qquad < (1-\alpha_j)\frac{\partial\pi_{i,c}}{\partial\alpha_j}+\alpha_i\frac{\partial\pi_{j,c}}{\partial\alpha_j} \end{aligned} \tag{14}$$

Proof. See Appendix C. $\square$

Proposition 5 establishes that under the assumption of profit function (11) an asymmetric increase of shareholdings α_j destabilizes collusion by increasing the critical discount factor $\tilde{\delta}_i^*$ of firm i if the unilateral effects are strong enough. Qualitatively, this is the same effect that also destabilized collusion under the assumption of profit function (2). However, profit function (11) imposes a further stabilizing effect on collusion. That is, if firm j acquires additional shares of firm i and extracts higher dividends, firm i benefits to a lesser extent from its own (deviation) profits net of dividend payments. This reduces the profitability of a deviation for firm i and thus stabilizes collusion.

This can be seen particularly well for Bertrand competition with homogeneous products. This assumption causes $\pi_{i,k} = \pi_{j,k}$, $\pi_{i,c} = \pi_{j,c} = \pi_{j,-d} = 0$, $\pi_{i,d} = 2\pi_{i,k}$, $\partial\pi_{i,c}/\partial\alpha_j = 0$, and $\partial\pi_{j,c}/\partial\alpha_j = 0$, such that inequality (14) simplifies to (15).

$$\frac{2\alpha_i \pi_{i,k}^2}{(1-\alpha_j-\alpha_i)\pi_{i,k}} < 0 \tag{15}$$

This inequality cannot be satisfied because its left-hand side is positive for minority shareholdings $0 < \alpha_i < 0.5$ and $0 < \alpha_j < 0.5$. Under the assumption of Bertrand competition with homogeneous products firm j's shareholdings lower the incentive of firm i whether to deviate from collusion, i.e., they raise the sustainability of collusion. This is because firm i receives a lower share of its deviation profits but – given the absence of unilateral effects – would be punished as harshly as before. In contrast, firm j's minority shares of firm i lower the sustainability of collusion if their unilateral effects (i.e., the right-hand side of inequality (14)) are high enough.

5.2 Punishments

Propositions 1, 2, 4, and 5 indicate that any procompetitive effects of minority shareholdings occur because of their unilateral effects, which soften the grim trigger punishments following a deviation from collusion. Lemma 6 shows that any such effects vanish if firm j punishes a deviation from collusion not by playing a grim trigger strategy, i.e., by playing its best response, but by setting the same price or quantity that it would set without any minority shareholdings. Hence, firm j sets its price or output independently of the shareholdings α_i (i.e., $\partial p_j/\partial\alpha_i = 0$ or $\partial q_j/\partial\alpha_i = 0$).

Lemma 6. *The equality* $(\partial\pi_{i,c}/\partial\alpha_i) + \alpha_i(\partial\pi_{j,c}/\partial\alpha_i) = 0$ *applies for* $\partial p_j/\partial\alpha_i = 0$ *(or* $\partial q_j/\partial\alpha_i = 0$*) if firm* i *maximizes profit function* (2)*. The equality* $(1-\alpha_j)(\partial\pi_{i,c}/\partial\alpha_i) + \alpha_i(\partial\pi_{j,c}/\partial\alpha_i) = 0$ *applies for* $\partial p_j/\partial\alpha_i = 0$ *(or* $\partial q_j/\partial\alpha_i = 0$*) if firm* i *maximizes profit function* (11)*.*

Proof. This proof is equivalent to the proofs of Lemma 7 in Appendix B and of Lemma 9 in Appendix C. □

Lemma 6 indicates that minority shareholdings α_i need not stabilize collusion if firm j can credibly commit to punish a deviation not only by a reversion to the competitive Nash equilibrium but by engaging in off-equilibrium conduct, i.e., setting the same price (or quantity) it would set for $\alpha_i = 0$.

6 Evidence

Our theoretical model shows that NCMS are likely to have a negative effect on the sustainability of explicit collusion. This section uses the model to derive hypotheses that may be analyzed by future, empirical work. As a first step towards such analyses, we present anecdotal evidence appearing to be in line with these hypotheses.

Proposition 1 in combination with equation (10) suggests that in the presence of an antitrust authority pursuing an effective anti-cartel policy minority shareholdings contribute to destabilizing collusion. This suggests the following hypotheses for empirical work: Explicit collusion among firms holding NCMS in each other could be observed in past times when antitrust enforcement had not been as effective as today (Hypothesis 1), in jurisdictions where antitrust enforcement is still relatively weak (Hypothesis 2), and in situations where firms are not aware of or do not pay attention to antitrust enforcement (Hypothesis 3).

Some evidence supporting Hypothesis 1 is provided by Leslie (2004) showing that cross-shareholdings among cartel firms were mainly a phenomenon of the first half of the 20th century. Leslie (2004, p. 581-583) names examples of cartels from this era such as aluminum or lamps: For example, Alcoa purchased an interest in Norsk Aluminium Company, Det Norske Nitrid, and Societa dell'Alluminio Italiano. Similarly, by 1935 General Electric possessed stocks of several other lamp producers such as Osram and Philips. When cartel enforcement became more effective in the second half of the 20th century, the colluding firms apparently refrained from acquiring such shares. Therefore, more recent examples of cartels among firms holding minority shares in each other are harder to find and may possibly be explained along the lines of Hypotheses 2 and 3.

The European Needles cartel of the 1990s may serve as an example for Hypothesis 3, i.e., the firms did not pay much attention to antitrust enforcement.[4] Three firms took part in the conspiracy: William Prym GmbH & Co. KG, Coats Holdings Ltd, and Entaco Ltd. In 1994 William Prym acquired a minority share of 10.1% in Entaco, which might serve as an example of minority shareholdings among colluding firms. However, rather than keeping the conspiracy secret the firms entered into a series of *written* market sharing agreements to partition both product and geographic markets. Interestingly, the lawyers of the firms had been involved in making at least some of these agreements. This may suggest that the firms were not aware of the illegality of their conduct.

Some evidence for Hypothesis 2 was provided by the OECD (2009): In Turkey, minority shareholdings played a role in the cartels among aerated concrete producers and scheduled maritime transportation by roll-on/roll-off vessels. In Chinese Taipei (Taiwan) cross-shareholdings could be observed among two cable TV service providers who engaged

[4]The description of the Needles case F-1/38.338 is based on the Commission Decision as of October 10, 2004: http://goo.gl/ZcUWv1, accessed on August 3, 2016

in market allocation in the years prior to 2003. Note that the Turkish competition authority was established in 1997, and the FTC of Taiwan in 1992. It may be hypothesized that competition enforcement in these countries is not as strong, yet, as, for example, in the United States or Europe and that firms may not always take antitrust laws into account when making business decisions. This was suggested by Hypothesis 2.

7 Conclusion

This article presents a framework for the analysis of collusion when the firms hold minority shares in each other. We combined the established models of Reynolds and Snapp (1986), Flath (1991, 1992), Malueg (1992), and Gilo *et al.* (2006). These earlier contributions are not always readily comparable, first, because they use different profit functions. Second, some combinations of assumptions had – so far – remained unexplored. Third, they sometimes rely on numerical proofs. We fill these gaps in the literature by studying the 'missing' combinations of assumptions and thus making the earlier papers better comparable. The properties of our model are proven analytically. Additionally, we extend this literature by adding an antitrust authority (Aubert *et al.* 2006).

Malueg (1992) pointed out a trade-off: The existence of unilateral effects may facilitate situations where the critical discount factor rises in the level of NCMS, which helps to prevent coordinated effects. Our study indicates that NCMS lower the sustainability of collusion under an even greater variety of circumstances than was acknowledged by this earlier literature. NCMS have a particularly detrimental effect on collusion in the presence of an antitrust authority that pursues an effective anti-cartel policy. Case evidence seems to support the prediction that in jurisdictions with effective anti-cartel enforcement minority shareholdings among cartel firms are rare even today.

Our analysis raises the question whether antitrust authorities should be concerned much with the coordinated effects of minority shareholdings as long as they maintain an effective enforcement of cartels. And – more provocatively – should antitrust authorities accept some acquisitions of NCMS even if they cause unilateral effects? This is because coordinated effects may be reduced in the presence of unilateral effects, and the net effect on consumer surplus may be positive. We are reluctant to answer these questions based on the present model only. This is because several extensions of the model should be studied before giving such policy advice.

For example, it will be interesting to endogenize firms' decision to acquire NCMS. The firms might also decide about splitting the collusive profits unequally or making side payments, which might be necessary if the firms were asymmetric in costs. Future work should also extend our duopoly model to the case of n firms. In order to give policy advice it will also be helpful to explicitly model the antitrust authority's enforcement costs.

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A Appendix to Section 3

The following proofs assume that the firms produce subject to constant marginal costs taking a value of zero.

Proof of Lemma 1. Lemma 1 proposes $\partial\pi_{i,c}/\partial\alpha_i < 0$ and $\partial\pi_{i,c}/\partial\alpha_j > 0$ for Cournot competition with homogeneous goods and profit function (2). To see this, reaction function (A.3) is determined by deriving profit function (A.1) for q_i and solving first-order condition (A.2) for q_i.

$$\hat{\pi}_i = \frac{1}{1-\alpha_i\alpha_j}\left[p(q_i, q_j)q_i + \alpha_i p(q_i, q_j)q_j\right] \tag{A.1}$$

$$\frac{\partial\hat{\pi}_i}{\partial q_i} = \frac{1}{1-\alpha_i\alpha_j}\left[\frac{\partial p}{\partial q_i}q_i + p(q_i, q_j) + \alpha_i\frac{\partial p}{\partial q_i}q_j\right] \stackrel{!}{=} 0 \tag{A.2}$$

$$\hat{q}_i^R(q_j) = q_i^R(q_j) - \alpha_i\theta q_j \tag{A.3}$$

The term $q_i^R(q_j)$ denotes firm i's reaction function if it maximizes product market profits π_i only. The term $\hat{q}_i^R(q_j)$ denotes firm i's reaction function if it maximizes accounting profits $\hat{\pi}_i$. The variable θ is a scaling factor with $0 < \theta < 1$.

For a specific demand curve $p = (1 - q_i - q_j)^x$ with $x > 0$ (Malueg 1992) one can show $-1 < \partial q_i^R(q_j)/\partial q_j < 0$ (i.e., quantities are strategic substitutes) and $\theta = x/(1+x)$. One finds $\partial\hat{q}_i^R/\partial\alpha_i < 0$, i.e., the best-response output of firm i falls in the value of the shareholdings α_i. Because q_i^R maximizes π_i this implies $\partial\pi_i(q_j)/\partial\alpha_i < 0$ for a given value of q_j, i.e., firm i's product market profits fall in the value of the shareholdings α_i. Therefore, firm j expands its best-response output: Using $\partial\hat{q}_j^R(q_i)/\partial q_i < 0$ and $\hat{q}_i^R(q_j) < q_i^R(q_j)$ one finds $\hat{q}_j^R(\hat{q}_i^R) > \hat{q}_j^R(q_i^R)$, i.e., $\partial\hat{q}_j^R/\partial\alpha_i > 0$. Because the output reduction of firm i exceeds the output expansion of firm j the equilibrium price rises: From $-1 < \partial\hat{q}_j^R(q_i)/\partial q_i$ it follows $\partial\hat{q}_j^R/\partial\alpha_i = (\partial\hat{q}_j^R/\partial q_i)\cdot(\partial\hat{q}_i^R/\partial\alpha_i) < -\partial\hat{q}_i^R/\partial\alpha_i$, such that $\partial(\hat{q}_i^R + \hat{q}_j^R)/\partial\alpha_i < 0$ and $\partial p(\hat{q}_i^R + \hat{q}_j^R)/\partial\alpha_i > 0$.

Combining $\partial p(\hat{q}_i^R + \hat{q}_j^R)/\partial\alpha_i > 0$ and $\partial\hat{q}_j^R/\partial\alpha_i > 0$ proves $\partial\pi_{j,c}/\partial\alpha_i > 0$, i.e., the higher output and price raise firm j's profit. Finding that the competitive profits of firm i fall in α_i for a given q_j (i.e., $\partial\pi_i(q_j)/\partial\alpha_i < 0$), and that q_j rises in α_i (i.e., $\partial\hat{q}_j^R/\partial\alpha_i > 0$ with $\partial\pi_i(q_j)/\partial q_j < 0$) proves $\partial\pi_{i,c}/\partial\alpha_i < 0$ for Cournot competition with homogeneous goods and profit function (2).

□

Proof of Lemma 2. Lemma 2 proposes $\partial\pi_{i,d}/\partial\alpha_i < 0$ and $\partial\pi_{j,-d}/\partial\alpha_i > 0$ for Cournot-competition with homogeneous goods. Firm j keeps its output $q_{j,-d} = q_{j,k}$ constant by assumption, i.e., $\partial q_{j,-d}/\partial\alpha_i = 0$. Reaction function (A.3) with $\partial\hat{q}_i^R(q_{j,-d})/\partial\alpha_i < 0$ causes $\partial(q_{j,-d} + \hat{q}_i^R(q_{j,-d}))/\partial\alpha_i < 0$ and thus $\partial p_d/\partial\alpha_i > 0$. This proves $\partial\pi_{j,-d}/\partial\alpha_i = (\partial p_d/\partial\alpha_i)\cdot q_{j,-d} > 0$. Finding $\partial\hat{q}_i^R(q_{j,-d})/\partial\alpha_i < 0$ also implies that with $\alpha_i > 0$ firm i sets a lower than the profit-maximizing output $q_i^R(q_{j,-d}) > \hat{q}_i^R(q_{j,-d})$ which causes $\pi_i(q_i^R, q_{j,-d}) > \pi_i(\hat{q}_i^R, q_{j,-d})$. This proves $\partial\pi_{i,d}/\partial\alpha_i < 0$. □

Proof of Lemma 3. Lemma 3 proposes for Bertrand competition with differentiated goods that $\partial\pi_{i,c}/\partial\alpha_i > 0$ if $(\alpha_i - \alpha_j) < \Delta\alpha^*$, $\partial\pi_{i,c}/\partial\alpha_i \leq 0$ if $(\alpha_i - \alpha_j) \geq \Delta\alpha^*$, and $\partial\pi_{i,c}/\partial\alpha_j > 0$. To see this, we determine the reaction function (A.6) by maximizing profit function (A.4) with respect to p_i. The term $\hat{p}_i^R(p_j)$ denotes the reaction function of firm i if it maximizes accounting profits $\hat{\pi}_i$. The term $p_i^R(p_j)$ denotes the reaction function if firm i maximizes product market profits π_i only.

$$\hat{\pi}_i = \frac{1}{1-\alpha_i\alpha_j}\left[q_i(p_i,p_j)p_i + \alpha_i q_j(p_i,p_j)p_j\right] \tag{A.4}$$

$$\frac{\partial\hat{\pi}_i}{\partial p_i} = \frac{1}{1-\alpha_i\alpha_j}\left[\frac{\partial q_i}{\partial p_i}p_i + q_i(p_i,p_j) + \alpha_i\frac{\partial q_j}{\partial p_i}p_j\right] \stackrel{!}{=} 0 \tag{A.5}$$

$$\hat{p}_i^R = p_i^R(p_j) + \alpha_i B p_j \tag{A.6}$$

For specific demand such as $q_i = 1 - p_i + \beta p_j$ with $0 < \beta < 1$ one can prove that prices are strategic complements (i.e., $0 < \partial p_i^R(p_j)/\partial p_j < 1$), and one finds $B = \beta/2 > 0$. Equation (A.6) indicates that firm i would want to raise its price ($\hat{p}_i^R(p_j) > p_i^R(p_j)$) after acquiring shares $\alpha_i > 0$ of firm j, which implies $\partial \hat{p}_i^R / \partial \alpha_i > 0$. Because of the strategic complementarity of prices, firm i will also raise its price if firm j acquires shares of firm i: Given $\partial \hat{p}_i^R / \partial \alpha_j = \left[(\partial p_i^R / \partial p_j) \cdot (\partial \hat{p}_j^R / \partial \alpha_j) + \alpha_i B (\partial \hat{p}_j^R / \partial \alpha_j) \right]$, $\partial p_i^R / \partial p_j) > 0$, and $\partial \hat{p}_j^R / \partial \alpha_j > 0$, one finds $\partial \hat{p}_i^R / \partial \alpha_j > 0$. For $q_i = 1 - p_i + \beta p_j$ one can show that firm i, after acquiring a share α_i of firm j, raises its own price more strongly than firm j, i.e., $\partial \hat{p}_j^R / \partial \alpha_i < \partial \hat{p}_i^R / \partial \alpha_i$, and the equilibrium output of firm i will fall, i.e., $\partial q_i(\hat{p}_i^R, \hat{p}_j^R) / \partial \alpha_i < 0$.

Finding that an increase in α_i raises the equilibrium price p_i and lowers the equilibrium quantity q_i suggests an ambiguous effect on the profits of firm i ($\partial \pi_{i,c} / \partial \alpha_i \lesseqgtr 0$) that is explored below. However, the profits of firm i will necessarily rise if firm j acquires shares α_j of it (i.e., $\partial \pi_{i,c} / \partial \alpha_j > 0$). Because firm j would want to raise its price ($\partial \hat{p}_j^R / \partial \alpha_j > 0$) after acquiring shares α_j, the output and profit of firm i would rise even when holding its own price constant. Firm i will only raise its price ($\partial \hat{p}_j^R / \partial p_j > 0$) if this raises its profit even further, which proves $\partial \pi_{i,c} / \partial \alpha_j > 0$.

Proving $\partial \pi_{i,c} / \partial \alpha_i > 0$ if $(\alpha_i - \alpha_j) < \Delta\alpha^*$ requires assuming a specific functional form for demand. While Flath (1991) provides proofs for a Hotelling model,[5] we use a demand function as follows: $q_i = 1 - p_i + \beta p_j$. The price and profit of firm i in the product market-equilibrium are then shown in (A.7) and (A.8). Defining $\Delta\alpha = \alpha_i - \alpha_j$ and solving the inequality $\partial \pi_{i,c} / \partial \alpha_i > 0$ for $\Delta\alpha$ yields inequality (A.9).

$$\hat{p}_i^R \left(\hat{p}_j^R \right) = \frac{2 + (1 + \alpha_i)\beta}{4 - (1 + \alpha_i)(1 + \alpha_j)\beta^2} \tag{A.7}$$

$$\pi_{i,c} = \pi_i \left(\hat{p}_i^R, \hat{p}_j^R \right) = (1 - \hat{p}_i^R + \beta \hat{p}_j^R) \hat{p}_i^R \tag{A.8}$$

$$\Delta\alpha < \frac{2\beta + 1}{2\beta \left[1 + (1 + \alpha_j)\beta \right]} - (1 + \alpha_j) \equiv \Delta\alpha^* \tag{A.9}$$

[5] A Salop circle model with three firms and asymmetric, controlling shareholdings was provided by Foros *et al.* (2011).

Inequality (A.9) indicates that the competitive profits $\pi_{i,c}$ rise in the value of the shareholdings α_i as long as α_i is sufficiently close to the shareholdings α_j of firm j. Because $\lim_{\beta\to 0} \Delta\alpha^* = \infty$, inequality (A.9) is more easily satisfied for goods that are more strongly differentiated. This proves Lemma 3. □

Proof of Lemma 2 for Bertrand competition with differentiated goods. Lemma 2 proposes $\partial\pi_{i,d}/\partial\alpha_i < 0$ and $\partial\pi_{j,-d}/\partial\alpha_i > 0$. The assumption $p_{j,-d} = p_{j,k}$ implies $\partial p_{j,-d}/\partial\alpha_i = 0$. Reaction function (A.6) causes a higher deviation price $(\partial\hat{p}_i^R(p_{j,-d})/\partial\alpha_i > 0)$ in the presence of minority shareholdings and thus less business stealing $(\partial q_{j,-d}/\partial\alpha_i > 0)$. This proves $\partial\pi_{j,-d}/\partial\alpha_i = (\partial q_{j,d}/\partial\alpha_i) \cdot p_{j,-d} > 0$. Finding $\partial\hat{p}_i^R(p_{j,-d})/\partial\alpha_i > 0$ also implies that with $\alpha_i > 0$ firm i sets a higher than the profit-maximizing price $p_i^R(p_{j,-d}) < \hat{p}_i^R(p_{j,-d})$, which causes $\pi_i(p_i^R, p_{j,-d}) > \pi_i(\hat{p}_i^R, p_{j,-d})$. This proves $\partial\pi_{i,d}/\partial\alpha_i < 0$. □

B Appendix to Section 4

The proof of Proposition 1 relies on Lemma 7.

Lemma 7. $\frac{\partial\pi_{i,d}}{\partial\alpha_i} + \alpha_i\frac{\partial\pi_{j,-d}}{\partial\alpha_i} = 0$

Proof. For *Cournot competition with homogeneous goods*, the equality stated in Lemma 7 can be restated as in (B.1).

$$\frac{\partial\pi_{i,d}}{\partial\alpha_i} + \alpha_i\frac{\partial\pi_{j,-d}}{\partial\alpha_i} = \frac{\partial q_i^R}{\partial\alpha_i}\left[\left(\frac{\partial p}{\partial q_i}q_i + p\right) + \alpha_i\left(\frac{\partial p}{\partial q_i}q_j\right)\right] \tag{B.1}$$

The bracketed term on the right-hand side of equation (B.1) is the same as the bracketed term in first order condition (A.2) that takes a value of zero in the optimum. For *Bertrand competition with differentiated goods*, the equality stated in Lemma 7 can be restated as in (B.2).

$$\frac{\partial\pi_{i,d}}{\partial\alpha_i} + \alpha_i\frac{\partial\pi_{j,-d}}{\partial\alpha_i} = \frac{\partial p_i^R}{\partial\alpha_i}\left[\left(\frac{\partial q_i}{\partial p_i}p_i + q_i\right) + \alpha_i\left(\frac{\partial q_j}{\partial p_i}q_j\right)\right] = 0 \tag{B.2}$$

The bracketed term on the right-hand side of equation (B.2) is the same as the bracketed term in first order condition (A.5) that takes a value of zero in the optimum. Section 3.2 established $\partial\pi_{i,d}/\partial\alpha_i = 0$ and $\partial\pi_{j,-d}/\partial\alpha_i = 0$ for *Bertrand competition with homogeneous goods*. This proves $(\partial\pi_{i,d}/\partial\alpha_i) + \alpha_i(\partial\pi_{j,-d}/\partial\alpha_i) = 0$. □

Proof of Proposition 1. To determine $\partial\delta_i^*/\partial\alpha_i$, re-write δ_i^* as follows.

$$\begin{aligned} \delta_i^* &= \frac{u(\alpha_i,\alpha_j)}{v(\alpha_i,\alpha_j)} \\ \text{with } u(\alpha_i,\alpha_j) &= (\pi_{i,d}-\pi_{i,k}) + \alpha_i(\pi_{j,-d}-\pi_{j,k}) > 0 \\ \text{and } v(\alpha_i,\alpha_j) &= (\pi_{i,d}-\rho F-\pi_{i,c}) + \alpha_i(\pi_{j,-d}-\rho F-\pi_{j,c}) > 0 \end{aligned} \tag{B.3}$$

Using $\partial\pi_{i,k}/\partial\alpha_i = \partial\pi_{j,k}/\partial\alpha_i = 0$ and $(\partial\pi_{i,d}/\partial\alpha_i) + \alpha_i(\partial\pi_{j,-d}/\partial\alpha_i) = 0$ from Lemma 7, $\partial\delta_i^*/\partial\alpha_i$ can be written as in (B.4).

$$\begin{aligned} \frac{\partial\delta_i^*}{\partial\alpha_i} &= \frac{\frac{\partial u(\alpha_i,\alpha_j)}{\partial\alpha_i}\cdot v(\alpha_i,\alpha_j) - \frac{\partial v(\alpha_i,\alpha_j)}{\partial\alpha_i}\cdot u(\alpha_i)}{v(\alpha_i,\alpha_j)^2} \\ \text{with } \frac{\partial u(\alpha_i,\alpha_j)}{\partial\alpha_i} &= \pi_{j,-d}-\pi_{j,k} \\ \text{and } \frac{\partial v(\alpha_i,\alpha_j)}{\partial\alpha_i} &= \pi_{j,-d}-\rho F-\pi_{j,c} - \left(\frac{\partial\pi_{i,c}}{\partial\alpha_i} + \alpha_i\frac{\partial\pi_{j,c}}{\partial\alpha_i}\right) \end{aligned} \tag{B.4}$$

Given $v(\alpha_i,\alpha_j) > 0$ the sign of $\partial\delta^*/\partial\alpha_i$ is the same as that of its numerator as is shown in (B.5).

$$\begin{aligned} &\frac{\partial u(\alpha_i,\alpha_j)}{\partial\alpha_i}\cdot v(\alpha_i,\alpha_j) - \frac{\partial v(\alpha_i,\alpha_j)}{\partial\alpha_i}\cdot u(\alpha_i,\alpha_j) = \dots \\ &(\pi_{i,k}-\rho F-\pi_{i,c})(\pi_{j,-d}-\pi_{i,d}) + \left(\frac{\partial\pi_{i,c}}{\partial\alpha_i} + \alpha_i\frac{\partial\pi_{j,c}}{\partial\alpha_i}\right)u(\alpha_i,\alpha_j) \end{aligned} \tag{B.5}$$

Equation (B.5), and thus $\partial\delta^*/\partial\alpha_i$, is positive if inequality (9) applies. This proves Proposition 1. □

Proof of Lemma 4. This Lemma proposes $(\partial\pi_{i,c}/\partial\alpha_i)+\alpha_i(\partial\pi_{j,c}/\partial\alpha_i) \geq 0$ for Bertrand competition with homogeneous or differentiated products, and for Cournot competition with homogeneous products.

Section 3.2 showed that in *Bertrand competition with homogeneous products* one finds $\partial\pi_{i,c}/\partial\alpha_i = 0$ and $\partial\pi_{j,c}/\partial\alpha_i = 0$. This proves $(\partial\pi_{i,c}/\partial\alpha_i) + \alpha_i(\partial\pi_{j,c}/\partial\alpha_i) = 0$ in this model of competition.

Now, assume *Cournot competition with homogeneous goods.* We derive the accounting profits $\hat{\pi}_{i,c}$ for α_i, as is shown in equation (B.6).

$$\begin{aligned}\frac{\partial\hat{\pi}_{i,c}}{\partial\alpha_i} &= \frac{\left(\frac{\partial\pi_{i,c}}{\partial\alpha_i} + \pi_{j,c} + \alpha_i\frac{\partial\pi_{j,c}}{\partial\alpha_i}\right)(1-\alpha_i\alpha_j) + \alpha_j(\pi_{i,c}+\alpha_i\pi_{j,c})}{(1-\alpha_i\alpha_j)^2} \\ &= \frac{\hat{\pi}_{j,c}}{1-\alpha_i\alpha_j} + \frac{\frac{\partial\pi_{i,c}}{\partial q_i}\frac{\partial\hat{q}_i^R}{\partial\alpha_i} + \alpha_i\frac{\partial\pi_{j,c}}{\partial q_j}\frac{\partial\hat{q}_j^R}{\partial q_i}\frac{\partial\hat{q}_i^R}{\partial\alpha_i}}{1-\alpha_i\alpha_j}\end{aligned} \tag{B.6}$$

The shareholdings α_i affect $\pi_{i,c}$ directly via their effect on $\hat{q}_i^R$. The shareholdings α_i affect $\pi_{j,c}$ only indirectly because the value of $\hat{q}_j^R$ is a function of $\hat{q}_i^R(\alpha_i)$ (see the proof of Lemma 1). The option to keep its output constant ($\partial q_{i,c}/\partial\alpha_i = 0$) allows firm i to ensure that the second summand of (B.6), which is the same as $(\partial\pi_{i,c}/\partial\alpha_i)+\alpha_i(\partial\pi_{j,c}/\partial\alpha_i)$, is not lower than zero. This proves $(\partial\pi_{i,c}/\partial\alpha_i)+\alpha_i(\partial\pi_{j,c}/\partial\alpha_i) \geq 0$ for Cournot competition with homogeneous goods.

Consider *Bertrand competition with differentiated goods* and derive $\hat{\pi}_{i,c}$ (see equation (A.4)) for α_i. This yields equation (B.7).

$$\begin{aligned}\frac{\partial\hat{\pi}_{i,c}}{\partial\alpha_i} &= \frac{\left(\frac{\partial\pi_{i,c}}{\partial\alpha_i} + \pi_{j,c} + \alpha_i\frac{\partial\pi_{j,c}}{\partial\alpha_i}\right)(1-\alpha_i\alpha_j) + \alpha_j(\pi_{i,c}+\alpha_i\pi_{j,c})}{(1-\alpha_i\alpha_j)^2} \\ &= \frac{\hat{\pi}_{j,c}}{1-\alpha_i\alpha_j} + \frac{\frac{\partial\pi_{i,c}}{\partial p_i}\frac{\partial\hat{p}_i^R}{\partial\alpha_i} + \alpha_i\frac{\partial\pi_{j,c}}{\partial p_j}\frac{\partial\hat{p}_j^R}{\partial p_i}\frac{\partial\hat{p}_i^R}{\partial\alpha_i}}{1-\alpha_i\alpha_j}\end{aligned} \tag{B.7}$$

The shareholdings α_i affect $\pi_{i,c}$ directly via their effect on $\hat{p}_i^R$. The shareholdings α_i affect $\pi_{j,c}$ only indirectly because the equilibrium value of $\hat{p}_j^R$ is a function of $\hat{p}_i^R(\alpha_i)$. The option to keep its price constant ($\partial p_{i,c}/\partial\alpha_i = 0$) allows firm i to ensure that the second summand of (B.7), which is the same as $(\partial\pi_{i,c}/\partial\alpha_i) + \alpha_i(\partial\pi_{j,c}/\partial\alpha_i)$, is not lower than zero. This proves $(\partial\pi_{i,c}/\partial\alpha_i) + \alpha_i(\partial\pi_{j,c}/\partial\alpha_i) \geq 0$ for Bertrand competition with differentiated goods. □

Proof of Lemma 5. Lemma 5 suggests that in Cournot competition one finds $\partial\delta_i^*/\partial\alpha_i > 0$ even for non-convex demand if $(\rho F)^* < \rho F$ (see condition (10)). Collusion would only be established under condition (6) ($\rho F < \pi_{i,k} - \pi_{i,c}$). Combining these inequalities yields (B.8).

$$\left[\frac{\partial\pi_{i,c}}{\partial\alpha_i} + \alpha_i\frac{\partial\pi_{j,c}}{\partial\alpha_i}\right] \cdot \frac{[(\pi_{i,d} + \alpha_i\pi_{j,-d}) - (\pi_{i,k} + \alpha_i\pi_{j,k})]}{\pi_{i,d} - \pi_{j,-d}} < \rho F - (\pi_{i,k} - \pi_{i,c}) < 0 \tag{B.8}$$

Assume a linear (i.e., non-convex) demand curve $p = 1 - q_i - q_j$ and a Cournot duopoly where the firms produce a homogeneous product at constant marginal costs of zero. Under these assumptions the profits of the firms can be written as follows.

$$\pi_{i,k} = \pi_{j,k} = 1/8 \tag{B.9}$$

$$\pi_{i,d} = \frac{(3-\alpha_i)(3+\alpha_i)}{64} \tag{B.10}$$

$$\pi_{j,-d} = \frac{3+\alpha_i}{32} \tag{B.11}$$

Plugging equations (B.9)-(B.11) in (B.8) and assuming $\rho F \to \pi_{i,k} - \pi_{i,c}$ yields (B.12).

$$\left[\frac{\partial\pi_{i,c}}{\partial\alpha_i} + \alpha_i\frac{\partial\pi_{j,c}}{\partial\alpha_i}\right] \cdot \frac{\alpha_i - 1}{\alpha_i + 3} < 0 \tag{B.12}$$

Lemma 4 proved that the bracketed term is positive in Cournot competition while $(\alpha_i - 1)/(\alpha_i + 3)$ is negative so that (B.12) is satisfied. Hence, for values of the sanction ρF sufficiently close to $\pi_{i,k} - \pi_{i,c}$ inequality (B.8) is satisfied, which proves Lemma 5. □

The proof of Proposition 2 relies on Lemma 8.

Lemma 8. $\frac{\partial \pi_{i,d}}{\partial \alpha_j} + \alpha_i \frac{\partial \pi_{j,-d}}{\partial \alpha_j} = 0$

Proof. For *Cournot competition with homogeneous goods*, the equality stated in Lemma 8 can be restated as in (B.13).

$$\frac{\partial \pi_{i,d}}{\partial \alpha_j} + \alpha_i \frac{\partial \pi_{j,-d}}{\partial \alpha_j} = \frac{\partial q_i}{\partial \alpha_j}\left[\left(\frac{\partial p}{\partial q_i} q_i + p\right) + \alpha_i \left(\frac{\partial p}{\partial q_i} q_j\right)\right] \tag{B.13}$$

The bracketed term on the right-hand side of equation (B.1) is the same as the bracketed term in first order condition (A.2) that takes a value of zero in the optimum. For *Bertrand competition with differentiated goods*, the equality stated in Lemma 8 can be restated as in (B.14).

$$\frac{\partial \pi_{i,d}}{\partial \alpha_j} + \alpha_i \frac{\partial \pi_{j,-d}}{\partial \alpha_j} = \frac{\partial p_i}{\partial \alpha_j}\left[\left(\frac{\partial q_i}{\partial p_i} p_i + q_i\right) + \alpha_i \left(\frac{\partial q_j}{\partial p_i} q_j\right)\right] = 0 \tag{B.14}$$

The bracketed term on the right-hand side of equation (B.2) is the same as the bracketed term in first order condition (A.5) that takes a value of zero in the optimum. Section 3.2 established $\partial \pi_{i,d}/\partial \alpha_j = 0$ and $\partial \pi_{j,-d}/\partial \alpha_j = 0$ for *Bertrand competition with homogeneous goods*. This proves $(\partial \pi_{i,d}/\partial \alpha_j) + \alpha_i(\partial \pi_{j,-d}/\partial \alpha_j) = 0$. □

Proof of Proposition 2. Proposition 2 suggests $\partial \delta_i^*/\partial \alpha_j \geq 0$. Equation (B.15) provides $\partial \delta_i^*/\partial \alpha_j$.

$$\begin{aligned} \frac{\partial \delta_i^*}{\partial \alpha_j} &= \frac{\frac{\partial u(\alpha_i,\alpha_j)}{\partial \alpha_j} \cdot v(\alpha_i,\alpha_j) - \frac{\partial v(\alpha_i,\alpha_j)}{\partial \alpha_j} \cdot u(\alpha_i,\alpha_j)}{v(\alpha_i,\alpha_j)^2} \\ \text{with } \frac{\partial u(\alpha_i,\alpha_j)}{\partial \alpha_j} &= \left(\frac{\partial \pi_{i,d}}{\partial \alpha_j} - \frac{\partial \pi_{i,k}}{\partial \alpha_j}\right) + \alpha_i \left(\frac{\partial \pi_{j,-d}}{\partial \alpha_j} - \frac{\partial \pi_{j,k}}{\partial \alpha_j}\right), \\ \text{and } \frac{\partial v(\alpha_i,\alpha_j)}{\partial \alpha_j} &= \left(\frac{\partial \pi_{i,d}}{\partial \alpha_j} - \frac{\partial \pi_{i,c}}{\partial \alpha_j}\right) + \alpha_i \left(\frac{\partial \pi_{j,-d}}{\partial \alpha_j} - \frac{\partial \pi_{j,c}}{\partial \alpha_j}\right) \end{aligned} \tag{B.15}$$

Using $\partial \pi_{i,k}/\partial \alpha_j = \partial \pi_{j,k}/\partial \alpha_j = 0$ and $(\partial \pi_{i,d}/\partial \alpha_j) + \alpha_i(\partial \pi_{j,-d}/\partial \alpha_j) = 0$ from Lemma 8, equation (B.15) can be simplified as is shown in equation (B.16).

$$\frac{\partial \delta_i^*}{\partial \alpha_j} = \frac{\left(\frac{\partial \pi_{i,c}}{\partial \alpha_j} + \alpha_i \frac{\partial \pi_{j,c}}{\partial \alpha_j}\right) \cdot u(\alpha_i, \alpha_j)}{v(\alpha_j)^2} \tag{B.16}$$

Given $u(\alpha_i, \alpha_j) > 0$ and $v(\alpha_i, \alpha_j) > 0$, the inequality $\partial \delta_i^* / \partial \alpha_j > 0$ applies when inequality (B.17) is satisfied.

$$\frac{\partial \pi_{i,c}}{\partial \alpha_j} \geq -\alpha_i \frac{\partial \pi_{j,c}}{\partial \alpha_j} \tag{B.17}$$

The weak inequality (B.17) is always satisfied: Lemma 4 implies $(\partial \pi_{j,c} / \partial \alpha_j) + \alpha_j (\partial \pi_{i,c} / \partial \alpha_j) \geq 0$, which can be combined with (B.17) as is shown in (B.18).

$$\frac{\partial \pi_{i,c}}{\partial \alpha_j} \geq -\frac{1}{\alpha_j} \frac{\partial \pi_{j,c}}{\partial \alpha_j} \geq -\alpha_i \frac{\partial \pi_{j,c}}{\partial \alpha_j} \tag{B.18}$$

Using $1/\alpha_j \geq 1 \geq \alpha_i$ shows that (B.18) is always satisfied. This proves Proposition 2. □

Proof of Proposition 3. Under the assumption of Bertrand competition with homogeneous goods ($\pi_{i,c} = 0$ and $\pi_{i,d} = 2\pi_{i,k}$) the critical discount factor δ_i^* can be written as in (B.19). Assuming $\partial \rho / \partial \alpha_i$ yields $\partial \delta_i^* / \partial \alpha_i$ as is shown in (B.20). If collusion is profitable ($\pi_{i,k} > \rho F$) the denominator of (B.20) is positive. The numerator is positive if inequality (B.21) is satisfied.

$$\delta_i^* = \frac{(1 - \alpha_i)\pi_{i,k}}{2\pi_{i,k} - (1 + \alpha_i)\rho F} \tag{B.19}$$

$$\frac{\partial \delta_i^*}{\partial \alpha_i} = \frac{-2\pi_{i,k}^2 + 2\rho F \pi_{i,k} + (1 - \alpha_i^2) \frac{\partial \rho}{\partial \alpha_i} F \pi_{i,k}}{(2\pi_{i,k} - (1 + \alpha_i)\rho F)^2} \tag{B.20}$$

$$\frac{\partial \rho}{\partial \alpha_i} > \frac{2(\pi_{i,k} - \rho F)}{(1 - \alpha_i^2) F} \tag{B.21}$$

This proves Proposition 3. □

C Appendix to Section 5

This appendix proves Propositions 4 and 5 for profit function (11). The profits $\tilde{\pi}_{i,c}$, $\tilde{\pi}_{i,k}$, and $\tilde{\pi}_{i,d}$ can be written as in (C.1) to (C.3). As before, we assume marginal costs of zero.

$$\tilde{\pi}_{i,c} = (1 - \alpha_j)\pi_{i,c} + \alpha_i \pi_{j,c} \tag{C.1}$$

$$\tilde{\pi}_{i,k} = (1 - \alpha_j + \alpha_i)(\pi_{i,k} - \rho F) \tag{C.2}$$

$$\tilde{\pi}_{i,d} = (1 - \alpha_j)(\pi_{i,d} - \rho F) + \alpha_i(\pi_{j,-d} - \rho F) \tag{C.3}$$

Lemma 9 is required to prove Proposition 4.

Lemma 9. $(1 - \alpha_j)\frac{\partial \pi_{i,d}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{j,-d}}{\partial \alpha_i} = 0$

Proof. In *Cournot competition with homogeneous products* $\tilde{\pi}_{i,d}$ can be written as in (C.4). The betrayed firm j sticks to the agreed-upon quantity, which implies $\partial q_j / \partial \alpha_i = 0$. The optimal deviation output of firm i satisfies first-order condition (C.5). Given $\partial q_j / \partial \alpha_i = 0$, one can write $\partial \pi_{i,d} / \partial \alpha_i$ and $\partial \pi_{j,-d} / \partial \alpha_i$ as is shown in (C.6) and (C.7). This allows to write the weighted sum of these first derivatives as is shown in equality (C.8), which equals zero given first-order condition (C.5), which proves Lemma 9 for Cournot competition with homogeneous products.

$$\tilde{\pi}_{i,d} = (1 - \alpha_j)\left[p\left(q_i(\alpha_i, \alpha_j), q_j\right) q_i - \rho F\right] + \alpha_i \left[p\left(q_i(\alpha_i, \alpha_j), q_j\right) q_j - \rho F\right] \tag{C.4}$$

$$\frac{\partial \tilde{\pi}_{i,d}}{\partial q_i} = (1 - \alpha_j)\left(\frac{\partial p}{\partial q_i} q_i + p\right) + \alpha_i \frac{\partial p}{\partial q_i} q_j \stackrel{!}{=} 0 \tag{C.5}$$

$$\frac{\partial \pi_{i,d}}{\partial \alpha_i} = \frac{\partial p}{\partial q_i}\frac{\partial q_i}{\partial \alpha_i} q_i + p\frac{\partial q_i}{\partial \alpha_i} \tag{C.6}$$

$$\frac{\partial \pi_{j,-d}}{\partial \alpha_i} = \frac{\partial p}{\partial q_i}\frac{\partial q_i}{\partial \alpha_i} q_j \tag{C.7}$$

$$(1-\alpha_j)\frac{\partial \pi_{i,d}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{j,-d}}{\partial \alpha_i} = \frac{\partial q_i}{\partial \alpha_i}\left[(1-\alpha_j)\left(\frac{\partial p}{\partial q_i} q_i + p\right) + \alpha_i \frac{\partial p}{\partial q_i} q_j\right] = 0 \tag{C.8}$$

Lemma 9 also applies in *Bertrand competition with homogeneous products* because of $\partial \pi_{i,d}/\partial \alpha_i = 0$ and $\partial \pi_{j,-d}/\partial \alpha_i = 0$. In *Bertrand competition with differentiated products* $\tilde{\pi}_{i,d}$ can be written as in (C.9). The betrayed firm j sticks to the agreed-upon price, which implies $\partial p_j/\partial \alpha_i = 0$. The optimal deviation price of firm i satisfies first-order condition (C.10). Given $\partial p_j/\partial \alpha_i = 0$, one can write $\partial \pi_{i,d}/\partial \alpha_i$ and $\partial \pi_{j,-d}/\partial \alpha_i$ as is shown in (C.11) and (C.12). This allows to write the weighted sum of these first derivatives as is shown in equality (C.13), which equals zero given first-order condition (C.5).

$$\tilde{\pi}_{i,d} = (1-\alpha_j)\left[q_i(p_i, p_j)\cdot p_i - \rho F\right] + \alpha_i \left[q_j(p_i, p_j)\cdot p_j - \rho F\right] \tag{C.9}$$

$$\frac{\partial \tilde{\pi}_{i,d}}{\partial p_i} = (1-\alpha_j)\left(\frac{\partial q_i}{\partial p_i} p_i + q_i\right) + \alpha_i \frac{\partial q_j}{\partial p_i} p_j \stackrel{!}{=} 0 \tag{C.10}$$

$$\frac{\partial \pi_{i,d}}{\partial \alpha_i} = \frac{\partial q_i}{\partial p_i}\frac{\partial p_i}{\partial \alpha_i} p_i + q_i \frac{\partial p_i}{\partial \alpha_i} \tag{C.11}$$

$$\frac{\partial \pi_{j,-d}}{\partial \alpha_i} = \frac{\partial q_j}{\partial p_i}\frac{\partial p_i}{\partial \alpha_i} p_j \tag{C.12}$$

$$(1-\alpha_j)\frac{\partial \pi_{i,d}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{j,-d}}{\partial \alpha_i} = \frac{\partial p_i}{\partial \alpha_i}\left[(1-\alpha_j)\left(\frac{\partial q_i}{\partial p_i} p_i + qi\right) + \alpha_i \frac{\partial q_j}{\partial p_i} p_j\right] = 0 \tag{C.13}$$

This proves Lemma 9 for Bertrand competition with differentiated products. $\square$

Proof of Proposition 4. Under the assumption of profit function (11) the critical discount factor is given by (12). Using $\partial\pi_{i,k}/\partial\alpha_i = 0$, $\partial\pi_{j,k}/\partial\alpha_i = 0$, $\pi_{i,k} = \pi_{j,k}$, and Lemma 9 the first derivative of $\tilde{\delta}_i^*$ with respect to α_i yields (C.14).

$$\begin{aligned}
\frac{\partial\tilde{\delta}_i^*}{\partial\alpha_i} =& \frac{\frac{\partial\tilde{u}(\alpha_i)}{\partial\alpha_i}\cdot\tilde{v}(\alpha_i) - \tilde{u}(\alpha_i)\cdot\frac{\partial\tilde{v}(\alpha_i)}{\partial\alpha_i}}{\tilde{v}(\alpha_i)^2} \\
\text{with } \tilde{u}(\alpha_i) =& (1-\alpha_j)(\pi_{i,d} - \pi_{i,k}) + \alpha_i(\pi_{j,-d} - \pi_{j,k}) > 0, \\
\tilde{v}(\alpha_i) =& (1-\alpha_j)(\pi_{i,d} - \rho F - \pi_{i,c}) + \alpha_i(\pi_{j,-d} - \rho F - \pi_{j,c}) > 0 \\
\frac{\partial\tilde{u}(\alpha_i)}{\partial\alpha_i} =& \pi_{j,-d} - \pi_{i,k}, \\
\text{and } \frac{\partial\tilde{v}(\alpha_i)}{\partial\alpha_i} =& \pi_{j,-d} - \rho F - \pi_{j,c} - \left((1-\alpha_j)\frac{\partial\pi_{i,c}}{\partial\alpha_i} + \alpha_i\frac{\partial\pi_{j,c}}{\partial\alpha_i}\right)
\end{aligned} \tag{C.14}$$

Given $\tilde{v}(\alpha_i) > 0$, the sign of $\partial\tilde{\delta}^*/\partial\alpha_i$ is the same as that of its numerator, which can be written as is shown in (C.15).

$$\begin{aligned}
\frac{\partial\tilde{u}(\alpha_i)}{\partial\alpha_i}\cdot\tilde{v}(\alpha_i) - \frac{\partial\tilde{v}(\alpha_i)}{\partial\alpha_i}\cdot\tilde{u}(\alpha_i) = \dots \\
(1-\alpha_j)\left[(\pi_{i,k} - \rho F - \pi_{i,c})\pi_{j,-d} - (\pi_{j,k} - \rho F - \pi_{j,c})\pi_{i,d} - (\pi_{i,c} - \pi_{j,c})\pi_{i,k}\right] \\
+ \left((1-\alpha_j)\frac{\partial\pi_{i,c}}{\partial\alpha_i} + \alpha_i\frac{\partial\pi_{j,c}}{\partial\alpha_i}\right)\tilde{u}(\alpha_i)
\end{aligned} \tag{C.15}$$

Equation (C.15), and thus $\partial\tilde{\delta}^*/\partial\alpha_i$, is positive if inequality (13) applies, which proves Proposition 4. $\square$

Lemma 10 is required to prove Proposition 5.

Lemma 10. $(1-\alpha_j)\frac{\partial\pi_{i,d}}{\partial\alpha_j} + \alpha_i\frac{\partial\pi_{j,-d}}{\partial\alpha_j} = 0$

Proof. For Bertrand competition with homogeneous goods, Lemma 10 applies because of $\partial\pi_{i,d}/\partial\alpha_j = 0$ and $\partial\pi_{j,-d}/\partial\alpha_j = 0$. For Cournot competition with homogeneous goods and $\partial q_j/\partial\alpha_j = 0$ (i.e., firm j adheres to the agreed-upon quantity), Lemma 10 can be restated as is shown by (C.16), which takes a value of zero because the bracketed term equals first-order condition (C.5) that is zero in the optimum.

$$(1-\alpha_j)\frac{\partial\pi_{i,d}}{\partial\alpha_j} + \alpha_i\frac{\partial\pi_{j,-d}}{\partial\alpha_j} = \frac{\partial q_i}{\partial\alpha_j}\left[(1-\alpha_j)\frac{\partial\pi_{i,d}}{\partial q_i} + \alpha_i\frac{\partial\pi_{j,-d}}{\partial q_i}\right] = 0 \tag{C.16}$$

For Bertrand competition with differentiated products and $\partial p_j/\partial\alpha_j = 0$, Lemma 10 can be restated as in (C.17). It takes a value of zero because the bracketed term equals first-order condition (C.10).

$$(1-\alpha_j)\frac{\partial\pi_{i,d}}{\partial\alpha_j} + \alpha_i\frac{\partial\pi_{j,-d}}{\partial\alpha_j} = \frac{\partial p_i}{\partial\alpha_j}\left[(1-\alpha_j)\frac{\partial\pi_{i,d}}{\partial p_i} + \alpha_i\frac{\partial\pi_{j,-d}}{\partial p_i}\right] = 0 \tag{C.17}$$

This proves Lemma 10. □

Proof of Proposition 5. Proposition 5 suggests $\partial\tilde{\delta}_i^*/\partial\alpha_j > 0$ if inequality (15) is satisfied. Equation (C.18) provides $\partial\tilde{\delta}_i^*/\partial\alpha_j$ when assuming profit function (11).

$$\begin{aligned}
\frac{\partial\tilde{\delta}_i^*}{\partial\alpha_j} &= \frac{\frac{\partial\tilde{u}(\alpha_j)}{\partial\alpha_j}\cdot\tilde{v}(\alpha_j) - \frac{\partial\tilde{v}(\alpha_j)}{\partial\alpha_j}\cdot\tilde{u}(\alpha_j)}{\tilde{v}(\alpha_j)^2} \\
\text{with } \frac{\partial\tilde{u}(\alpha_j)}{\partial\alpha_j} &= -(\pi_{i,d}-\pi_{i,k}) + (1-\alpha_j)\left(\frac{\partial\pi_{i,d}}{\partial\alpha_j} - \frac{\partial\pi_{i,k}}{\partial\alpha_j}\right) + \alpha_i\left(\frac{\partial\pi_{j,-d}}{\partial\alpha_j} - \frac{\partial\pi_{j,k}}{\partial\alpha_j}\right), \\
\text{and } \frac{\partial\tilde{v}(\alpha_j)}{\partial\alpha_j} &= -(\pi_{i,d}-\rho F-\pi_{i,c}) + (1-\alpha_j)\left(\frac{\partial\pi_{i,d}}{\partial\alpha_j} - \frac{\partial\pi_{i,c}}{\partial\alpha_j}\right) + \alpha_i\left(\frac{\partial\pi_{j,-d}}{\partial\alpha_j} - \frac{\partial\pi_{j,c}}{\partial\alpha_j}\right)
\end{aligned} \tag{C.18}$$

Using $\partial\pi_{i,k}/\partial\alpha_j = 0$, $\partial\pi_{j,k}/\partial\alpha_j = 0$, and Lemma 10, the partial derivatives can be simplified as is shown in (C.19).

$$\begin{aligned}
\frac{\partial\tilde{u}(\alpha_j)}{\partial\alpha_j} &= -(\pi_{i,d}-\pi_{i,k}) \\
\frac{\partial\tilde{v}(\alpha_j)}{\partial\alpha_j} &= -(\pi_{i,d}-\rho F-\pi_{i,c}) - (1-\alpha_j)\frac{\partial\pi_{i,c}}{\partial\alpha_j} - \alpha_i\frac{\partial\pi_{j,c}}{\partial\alpha_j}
\end{aligned} \tag{C.19}$$

Plugging $\partial\tilde{u}(\alpha_j)/\partial\alpha_j$, $\partial\tilde{v}(\alpha_j)/\partial\alpha_j$, $\tilde{u}(\alpha_j)$, and $\tilde{v}(\alpha_j)$ (see (C.14)) in $\partial\tilde{\delta}_i^*/\partial\alpha_j > 0$ yields inequality (15) after re-arranging. This proves Proposition 5. $\square$