Cost Shifting Incentives
in Network Industries

Benjamin Pakula*, Clemens Löffler† and Georg Götz*

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Abstract

In this paper we discuss cost shifting incentives in network industries under different organisational structures. We find that cost shifting occurs under vertical integration (VI) but not under ownership unbundling (OU) and legal unbundling (LU). The reason for this is that the network operators under OU and LU act independently and do not accept any shifted costs from the downstream segment. The welfare implications are still not unambiguous since a VI regime often features economies of scope, which lead to lower unit costs in the industry. Accordingly, the decision on what organisational form should optimally be chosen depends on the interplay of economies of scope under VI and under LU as well as on the monitoring intensity of the regulator. If the regulator has a high monitoring intensity and if economies of scope are much less effective under LU, VI may perform best from a welfare perspective. However, if economies of scope are (nearly) as effective under LU as under VI, LU seems to be the best option. This holds equally if economies of scope are absent in the industry. Only in this latter case OU performs equally well as LU, whereas VI is the worst alternative.

JEL classification: D20, D40, L43, L51, L90

Key words: Vertical Integration, Discrimination, Regulation, Foreclosure

Contact:
* Professur VWL I, Justus-Liebig-Universität Gießen, Licher Strasse 62, 35394 Gießen, Germany
  benjamin.pakula@wirtschaft.uni-giessen.de
† Lehrstuhl für Controlling, Universität Wien, Brünner Straße 72, 1210 Wien, Austria
  clemens.loeffler@univie.ac.at

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1 Introduction

Since the liberalisation of the formerly state-owned and vertically integrated monopolies of the railway, energy, and telecoms sectors in the 1990ies, the question of what constitutes the best organisational structure has been paid particular attention to. Some form of unbundling regulation has been established in all industries in order to facilitate regulation of the network. With respect to the railway industry, accounting separation is compulsory, but full vertical separation is often recommended. The debate has gained new relevance with the the infringement proceedings the European Commission has launched in June 2010 against 13 member states. The Commission claims that member states had infringed the implementation of the first railway package “most often by not sufficiently ensuring the independence of the rail infrastructure manager [...]”. However, further disintegration may destroy economies of joint production that stem from integration. Although – cross-sectoral and cross-country – different vertical regimes between full vertical integration and full vertical separation have emerged during the last two decades, economic literature mainly considers these two polar cases. The advocates of vertical separation claim that discrimination and sabotage incentives could be wiped out under this organisational regime. Accordingly, under vertical separation or ownership unbundling, competition could be fiercer and welfare higher. However, network investment incentives may be distorted since the network operator does not fully internalise the benefits of its investment. Moreover, economies of scope may exist among the network operator's and the downstream activities. These could be destroyed in case of vertical separation. Both, proponents and opponents of vertical separation get theoretical and empirical support for their arguments. Rey/ Tirole (2007) provide an overview on foreclosure under different organisational settings. Economides (1998) shows incentives for non-price discrimination for vertically integrated network companies. Mandy/ Sappington (2007) analyse cost-increasing and demand-reducing incentives to sabotage under different competition regimes. They conclude that the incentives vary with the type of sabotage and the nature of competition in the downstream market. Buehler/ Schmutzler/ Benz (2004) analyse investment incentives and find that vertical integration is superior to vertical separation in most cases since the benefits are fully internalised. Pakula/ Götz (2010) also include a legal unbundling regime with an independently-acting network operator, and confirm that vertical separation provides low investment incentives.

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2 cf. e.g. Monopolkommission (2009) with respect to the German market.
3 cf. EU (2010).
This paper aims at highlighting the benefits of different organisational structures with respect to the incentives to shift costs from the competitive downstream sector into the upstream sector, which is subject to a cost-oriented regulation. It is often argued that a vertically integrated company may have an incentive to shift costs from the downstream sector to the upstream sector (Armstrong/Sappington (2006: 358)). This makes its downstream activities more competitive compared to its rivals and may even discourage potential entrants from entering the market (e.g. Brennan (1990)). If the upstream sector is subject to a cost-oriented regulation, there seems to be a strong incentive to shift costs.\(^4\)

However, the problem is not so obvious for the regulator since a competitive advantage of the integrated company may also stem from economies of scope, i.e. joint production generates benefits in terms of lower costs. Growitsch/Wetzel (2007) analyse 54 railway companies from 27 European countries and find efficiency advantages and economies of scope for integrated companies. Ivaldi/McCullough (2008: 167) find vertical and horizontal economies of scope for integrated U.S. freight railroads: “The projections suggest that the fully integrated firm would have a 20-40 percent cost advantage over a vertically separated system where the operating company provided bulk and general freight services.” In a previous study of the U.S. freight railroad market, Bitzan (2003: 222-224) even concludes that “[...] policies introducing railroad competition through “open access” or on bottleneck segments would not be beneficial from a cost perspective. Moreover, the price decreases necessary for the introduction of such competition to be beneficial would be large.” Pittman (2005: 187) quotes from a report commissioned by the UK’s Office of the Rail Regulator: “[...] in a railway system where vehicle owners and maintainers are insulated from direct track damage costs (such as the situation that now exists in Britain), there is less pressure on the mechanical side to maintain wheels in good condition.” He concludes from this statement that a great deal of the short-term and long-term efficiency depends on the successful coordination of this intersection of steel wheels and steel rails.

Hence, there is a trade-off between more transparency through separation and potentially lower cost under integration with an open access regime. This makes it particularly hard for the regulator to set a non-discriminatory access charge since he can hardly observe whether the competitive advantage of the downstream company is due to cost shifting or to economies of scope. For transparency reasons (and against sabotage) Neelie Kroes (2007), former Commissioner for Competition Policy,

\(^4\) This is a similar argument as to what is called Bell Doctrine. According to the Bell Doctrine a vertically integrated firm whose upstream activities are regulated has an incentive to discriminate against its downstream competitors, cf. Joskow/Noll (1999).
argues for more transparency: “Such transparency is also crucial for market participants. If authorities struggle to get the information they need, competitors of vertically integrated incumbents must be really desperate for more transparency!” In many cases vertical separation is considered as remedy for anticompetitive behaviour. Such was the case when AT&T was broken up into the Baby Bells.\footnote{3}

The central question that we address in this paper is whether there are alternative organisational forms – adjacent to vertical integration and ownership unbundling – that may combine the positive aspects of both without featuring the undesirable properties. From an organisational perspective, several remedies are conceivable and are observed in practice: Accounting separation within a vertically integrated company is one option, surely the least strong. Giving the network operator a wide scope for decision-making and own profit maximisation is a second possibility (which we call “legal unbundling” in this paper). This model is particularly relevant in the European context. The most fundamental remedy is ownership unbundling. But this does not seem to be a panacea since breaking up network companies destroys economies of scope if there are any.

In this paper, we discuss cost shifting incentives in network industries under different organisational structures. We model a vertically related industry where the monopolistic network operator supplies an essential input for downstream production. In the downstream market both companies are in Cournot competition. We distinguish vertical integration (VI) and legal unbundling (LU) as two forms of (partial) integration open for third party access, as well as ownership unbundling (OU). We find that cost shifting occurs under VI but not under OU and LU. The reason for this is that the network operators under OU and LU act independently and do not accept any shifted costs from the downstream segment. The welfare implications are still not unambiguous since a VI regime often features higher economies of scope than LU. Accordingly, the decision on what organisational form should optimally be chosen depends on the interplay of economies of scope under VI and under LU as well as on the exogenous monitoring intensity of the regulator, which influences cost shifting.

A paper that also models the trade-off between sabotage and economies of scope is that of Crew/ Kleindorfer/ Sumpter (2005). They find that vertical separation is superior from a welfare perspective if economies of scope are low. The two main differences to our analysis are that they model a costly sabotage technology and that they only consider vertical separation and vertical integration, but not legal unbundling. Sappington (2006) builds on Crew/ Kleindorfer/ Sumpter and finds that the merits of vertical divestiture vary with the intensity of competition in the downstream

\footnote{cf. Joskow/ Noll (1999).}
market and with the locus of economies of scope. His analysis is also limited to vertical separation and vertical integration.

Besides the literature on discrimination as mentioned above, this paper is related to the literature on organisational structures. There is little work done to include more than vertical separation and vertical integration. Exceptions are Cremer/ Crémer/ De Donder (2006), Höffler/ Kranz (2011a/b) and Pakula/ Goetz (2010) which all relate to investments into the network and quantities supplied on the network. Moreover, they have in common the way they model legal unbundling: Compared to vertical integration, where the integrated company maximises the integrated profit, under legal unbundling, the legally unbundled (network) entity maximises its own (accounting) profit\(^6\) whereas the parent company maximises the integrated profit. This is aimed at preventing the network company from discriminating against downstream entrants.

Compared to the existing literature, the contribution of this paper is twofold: First, we extend the existing literature on cross-subsidisation and cost shifting by an organisational perspective. Instead of proposing sophisticated new regulatory mechanisms, we suggest an alternative organisational form next to vertical integration and ownership unbundling that has not been modelled in this context yet. Second, until now, the recent legal unbundling models were focused on investment decisions. We contribute to this literature the new aspect of cost shifting.

The remainder of the paper is organised as follows. In Section 2, we elaborate the model and derive the level of cost shifting under different organisational structures. Moreover, we make a welfare analysis. Section 3 summarises and concludes.

2 The Model

This section contains three parts: First, we describe the different components of the model. Second, we solve the for the equilibrium of the model. Third, we perform a welfare analysis and discuss the results of the model.

2.1 Components of the model

In this section we develop a model of a network industry in which the network operator provides an essential input for the final customer products of the downstream companies. One unit of network

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\(^6\) In Cremer/ Crémer/ De Donder (2006), the downstream companies are legally unbundled. To us, this seems at odds with reality in network industries. This is why, we proceed as Höffler/ Kranz (2011a/b) and Pakula/ Goetz (2010) do.
input is needed to produce one unit of the final product. The charge for the network input \( a \) is subject to a cost-oriented regulation. This is not only a common assumption in economic theory but reflects also the fact that linear access charges are not discriminatory as two-part tariffs may be.\(^7\)

The downstream market consists of two companies, called incumbent \((I)\) and entrant \((E)\) that compete à la Cournot. Imagine e.g. Thalys and DB which will soon compete for passengers for the journey from the Continent to London via the Eurotunnel. It is reasonable to assume that the companies are in quantity competition since the journey through the Eurotunnel needs technical adaptations of the trains for security reasons among others. Hence, a fast adjustment to higher demand is not possible.\(^8\)

**Demand Side**

We assume a simple linear demand function. The inverse demand function takes the following form:

\[
p = 1 - (q_i + q_E)
\]

The maximum willingness to pay is normalised to one. \( q_i \) and \( q_E \) are the quantities of the incumbent and of the entrant respectively.

**Supply Side**

The supply side consists of a network operator and two competing downstream companies. The network operator charges a regulated and linear access charge \( a \) to the downstream companies. We assume for simplicity that the network operator incurs no operating and no fixed costs. This assumptions is far away from reality but reduces complexity without limiting the conclusions drawn from the model. Therefore this simplification seems adequate for the purpose of our paper.\(^9\)

The downstream market is characterised by two competing companies. Both charge a final customer price \( p \) to their customers. Both companies have a (downstream) per unit cost of \( c \), but under VI and LU, there may be economies of joint production between the incumbent and the network operator. These economies of scope are represented by \( b \), where \( b < c \) holds. The larger these economies are, the higher is the advantage of the incumbent and the higher is welfare in this model. For simplicity

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\(^7\) cf. Nash (2005). Knieps (2006) discusses the Deutsche Bahn tariff menu between 1998 and 2001. Track user could choose among a two-part tariff and a linear tariff. If the track user opted for the two part tariff it became holder of the “Infracard” which entitled it to use the tracks at a lower linear tariff compared to the standard uniform charge. This access tariff was deemed to be anti-competitive because it favoured the own downstream subsidiary.

\(^8\) This is also in line with previous literature, e.g. Baumol (1983), Quinet/ Vickerman (2004), and Friebel/ Gonzalez (2005).

\(^9\) e.g. Laffont/ Rey/ Tirole (1998) equally abstract from fixed costs for a similar reason.
we assume that economies of scope can be fully realised within the downstream sector. However, (partial) integration may also provoke the potentially harmful conduct of cost shifting in order to get a competitive advantage over the entrant. Since we assume that network costs are regulated in a cost-oriented manner and that the regulator does not observe the level of original upstream and downstream costs, there may be incentives to shift costs to the upstream company because it is subject to a cost-oriented regulation. If the (partially) integrated company shifts costs from the competitive downstream sector into the network and if the regulator sets an access charge based on this higher level of cost, the downstream incumbent gains a competitive advantage over the entrant. This is due to the fact that the access charge does not affect the overall profit of the (partially) integrated company whereas it constitutes a real cost for the entrant.

Formally, we model this cost shifting possibility by a factor $\beta$, that is multiplied with the marginal cost $c$. $\beta$ takes values between zero and one, where one means that no costs are shifted (100% of the costs remains within the downstream company) and zero that all costs are shifted to the upstream company (i.e. zero cost remains within the downstream company). Accordingly $\beta c$ is that amount of the downstream cost which remains within the downstream company, and $(1-\beta)c$ is the amount that is shifted into the network. Shifting all costs may not be a good strategy yet because the more costs are shifted the higher is the probability that the regulator finds out that costs have been shifted. If he figures out the true level of cost, he only accepts this level of upstream cost and imposes a fine.\(^{10}\) This is a realistic assumption since the regulator is not as well informed as the company's management. Hence, it is conceivable that the company may decide to shift some costs, which cannot easily be detected in the reports submitted to the regulator. But the more costs are shifted and the more the downstream companies are asymmetric, the higher is the probability that the regulator realises that he is fooled. Note that the regulator cannot derive the level of the incumbent's upstream and downstream costs even if he receives truthful cost information from the downstream entrant. The reason for this is that the regulator is unaware of the amount of economies of scope.

Turning to the profit functions, the downstream profit function under OU reads as follows:

$$\Pi_i = (p-a-c)q_i$$

where $i = I, E$. Under OU, both companies are identical, and there is no possibility to shift cost because all companies are independent.

Under VI and LU, the downstream segment of the incumbent faces a slightly different profit function:

\(^{10}\) We do not model the fine explicitly. The results of the model hold even if there is no fine.
\[ \Pi_I = (p - a - \beta c + \alpha b) q_I \]  
(3)

Where \( 0 \leq \beta \leq 1, 0 \leq \alpha \leq 1 \). \( \beta c \) is the part of downstream cost that remains within the (partially) integrated downstream unit \( \alpha \) is a factor that indicates how effective the economies of scope are under LU compared to VI. \( \alpha \) takes a value of 1 under VI meaning that economies of scope are fully effective, and may take a value smaller than 1 under LU meaning that partial disintegration may reduce economies stemming from joint production.

Formally, the network operator's profit function takes the following form:

\[ \Pi_U = a q_e + (a - (1 - \beta) c) q_I \]  
(4)  
\[ \text{shifted cost} \]

The term \((1 - \beta) c\) represents the level of shifted cost from the downstream level to the upstream level.

**The Regulator**

We assume that the regulator sets the access charge for the network operator by maximising total welfare under a zero profit constraint for the network operator. Welfare is the sum of the profits and consumer surplus (CS):

\[ \text{Welfare} = \Pi_I + \Pi_E + CS \]
\[ \text{s.t.} \]
\[ \Pi_U = 0 \]  
(5)

**Organisational Forms**

We consider three organisational forms: vertical integration (VI), ownership unbundling (OU), and legal unbundling (LU). Under VI, the integrated company maximises the integrated profit and the entrant its own profit. Under OU, all companies act independently. Under LU, the network operator is legally unbundled from its parent company and maximises its accounting profit whereas the parent company that acts in the downstream market maximises the integrated profit.

*Figure 1* shows the different organisational forms and the resulting industry structure:

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11 It is often assumed that consumer surplus has a higher weight in the social welfare function than firms profits. Below, we consider total welfare and consumer surplus separately as two cases where profits have an equal weight and no weight respectively.
The Game
The structure of the game is as follows:

1. The incumbent and the upstream company decide whether to shift cost and, if so, how much
2. The regulator monitors costs and sets the access charge
3. The downstream companies compete in quantities

We analyse the game for the three organisational structures and solve it recursively. The first stage of the game only applies to VI and LU.

2.2 Quantity Competition Stage

The downstream companies act in a Cournot competition regime so that they maximise profits with respect to their quantity.

Under OU, both downstream companies maximise the profit function stated as equation (2).

\[
\max_{q_i} \Pi_i = (p - a - c)q_i
\]

where \( i = I, E \).

Under VI and LU, the incumbent and the network operator maximise the integrated profit function stated as (3) and (4):

\[
\max_{q_i} \Pi_I + \Pi_U = ((p - a - \beta c + \alpha b)q_i)
\]

The derivation of the equilibrium is straightforward. The resulting equilibrium quantities at the third stage of the game are shown in the following Table 1:
<table>
<thead>
<tr>
<th></th>
<th>$q_I$</th>
<th>$q_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>VI=LU</td>
<td>$\frac{1}{3}(1+a+2b\alpha-c)$</td>
<td>$\frac{1}{3}(1-2a-b\alpha-c)$</td>
</tr>
<tr>
<td>OU</td>
<td>$\frac{1}{3}(1-a-c)$</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Equilibrium Quantities at the Third Stage of the Game

Under OU, the quantities of the entrant and of the incumbent are identical because we assume in this scenario that all companies act independently. As standard result in a Cournot model, quantities are positively related to the willingness to pay and negatively related to the access price and the marginal cost of the downstream companies.

Under VI and LU, the equilibrium quantity of the incumbent is positively related to the willingness to pay, the access charge, and the amount of economies of scope while marginal costs affect the incumbent's quantity negatively. The access charge has a positive effect on the quantity of the incumbent. This results from the fact that the access charge does not constitute a real cost for the (partially) integrated company whereas it is a cost for the entrant. This means that a higher access charge creates a competitive advantage of the incumbent over the entrant under VI and LU.

Note that the quantities at this stage of the game do not depend directly on the amount of shifted costs as the opportunity cost of the (partially) integrated company does not change. However, the access charge $a$ depends on the choice of $\beta$. How this materialises is part of the second stage of the game.

2.3 Access Charge Setting Stage

At this stage of the game, we consider the task of the regulator. First, the regulator monitors the network operator's cost structure. With a probability of $\beta'$ the regulator finds no verifiable evidence for cost shifting, and with a probability of $(1-\beta')$ the regulator gets documentary evidence. Second, the regulator sets the access charge so that the network operator makes zero profits. With a probability of $\beta'$ the regulator sets the access charge based on the amount of upstream cost that the network operator reports. This access charge includes potentially shifted costs. Otherwise, the regulator finds out the true level of costs and sets the access charge by only accounting for the genuine cost of the network operator, i.e. $c^U = 0$. This happens with a probability $(1-\beta')$.

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12 This is economically equivalent to a welfare or and consumer surplus maximisation under a zero profit constraint for the network operator.
Note that we use the cost shifting parameter $\beta$ here to model also the probability. The specification guarantees that the probability to be discovered increases the more costs are shifted, i.e. the lower $\beta$. It is reasonable and realistic to assume this since the regulator detects cost shifting more easily if more costs are wrongly attributed. $\gamma$ is an exogenous parameter that indicates the monitoring intensity of the regulatory authority. The higher $\gamma$ the more effective is the analysis performed by the regulator for any level of $\beta$. $\gamma$ might depend on the financial resources available to the regulator. These are outside the regulator's control, and one may assume that they are determined by policy makers.

Accordingly the problem of the regulator can be stated as follows:

$$Welfare = \Pi_i + \Pi_E + CS$$

s.t. $$\Pi_U = 0$$ (8)

With a probability of $\beta^r$ $\Pi_U = \Pi_U^{shift}$ holds, and with a probability $(1-\beta^r)$ $\Pi_U = \Pi_U^\epsilon$ applies.

The resulting access charges under VI, OU, and LU are summarised in the following Table 2:

<table>
<thead>
<tr>
<th></th>
<th>$a^{shift}$</th>
<th>$a^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>VI=LU</td>
<td>$\frac{c(1-\beta)q^*_i}{q_i + q_E}$</td>
<td>0</td>
</tr>
<tr>
<td>OU</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2: Equilibrium Access Charge at the Second Stage of the Game
(an asterisk denotes the quantities derived in the previous stage of the game)

The interpretation of the access charge under OU is simple: As the regulator has full transparency, he sets the access charge equivalently to the average costs of the network operator, which are zero by assumption. The same holds if the regulator's monitoring intensity $\gamma$ reaches infinity.

$a^{shift}$ is also easy to interpret: the access charge is the amount of shifted costs divided by industry output, i.e. the per unit cost. If all the costs of the downstream unit remain within that company ($\beta = 1$), the access charge equals zero.

2.4 Cost Shifting Stage

At the first stage of the game, the incumbent and the network operator decide on the amount of cost to shift to the regulated network sector. Under OU, this stage is dropped.
Vertical Integration

In the VI scenario, the vertically integrated incumbent maximises the expected profit taking into account the decisions in stages 2 and 3. The expected profit is the probability-weighted average of the profit containing the cases that cost shifting is not detected and that cost shifting is detected:

$$\max_{\beta} : \Pi_I + \Pi_U = \beta^\gamma (\Pi_{I}^{\text{shift}} + \Pi_{U}^{\text{shift}}) + (1 - \beta^\gamma) (\Pi_{I}^\theta + \Pi_{U}^\theta)$$

(9)

where the superscripts shift and $\theta$ refer to the cases when cost shifting is detected or not detected respectively, i.e. $a^{\text{shift}}$ and $a^\theta$ are used.

We can make use of $\beta$ for weighting the profits because $\beta$ lies in the range between zero and one. The specification that we model here guarantees that the probability to be discovered increases, the more costs are shifted. Looking at the two polar cases, we see that the profit is the same if $\beta = 1$ and if $\beta = 0$. This means that if the incumbent is honest, he will not be punished. If he wanted to shift all costs, the regulator would certainly detect the true level of cost. As stated above, $\gamma$ is the exogenous monitoring intensity. Note at this point that we do not consider cost of monitoring in this paper. They may be quite high and create further distortions that go beyond this analysis.\(^{13}\) As a sufficient condition to get real number results we make

**Assumption 1**: $\gamma > 2$.

The first order condition of the maximisation problem stated in equation (9) takes the following form:

$$\frac{\partial (\Pi_I + \Pi_U)}{\partial \beta} = 0$$

(10)

A closed form solution of equation (10) for $\beta$ is not available. However, Proposition 1 shows that there exists an interior solution to the maximisation problem for the vertically integrated company.

**Proposition 1**: In equilibrium the incumbent engages in cost shifting but does not shift all costs, i.e. $\exists \beta^* \in (0, 1)$.

**Proof**: See Appendix.\(^{\Box}\)

In order to interpret the results more easily, we depict the equilibrium values for certain parameter values. We also present some comparative statics based on these simulations.

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\(^{13}\) For an overview cf. Mägli/ Jaag/ Finger (2010).
Figure 2: First Order Condition for the Optimal Level of $\beta$ under VI

LHS: $\gamma$ takes the values $\{2, 3, 4\}$ at $c = 1/10$ and $b = 0$

Middle: $c$ takes values $\{1/10, 1/4, 1/2\}$ at $\gamma = 2$ and $b = 0$

RHS: $b$ takes values $\{0, 1/10, 1/5\}$ at $c = 1/4$ and $\gamma = 2$

We have plotted on the vertical axis the variable $\beta$, i.e. the part of the downstream cost that remains within in the incumbent's downstream service provider.\textsuperscript{14} As the left hand side of Figure 2 shows, $\beta$ increases \textit{ceteris paribus} when the monitoring intensity of the regulator is higher ($\gamma$ increases). This is intuitive because the higher $\gamma$ the higher the probability to be discovered, which reduces profits for a given level of $\beta$.

The middle and the right hand side of Figure 2 show different levels of costs. As observable by inspection of the graphs, the optimal value of $\beta$ varies slightly around $2/3$ (i.e. $\frac{\gamma}{1+\gamma}$) for the chosen specifications. It is evident that the effect on the optimal choice of $\beta$ depends much more on $\gamma$ than on the costs of the incumbent's downstream operations. We see that VI leads not only to cost shifting but also to higher access prices (cf. Table 2) which translates into a distorted allocation in the downstream market. This is in line with Brennan (1990).

This simulated result holds as long as there is competition in the market, i.e. the entrant offers positive quantities. Otherwise there will be a downstream monopoly. A convenient way to identify the threshold between competition and monopoly is to set $q_e = 0$ and to solve for a critical value $c^{VI}$.

For we do not have an analytical solution for $\beta$ we express $c^{VI}$ as a function of $\beta$: $c < c^{VI} = \frac{1-b}{3-2\beta}$.

This expression constitutes the threshold level that separates a competitive market outcome from a monopoly outcome. $c^{VI}$ is negatively related to the amount of economies of scope and the level of cost shifting, i.e. the higher the competitive advantage of the incumbent (whether by economies of joint production or by cost shifting), the more likely the entrant is driven out of the market.

At the right of the $c^{VI}$, if marginal cost is larger than the critical value, the competitive market turns into a monopoly market. The equilibrium level of cost shifting is chosen so that that entry is

\textsuperscript{14} We have checked that for all chosen values the entrant is in the market.
deterred. To understand this, consider the monopoly profit of the vertically integrated company:

$$\Pi_{IM}^{VI} + \Pi_{UM}^{VI} = \frac{1}{4}(1+b-c)^2$$

This profit is independent from \( \beta \). In order to make sure that the monopoly profit is realised the integrated company must shift at least a proportion of the cost that is large enough to keep the entrant out of the market, i.e. \( \beta \leq \frac{3c+b-1}{2c} \). \(^{15}\) If the integrated company were to shift no cost the entrant would offer positive quantities so that the monopoly outcome could not be sustained.

**Result 1:** Under VI, if downstream costs are low \( (c < c^{VI}) \), there is competition in the market. Foreclosure of the entrant is driven by economies of scope and the level of cost shifting:

$$\frac{\partial c^{VI}}{\partial b} < 0$$

and

$$\frac{\partial c^{VI}}{\partial \beta} > 0$$

hold.

**Legal Unbundling**

Under LU, the incumbent maximises also the profit as stated as equation (9). The difference to the VI regime is that the network operator acts independently. The network operator maximises its accounting profit. Hence, he will accept cost shifting if this leads to higher profits, but will reject it if this reduces its profits. As the profits of the network operator are regulated on a cost-oriented basis, any \( \beta \) deviating from 1 causes an expected loss of

$$\frac{(1-\beta) c q_i}{\text{shifted cost}} \cdot \frac{(1-\beta^{VI})}{\text{probability to be discovered}}$$

so that the network operator only accepts:

$$\beta^{LU} = 1$$

\(^{(11)}\)**

Under LU, there is competition in the market for all \( c < c^{LU} = 1 - b \alpha \). Otherwise, only the incumbent is active. Comparing \( c^{VI} \) and \( c^{LU} \) shows that the entrant is only foreclosed due to a lack of efficiency under LU whereas under VI the cost shifting decision also plays a role. This leads to the following Proposition:

**Proposition 2:** Under LU, if downstream costs are low \( (c < c^{LU}) \), there is competition in the market. Foreclosure of the entrant only depends on the extent of economies of scope.

**Proof:** Straightforward from inspection of \( c^{LU} \).

\(^{15}\) This threshold is derived by solving \( c^{VI} \) for \( \beta \).
**Ownership Unbundling**

For the sake of completeness, we report here the equivalent Proposition for OU:

**Proposition 3:** Under OU, for any defined level of $c$, both companies compete in the market.

**Proof:** Straightforward.

Summarising the outcome of the cost shifting stage of the game, we retain that cost shifting will only take place under VI. How this affects welfare will be shown in the next section.

### 2.5 Model Results and Welfare

In this section, we compare the outcomes of the three different regimes. Since we can only simulate the results under VI, we start by comparing quantities and welfare under OU and LU; both allow for analytical solutions. In a second step we complement this analysis with some simulations of the results under VI.

Inserting the equilibrium values into the respective quantity and price equations we get the following results:

<table>
<thead>
<tr>
<th></th>
<th>Ownership Unbundling</th>
<th>Legal Unbundling</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_i$</td>
<td>$\frac{1-c}{3}$</td>
<td>$\frac{4+5\alpha b-4c-\sqrt{(2+\alpha b-2c)^2}}{6}$</td>
</tr>
<tr>
<td>$q_E$</td>
<td>$\frac{1-c}{3}$</td>
<td>$\frac{c-2\alpha b-1+\sqrt{(2+\alpha b-2c)^2}}{3}$</td>
</tr>
<tr>
<td>$p^*$</td>
<td>$\frac{1+2c}{3}$</td>
<td>$\frac{4+2c-\alpha b-\sqrt{(2+\alpha b-2c)^2}}{6}$</td>
</tr>
</tbody>
</table>

**Table 3:** Quantities and Prices under OU and LU at the First Stage of the Game

Under LU, the quantities and the price depend on the marginal cost of the downstream companies, the amount of economies of scope (weighted with $\alpha$), and the willingness to pay. Under OU, economies of scope do not play any role. Quantities of the entrant and of the incumbent are the same because both companies are identical. In the LU regime, quantities of the incumbent are higher and those of the entrant are lower compared to OU if the cost function of the legally unbundled incumbent exhibits economies of scope ($b>0 \wedge \alpha>0$). Total quantity is also higher in this case compared to OU. If there are no economies of scope under LU ($b=0 \vee \alpha=0$), market outcomes under OU and LU are the same.
Proposition 4 summarises this result:

**Proposition 4:** Under LU total quantity in the market is always higher and prices always lower compared to OU if the legally unbundled company exhibits economies of scope (i.e. \( b > 0 \land \alpha > 0 \)).

**Proof:** Straightforward from inspection of the results displayed in Table 3.\( \square \)

The intuition behind Proposition 4 is that higher quantities of the incumbent more than outweighs lower quantities of the entrant under LU compared to OU. This result translates directly into higher consumer surplus and higher total welfare under LU. If there are no economies of scope under LU (i.e. \( b = 0 \lor \alpha = 0 \)) the outcome is identical to OU for quantities, the price, consumer surplus, and welfare.

Summarising the discussed results for OU and LU we show that the LU regime performs always at least as well as the OU regime does. This holds similarly for the performance indicators of welfare, consumer surplus, and total quantity.

We turn now to some simulations in order to compare the VI regime to the OU and LU regimes. We start by plotting quantities. Following Bitzian (2003) who estimated economies of joint production amounting from 20% to 40% we choose \( b = c/3 \). In what follows we proceed in three steps. First, we look at quantities for different values of the marginal cost \( c \). Second, we compare the market equilibrium for varying \( \alpha \), i.e. the strength of economies of scope under LU compared to VI. Third, we report evidence from extensive simulations and conclude this section.

![Graphs showing quantities for different regimes and values of c](image)

**Figure 4:**

**LHS:** \( q_{I} \) (\( b = c/3, \gamma = 2, \alpha = 1 \))

**Middle:** \( q_{E} \) (\( b = c/3, \gamma = 2, \alpha = 1 \))

**RHS:** \( q_{I} + q_{E} \) (\( b = c/3, \gamma = 2, \alpha = 1 \))

The three graphs of Figure 4 show the quantities of the incumbent, the entrant and both taken together, which is total quantity. They are plotted over the range for all possible values of \( c \) if economies of scope are positive and equally effective under VI and LU. Under OU the quantities of the entrant and of the incumbent are the same because both companies are identical. Under VI and LU, the cost function of the incumbent exhibits economies of scope, which translates into higher quantities of the incumbent and lower quantities of the entrant because the incumbent has a...
competitive advantage. For the chosen specification, the quantity of the incumbent (entrant) under 
VI is higher (lower) than under LU since costs are shifted which increases the competitive 
advantage of the incumbent over the entrant. Total quantity is also higher compared to OU. The 
kink in the cost functions of vertically integrated and the legally unbundled downstream company 
mark the threshold level of downstream cost between a competitive and a monopolised market. 
When there is a downstream monopoly under VI and LU, the market outcome is the same. 
In this second step we compare the market outcome for varying levels of \( \alpha \), which indicates the 
effectiveness of economies of joint production under LU compared to VI.\(^{16}\) All other variables are 
fixed as indicated.

As in the previous figure, quantities of the the incumbent and of the entrant as well as total quantity 
is displayed in Figure 5. Looking at the polar case when economies of scope are not effective under 
LU \( \alpha = 0 \) we see that the OU and the LU regimes produce the same output. The output under VI is 
driven by two effects: First, there are economies of scope which provide a competitive advantage to 
the incumbent and which reduce average unit cost in the industry. Second, only about two third of 
the original downstream cost remain within the company after cost shifting, which also translates 
into a competitive advantage of the incumbent, but reduces quantity of the entrant. As the first effect 
dominates, total quantity is higher under VI if there are no economies of scope under LU. 
If economies of scope are equally effective under LU as under VI \( \alpha = 1 \) total quantity is highest 
under LU. To understand this, we refer again to the two effects which we have just described: The 
first effect means that economies of scope reduce average unit cost within the industry and, thus, 
lead to higher quantity. The second effect, the cost shifting effect, brings a distortion into the

\(^{16}\) Quantities under VI and OU do not depend on \( \alpha \). Therefore, the graphs of these regimes are a flat line in the 
diagram.
industry and leads to a reallocation of profits within the industry. This effect reduces $q_e$ more than it increases $q_I$. The second effect is absent under LU because there is no cost shifting. Therefore total quantity is higher under LU if economies of joint production are (nearly) as effective under LU than under VI.

This result translates directly into total welfare and consumer surplus as shown in *Figure 6* below, which needs no further discussion:

![Figure 6](image)

### Figure 6

- **LHS: Welfare**
  - $(c = 1/10, b = 1/30, \gamma = 2, \beta = 0.664838)$

- **RHS: Consumer Surplus**
  - $(c = 1/10, b = 1/30, \gamma = 2, \beta = 0.664838)$

The results of further simulations of the different model parameters provide the following conclusions:

- Comparative statics for VI and LU with respect to $b$ show that the higher the economies of scope are, the higher is $q_I$ and the lower $q_E$. The effect of $b$ on total quantity is positive since economies of scope lead to lower unit cost in the industry: $\frac{\partial (q_I + q_E)}{\partial b} > 0$. However, if there are no economies of scope ($b = 0$), quantities under LU equal quantities under OU, and total quantities under LU are higher than under VI. The reason for that is that cost shifting increases the access charge which leads to lower quantities of the entrant and only to slightly higher quantities of the incumbent. Hence if there are no economies of scope under either regime VI only leads to distortions and is therefore least favourable.

- Allowing for different levels of $\alpha$, i.e. indicating the effectiveness of economies of joint production under LU compared to VI, we derive the following results: If the economies of scope are equally effective under LU than under VI ($\alpha = 1$) and if there are positive downstream costs, $c$, the LU regime produces unambiguously highest welfare and consumer

---

17 They hold if $c$ is below 0.5. Simulations can be provided upon request.
surplus. Welfare under LU is negatively related to $a$ and converges to welfare under OU if $a = 0$. If welfare is higher under LU or VI depends on $a$. For intermediate values of $a$ we see that $qI$ ($qE$) decreases (increases) the lower the value of $a$. For $a = 1$, consumer surplus under LU is at least as high as under OU (if $b = 0$) and always superior to VI. The reason for this is the following: Even if there are no economies of scope, consumer surplus under LU is higher compared to VI because under VI, the incumbent has an incentive to shift costs. This leads to higher upstream costs, a higher access charge, and finally translates into a higher price. If $b > 0$, $CS^{LU} > CS^{VI}$ always holds if there are positive marginal costs $c$. If the economies of scope under LU are not as effective as under VI, consumer surplus reduces the lower $a$ and converges to consumer surplus under OU for $a = 0$.

- With respect to the monitoring intensity of the regulator, $\gamma$, we can state that the higher the monitoring intensity is, the lower is the amount of shifted costs and the more the outcome under VI converges to that under LU.

Table 4 summarises the model outcome for some corner values if there are positive marginal costs:

<table>
<thead>
<tr>
<th></th>
<th>$b=0$</th>
<th>$0 &lt; b &lt; c$</th>
<th>$b=0$</th>
<th>$0 &lt; b &lt; c$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$y \to \infty$</td>
<td>$2 \leq y &lt; \infty$</td>
<td>$2 \leq y &lt; \infty$</td>
<td></td>
</tr>
<tr>
<td>$\alpha = 1$</td>
<td>VI = LU = OU</td>
<td>VI = LU &gt; OU</td>
<td>LU = OU &gt; VI</td>
<td>LU &gt; VI</td>
</tr>
<tr>
<td>$\alpha = 0$</td>
<td>VI = LU = OU</td>
<td>LU = OU</td>
<td>LU = OU</td>
<td>LU = OU</td>
</tr>
</tbody>
</table>

Table 4: Model Outcome for Corner Values of the Exogenous Variables

3 Discussion and Conclusion

In this paper we have discussed cost shifting incentives in network industries under different organisational structures. We find that cost shifting occurs under VI but not under OU or LU. The reason for this is that the network operators under OU and LU act independently and do not accept any shifted costs from the downstream segment. The welfare implications are still not unambiguous since a VI regime often features economies of scope, which lead to lower unit costs in the industry. Accordingly, the decision on what organisational form should optimally be chosen depends on the interplay of economies of scope under VI and under LU as well as on the monitoring intensity of the regulator. If the regulator has a high monitoring intensity and if economies of scope are much less effective under LU, VI may perform best from a welfare perspective. However, if economies of scope are (nearly) as effective under LU as under VI, LU seems to be the best option. This holds
equally if economies of scope are absent in the industry. Only in this latter case OU performs equally well as LU, whereas VI is the worst alternative.

It is necessary to indicate that we assume that LU works as intended. This means that the network operator acts fully independently. If this reflects reality appropriately is doubted. To quote Neelie Kroes (2006) once again: “[...] I see only one way forward if we are to restore credibility and faith in the market. Europe has had enough of “Chinese walls” and quasi-independence. There has to be a structural solution that once and for all separates infrastructure from supply and generation. In other words: ownership unbundling. Then we will finally see an end to discrimination [...].” However, we show that even if Chinese walls do not work correctly, it is not clear that separating infrastructure from operations is a good strategy because economies of scope are destroyed.

Therefore the following policy implication should be considered tentatively. The model shows that an OU regime seems to be always dominated by LU and often by VI. Further general conclusions cannot be drawn from the analysis. Whether VI or LU is more advantageous depends on how effective economies of scope are under LU. This should be analysed for each network industry separately and remains an interesting field of research for empirical analysis.

Moreover, for an overall assessment of the optimal industry structure, investment incentives and much more have to be included into the analysis. This is also a promising task for future research.
4 Appendix

Proposition 1: In equilibrium the incumbent engages in cost shifting but does not shift all costs, i.e. \( \exists \beta^* \in (0,1) \).

Proof: To prove Proposition 1 we show that the equilibrium value for \( \beta \) lies in the range between zero and one. The first order condition of equation (9) for the boundary value \( \beta = 1 \) is negative:

\[
\frac{\partial (\Pi_I + \Pi_U)}{\partial \beta} \bigg|_{\beta=1} < 0.
\]

With this result we know that the optimal value for \( \beta \) is lower than 1, i.e. there is some level of cost shifting.

In order to show that maximum cost shifting is not optimal either, we consider the first and second order conditions for the lower bound \( \beta = 0 \):

\[
\frac{\partial (\Pi_I + \Pi_U)}{\partial \beta} \bigg|_{\beta=0} \geq 0 \quad \text{and} \quad \frac{\partial^2 (\Pi_I + \Pi_U)}{\partial \beta^2} \bigg|_{\beta=0} > 0.
\]

As both conditions are non-negative, the optimal value for \( \beta \) is in the range between zero and one.■

At the following pages the Mathematica output is provided in order show the proofs for the aforementioned inequalities.
Proof that there exists an interior solution for $\beta$

- The first order condition of equation (9) takes the following non simplified form:

\[
\operatorname{FOC} := \frac{1}{3} (1 + 2 b - c) c (-1 + \beta^\gamma) + \frac{1}{3} c (-1 - 2 b + c) (-1 + \beta^\gamma) - \\
\frac{1}{18} \beta^\gamma \left( -c + \frac{8 c + 10 b c + c^2 (-10 + 2 \beta)}{2 \sqrt{4 + b^2 + 8 c (-2 + \beta) + c^2 (13 + (-10 + \beta) \beta) + 2 b (2 + c (-7 + 5 \beta))}} \right) \\
\left( -1 + b + c (5 - 4 \beta) + \sqrt{4 + b^2 + 8 c (-2 + \beta) + c^2 (13 + (-10 + \beta) \beta) + 2 b (2 + c (-7 + 5 \beta))} \right) - \\
\frac{1}{18} \beta^\gamma \left( -4 c + \frac{8 c + 10 b c + c^2 (-10 + 2 \beta)}{2 \sqrt{4 + b^2 + 8 c (-2 + \beta) + c^2 (13 + (-10 + \beta) \beta) + 2 b (2 + c (-7 + 5 \beta))}} \right) \\
\left( -4 - 5 b - c (-5 + \beta) + \sqrt{4 + b^2 + 8 c (-2 + \beta) + c^2 (13 + (-10 + \beta) \beta) + 2 b (2 + c (-7 + 5 \beta))} \right) - \\
\frac{1}{9} \beta^{-1+\gamma} \left( -1 + b + c (5 - 4 \beta) + \sqrt{4 + b^2 + 8 c (-2 + \beta) + c^2 (13 + (-10 + \beta) \beta) + 2 b (2 + c (-7 + 5 \beta))} \right) \\
\left( -4 - 5 b - c (-5 + \beta) + \sqrt{4 + b^2 + 8 c (-2 + \beta) + c^2 (13 + (-10 + \beta) \beta) + 2 b (2 + c (-7 + 5 \beta))} \right) \gamma
\]
The second order condition of equation (9) takes the following non simplified form:

\[
\text{SOC} := - \frac{1}{\beta} \left( \begin{array}{c}
-4 c + \frac{8 c + 10 b c + c^2 (-10 + 2 \beta)}{2 \sqrt{4 + b^2 + 8 c (-2 + b) + c^2 (13 + (-10 + \beta) \beta) + 2 b (2 + c (-7 + 5 \beta))}} \\
-c + \frac{8 c + 10 b c + c^2 (-10 + 2 \beta)}{2 \sqrt{4 + b^2 + 8 c (-2 + b) + c^2 (13 + (-10 + \beta) \beta) + 2 b (2 + c (-7 + 5 \beta))}} \\
\frac{1}{18} \beta^2 \left( - \frac{(8 c + 10 b c + c^2 (-10 + 2 \beta))^2}{4 (4 + b^2 + 8 c (-2 + b) + c^2 (13 + (-10 + \beta) \beta) + 2 b (2 + c (-7 + 5 \beta)))^{3/2}} + \right) \\
\frac{c^2}{\sqrt{4 + b^2 + 8 c (-2 + b) + c^2 (13 + (-10 + \beta) \beta) + 2 b (2 + c (-7 + 5 \beta))}} \\
\left(-4 - 5 b - c (-5 + \beta) + \sqrt{4 + b^2 + 8 c (-2 + b) + c^2 (13 + (-10 + \beta) \beta) + 2 b (2 + c (-7 + 5 \beta))} \right) \cdot \\
\frac{2}{3} (1 + 2 b - c) c \beta^{-1+\gamma} + \frac{2}{3} c (-1 - 2 b + c) \beta^{-1+\gamma} \gamma - \\
\frac{1}{9} \beta^{-1+\gamma} \left( -c + \frac{8 c + 10 b c + c^2 (-10 + 2 \beta)}{2 \sqrt{4 + b^2 + 8 c (-2 + b) + c^2 (13 + (-10 + \beta) \beta) + 2 b (2 + c (-7 + 5 \beta))}} \\
\left(-4 - 5 b - c (-5 + \beta) + \sqrt{4 + b^2 + 8 c (-2 + b) + c^2 (13 + (-10 + \beta) \beta) + 2 b (2 + c (-7 + 5 \beta))} \right) \gamma - \\
\frac{1}{9} \beta^{-1+\gamma} \left( -4 c + \frac{8 c + 10 b c + c^2 (-10 + 2 \beta)}{2 \sqrt{4 + b^2 + 8 c (-2 + b) + c^2 (13 + (-10 + \beta) \beta) + 2 b (2 + c (-7 + 5 \beta))}} \\
\left(-4 - 5 b - c (-5 + \beta) + \sqrt{4 + b^2 + 8 c (-2 + b) + c^2 (13 + (-10 + \beta) \beta) + 2 b (2 + c (-7 + 5 \beta))} \right) \gamma - \\
\frac{1}{9} \beta^{-2+\gamma} \left(-1 + b + c (5 - 4 \beta) + \sqrt{4 + b^2 + 8 c (-2 + b) + c^2 (13 + (-10 + \beta) \beta) + 2 b (2 + c (-7 + 5 \beta))} \right) \gamma - \\
\frac{1}{18} \beta^{-2+\gamma} \left(-1 + b + c (5 - 4 \beta) + \sqrt{4 + b^2 + 8 c (-2 + b) + c^2 (13 + (-10 + \beta) \beta) + 2 b (2 + c (-7 + 5 \beta))} \right) \gamma - \\
\frac{1}{3} c (-1 - 2 b + c) \left( -1 + \gamma \right) \gamma -
\end{array} \right)
\]
In order to prove that the optimal $\beta$ is smaller than 1, we show that the FOC < 0 hold for $\beta = 1$

$$\text{Simplify}[\text{FOC} /. \beta \to 1]$$

$$\frac{1}{18 \sqrt{(2 + b - 2 c)^2}} \left( -20 c^3 (-2 + \gamma) - \left( 10 + 13 b + 4 b^2 \right) \left( -2 - b + \sqrt{(2 + b - 2 c)^2} \right) \gamma - c^2 \left( b (61 - 36 \gamma) + 5 \left( 16 - 5 \sqrt{(2 + b - 2 c)^2} + 2 \left( -6 + \sqrt{(2 + b - 2 c)^2} \right) \gamma \right) \right) + c \left( b^2 (25 - 21 \gamma) + 5 \left( 8 - 5 \sqrt{(2 + b - 2 c)^2} + 4 \left( -3 + \sqrt{(2 + b - 2 c)^2} \right) \gamma \right) \right) + b \left( 61 - 29 \sqrt{(2 + b - 2 c)^2} + \left( 72 - 13 \sqrt{(2 + b - 2 c)^2} \right) \gamma \right) \right)$$

With the following inequality we show that there exists no real number solution for FOC >= 0 if $\beta = 1$

$$\text{FindInstance}[\text{Simplify}[\text{FOC} /. \beta \to 1] \geq 0 \&\& 1 > c > a \&\& c > 0 \&\& b > 0 \&\& \gamma > 1, \{a, b, \gamma, c\}]$$

\{
\}

With the following inequality we show that FOC < 0 holds if $\beta = 1$

$$\text{Assuming}[\{b > 0 \&\& a < c \&\& a > 0 \&\& b < c \&\& 1 > c > 0 \&\& b \leq a \&\& \gamma > 1 \&\& \gamma \leq 1 \&\& \text{Reals}\}, \text{Simplify}[\{\text{Reduce}[\text{Simplify}[\text{FOC} /. \beta \to 1] < 0, \text{Reals}]\}]$$

\{True\}

In order to prove that the optimal $\beta$ is larger than 0, we show that FOC = 0 and SOC > 0 holds for $\beta = 0$

$$\text{Simplify}[\text{FOC} /. \beta \to 0]$$

$$\frac{1}{18} \left[ 6 (-1 + 0^\gamma) (1 + 2 b - c) c + 6 (-1 + 0^\gamma) c (-1 + 2 b + c) - 0^\gamma c \right.$$

$$\left( -4 + \frac{4 + 5 b - 5 c}{\sqrt{4 + b^2 + b (4 - 14 c) - 16 c + 13 c^2}} \right) \left( -4 - 5 b + 5 c + \sqrt{4 + 4 b + b^2 - 16 c - 14 b c + 13 c^2} \right) \left( -1 + \frac{4 + 5 b - 5 c}{\sqrt{4 + b^2 + b (4 - 14 c) - 16 c + 13 c^2}} \right) \left( -1 + b + 5 c + \sqrt{4 + 4 b + b^2 - 16 c - 14 b c + 13 c^2} \right) -$$

$$\left. 6 0^\gamma c (-1 - 2 b + c) \gamma - 2 0^\gamma (1 + 2 b - c) (1 + 2 b + 2 c) \gamma - 0^\gamma \left( -4 - 5 b + 5 c + \sqrt{4 + 4 b + b^2 - 16 c - 14 b c + 13 c^2} \right) \left( -1 + b + 5 c + \sqrt{4 + 4 b + b^2 - 16 c - 14 b c + 13 c^2} \right) \right) \right.$$
With the following inequality we show that there exists no real number solution for $SOC < 0$ if $\beta \to 0$

\[ \text{FindInstance}[\text{Simplify}[\text{FOC} /. \beta \to 0] < 0 \&\& 1 > c > a > 0 \&\& c > 0 \&\& c > b > 0 \&\& \gamma > 1, \{a, b, \gamma, c\}] \]

\{

With the following inequality we show that $FOC \geq 0$ holds if $\beta = 0$

\[ \text{Assuming}[\{b > 0 \&\& a < c \&\& a > 0 \&\& b < c \&\& 1 > c > 0 \&\& qI > 0 \&\& qE > 0 \&\& c < 1 \&\& \gamma > 1 \&\& \text{Reals}\}, \text{Simplify}[\{\text{Reduce}[\text{Simplify}[\text{FOC} /. \beta \to 0] \geq 0, \text{Reals}]\}]] \]

\{True\}

With the following inequality we show that there exists no real number solution for $SOC < 0$ if $\beta = 0$

\[ \text{FindInstance}[\text{Simplify}[\text{SOC} /. \beta \to 0] < 0 \&\& 1 > c > a > 0 \&\& c > 0 \&\& c > b > 0 \&\& \gamma > 1, \{a, b, \gamma, c\}] \]

\{

\]
With the following inequality we show that the slope of the SOC is positive for $\beta = 0$ if $\gamma > 2$

```math
Assuming[ {b > 0 && a < c && a > 0 && b < c && 1 > c > 0 && qI > 0 && qE > 0 && c < 1 && \gamma > 1},
Simplify[{Reduce[Simplify[SOC /. \beta \to 0] \geq 0, Reals ]}]]
```

$\{\gamma > 2\}$
5 References


