Location, Technology, and Competitive Strategy

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Starting from Neven 1987, p. 422: The consumer who is indifferent. The notation is like in the paper except that I always use small letters. Instead of superscripts I put the firm indices in brackets.

\[
a[i_, j_] := \frac{p[j] - p[i]}{2(x[j] - x[i])} + \frac{x[i] + x[j]}{2}
\]

\[
a[1, 2] = \frac{-p[1] + p[2]}{2} - \frac{1}{2} (x[1] + x[2])
\]

\[
demand[i_] := (a[i, i+1] - a[i, i-1])n
\]

\[
profit[i_] := (p[i] - c[i])demand[i] - f[i]
\]

\[
\]

\[
foc[i_] := D[profit[i], p[i]]
\]

■ The game with 2 firms

In this and the next section I evaluate the benchmark case in which only one technology is available. This case is dealt with in Neven (1987) and Götz (2005). It is included since it both is a special case of the more general model with technology choice. It also serves to make many definitions.

■ Equilibrium of the price-setting game with 2 firms 1 and 2, locations given as well as technology

The definition of the boundaries of the market.

\[
a[1, 0] = 0; a[2, 3] = 1;
\]

\[
\]

n
The reduced profit function taking into account the pricing rules.

\[
\text{redProfit}[1] = \frac{\text{profit}[1]}{\text{prices2Firms} // \text{Simplify}}[1]
\]

\[
\]

Operating profits of firm 1 with locations x[1] and x[2]

\%
a // FullSimplify

\[
\text{redProfit}[1] = \frac{\text{profit}[1]}{\text{prices2Firms} // \text{Simplify}}[1]
\]

\[
\]

The reduced profit function taking into account the pricing rules.

\[
\text{redProfit}[2] = \frac{\text{profit}[2]}{\text{prices2Firms} // \text{Simplify}}[1]
\]

\[
\]
\[ \text{Different profit configurations} \]

The following calculations present the profits for various constellations when two firms are active. The first element gives the profit of firm 1, the second that of firm 2. Locations are 0 and 1, respectively. Both firms' marginal costs are the same.

\[
\{ \text{redProfit}[1], \text{redProfit}[2] \} / . \{ x[1] \rightarrow 0, c[1] \rightarrow c[2], x[2] \rightarrow 1 \} / . \text{Expand}
\]

\[
\{ \frac{n}{2} - f[1], \frac{n}{2} - f[2] \}
\]

Locations are 1/2 and 1

\[
\{ \text{redProfit}[1], \text{redProfit}[2] \} / . \{ x[1] \rightarrow 1/2, c[1] \rightarrow c[2], x[2] \rightarrow 1 \} / . \text{Expand}
\]

\[
\{ \frac{49 n}{144} - f[1], \frac{25 n}{144} - f[2] \}
\]

Locations are 1/2 and 1. The second firms marginal costs are higher. The cost difference is 1/2.

\[
\{ \text{redProfit}[1], \text{redProfit}[2] \} / . \{ x[1] \rightarrow 0, c[1] \rightarrow c[2] - 1/2, x[2] \rightarrow 1 \} / . \text{Expand}
\]

\[
\{ \frac{49 n}{72} - f[1], \frac{25 n}{72} - f[2] \}
\]

\[
\{ 0.680556 n - 1. f[1], 0.347222 n - 1. f[2] \}
\]

\[
\{ \text{redProfit}[1], \text{redProfit}[2] \} / . \{ x[1] \rightarrow 1/2, c[1] \rightarrow c[2] - 1/2, x[2] \rightarrow 1 \} / . \text{Expand}
\]

\[
\{ \frac{9 n}{16} - f[1], \frac{n}{16} - f[2] \}
\]

\[
\{ 0.5625 n - 1. f[1], 0.0625 n - 1. f[2] \}
\]

\[
\{ \text{redProfit}[1], \text{redProfit}[2] \} / . \{ x[1] \rightarrow 1/2, c[1] \rightarrow c[2] - 1/2, x[2] \rightarrow 1 \} / . \text{Expand}
\]

A useful definition

\[
\text{profits2Firms}[x1_, x2_, \text{costdif}_] := \{ \text{redProfit}[1], \text{redProfit}[2] \} / . \{ x[1] \rightarrow x1, c[1] \rightarrow c[2] - \text{costdif}, x[2] \rightarrow x2 \}
\]

Maximum variable cost savings due to investment in large scale technology

\[
\frac{3}{4} \frac{1}{2} n
\]

\[
\frac{3 n}{8}
\]

\[
\text{Solve}\{ n = 25, n \}
\]

\[
\{ \{ n \rightarrow \frac{200}{3} \} \}
\]
The equilibrium prices when the marginal costs are 0 for all firms.

\[ \frac{1}{4}, \frac{1}{4}, n \]

The game with 3 firms

Equilibrium of the price-setting game with 3 firms 1, 2 and 3, locations given as well as technology

\[ a[1,0]=0; a[3,4]=1; a[2,3]=. \]

\[ \text{demand}[1] + \text{demand}[2] + \text{demand}[3]//\text{Simplify} \]

\[ n \]

\[ \text{foc}[2] \]


The equilibrium prices depending on locations and technologies.

\[ \text{prices3Firms} = \text{Solve}[[\text{foc}[1]==0, \text{foc}[2]==0, \text{foc}[3]==0], \]

\[ \{p[1], p[2], p[3]\}]//\text{Simplify} \]

\[ \left\{ \begin{array}{l}
    \{p[1] \rightarrow \frac{1}{6} (\text{x}[1] - \text{x}[3]) (\text{c}[1] (3 \text{x}[1] + \text{x}[2] - 4 \text{x}[3]) + 2 \text{c}[2] (\text{x}[1] - \text{x}[3]) + (\text{x}[1] - \text{x}[2])
    
    (\text{c}[3] - 3 \text{x}[1]^2 - 2 \text{x}[2] - 2 \text{x}[1] \text{x}[2] + 2 \text{x}[3] + 2 \text{x}[1] \text{x}[3] + 2 \text{x}[2] \text{x}[3] + \text{x}[3]^2)\},

    \{p[2] \rightarrow \frac{1}{3} (\text{x}[1] - \text{x}[3]) (\text{c}[3] (\text{x}[1] - \text{x}[2]) + 2 \text{c}[2] (\text{x}[1] - \text{x}[3]) +
    
    (\text{x}[2] - \text{x}[3]) (\text{c}[1] - 2 \text{x}[1] + \text{x}[1]^2 + 2 \text{x}[2] - \text{x}[1] \text{x}[2] - \text{x}[1] \text{x}[3] + \text{x}[2] \text{x}[3]))\},

    \{p[3] \rightarrow \frac{1}{6} (\text{x}[1] - \text{x}[3]) (\text{c}[3] (4 \text{x}[1] - \text{x}[2] - 3 \text{x}[3]) + 2 \text{c}[2] (\text{x}[1] - \text{x}[3]) +
    
    (\text{x}[2] - \text{x}[3]) (\text{c}[1] - 8 \text{x}[1] + \text{x}[1]^2 + 2 \text{x}[2] + 2 \text{x}[1] \text{x}[2] +
    
    6 \text{x}[3] + 2 \text{x}[1] \text{x}[3] - 2 \text{x}[2] \text{x}[3] - 3 \text{x}[3]^2))\}\} \]

The equilibrium prices when the marginal costs are 0 for all firms.

\[ \text{priceC0} = \text{prices3Firms} /. \{\text{c}[1] \rightarrow 0, \text{c}[2] \rightarrow 0, \text{c}[3] \rightarrow 0\} //\text{Simplify} \]

\[ \left\{ \begin{array}{l}
    \{p[1] \rightarrow \frac{1}{4} (\text{x}[1] - \text{x}[2]) (3 \text{x}[1]^2 + 2 \text{x}[1] (\text{x}[2] - \text{x}[3]) - 2 \text{x}[2] (-1 + \text{x}[3]) - \text{x}[3] (2 + \text{x}[3])) ,

    \{p[2] \rightarrow \frac{(\text{x}[1] - \text{x}[2]) (-2 + \text{x}[1] - \text{x}[3]) (\text{x}[2] - \text{x}[3])}{3 (\text{x}[1] - \text{x}[3])} ,

    \{p[3] \rightarrow \frac{1}{6} (\text{x}[1] - \text{x}[3]) (\text{x}[1]^2 - 2 \text{x}[2] (-1 + \text{x}[3]) - 3 (-2 + \text{x}[3]) \text{x}[3] + 2 \text{x}[1] (-4 + \text{x}[2] + \text{x}[3]))\}\} \]

\[ \text{priceC0} /. \{\text{x}[1] \rightarrow 0, \text{x}[2] \rightarrow 1/2, \text{x}[3] \rightarrow 1\} \]

\[ \left\{ \begin{array}{l}
    \{p[1] \rightarrow \frac{1}{4} , p[2] \rightarrow \frac{1}{4} , p[3] \rightarrow \frac{1}{4}\}\} \]
The equilibrium with simultaneous locational choice.

Evaluation of the first order condition of the firm located at the left edge of the market. This firm is the last entrant in the center of the market only depends on the distance between the other two firms, not on the location!

This result is different from that reported by Neven, p. 425! The order and the magnitudes fit, however.

The profit of the firm which is located in the center of the market.

\[
\text{redProfit3}[i_\text{]} := 
\text{redProfit3}[i_\text{]} = (\text{profit}[i_\text{]} / .\text{prices3Firms} /. \text{Simplify})[[1]] \\
\text{redProfit3}[1] \\
\text{redProfit3}[2] \\
\text{redProfit3}[3] \\
\]

The profit of the firm which is located in the center of the market only depends on the distance between the other two firms, not on the location!

\[
\text{redProfitSym}[i_\text{]} := \text{redProfitSym}[i_\text{]} = \text{redProfit3}[i_\text{]} /. \{c[1] \rightarrow 0, c[2] \rightarrow 0, c[3] \rightarrow 0\} /. \text{Simplify} \\
\text{redProfitSym}[1] \\
\]

Evaluation of the first order condition of the firm located at the left edge of the market. This firm is the last entrant in Neven's paper. The positive value is indicating that the locations cannot be equilibrium locations.

\[
\text{D}[\text{redProfitSym}[1], x[1]] /. \{x[1] \rightarrow 0.0685, x[2] \rightarrow 0.42, x[3] \rightarrow 0.891\} /. \text{Simplify} \\
\]

The equilibrium with simultaneous locational choice.
The third element of the list gives the result for the simultaneous choice of locations! Mentioned by Neven!

\[
\text{symEqu} = \text{Solve}\left[\{D[\text{redProfitSym}[1], x[1]] == 0, D[\text{redProfitSym}[2], x[2]] == 0, D[\text{redProfitSym}[3], x[3]] == 0\}, \{x[1], x[2], x[3]\}\right]
\]

\[
\left\{\left\{x[2] \rightarrow \frac{1}{15}, x[1] \rightarrow -\frac{19}{30}, x[3] \rightarrow \frac{23}{30}\right\}, \left\{x[2] \rightarrow \frac{1}{2}, x[1] \rightarrow -\frac{3}{2}, x[3] \rightarrow \frac{5}{2}\right\}\right\}
\]

\[
\text{N[\%]}
\]

\[
\left\{\left\{x[2] \rightarrow 0.0666667, x[1] \rightarrow -0.633333, x[3] \rightarrow 0.766667\right\}, \left\{x[2] \rightarrow 0.5, x[1] \rightarrow -1.5, x[3] \rightarrow 2.5\right\}, \left\{x[2] \rightarrow 0.5, x[1] \rightarrow 0.125, x[3] \rightarrow 0.875\right\}, \left\{x[2] \rightarrow 0.933333, x[1] \rightarrow 0.233333, x[3] \rightarrow 1.63333\right\}, \left\{x[2] \rightarrow 2, x[1] \rightarrow -1.41421, x[3] \rightarrow 1.41421\right\}, \left\{x[2] \rightarrow 2, x[1] \rightarrow -1.41421, x[3] \rightarrow 1.41421\right\}, \left\{x[2] \rightarrow 3, x[1] \rightarrow 1.41421, x[3] \rightarrow 1.41421\right\}, \left\{x[2] \rightarrow -1.41421, x[1] \rightarrow -1.41421, x[3] \rightarrow -1.41421\right\}, \left\{x[2] \rightarrow -1.41421, x[1] \rightarrow -1.41421, x[3] \rightarrow -1.41421\right\}\right\}
\]

The third element of the list gives the result for the simultaneous choice of locations! Mentioned by Neven!

\[
\text{symEqu}[3]
\]

\[
\left\{x[2] \rightarrow \frac{1}{2}, x[1] \rightarrow \frac{1}{8}, x[3] \rightarrow \frac{7}{8}\right\}
\]

Profits in the case of the simultaneous choice:

\[
\text{\{redProfitSym[1], redProfitSym[2], redProfitSym[3]\} / \text{symEqu}[3]}
\]

\[
\left\{\left\{\frac{169}{3072} - f[1], \frac{121 n}{1536} - f[2], \frac{169 n}{3072} - f[3]\right\}\right\}
\]

\[
\text{priceCO / \text{symEqu}[3]}
\]

\[
\left\{\left\{p[1] \rightarrow \frac{13}{64}, p[2] \rightarrow \frac{11}{64}, p[3] \rightarrow \frac{13}{64}\right\}\right\}
\]
The profit is highest at the centre location, the price of the respective firm is lower than that of its rivals.

- **Sequential entry**

For the sequential entry scenario the reaction functions must be determined. The sequence of entry is as follows: 2,3,1 as used by Neven.

Derivation of the reaction functions. The first order condition with respect to locational choice. At the moment I use the condition for firm 1 only. Later on, I need the other conditions when evaluating entry deterrence in the case of two active firms.

\[
\begin{align*}
\text{focLoc[1]} &= D[\text{redProfit[1]}, x[1]] // \text{Simplify} \\
\text{focLoc[2]} &= D[\text{redProfit[2]}, x[2]] // \text{Simplify} \\
\text{focLoc[3]} &= D[\text{redProfit[3]}, x[3]] // \text{Simplify}
\end{align*}
\]

\[
\begin{align*}
\}
\]

\[
(72 \cdot (x[1] - x[2])^2 \cdot (x[1] - x[3])^2)
\]

\[
\}

\[
\}

\[
(18 \cdot (x[1] - x[2])^2 \cdot (x[1] - x[3]) \cdot (x[2] - x[3])^2)
\]

The reaction function of firm 1 determines the location of this firm as a function of the other firms’ locations and of the technologies.

\[
\text{react1} = \text{Solve}[\text{focLoc[1]} == 0, x[1]];
\]

\[
\]

\[
\{\{x[1] -> 1.18046 - 2.22045 \times 10^{-16} \text{ I}, \{x[1] -> 0.847127 - 1.11022 \times 10^{-16} \text{ I},
\}
\{x[1] -> 0.5 + 4.44089 \times 10^{-16} \text{ I}, \{x[1] -> 0.0977664, \{x[1] -> 0.5,
\}
\{x[1] -> 1.00667 - 0.350866 \text{ I}, \{x[1] -> 1.00667 + 0.350866 \text{ I}
\}
\]

As Mathematica finds not only the (real valued) maxim, I pick out the solution in the next step (element 4).

\[
\text{react1} = \text{react1}[[4]]
\]
Consistency check: The values from the symmetric equilibrium.

Further analytical results like the reaction function of firm 3 taking into account the reaction function of firm 1 are not available. Therefore I start with numerical solutions. First I consider the benchmark case.

- The benchmark case: Only large firms.

All firms use the same technology. Fixed costs are 25, marginal costs are 0.

\[ f[\_] = 25; c[\_] = 0; \]

- The equilibrium locations when three firms are active and there is no further entry

First I derive the equilibrium locations for the case with 3 firms active. To do this I must specify the market size as well. The market size has only an effect on profits, not on the equilibrium locations.

\[ n = 1000; \]

The following function gives the first order constraint for the locational choice of firm 3, taking into account the reaction function of firm 1. It is a function of \( i \), the locational choice of firm 2 (Firm 2 is the first entrant!).

\[ \text{function}[i\_] := \text{function}[i] = \left( (\text{D}[\text{redProfit3}[3]/\text{react1}, x[3]])/\left(\{x[3] \to x3, x[2] \to i\}[[1]]\right) \right); \]

Given the location of firm 2, the function can be solved for the location of firm 3 (here \( x3 \)), and also for \( x[1] \)

\[ \text{FindRoot}[\text{function}[.5] == 0, \{x3, .9\}] \]

\[ \{x3 \to 0.918677\} \]

\[ \text{react1}/\left(\{x[2] \to .5, x[3] \to .918677\}\right) \]

\[ \{x[1] \to 0.115256\} \]

The solution is defined in the next routine.

\[ \text{reactNum3}[i\_] := \text{reactNum3}[i] = \text{FindRoot}[\text{function}[i] == 0, \{x3, .9\}] \]

\[ \text{reactNum3}[.4] \]

\[ \{x3 \to 0.878158\} \]

The next line (and the plot following) gives the profit of the first entrant as a function of her location (here .5) taking into account the reaction functions of firms 1 and 3 (that is entrants 2 and 3).
It takes quite long to calculate the next plot.

Join[reactNum3[j], \{x[2] \[Rule] j\}], \{j, .4, .5\}]}

- Graphics -

InputForm[%]

valuesSym = %[[1, 1, 1, 1]]

\{\{0.4, 62.9774\}, \{0.404057, 62.9959\}, \{0.408481, 63.0125\}, \{0.412636, 63.0248\},
\{0.416632, 63.0334\}, \{0.418687, 63.0367\}, \{0.420885, 63.0394\},
\{0.421958, 63.0403\}, \{0.422523, 63.0407\}, \{0.423122, 63.0411\},
\{0.423646, 63.0414\}, \{0.424222, 63.0416\}, \{0.424474, 63.0417\}, \{0.424746, 63.0418\},
\{0.425004, 63.0418\}, \{0.425237, 63.0419\}, \{0.425503, 63.0419\}, \{0.42565, 63.0419\},
\{0.425787, 63.0419\}, \{0.425915, 63.042\}, \{0.426033, 63.042\}, \{0.426162, 63.042\},
\{0.426298, 63.042\}, \{0.426436, 63.042\}, \{0.426513, 63.0419\}, \{0.426584, 63.0419\},
\{0.42685, 63.0419\}, \{0.426999, 63.0419\}, \{0.427161, 63.0419\}, \{0.427456, 63.0418\},
\{0.427987, 63.0416\}, \{0.428552, 63.0414\}, \{0.429566, 63.0409\},
\{0.430549, 63.0401\}, \{0.43161, 63.0392\}, \{0.433529, 63.0369\}, \{0.435634, 63.0335\},
\{0.437849, 63.0291\}, \{0.44185, 63.0189\}, \{0.446109, 63.0048\}, \{0.450209, 62.988\},
\{0.454566, 62.9668\}, \{0.458765, 62.943\}, \{0.462803, 62.9172\}, \{0.4671, 62.8864\},
\{0.471237, 62.8536\}, \{0.475215, 62.8192\}, \{0.479451, 62.7794\}, \{0.483528, 62.7381\},
\{0.487862, 62.6909\}, \{0.492037, 62.6423\}, \{0.496052, 62.5927\}, \{0.5, 62.5412\}\}

Max[valuesSym]

63.042

N[\%, 16]

63.04195934723518

Position[valuesSym, Max[valuesSym]]

\{\{22\}, \{2\}\}

N[valuesSym[[22]], 16]

\{0.4261616680713123, 63.04195934723518\}
These are the equilibrium locations in the benchmark case when three firms are active and further entry is blocked.

\[
\text{locationsSym} = \text{Flatten}[\text{react1} /\{\x[2] \to 0.42615, \x[3] \to x3 /\text{reactNum3}[0.42615], \\
x[2] \to 0.42615, \x[3] \to x3 /\text{reactNum3}[0.42615]]]
\]

\[
\{x[1] \to 0.0736691, x[2] \to 0.42615, x[3] \to 0.888526\}
\]

\[
\text{locationsSym} = (x[1] \to 0.07367, x[2] \to 0.42615, x[3] \to 0.88853)
\]

\[
\{x[1] \to 0.07367, x[2] \to 0.42615, x[3] \to 0.88853\}
\]

Compared to Neven's equilibrium the last entrant has a higher profit in the equilibrium I derive, the two earlier entrants have lower profits. The same holds for the prices. Curiously, the prices I get for the Neven locations are greater by an order of at least 0.005 than the prices Neven reports (p.429, Table 2). In general, however, the magnitudes are not too different.

The results really differ when it comes to entry deterrence in the 2 firms case.
Entry deterrence in the case of two active firms

At first I undo the definition of the parameter $n$.

\[ n = \ldots \]

Here I show first that the profit a potential entrant at the center is not changed with a change in the locations of the two incumbents as long as the distance between the two incumbents stays the same.

\[
\text{Solve}[(\text{focLoc}[2][[1]] / . \{x[1] -> i, x[3] -> k\}) == 0, x[2]]
\]

\[
\{\{x[2] -> \frac{i + k}{2}\}\}
\]

\[
(\text{redProfit3}[2] / . \{x[1] -> i, x[2] -> (1 + i)/2, x[3] -> 1\}) -
\]

\[
\{0\}
\]

\[
\text{profits2Firms}[i, 1, 0] - \text{profits2Firms}[i/2, 1 - i/2, 0] // \text{Simplify}
\]

\[
\left\{-\frac{1}{18} (-1 + i) (6 + i) n, -\frac{1}{18} (-6 + i) (-1 + i) i n\right\}
\]

The profit difference for the first entrant (the first element) is positive. This result proofs that the first entrant will deter further entry alone as long as it is possible.

\[
x2opt[i_] = x[2] / . \text{Solve}[(\text{focLoc}[2][[1]] / . \{x[1] -> i, x[3] -> 1\}) == 0, x[2]][[-1]]
\]

\[
\frac{1 + i}{2}
\]

\[
\]

\[
\text{Solve}[\text{function1}[i, n][[1]] == 0, i]
\]

\[
\left\{\begin{array}{l}
i \rightarrow \frac{7}{3} + \frac{2^{2/3} n^{1/3}}{3 (-6075 - 2 n + 45 \sqrt[3]{\sqrt[3]{6075 + 4 n}})^{1/3}} + \frac{2^{2/3} (-6075 - 2 n + 45 \sqrt[3]{\sqrt[3]{6075 + 4 n}})^{1/3}}{3 n^{1/3}}, \\
i \rightarrow \frac{7}{3} - \frac{2^{1/3} (1 + I \sqrt[3]{3}) n^{1/3}}{3 (-6075 - 2 n + 45 \sqrt[3]{\sqrt[3]{6075 + 4 n}})^{1/3}} - \frac{(1 - I \sqrt[3]{3}) (-6075 - 2 n + 45 \sqrt[3]{\sqrt[3]{6075 + 4 n}})^{1/3}}{3 2^{1/3} n^{1/3}}, \\
\end{array}\right\}
\]

\[
\text{N}[% / . n -> 300]
\]

\[
\left\{\begin{array}{l}
i \rightarrow 3.38883 - 1.41736 I, \\
i \rightarrow 0.222336 + 4.57852 \times 10^{-16} I, \\
i \rightarrow 3.38883 + 1.41736 I
\end{array}\right\}
\]

The second solution is the right one, it yields the real numbers.

\[
deter[j_] = i /. \text{Solve}[\text{function1}[i, j][[1]] == 0, i][[2]]
\]

\[
\frac{7}{3} - \frac{2^{1/3} (1 + I \sqrt[3]{3}) j^{1/3}}{3 (-6075 - 2 j + 45 \sqrt[3]{\sqrt[3]{6075 + 4 j}})^{1/3}} - \frac{(1 - I \sqrt[3]{3}) (-6075 - 2 j + 45 \sqrt[3]{\sqrt[3]{6075 + 4 j}})^{1/3}}{3 2^{1/3} j^{1/3}}
\]
This the pattern of entry deterrence for $n > 200$.
Now I calculate the value of $n$ for which the entry deterrence constraint for an entrant at the left edge becomes binding.
As the following calculation shows the potential entrant will not always choose maximum product differentiation that is the location 0.
Deterring an entrant at the center and at 0.

\[ x_{\text{OptAt0}}[.4, 1] \]

\[ 0.033366 - 6.245 \times 10^{-17} \, \text{I} \]

\[ \text{deter[350.]} \]

\[ 0.296407 + 1.38778 \times 10^{-17} \, \text{I} \]

\[ x_{\text{OptAt0}}[\text{deter[440.], 1}] \]

\[ 0.0310802 - 5.39692 \times 10^{-17} \, \text{I} \]

\[ \text{FindRoot[} \]

\[ \left( \text{redProfit3[1][1]} / . \{ x[1] \rightarrow x_{\text{OptAt0}}[\text{deter[n], 1}], x[2] \rightarrow \text{deter[n]}, x[3] \rightarrow 1) \} == 0, \{ n, 600., 610 \}, \text{MaxIterations} \rightarrow 50 \]}

\[ \{ n \rightarrow 468.946 \} \]

\[ n_{\text{Deter0}} = n / . \% \]

\[ 468.946 \]

\[ (\text{redProfit3[1][1]} / . \{ x[1] \rightarrow x_{\text{OptAt0}}[\text{deter[n], 1}], x[2] \rightarrow \text{deter[n]}, x[3] \rightarrow 1) / . \]

\[ n \rightarrow n_{\text{Deter0}} \]

\[ 8.23104 \times 10^{-9} + 5.1769 \times 10^{-30} \, \text{I} \]

\[ \text{deter[n}_{\text{Deter0}} \]

\[ 0.422316 + 0 \, \text{I} \]

\[ x_{\text{OptAt0}}[\text{deter[n}_{\text{Deter0}], 1}] \]

\[ 0.0478158 - 6.93889 \times 10^{-18} \, \text{I} \]

Deterring an entrant at the center and at 0.

\[ \text{function2[i, j, k] := redProfit3[2] / . \{ x[1] \rightarrow i, x[2] \rightarrow k/2 + i/2, x[3] \rightarrow k, n \rightarrow j \} \]

\[ \text{deterCenter[i, n_] = k / . \text{Solve[function2[i, n, k] == 0, k][[1]]} / / \text{Simplify} \]

\[ \left( (-4 + 3 \, i) \, n + \frac{2 \, 2^{1/3} \, n^2}{(n^2 \, (2 \, n + 45 \, (135 + \sqrt{18225 + 12 \, n})))^{1/3}} + 2^{2/3} \left( n^2 \left( 2 \, n + 45 \left( 135 + \sqrt{18225 + 12 \, n} \right) \right) \right) \right)^{1/3} \]

\[ 3 \, n \]

\[ \text{deterLeft[n1_] :=} \]

\[ i / . \text{FindRoot[} \left( \text{redProfit3[1]} / . \{ x[1] \rightarrow x_{\text{OptAt0}}[\text{i, deterCenter[i, n1]], x[2] \rightarrow \text{i}, x[3] \rightarrow \text{deterCenter[i, n1], n \rightarrow n1)}[[1]] == 0, \{ i, .35, .4 \}[[1]] \right] \]
As I show below in the section with four firms, entry of a fourth entrant is blockaded up to \( n=1136.86 \). I do not consider the industry structure for greater values of \( n \).

As I show below in the section with four firms, entry of a fourth entrant is blockaded up to \( n=1136.86 \). I do not consider the industry structure for greater values of \( n \).
Plot[deter1stEntrant[n], {n, 10, 1160}, PlotStyle -> GrayLevel[0], AxesLabel -> 
{FontForm["N", {"Times-Italic", 12}], FontForm["Location", 
{"Times-Italic", 12}]}]
Plot[deter2ndEntrant[n], {n, 144, 1160}, 
PlotStyle -> Dashing[{.05, .01}], AxesLabel -> 
{FontForm["N", {"Times-Italic", 12}], FontForm["Location", 
{"Times-Italic", 12}]}]
Plot[x[1] / . locationsSym, {n, nNoDeter, 1160}, 
PlotStyle -> Dashing[{.01, .01}], AxesLabel -> 
{FontForm["N", {"Times-Italic", 12}], FontForm["Location", 
{"Times-Italic", 12}]}]
figurEntry = Show[%, %%, %%%]
The profits:

\[
\begin{align*}
\text{prof1}[n_\_] := & \text{redProfit1} / (x[1] \rightarrow \text{deter}[n], x[2] \rightarrow 1) \\
\text{prof2}[n_\_] := & \text{redProfit2} / (x[1] \rightarrow \text{deter}[n], x[2] \rightarrow 1) \\
\text{prof3}[n_\_] := & \text{redProfit1} / (x[1] \rightarrow \text{deterLeft}[n], x[2] \rightarrow \text{deterCenter}[\text{deterLeft}[n], n]) \\
\text{prof4}[n_\_] := & \text{redProfit2} / (x[1] \rightarrow \text{deterLeft}[n], x[2] \rightarrow \text{deterCenter}[\text{deterLeft}[n], n]) \\
\text{profit1stEntrant}[n_\_] := & \text{Which}[n < 200, (\text{redProfit1} / (x[1] \rightarrow 0, x[2] \rightarrow 1)), \\
& n < n\text{NoDeter}, \text{prof1}[n], \\
& n < n\text{NoDeter}, \text{prof3}[n], \\
& \text{True}, (\text{redProfit3}[2] / . \text{locationsSym})[[1]]] \\
\text{profit2ndEntrant}[n_\_] := & \text{Which}[n < 200, (\text{redProfit2} / (x[1] \rightarrow 0, x[2] \rightarrow 1)), \\
& n < n\text{NoDeter}, \text{prof2}[n], \\
& n < n\text{NoDeter}, \text{prof4}[n], \text{True}, (\text{redProfit3}[3] / . \text{locationsSym})[[1]]] \\
\end{align*}
\]

Plot[\text{profit1stEntrant}[n], \{n, 144, 1160\}, \text{PlotStyle} \to \text{GrayLevel}[0], \text{AxesLabel} \to \\
\{\text{FontForm["N"}, \{"Times-Italic", 12\}], \text{FontForm["Profits"}, \\
\{"Times-Italic", 12\}\}] \\
\text{Plot[profit2ndEntrant}[n], \{n, 144, 1160\}, \\
\text{PlotRange} \to \text{All}, \text{PlotStyle} \to \text{Dashing}[[.02, .01]], \text{AxesLabel} \to \\
\{\text{FontForm["N"}, \{"Times-Italic", 12\}], \text{FontForm["Profits"}, \\
\{"Times-Italic", 12\}\}] \\
\text{Plot}[(\text{redProfit3}[1] / . \text{locationsSym})[[1]], \\
\{n, n\text{NoDeter}, 1160\}, \text{PlotStyle} \to \text{Dashing}[[.01, .01]], \text{AxesLabel} \to \\
\{\text{FontForm["N"}, \{"Times-Italic", 12\}], \text{FontForm["Profits"}, \\
\{"Times-Italic", 12\}\}] \\
\text{figurEntryProfits} = \\
\text{Show}[\%, \%, \%, \text{PlotRange} \to \{(100, 1160), (0, 160)\}, \text{AxesOrigin} \to (144, 0)]
The prices

Attention! The price of firm 3 is the price of the 2nd entrant!! Therefore I redefine manually the prices! That is, in the case of 3 firms in equPrices I \( p[1|2,3] \) denotes the price the 1st(2nd, 3rd) entrant charges.

```
prices3Firms /. locationsSym

{\{p[1] \to 0.181921, p[2] \to 0.187666, p[3] \to 0.252272\}}

equPrices[n_] := Which[n < 200, (prices2Firms /. \{x[1] \to 0, x[2] \to 1\}),
            n < nDeter0, prices2Firms /. \{x[1] \to \text{deter}[n], x[2] \to 1\}, n < nNoDeter,
            prices2Firms /. \{x[1] \to \text{deterLeft}[n], x[2] \to \text{deterCenter}[\text{deterLeft}[n], n\},
            True, \{(p[1] \to 0.187666, p[2] \to 0.252272, p[3] \to 0.181921)\}]

Plot[{p[1] /. equPrices[n][[1]]},
     \{n, 144, 1160\}, PlotStyle -> GrayLevel[0], AxesLabel ->
     \{FontForm["N"], \{"Times-Italic", 12\}, FontForm["Prices",
     \{"Times-Italic", 12\}]\}]
Plot[{p[2] /. equPrices[n][[1]]}, \{n, 144, 1160\},
     PlotStyle -> Dashing[\{.02, .01\}], AxesLabel ->
     \{FontForm["N"], \{"Times-Italic", 12\}, FontForm["Prices",
     \{"Times-Italic", 12\}]\}]
Plot[{p[3] /. equPrices[n][[1]]}, \{n, 144, 1160\},
     PlotStyle -> Dashing[\{.005, .01\}], AxesLabel ->
     \{FontForm["N"], \{"Times-Italic", 12\}, FontForm["Prices",
     \{"Times-Italic", 12\}]\}]

figurEntryPrices =
Show[%, %%, %%%, PlotRange -> \{(144, 1160), \{0, 1.1\}\}, AxesOrigin -> \{144, 0\}]
```

`figurEntryPrices`
Plot::plnr:
\[ p[3]/.\text{equPrices}[n][1] \text{ is not a machine-size real number at } n = 144.0000423333334^\circ. \]

Plot::plnr:
\[ p[3]/.\text{equPrices}[n][1] \text{ is not a machine-size real number at } n = 185.216063438082478^\circ. \]

Plot::plnr:
\[ p[3]/.\text{equPrices}[n][1] \text{ is not a machine-size real number at } n = 230.165740657123718^\circ. \]

General::stop: Further output of Plot::plnr will be suppressed during this calculation.

**Case with 4 firms**

\[ a[3, 4] = . \]
\[ a[3, 4] \]
\[ a[4, 5] = 1 \]
\[ \text{foc}[3] \]
\[ \text{foc}[4] \]
The 4th entrant would locate quite close to the center firm, not halfway between the two adjacent firms.
From this value onwards entry of a forth firm is no longer blockaded.

\[ f[2] / n / . \%

0.0219904

Result differs from Neven! He gets 0.0245.

**The case with two different technologies**

**General**

Before carrying on with the calculations in this case all the definitions made above are undone.

\[ \text{Clear["Global`*"\]} \]

Now the initialization cells have to be evaluated again.

The profits of the first and the second entrant if the locations are \( i \) and \( j \), respectively, and the cost advantage of the first entrant is equal to \( c \).

\[
\text{profits2Firms}[i, j, c] // \text{Simplify}
\]

\[ \left\{ -\frac{(c-2i-i^2+j)(2+j)^2n+18(i-j)f[1]}{18(i-j)}, -\frac{(c+4i-i^2+(-4+j)j^2n+18(i-j)f[2])}{18(i-j)} \right\} \]

The difference in the profits of the first entrant between the situation with a large and with a small rival.

\[
\text{profits2Firms}[i, j, 0][[1]] - \text{profits2Firms}[i, j, c][[1]] // \text{Simplify}
\]

\[ \frac{c(c-4i-2i^2+2j(2+j))n}{18(i-j)} \]
This expression is negative.

A comparison of the profits of the second entrant with high and low costs, resp.

\[
\text{profits2Firms}[i, j, 0][[2]] - \text{profits2Firms}[i, j, c][[2]] // \text{Simplify}
\]

\[
\frac{c (c - 2 (-4 i + i^2 - (-4 + j) j)) n}{18 (1 - j)}
\]

\[
\frac{c (c - 2 (i - j) (-4 + i + j)) n}{18 (1 - j)}
\]

\[D[%, i] // \text{Simplify}\]

\[-\frac{c (c + 2 (1 - j)^2) n}{18 (1 - j)^2}\]

The derivative with respect to i is negative. The derivative is used in the paper.

The pricing rules and the restriction on c for an interior solution to apply.

\[\text{prices2Firms} /. \{x[1] \to i, x[2] \to j, c[2] \to c[1] + c\} // \text{Simplify}\]

\[
\left\{
\begin{array}{l}
p[1] \to \frac{1}{3} (c - 2 i - i^2 + 2 j + j^2 + 3 c[1]),
p[2] \to \frac{1}{3} (2 c - 4 i + i^2 + 4 j - j^2 + 3 c[1])
\end{array}
\right.
\]

\[\frac{1}{3} (c - 2 (1 - i^2 + j - j^2))\]

\[\text{Solve}[%[[1]] == (j - i)^2, c]\]

\[
\left\{
\begin{array}{l}
c \to 2 i + i^2 - 2 j - 6 i j + 5 j^2
\end{array}
\right.
\]

\[\text{Factor}[2 i + i^2 - 2 j - 6 i j + 5 j^2]\]

\[
\left\{
\begin{array}{l}
c \to (2 + i - 5 j) (1 - j)
\end{array}
\right.
\]

Is it better to have a large rival and locate at the edges or to have a small one and to locate at the center? Assumption: Further entry is blockaded.

\[\text{Solve}[	ext{profits2Firms}[0, 1, 0][[1]] - \text{profits2Firms}[1/2, 1, c][[1]] == 0, c]\]

\[
\left\{
\begin{array}{l}
c \to \frac{1}{4} (-7 - 6 \sqrt{2}),
c \to \frac{1}{4} (-7 + 6 \sqrt{2})
\end{array}
\right.
\]

\[\text{N}[%]\]

\[
\left\{
\begin{array}{l}
c \to -3.87132,
c \to 0.37132
\end{array}
\right.
\]

For \(c_S \geq 0.37132\), the first entrant will try to prevent the second firm from using L-tech as long as it is possible, that is, until she is located at 1/2.

- The competitive fringe: S-technology with zero fixed costs
The equilibria for c=1/2 as a function of the fixed costs and of the market size.

Below I determine the values of n and \( f_s/n \) for which the various industry structures arise. For \( f_s/n \) I use \( f_2 \) as an abbreviation.

**A monopolist using the small scale technology**

\[
\text{profits2Firms}[1/2, 1, 0] // \text{Simplify}
\]

\[
\left\{ \frac{49 n}{144} - f[1], \frac{25 n}{144} - f[2] \right\}
\]

\[ 25/144. \]

\[ 0.173611 \]

By using the small scale technology entry of a small scale producer be deterred for \( f_2 \geq 25/144 \).

\[
\text{profits2Firms}[0, 1/2, 1/2] // \text{Simplify}
\]

\[
\left\{ \frac{49 n}{144} - f[1], \frac{25 n}{144} - f[2] \right\}
\]

\[ \text{Solve[profits2Firms[0, 1/2, 1/2][[1]] /. f[1] -> 25] == 0, n] \]

\[ \{\{n \rightarrow 3600/49\}\} \]

\[ N[\%] \]

\[ \{\{n \rightarrow 73.4694\}\} \]

When the 1st entrant uses the S-tech entry of a large rival is blockaded for all \( n < 3600/49 \). This constraint is not binding as becomes clear below.

The combination of \( f_2 \geq 25/144 \) and of the condition \( 25 - f_s = n/2 \) derived in the text yields the small monopoly region depicted in the next diagram. I define the functions in the (n,f2)-space, and invert the result.

\[
\text{Show[Plot[((50 - n)/(2 n), (n, 3, 50)) /. \{x_, y_\} \rightarrow \{y, x\}\]}
\]

\[
\text{Show[Plot[(25/144), (n, 3, 50)] /. \{x_, y_\} \rightarrow \{y, x\}\]}
\]
Show[%, %%, AxesLabel -> {FontForm["f_s/N", {"Times-Italic", 12}], FontForm["N", {"Times-Italic", 12}]]]
The area between to the right of the vertical line and below the hyperbola gives the respective range of parameters in the \((f_2,n)\)-space.

\[
\text{Solve}\left\{ \frac{50 - n}{2n} = f_2, n \right\}
\]

\[
\{\{n \rightarrow \frac{50}{1 + 2 f_2}\}\}
\]

\[
\text{fig0a = Plot}\left[ \frac{50}{1 + 2 f_2}, \{f_2, 1/16, .068\}, \text{PlotStyle} \rightarrow \text{Dashing}[\{.01, .01\}] \right]
\]

- Graphics -

\[
\text{Solve}\left\{ \frac{50 - n}{2n} = \frac{25}{144}, n \right\}
\]

\[
\{\{n \rightarrow \frac{3600}{97}\}\}
\]

\[
\text{N[\%]}
\]

\[
\{\{n \rightarrow 37.1134\}\}
\]

\[
25/144.
\]

0.173611

\[
\text{Solve}\left\{ \frac{50 - n}{2n} = \frac{1}{16.}, n \right\}
\]

\[
\{\{n \rightarrow 44.4444\}\}
\]

\[
\text{Solve}\left\{ \frac{50 - n}{2n} = 0, n \right\}
\]

\[
\{\{n \rightarrow 50\}\}
\]

- A monopolist using the large scale technology

\[
\text{profits2Firms}[1/2, 1, 1/2] \text{ // Expand}
\]

\[
\left\{ \frac{9 n \ f[1]}{16} - f[2], \frac{n \ f[1]}{16} - f[2] \right\}
\]

\[
1/16.
\]

0.0625

Small entrants are blocked for \(f_2 \geq 1/16\). Large entrants for \(n \leq 144\).
For a discussion of restrictions on the reservation price, see the text.

The case with two active firms

The locational choice of a third entrant locating between the two incumbents when the first entrant uses the large scale technology and is located at i ('close' to 0).

\[ x_{2\text{opt}}[i_] = x[2] / . \text{Solve}\left[ (\text{focLoc}[2][[1]] / . \{x[1] \to i, x[3] \to 1, c[1] \to 0, c[2] \to 1/2, c[3] \to 1/2\}) == 0, x[2]\right] \]

\[ \frac{-3 - 8i + 3i^2 - \sqrt{21 - 40i + 26i^2 - 8i^3 + i^4}}{4(-3 + i)} \]

\[ x_{2\text{opt}}[.1] \]

0.683066

The same for the case of two small incumbents.

\[ x_{2\text{optSym}}[i_] = x[2] / . \text{Solve}\left[ (\text{focLoc}[2][[1]] / . \{x[1] \to i, x[3] \to 1, c[1] \to 1/2, c[2] \to 1/2, c[3] \to 1/2\}) == 0, x[2]\right] \]

\[ \left\{ \frac{1 + i}{2} \right\} \]

\[ \text{function1}[i_, j_] := \text{redProfit3}[2] / . \{x[1] \to i, x[2] \to x_{2\text{optSym}}[i], x[3] \to 1, n \to j, c[1] \to 1/2, c[2] \to 1/2, c[3] \to 1/2\} \]

\[ \left\{ \frac{(-\frac{i}{2} (-1 + i) + \frac{i}{2} (-1 + \frac{i}{2}) + (-1 + \frac{i}{2}) \left(\frac{3}{2} - 2i + i^2 + \frac{i}{2} - \frac{i}{2} (1 + i))\right)^2 n}{18 (-1 + i) (\frac{i}{2} (-1 - i) + i) (-1 + \frac{i}{2})} - f[2] \right\} \]

To get rid of the n, I define f2 as the fraction f[2]/n
Solve[function1[i, n][[1]] + f[2] == (f2 n), i]

\[
\begin{align*}
\{ & i \rightarrow 7/3 + \frac{2^{1/3}}{3} \left( -2 - 243 f2 + 9 \sqrt[3]{f2} \frac{\sqrt{4 + 243 f2}}{\sqrt{4 + 243 f2}} \right)^{1/3} + \\
& \frac{1}{3} 2^{2/3} \left( -2 - 243 f2 + 9 \sqrt[3]{f2} \frac{\sqrt{4 + 243 f2}}{\sqrt{4 + 243 f2}} \right)^{1/3} \\
& i \rightarrow 7/3 - \frac{2^{1/3} \left( 1 + \sqrt[3]{3} \right)}{3} \left( -2 - 243 f2 + 9 \sqrt[3]{f2} \frac{\sqrt{4 + 243 f2}}{\sqrt{4 + 243 f2}} \right)^{1/3} - \\
& \frac{1}{3} 2^{2/3} \left( 1 - \sqrt[3]{3} \right) \left( -2 - 243 f2 + 9 \sqrt[3]{f2} \frac{\sqrt{4 + 243 f2}}{\sqrt{4 + 243 f2}} \right)^{1/3} \\
& i \rightarrow 7/3 - \frac{2^{1/3} \left( 1 - \sqrt[3]{3} \right)}{3} \left( -2 - 243 f2 + 9 \sqrt[3]{f2} \frac{\sqrt{4 + 243 f2}}{\sqrt{4 + 243 f2}} \right)^{1/3} - \\
& \frac{1}{3} 2^{2/3} \left( 1 + \sqrt[3]{3} \right) \left( -2 - 243 f2 + 9 \sqrt[3]{f2} \frac{\sqrt{4 + 243 f2}}{\sqrt{4 + 243 f2}} \right)^{1/3} \}
\end{align*}
\]

% /. f2 -> .07

\[
\{ i \rightarrow 3.34715 - 1.32292 \}, \{ i \rightarrow 0.305709 + 2.77556 \times 10^{-17} \}, \{ i \rightarrow 3.34715 + 1.32292 \} \}
\]

deter[f2_] = i /. Solve[function1[i, n][[1]] + f[2] == (f2 n), i][[2]] // FullSimplify

\[
\frac{1}{6} \left( 14 - \frac{2^{1/3} \left( 1 + \sqrt[3]{3} \right)}{\left( -2 - 243 f2 + 9 \sqrt[3]{f2} \frac{\sqrt{4 + 243 f2}}{\sqrt{4 + 243 f2}} \right)^{1/3}} + \right. \\
\left. 12^{2/3} \left( 1 + \sqrt[3]{3} \right) \left( -2 - 243 f2 + 9 \sqrt[3]{f2} \frac{\sqrt{4 + 243 f2}}{\sqrt{4 + 243 f2}} \right)^{1/3} \right)
\]

deter[1/16.]

0.356196 + 3.70074 \times 10^{-17} \text{I}

fig1 = Plot[{deter[f2], 1}, {f2, 0, 1/16}, PlotRange -> (0.356, .5)]

\[
\text{function2}[i_, j_, k_] := \text{redProfit3}[2] /.
\]
deterCenter[i_, f2_] = k /. Solve[function2[i, n, k] + f[2] == (f2 n), k][[1]] // Simplify

\[
\begin{align*}
\frac{1}{3} \left(-4 + \frac{2^{1/3}}{(2 + 243 f2 + 9 \sqrt{3} \sqrt{f2 (4 + 243 f2)})^{1/3}} + \\
2^{2/3} (2 + 243 f2 + 9 \sqrt{3} \sqrt{f2 (4 + 243 f2)})^{1/3} + 3 \right)
\end{align*}
\]

xOptAt0[j_, k_] = 

\[
\begin{align*}
\frac{1}{27} (4 j + 17 k) + \left((-1 + \sqrt{3})(-54 j - 16 j^2 + 54 k + 80 j k - 100 k^2)\right) / \\
(54 2^{1/3} (-2025 j^2 + 32 j^3 + 3807 j k - 240 j^2 k - 1782 k^2 + 600 j k^2 - 500 k^3 + 27 \sqrt{3} (-18 j^3 + 1859 j^4 - 64 j^5 + 54 j^2 k - 6938 j^3 k + 608 j^4 k - 54 j k^2 + 9651 j^2 k^2 - 2224 j^3 k^2 + 18 k^3 - 5924 j k^3 + 3880 j^2 k^3 + 1352 k^3 - 3200 j k^4 + 1000 k^5))^{(1/3)} - \\
\frac{1}{27 2^{1/3}} ((1 - \sqrt{3}) (-2025 j^2 + 32 j^3 + 3807 j k - 240 j^2 k - 1782 k^2 + 600 j k^2 - 500 k^3 + 27 \sqrt{3} (-18 j^3 + 1859 j^4 - 64 j^5 + 54 j^2 k - 6938 j^3 k + 608 j^4 k - 54 j k^2 + 9651 j^2 k^2 - 2224 j^3 k^2 + 18 k^3 - 5924 j k^3 + 3880 j^2 k^3 + 1352 k^3 - 3200 j k^4 + 1000 k^5))^{(1/3)}
\end{align*}
\]

xOptAt0[.35, .9] = 0.0206736 + 6.93889 \times 10^{-18} I

deterLeft[1/20.] = 
  i /. FindRoot[(redProfit3[1] /. {x[1] -> xOptAt0[i, deterCenter[i, f2]], 
                        c[3] -> 1/2, n -> 1})[[1]] + f[1] == f2, 
                        {i, .35, .4}][[1]]

deterLeft[1/20.]

0.413103

fig2 = Plot[{deterCenter[deterLeft[f2], f2], deterLeft[f2]}, 
               {f2, 0.01, 1/16}, PlotRange -> (0, 1)]
Show[fig1, fig2, PlotRange -> {0, 1}]

\[ \text{f2Deter1stEntr} \] determines the fixed costs from which on both firms must move in order to deter entry.

\{f2, 0.05, 0.052\}, \text{MaxIterations} -> 50, \text{WorkingPrecision} -> 25\}\]

\[ \{f2 \to 0.05331106166397552288738235\} \]

\[ \text{f2Deter1stEntr} = f2 / . \%
\]

\[ 0.05331106166397552288738235 \]

\[ \text{xOptAt0[\text{deter}[f2Deter1stEntr], 1]}
\]

\[ 0.0478157817620169125 + 0. \times 10^{-29} I \]

\[ 25/468.94582894651782 \]

\[ 0.0533111 \]

Above I have determined that no more deterrence is possible if \( f2 \) is smaller than 0.0258377

Now I go on to the case of a large and a small firm. If the first entrant chooses the large scale technology, it will also move to deter entry of a third firm at the center. \( \text{deterLargeIncumbent} \) determines the deterring location.

\[ \text{deterLargeIncumbent}[f2_] =
\]


\[ \text{Root}\{-125 + 1656 f2 + 5184 f2^2 + 600 \#1 - 2808 f2 \#1 - 10368 f2^2 \#1 - 1110 \#1^2 + 288 f2 \#1^2 + 5184 f2^2 \#1^2 + 992 \#1^3 + 1440 f2 \#1^3 - 444 \#1^4 - 648 f2 \#1^4 + 96 \#1^5 + 72 f2 \#1^5 - 8 \#1^6 \&, 1\}
\]

\[ \text{deterLargeIncumbent}[0.022] \]

\[ 0.31052 \]
Plot[{deterLargeIncumbent[f2]}, {f2, 0, 1/16}, PlotRange -> {0, .5}]

-Solve-

Solve[deterLargeIncumbent[f2] == 0, f2]

\{
\{f2 -> 1/144 (-23 - 7 \sqrt{21})\},
\{f2 -> 1/144 (-23 + 7 \sqrt{21})\}\}

N[\%

\{
\{f2 -> -0.382486\},
\{f2 -> 0.0630419\}\}

Entry of a small entrant at the center must always be deterred; deterLargeIncumbent is positive for f2 equal to 1/16.

redProfit[3][]. \{x[1] -> 0, x[2] -> deterLargeIncumbent[0.022],


\{0.00020951 - f2\}

Entry at 0 is clearly deterred.

Now I have to compare the profits from using the two different technologies.

profSymLargeF[f2] = (profits2Firms[deter[f2], 1, 0][[1]] /. f[1] -> 0) - f2 n // Simplify

\(-f2 n + \left(-2^{1/3} - 2 \ 2^{1/3} \sqrt{3} +
8 \left(-2 - 243 f2 + 9 \sqrt{3} \ \sqrt{f2} \ \sqrt{4 + 243 f2}\right)^{1/3} - \left(-4 - 486 f2 + 18 \sqrt{3} \ \sqrt{f2} \ \sqrt{4 + 243 f2}\right)^{2/3} +
I \sqrt{3} \ \left(-4 - 486 f2 + 18 \sqrt{3} \ \sqrt{f2} \ \sqrt{4 + 243 f2}\right)^{2/3} +
\right)\left(2 \ 2^{1/3} - 2 \ 2^{1/3} \sqrt{3} - 32 I \ \left(-2 - 243 f2 + 9 \sqrt{3} \ \sqrt{f2} \ \sqrt{4 + 243 f2}\right)^{1/3} +
I \ \left(-4 - 486 f2 + 18 \sqrt{3} \ \sqrt{f2} \ \sqrt{4 + 243 f2}\right)^{2/3} +
\\sqrt{3} \ \left(-4 - 486 f2 + 18 \sqrt{3} \ \sqrt{f2} \ \sqrt{4 + 243 f2}\right)^{2/3}\right)^n /\n\left(3888 \ \left(-2 - 243 f2 + 9 \sqrt{3} \ \sqrt{f2} \ \sqrt{4 + 243 f2}\right)^{3}\right)\)

%//. f2 -> .06

(0.33603 - 1.11022 \times 10^{-16} I) n

profits2FirmsOfN[x1_, x2_, costdif_, n_] = profits2Firms[x1, x2, costdif]

\{-\frac{n \ (-\text{costdif} + 2 x1 + x1^2 - 2 x2 - x2^2)^2 + 18 (x1 - x2) f[1]}{18 (x1 - x2)},
\frac{n \ (-\text{costdif} - 4 x1 + x1^2 + 4 x2 - x2^2)^2 + 18 (x1 - x2) f[2]}{18 (x1 - x2)}\}
This a strange result: Increasing the fixed costs will increase the profits, given n!!
Plot[profSymSmallF[f2, 1], {f2, 0.01, 1/16}]

Show[%, %]

Plot[profAsym[f2, 1], {f2, 0, 1/16}]
nOfLargeF2[f2_] = n /. Solve[{profAsym[f2, n] == profSymLargeF[f2, n][[1]]}
25/
(\(f2 - \left( (2 - \frac{2}{\sqrt{3}} - 2 I 2^{1/3} \sqrt[3]{3} + 8 \left( 2 - 243 f2 + 9 \sqrt{3} \sqrt{f2} \sqrt{4 + 243 f2} \right)^{1/3} - (4 - 486 f2 + 18 \sqrt{3} \sqrt{f2} \sqrt{4 + 243 f2})^{2/3} \right) + 1 \sqrt{3} \left( -4 - 486 f2 + 18 \sqrt{3} \sqrt{f2} \sqrt{4 + 243 f2} \right)^{1/3} + 1 \left( -4 - 486 f2 + 18 \sqrt{3} \sqrt{f2} \sqrt{4 + 243 f2} \right)^{2/3} + \sqrt{3} \left( -4 - 486 f2 + 18 \sqrt{3} \sqrt{f2} \sqrt{4 + 243 f2} \right)^{1/3} + 1 \right) - 
(\left( 3888 \left( 2 - 243 f2 + 9 \sqrt{3} \sqrt{f2} \sqrt{4 + 243 f2} \right) \right) + 
(\left( -\frac{7}{2} + 2 \text{Root}[\{-125 + 1656 f2 + 5184 f2^2 + 600 \#1 - 2808 f2 \#1 - 10368 f2^2 \#1 - 1110 \#1^2 + 288 \#1^2 + 5184 \#1^2 + 5184 \#1^2 + 992 \#1^3 + 1440 \#1^3 + 444 \#1^4 + 648 \#1^4 + 96 \#1^5 + 72 f2 \#1^5 - 8 \#1^6 & ] & 1 \} + Root[\{-125 + 1656 f2 + 5184 f2^2 + 600 \#1 - 2808 f2 \#1 - 10368 f2^2 \#1 - 1110 \#1^2 + 288 \#1^2 + 5184 \#1^2 + 992 \#1^3 + 1440 \#1^3 + 444 \#1^4 + 648 \#1^4 + 96 \#1^5 + 72 f2 \#1^5 - 8 \#1^6 & ] & 1 \} \right)^2) + 
(\left( 18 \left( -1 + \text{Root}[\{-125 + 1656 f2 + 5184 f2^2 + 600 \#1 - 2808 f2 \#1 - 10368 f2^2 \#1 - 1110 \#1^2 + 288 \#1^2 + 5184 \#1^2 + 992 \#1^3 + 1440 \#1^3 + 444 \#1^4 + 648 \#1^4 + 96 \#1^5 + 72 f2 \#1^5 - 8 \#1^6 & ] & 1 \} \right)^2) + 2 
)^2 
)^2 
)^2
)^2}

<< NumericalMath`InterpolateRoot`
nOfSmallF2[f2_] := 
n /. \text{InterpolateRoot}[\{\text{profAsym}[f2, n] == \text{profSymSmallF}[f2, n], \{n, 40., 40.1\}]\n
nOfSmallF2[.033]
54.486794633587387976746

profAsym[.04, 58.88]
profSymSmallF[.04, 58.88]
13.8024
13.8041

nOfLargeF2[.06]
72.9666 - 2.36439 \times 10^{-14} I
Plot[nOfSmallF2[f2], {f2, 0.01, f2Deter1stEntr}]

- Graphics -

profAsym[.04, 58.88]
profSymSmallF[.04, 58.88]

13.8024
13.8041

fig1a = Plot[nOfSmallF2[f2], {f2, 49/1944, f2Deter1stEntr}]

General::spell1 :
Possible spelling error: new symbol name "fig1a" is similar to existing symbol "fig1".
Above I have determined the locus where the first entrant is indifferent between using the small and using the large scale technology. In both cases one rival firm is active using S-tech. Next I derive the locus where the first entrant uses the small scale technology and is indifferent between an L- and an S-rival.

First, I derive the value of $f_2$ from which on the first entrant would like to induce the second entrant to choose the large scale technology. Assumption: the first entrant moves towards the center.
Location of the potential entrant if the first entrant is small and the second entrant is large:

\[
x_{20pt1stSmall[i]} = x[2] \ /
\]

\[
\text{Solve}\left[\left.\left(\text{FocLoc}[2][[1]] \rightarrow i, \ x[3] \rightarrow 1, \ c[1] \rightarrow 1/2, \ c[2] \rightarrow 1/2, \ c[3] \rightarrow 0\right) \right\} == 0, \ x[2]]\right)[[3]]
\]

\[
-9 + i^2 + \sqrt{21 - 40 i + 26 i^2 + 4 i^3 + i^4} \quad 4 (-3 + i)
\]

\[
\text{Solve}\left[\left.\left(\text{redProfit3}[2] \rightarrow i, \ x[2] \rightarrow x_{20pt1stSmall[i]}, \ x[3] \rightarrow 1, \ c[1] \rightarrow 1/2, \ c[2] \rightarrow 1/2, \ c[3] \rightarrow 0\right)\right\}[[1]] + f[2] == f2 \_ n, \ i\right]
\]

\[
\{i \rightarrow \text{Root}[-125 + 1656 f2 + 5184 f2^2 + 600 \_1 - 2808 f2 \_1 - 10368 f2^2 \_1 - 1110 \_1^2 + 288 f2 \_1^2 + 5184 f2^2 \_1^2 + 992 \_1^3 + 1440 f2 \_1^3 - 444 \_1^4 - 648 f2 \_1^4 + 96 \_1^5 + 72 f2 \_1^5 - 8 \_1^6 \_&, 1]],
\{i \rightarrow \text{Root}[-125 + 1656 f2 + 5184 f2^2 + 600 \_1 - 2808 f2 \_1 - 10368 f2^2 \_1 - 1110 \_1^2 + 288 f2 \_1^2 + 5184 f2^2 \_1^2 + 992 \_1^3 + 1440 f2 \_1^3 - 444 \_1^4 - 648 f2 \_1^4 + 96 \_1^5 + 72 f2 \_1^5 - 8 \_1^6 \_& \_2], 1\}]
\]

\[
\{i \rightarrow \text{Root}[-125 + 1656 f2 + 5184 f2^2 + 600 \_1 - 2808 f2 \_1 - 10368 f2^2 \_1 - 1110 \_1^2 + 288 f2 \_1^2 + 5184 f2^2 \_1^2 + 992 \_1^3 + 1440 f2 \_1^3 - 444 \_1^4 - 648 f2 \_1^4 + 96 \_1^5 + 72 f2 \_1^5 - 8 \_1^6 \_& \_3], 3\}]
\]

\[
\{i \rightarrow \text{Root}[-125 + 1656 f2 + 5184 f2^2 + 600 \_1 - 2808 f2 \_1 - 10368 f2^2 \_1 - 1110 \_1^2 + 288 f2 \_1^2 + 5184 f2^2 \_1^2 + 992 \_1^3 + 1440 f2 \_1^3 - 444 \_1^4 - 648 f2 \_1^4 + 96 \_1^5 + 72 f2 \_1^5 - 8 \_1^6 \_& \_4], 4\}]
\]

\[
\{i \rightarrow \text{Root}[-125 + 1656 f2 + 5184 f2^2 + 600 \_1 - 2808 f2 \_1 - 10368 f2^2 \_1 - 1110 \_1^2 + 288 f2 \_1^2 + 5184 f2^2 \_1^2 + 992 \_1^3 + 1440 f2 \_1^3 - 444 \_1^4 - 648 f2 \_1^4 + 96 \_1^5 + 72 f2 \_1^5 - 8 \_1^6 \_& \_5], 5\}]
\]

\[
\{i \rightarrow \text{Root}[-125 + 1656 f2 + 5184 f2^2 + 600 \_1 - 2808 f2 \_1 - 10368 f2^2 \_1 - 1110 \_1^2 + 288 f2 \_1^2 + 5184 f2^2 \_1^2 + 992 \_1^3 + 1440 f2 \_1^3 - 444 \_1^4 - 648 f2 \_1^4 + 96 \_1^5 + 72 f2 \_1^5 - 8 \_1^6 \_& \_6], 6\}]
\]

% /. f2 -> 0.05

\[
\{i \rightarrow 0.0834251, \{i \rightarrow 3.29209\}, \{i \rightarrow 0.981157 - 0.0864171 i\}, \{i \rightarrow 0.981157 + 0.0864171 i\}, \{i \rightarrow 3.55608 - 1.03542 i\}, \{i \rightarrow 3.55608 + 1.03542 i\}]
\]

deterBySmall1stEntrant[f2_] =

\[
\text{Solve}\left[\left.\left(\text{redProfit3}[2] \rightarrow i, \ x[2] \rightarrow x_{20pt1stSmall[i]}, \ x[3] \rightarrow 1, \ c[1] \rightarrow 1/2, \ c[2] \rightarrow 1/2, \ c[3] \rightarrow 0\right)\right\}[[1]] + f[2] == f2 \_ n, \ i\right][[1]]
\]

\[
\text{Root}[-125 + 1656 f2 + 5184 f2^2 + 600 \_1 - 2808 f2 \_1 - 10368 f2^2 \_1 - 1110 \_1^2 + 288 f2 \_1^2 + 5184 f2^2 \_1^2 + 992 \_1^3 + 1440 f2 \_1^3 - 444 \_1^4 - 648 f2 \_1^4 + 96 \_1^5 + 72 f2 \_1^5 - 8 \_1^6 \_& \_1\]
\]

deterBySmall1stEntrant[.05]

0.0834251

Proof that I can use my algorithms to determine the profits with negative cost differences as well.

\[
\text{profits2Firms}[0, 1 - \text{deterBySmall1stEntrant}[f2], 1/2][[2]] - \text{profits2Firms}[\text{deterBySmall1stEntrant}[f2], 1, -1/2][[1]] // \text{Simplify}
\]

Plot[Evaluate[profits2Firms[deterBySmall1stentrant[f2], 1, -1/2][[1]] /. 
{f[1] -> f2, n -> 1}, {f2, 0, 1/16}]

- Graphics -

Plot[profSymSmallF[f2, 1], {f2, 0.01, 1/16}]

- Graphics -

Show[% , %]

- Graphics -

FindRoot[
(profits2Firms[deterBySmall1stentrant[f2], 1, -1/2][[1]] /. 
{f[1] -> f2, n -> 1}) == 
(profSymSmallF[f2, 1]), {f2, 0.04, .05}]

{f2 -> 0.0461577}
For $f_2 \leq 0.046$, the first entrant, being small, prefers a large rival if the first entrant can choose the entry deterring location. Check whether the 2nd entrant could deter a third one even if she uses the S-Tech given the first entrant locates according to deterBySmall1stentrant.

\[
\text{Plot}\left[\{1 - \text{deterCenter}[\text{deterLeft}[f_2], f_2], \text{deterBySmall1stentrant}[f_2]\}, \{f_2, 0.02, 0.05\}, \text{PlotRange} \to \{0, 1\}, \text{PlotStyle} \to \{, \text{Hue}[0.8]\}\right]
\]


- Graphics -

\[
\text{FindRoot}\left[1 - \text{deterCenter}[\text{deterLeft}[f_2], f_2] == \text{deterBySmall1stentrant}[f_2], \{f_2, 0.035, 0.036\}\right]
\]

\[
f_{20f2ndEntrantSmallNotBinding} = f_2 / \%
\]

Deterrence of third entrant is possible for $f_2 > 0.037$ if the first entrant locates according to deterBySmall1stentrant and at the same time the second entrant uses S-tech. Therefore the location of the second entrant in the case of two small firms is a binding constraint if the second entrant prefers to choose the small technology in this case! As the following graphs show, the second entrant would choose the small scale technology.

\[
\text{Plot}\left[\text{profits2Firms}[0, 1 - \text{deterBySmall1stentrant}[f_2], 1/2][[1]] / . \{f[1] \to 25, f[2] \to f_2, n \to nOfSmallF2[f_2]\}, \{f_2, 0.03, 0.05\}, \text{PlotStyle} \to \text{Hue}[0.8]\right]
\]

- Graphics -
I now check the locations the first entrant is willing to accept in order to induce the second to invest in entry deterrence.

\[ x_{2opt1stSmallBothMove}[i, k] = x[2] / . \]
\[ \text{Solve}\left[\frac{1}{2 \left(-8 + 4 i - 4 k\right)} \left(-4 i + 2 i^2 - 12 k + 4 i k - 6 k^2 + \sqrt{(4 i - 2 i^2 + 12 k - 4 i k + 6 k^2)^2 - 4 \left(-8 + 4 i - 4 k\right) (-i + k - 4 i k + 2 i^2 k - 4 k^2 - 2 k^3)}\right)\right] = 0 \]

\[ x_{2opt1stSmallBothMove}[.1, .9] \]
0.366191

\[ \text{function2MR}[i, k] := \]
\[ \text{deterCenterMR}[i_, f2_] = k / . \text{Solve}[\{\text{function2MR}[i, k] == f2, k\}] \]
deterCenterMR[.1, .04]
{-2.10682, 0.945039, -2.42768 - 0.942796 I, -2.42768 + 0.942796 I, 0.128571 - 0.0916164 I, 0.128571 + 0.0916164 I}

deterCenterMR[i_, f2_] = k /. Solve[(function2MR[i, k] == f2, k)][[2]]
Root[{-1 - 12 i + 144 f2 i - 42 i^2 - 1440 f2 i^2 - 5184 f2^2 i^2 - 16 i^3 + 432 f2 i^3 + 84 i^4 + 288 f2 i^4 - 48 i^5 - 72 f2 i^5 + 8 i^6 + 12 i^7 + 84 i^8 + 1 + 2880 f2 i^9 + 1 + 10368 f2^2 i^10 + 1296 f2 i^11 - 1152 f2 i^12 + 360 f2 i^13 - 48 i^14 - 42 i^15 - 1440 f2 i^16 - 5184 f2^2 i^17 - 48 i^18 + 1296 f2 i^19 + 504 i^20 + 1728 f2 i^21 - 480 i^22 i^1 - 720 f2 i^23 i^1 - 120 i^24 i^1 + 16 i^25 - 432 f2 i^26 i^1 - 336 i^27 i^1 - 1152 f2 i^28 i^1 - 480 i^29 i^1 - 720 f2 i^30 i^1 - 160 i^31 i^1 - 84 i^32 i^1 + 288 f2 i^33 i^1 - 240 i^34 i^1 - 360 f2 i^35 i^1 + 120 i^36 i^1 + 48 i^37 i^1 - 48 i^38 i^1 + 8 i^39 i^1 &}], 2]

deterCenterMR[.1, .04]
0.945039

iof2ndCannotDeter[0.04]
0.145814

1 - xOptAt0[1 - deterCenter[i, f2], 1 - i] /. {f2 -> 0.04, i -> .1458}
0.947787 + 2.77556\times10^{-17} I

Plot[iOf2ndCannotDeter[f2], {f2, .02, .06}]
Plot[(1 - deterCenter[deterLeft[f2], f2], deterBySmall1stentrant[f2]),
    {f2, 0.02, 0.06}, PlotRange -> {0, 1}, PlotStyle -> {Hue[.4], Hue[.8]}]

Of course, the curves for iOf2ndCannotDeter[f2] and 1 - deterCenter[deterLeft[f2], f2] coincide!

iOf2ndCannotDeter[.04]
1 - deterCenter[deterLeft[0.04], 0.04]
0.145814
0.145814

iOfIndifferentLargeAndSmallRival1stFirmSmall[f2_] :=
iOfIndifferentLargeAndSmallRival1stFirmSmall[f2] =
  i /. FindRoot[{
    (profits2Firms[i, deterCenterMR[i, f2], -1/2][[1]]) /. 
    {f[1] -> f2, n -> 1} == (profSymSmallF[f2, 1], {i, .05, 0.051})
  }

iOfIndifferentLargeAndSmallRival1stFirmSmall[0.033]
0.0471991

iOfIndifferentLargeAndSmallRival1stFirmSmall[0.044]
0.10056
FindRoot[
  iOf2ndCannotDeter[f2] - iOfIndifferentLargeAndSmallRival1stFirmSmall[f2] == 0,
  {f2, 0.035, 0.04}]

{f2 \to 0.0439468}

f2IndifferentLargeAndSmallRival1stFirmSmall = f2 /. %

0.0439468

iOfIndifferentLargeAndSmallRival1stFirmSmall[
  f2IndifferentLargeAndSmallRival1stFirmSmall]

0.100324

From the benchmark case:
1. the value of f2 for which two firms with the same marginal costs can no longer deter entry of a third firm.
2. the value of f2 for which entry of a forth entrant is no longer blockaded if three firms are active.

f2noDeter3rdEntrant = 0.025837653961985616`

0.0258377

f2block4thEntrant = 0.0219904462869803651`

0.0219904

Plot[iOf2ndCannotDeter[f2] - iOfIndifferentLargeAndSmallRival1stFirmSmall[f2],
  {f2, 0.035, 0.046}]

- Graphics -
The above calculations and the diagram yield the location of the first entrant for the case where the first entrant uses the S-tech while the second uses the L-tech. For $f_2 \geq 0.0439469$, the first entrant will not induce the second to use the L-tech as $i_{Of2nd\text{CannotDeter}}$ is smaller than $i_{OfIndifferentLargeAndSmallRival1stFirmSmall}$. Therefore, the profit associated with $i_{Of2nd\text{CannotDeter}}$ is smaller than in the case where the first entrant switches to the large scale technology.

For $f_2 < 0.0439469$, we get the case where the first entrant is small and the second large.

The upper bound of the region where the first entrant is small and the second is large is derived from the locations defined by $deterBySmall1stentrant$. These locations give the maximum profit the first entrant can earn if it is small and the second firm is large. I solve for the values of $n$ such that the first entrant is indifferent between the two asymmetric situations. As the diagram above shows, $deterBySmall1stentrant$ can only be realized for $f_2 \leq 0.0439469$. 
f2Of2ndEntrantSmallNotBinding (0.037). For larger values of f2, the constraint iOf2ndCannotDeter becomes binding. I use this constraint to determine the pivotal values of n for the respective values of f2.

\[
\text{react3MR} = \text{Solve}[[\text{focLoc[3]} /. \{c[1] \to 1/2, c[2] \to 1/2, c[3] \to 1/2\}] == 0, x[3]];
\]

\[
\text{react3MR} /. \{x[1] \to 0, x[2] \to 1/2.\}
\]

\[
\{\{x[3] \to -0.18046\}, \{x[3] \to 1.84713\}, \{x[3] \to 0.902234\}, \{x[3] \to -0.00667235 + 0.350866 I\}, \{x[3] \to -0.00667235 - 0.350866 I\}\}
\]

\[
\text{react3MR} /. \{x[1] \to 0.2, x[2] \to .7\}
\]

\[
\{\{x[3] \to -0.023664\}, \{x[3] \to 1.69033\}, \{x[3] \to 0.992315\}, \{x[3] \to 0.18162 + 0.37403 I\}, \{x[3] \to 0.18162 - 0.37403 I\}\}
\]

\[
\text{react3MR} = \text{Solve}[[\text{focLoc[3]} /. \{c[1] \to 1/2, c[2] \to 1/2, c[3] \to 1/2\}] == 0, x[3]][[3]]
\]

\[
\]

\%

\{x[1] \to 0, x[2] \to 1/2.\}

\{x[3] \to 0.902234\}

\[
\text{react2MR[i_] = ((D[redProfit3[2] /. react3MR, x[2]]) /. \{x[1] \to i, x[2] \to x[2], c[1] \to 1/2, c[2] \to 1/2, c[3] \to 1/2, n \to 100\})[[1]]};
\]

\[
\text{Short[react2MR[i_]]}
\]

\[
-\frac{100 \ll 1} {9 (0.1 - x[2]) (0.1 + \ll 4\rr) (\ll 1\rr)} \ll 4\rr)
\]

\[
\text{reactNum2MR[i_]} := \text{reactNum2MR[i]} = \text{FindRoot}[\text{react2MR[i]} == 0, \{x[2], .4\}]
\]

\[
\text{reactNum2MR[.211]}
\]

\{x[2] \to 0.842054\}

An analytic solution is not available.

\[
(\ast \ \text{Solve}[\{x[3]/.\text{react3MR}=1,x[2]\} \ast])
\]
react3MReq1[i_] := 

react3MReq1[.15]
{x2 -> 0.696031}

location2ndEntrant3rdAt1[i_] := location2ndEntrant3rdAt1[i] = x2 /. react3MReq1[i]

location2ndEntrant3rdAt1[.15]
0.696031

1.

Plot[x2 /. react3MReq1[i], {i, 0, .4}]

- Graphics -

iOf2ndCannotDeter[0.04]
deterCenter[iOf2ndCannotDeter[0.04], 0.04]
react3MR /. 
{x[1] -> iOf2ndCannotDeter[0.04], x[2] -> deterCenter[iOf2ndCannotDeter[0.04], 0.04]}
0.145814
0.617306

{x[3] -> 0.947793}

reactNum2MR[iOf2ndCannotDeter[0.04]]
{x2 -> 0.816997}

react3MR /. Flatten[{x[1] -> iOf2ndCannotDeter[0.04],
  reactNum2MR[iOf2ndCannotDeter[0.04]] /. x2 -> x[2]}]
{x[3] -> 1.08253}
Plot[x[3]/. (react3MR /. Flatten[{x[1] -> iOf2ndCannotDeter[f2], reactNum2MR[iOf2ndCannotDeter[f2]] /. x2 -> x[2]]}, {f2, 0.03, 0.045}]

- Graphics -

Plot[x2 /. reactNum2MR[iOf2ndCannotDeter[f2]], {f2, 0.03, 0.045}]

- Graphics -

Plot[x[3]/. (react3MR /. {x[1] -> iOf2ndCannotDeter[f2], x[2] -> deterCenter[iOf2ndCannotDeter[f2], f2]}), {f2, 0.03, 0.045}]

- Graphics -
Plot[deterCenter[iOf2ndCannotDeter[f2], f2], {f2, 0.03, 0.045}]

Plot[iOf2ndCannotDeter[f2], {f2, 0.03, 0.045}]

c[1] -> 1/2, c[2] -> 1/2, c[3] -> 1/2, n -> 100, f[2] -> 0\}, \{x2, 0.5, 0.9\}]

- Graphics -

Show[%, %%]

c[2] -> 1/2, c[3] -> 1/2, n -> 100, f[2] -> 0\}, \{x2, 0.5, 0.8\}]

- Graphics -
I check the locus dividing the 2 asymmetric cases first. For \( f2O2f2n2dEntrantSmallNotBinding \), the condition that the second firms must not be able to deter further entry by using S-tech becomes binding (see above). The first entrant will therefore locate according to \( iOf2ndCannotDeter \) for these values.

\[
\text{redProfit3ofN2ndFirm[n_] = redProfit3[2]}
\]

\[
\]

\( \text{prof1stEntrant2ndEntrantLarge1stSmall[f2, n_] :=}
\)

\( \text{profits2FirmsOfN[iOf2ndCannotDeter[f2],}
\)

\( \text{deterCenterMR[Chop[iOf2ndCannotDeter[f2]], f2], -1/2, n][[1]] /. f[1] \rightarrow f2 n} \)
Plot[{deterLeft[f2], iOf2ndCannotDeter[f2]}, 
{f2, 0.03, 0.045}, PlotStyle -> {, Hue[.8]}]


- Graphics -

nOf1stEntrantIndifferent2AsymSituations[f2_] := n /. 
FindRoot[prof1stEntrant2ndEntrantLarge1stSmall[f2, n] == profAsym[f2, n], {n, 50}]
nOf1stEntrantIndifferent2AsymSituations[0.04]
63.301

fig1MR = Plot[nOf1stEntrantIndifferent2AsymSituations[f2], 
{f2, f2Of2ndEntrantSmallNotBinding, f2IndifferentLargeAndSmallRival1stFirmSmall}]

The locus for f2 ≤ f2Of2ndEntrantSmallNotBinding. The first firm locates according to deterBySmall1stentrant.

prof1stEntrant2ndEntrantLarge1stSmallDeterBySmall1stEntrant[f2_, n_] :=
profits2FirmsOfN[deterBySmall1stEntrant[f2],
  deterCenterMR[Chop[deterBySmall1stEntrant[f2]], f2], -1/2, n][[1]] /. f[1] -> f2

nOf1stEntrantIndifferent2AsymSituationsDeterBySmall1stEntrant[f2_] :=
n /. FindRoot[prof1stEntrant2ndEntrantLarge1stSmallDeterBySmall1stEntrant[f2, n] ==
  profAsym[f2, n], {n, 50}]
nOf1stEntrantIndifferent2AsymSituationsDeterBySmall1stEntrant[0.03]
64.1785
fig2MR = Plot[nOf1stEntrantIndifferent2AsymSituationsDeterBySmall1stentrant[f2], 
{f2, f2block4thEntrant, f2of2ndEntrantSmallNotBinding}]

Show[fig1a, fig1MR, fig2MR]

profits2Firms[0, 1 - deterBySmall1stentrant[.04], 1/2]

{0.0657432 (8.4342 n - 15.2107 f[1]), 0.0657432 (4.69184 n - 15.2107 f[2])}

Plot[ 
profits2Firms[0, 1 - deterBySmall1stentrant[f2], 1/2][[1]] /. {f[1] -> 25, f[2] -> f2, 
n -> nOf1stEntrantIndifferent2AsymSituationsDeterBySmall1stentrant[f2]}, 
{f2, 0.03, 0.05}, PlotStyle -> Hue[.8]]
The condition that the second entrant must be deterred from choosing the two small firms case is still binding.

The market shares in this case:

```
deterBySmall1stentrant[f2block4thEntrant]
```

```
0.310616
```

```
    x[1] -> deterBySmall1stentrant[f2block4thEntrant], x[2] -> 1}) / n) [[1]]
```

```
0.430888
```

```
((prices2Firms) /. {c[1] -> 1/2, c[2] -> 0,
```

```
```

```
    deterBySmall1stentrant[f2Of2ndEntrantSmallNotBinding], x[2] -> 1}) / n) [[1]]
```

```
0.427991
```
The prices in the region where the first entrant is large.

\[
((\text{prices2Firms}) \, /\, \{c[1] \rightarrow 0, c[2] \rightarrow 1/2, x[1] \rightarrow \text{deterLargeIncumbent[f2block4thEntrant], x[2] \rightarrow 1}\})[[1]]
\]
FindRoot[iOfIndifferentLargeAndSmallRival1stFirmSmall[f2] == 0, {f2, .02, .021}]

\{f2 -> 0.0247997\}

The locational constraint that each firm must locate inside the market would become binding for the first firm at the above value. However, for f2 \(\leq f2\) noDeter3rdEntrant, the alternatives for the first entrant are one large or two small rivals. That is, we are in a different regime. As iOfIndifferentLargeAndSmallRival1stFirmSmall3Firms shows, the first entrant will choose the location 0 for f2 \(\leq f2\) noDeter3rdEntrant.

f2noDeter3rdEntrant

0.0258377

locationsSym = \{(x[1] \to 0.07367, x[2] \to 0.42615, x[3] \to 0.88853)\}

\{x[1] \to 0.07367, x[2] \to 0.42615, x[3] \to 0.88853\}

{(c[1] \to 1/2, c[2] \to 1/2, c[3] \to 1/2, f[1] \to f2 n, f[2] \to f2 n, f[3] \to f2 n)}

\{0.0469465 n - f2 n, 0.0880423 n - f2 n, 0.0688192 n - f2 n\}

iOfIndifferentLargeAndSmallRival1stFirmSmall3Firms[f2noDeter3rdEntrant]
iOfIndifferentLargeAndSmallRival1stFirmSmall3Firms[f2noDeter3rdEntrant]

0.00641473

-0.280934

iOfIndifferentLargeAndSmallRival1stFirmSmall3Firms[f2block4thEntrant]

-0.223956

The negative values imply that the first entrant chooses the location 0 for f2 \(\leq f2\) noDeter3rdEntrant.

{x[1] \to iOfIndifferentLargeAndSmallRival1stFirmSmall3Firms[f2], x[2] \to 
location2ndEntrant3rdEntrant3At1[iOfIndifferentLargeAndSmallRival1stFirmSmall3Firms[f2]],
{x[3] \to 1, c[1] \to 1/2, c[2] \to 1/2, c[3] \to 1/2}][[1]] + f2 - f2 n]

prof2ndEntrant2ndEntrantLarge1stSmall[f2_, n_] := 
profits2FirmsOfN[f2, n] /. deterCenterMR[Chop[iOfIndifferentLargeAndSmallRival1stFirmSmall3Firms[f2]], f2],
-1/2, n] [[2]] /. f[2] \to 25

nOf2ndEntrantIndifferentLargeAndSmallTech1stEntrantSmall[f2_] := 
n /. InterpolateRoot[prof2ndEntrant2ndEntrantLarge1stSmall[f2, n] - 
prof2ndEntrant3firms[f2, n] == 0, \{n, 40, 40.001\}]
Now I determine \( f_2 \) of 2ndEntrantMustDeter4thEntrant, the value of \( f_2 \) such that a forth entrant is no longer blockaded at the locations underlying \( n_{\text{Of2ndEntrantIndifferentLargeAndSmallRival1stFirmSmall}} \). To do this I first evaluate the respective sections in the four firms case. Some definitions must be removed afterwards. (like \( c[\_] \)).

\[
\begin{align*}
a[3, 4] &= . \\
a[4, 5] &= 1 \\
\text{prices4Firms} &= \text{Solve}[\{\text{foc}[1] == 0, \text{foc}[2] == 0, \text{foc}[3] == 0, \text{foc}[4] == 0\}, \\
&\{\text{p}[1], \text{p}[2], \text{p}[3], \text{p}[4]\}] / / \text{Simplify}; \\
\text{redProfit4[i]} &:= \\
\text{redProfit4[i]} &= \text{Simplify}[\{\text{profit}[i] / \text{prices4Firms} / . \text{c}[\_] -> 1/2\}]; \\
\text{redProfit4[2]} &= \\
\{\text{Solve}[\{\text{D[redProfit4[2], x[2]] == 0, x[2]}\}]; \\
\end{align*}
\]
Problem: For different values I have to put different solutions!

$$\text{x2opt4firms[i_, j_] = x[2] /.}

Root[
-24 i j + 16 i^3 + 48 i^2 j - 12 i^3 j + 24 i j^2 - 28 i^2 j^2 + 8 i^3 j^2 - 6 i^3 j + 32 i j^3 + i^2 j^3 - 7 i j^4 +
48 i j^2 #1 - 56 i^2 j 1 + 4 j #1 - 4 i j #1 + 24 i j #1 - 1 + 60 i j^2 #1 - 15 i^2 j^2 #1 - 36 i^2 j #1 - 8 i^3 j #1 + 19 j #1 + 18 i j #1^2 + 96 i^2 j #1^2 + 12 i^3 j^2 + 18 j #1^2 -
108 i j #1^2 + 15 i^2 j #1^2 - 12 i j^2 #1^2 + 9 j^3 #1 - 12 #1^3 - 64 i #1^3 -
57 i^2 #1^3 + 72 j #1^3 + 16 i j #1^3 - 15 i j #1^3 + 31 j #1^3 - 21 j #1^3 - 6 #1^5 &. 3]

x2opt4firms[0, location2ndEntrant3rdAt1[0]]

0.372696


\{0.0365556 n - 1. f[2]\}

REDPROFIT4[2] /. \{x[1] -> iOfIndifferentLargeAndSmallRival1stFirmSmall[.03],
\{x[2] -> x2opt4firms[iOfIndifferentLargeAndSmallRival1stFirmSmall[.03],
location2ndEntrant3rdAt1[iOfIndifferentLargeAndSmallRival1stFirmSmall[.03]],

\{0.00512481\}

\{x[1] -> iOfIndifferentLargeAndSmallRival1stFirmSmall[f2], x[2] -> x2opt4firms[
\{iOfIndifferentLargeAndSmallRival1stFirmSmall[f2], location2ndEntrant3rdAt1[
\{iOfIndifferentLargeAndSmallRival1stFirmSmall[f2]], x[3] ->
location2ndEntrant3rdAt1[iOfIndifferentLargeAndSmallRival1stFirmSmall[f2]],
\{x[4] -> 1, n -> 1, f[2] -> f2\}\], [1]] == 0, \{f2, 0.035, .036\}]}

\{f2 -> 0.0341063\}

2of2ndEntrantMustDeter4Entrant = f2 /. %

0.0341063
Next I determine the entry deterring location of the second entrant.

\[
\text{redProfit4[2] /.
\{x[1] -> iOfIndifferentLargeAndSmallRivalistFirmSmall[.0341], x[2] -> x2opt4firms[
iOfIndifferentLargeAndSmallRivalistFirmSmall[.0341], location2ndEntrant3rdAt1[
iOfIndifferentLargeAndSmallRivalistFirmSmall[.0341]], x[3] ->
location2ndEntrant3rdAt1[iOfIndifferentLargeAndSmallRivalistFirmSmall[.0341]],
\]

\{7.79278 \times 10^{-6}\}

\]
\[
x[4] \rightarrow \frac{1}{27} (6 + 17 i + 4 j) - \\
(-36 + 66 i - 100 i^2 + 6 j + 80 i j - 16 j^2) / \left(27^{2/3} \left[108 - 297 i + 2682 i^2 - 500 i^3 - 27 j - \\
4527 i j + 600 i^2 j + 2169 j^2 - 240 i^3 j + 32 j^3 + 27 \sqrt{3} \sqrt{(207 i^2 - 582 i^3 + 3152 i^4 - \\
1000 i^5 - 414 i^5 j + 1116 i^6 j - 10964 i^7 j + 3200 i^8 j + 207 j^2 - 486 i j^2 + 14619 i^2 j^2 - \\
3880 i^3 j^2 - 48 j^3 - 8954 i^2 j^3 + 2224 i^3 j^3 + 2147 j^4 - 608 i j^4 + 64 j^5) \right] \right)^{-1} / 3) + \\
\frac{1}{27} 2^{1/3} \left[108 - 297 i + 2682 i^2 - 500 i^3 - 27 j - 4527 i j + 600 i^2 j + 2169 j^2 - 240 i^3 j + \\
32 j^3 + 27 \sqrt{3} \sqrt{(207 i^2 - 582 i^3 + 3152 i^4 - 1000 i^5 - 414 i^5 j + 1116 i^6 j - \\
10964 i^7 j + 3200 i^8 j + 207 j^2 - 486 i j^2 + 14619 i^2 j^2 - 3880 i^3 j^2 - \\
48 j^3 - 8954 i^2 j^3 + 2224 i^3 j^3 + 2147 j^4 - 608 i j^4 + 64 j^5) \right] \right)^{-1} / 3)
\]

\(x2ndEntrantDeter4th[i_, f2_] := x2ndEntrantDeter4th[i, f2] = \\
n -> 1, f[2] -> f2)\}[[1]] == 0, \{j, 0.5, .51\})
\]

\(x2ndEntrantDeter4th[iOfIndifferentLargeAndSmallRivalistFirmSmall[.03], 0.03]
\]

0.607086

\(iOfIndifferentLargeAndSmallRivalistFirmSmall[.03]
\]

\(\text{react3MR / . \{x[1] -> iOfIndifferentLargeAndSmallRivalistFirmSmall[.03], \}
\{x[2] -> x2ndEntrantDeter4th[
\text{iOfIndifferentLargeAndSmallRivalistFirmSmall[.03], .03])\}[[1]]\)
\]

0.0308036

\(x[3] \rightarrow 0.965085
\]

\(\text{prof2ndEntrant3firmsDeter4thEntrant[f2_, n_] := ((\text{redProfit3of2ndFirm[n]} / . \)
\{x[1] -> iOfIndifferentLargeAndSmallRivalistFirmSmall[f2], x[2] -> 
\text{x2ndEntrantDeter4th[iOfIndifferentLargeAndSmallRivalistFirmSmall[f2], f2], \}
\{\text{react3MR / . \{x[1] -> iOfIndifferentLargeAndSmallRivalistFirmSmall[f2], \}
\{x[2] -> x2ndEntrantDeter4th[
\text{iOfIndifferentLargeAndSmallRivalistFirmSmall[f2], f2])\}[[1]], \}
\]

\(nOf2ndEntrantIndifferentAndSTech1stENTrantSmallDeter4thEntrant[f2_] := \\
n /. \text{InterpolateRoot}[\text{prof2ndEntrant2ndEntrantLarge1stSmall[f2, n] - }
\text{prof2ndEntrant3firmsDeter4thEntrant[f2, n]} == 0, \{n, 40, 40.01\})
\]

\(nOf2ndEntrantIndifferentAndSTech1stENTrantSmallDeter4thEntrant[0.03]
\]

45.733660005192183124242
Now I derive the situation for $f_2 < f_{2 \text{noDeter3rdEntrant}}$, the value at which the first entrant locates at 0. First, I check whether entry deterrence works like before.

```math
\text{Solve}[D[\text{redProfit4}[3], x[3]] == 0, x[3]];
```

I renew the definition of $a[3,4]$ for the three firms case.

```math
a[3, 4] = 1
```

No further entry deterrence constraint is binding. The locus in the respective domain is determined next.

```math
\text{profits2FirmsOfN}[0, \text{deterCenterMR}[0, f_2], -1/2, n][[2]] / f_2 \to 25
```
An additional area with the first entrant large, the second small below the area with the same structure and the roles switched.

{x[1] -> 0, x[2] -> x2ndEntrantDeter4th[0, f2block4thEntrant],
 (react3MR /. \{x[1] -> 0, x[2] -> x2ndEntrantDeter4th[0, f2block4thEntrant]\})}
\[
\text{nOf3smallFirms}[f2_\_] = n /. \text{Solve}[\{\text{profAsym}[f2, n] == \text{prof3sym}[f2, n][[2]], n][[1]]
\]

25.
\[
(-0.0880423 + f2 - (0.0555556 (-3.5 + 2. \text{Root}[-125 + 1656 f2 + 5184 f2^2 + 600 \#1 - 2808 f2 \#1 - 10368 f2^2 \#1 - 1110 \#1^2 + 288 f2 \#1^2 + 5184 f2^2 \#1^2 + 992 \#1^3 + 1440 f2 \#1^3 - 444 \#1^4 - 648 f2 \#1^4 + 96 \#1^5 + 72 f2 \#1^5 - 8 \#1^6 & , 1] + 
\text{Root}[-125 + 1656 f2 + 5184 f2^2 + 600 \#1 - 2808 f2 \#1 - 10368 f2^2 \#1 - 1110 \#1^2 + 288 f2 \#1^2 + 5184 f2^2 \#1^2 + 992 \#1^3 + 1440 f2 \#1^3 - 444 \#1^4 - 648 f2 \#1^4 + 96 \#1^5 + 72 f2 \#1^5 - 8 \#1^6 & , 1])^2)^2)/
(-1. + \text{Root}[-125 + 1656 f2 + 5184 f2^2 + 600 \#1 - 2808 f2 \#1 - 10368 f2^2 \#1 - 1110 \#1^2 + 288 f2 \#1^2 + 5184 f2^2 \#1^2 + 992 \#1^3 + 1440 f2 \#1^3 - 444 \#1^4 - 648 f2 \#1^4 + 96 \#1^5 + 72 f2 \#1^5 - 8 \#1^6 & , 1]^2 ))
\]

\text{fig6MR} = \text{Plot}[\text{nOf3smallFirms}[f2], \{f2, f2block4thEntrant, f2noDeter3rdEntrant}\]

- Graphics -

\text{Show}[\text{fig6MR}, \text{fig5MR}]

- Graphics -

\text{deterLargeIncumbent}[f2block4thEntrant]

0.310616

\text{deterCenterMR}[0, f2block4thEntrant]

0.689384

\text{profAsym}[f2block4thEntrant, n] // \text{Expand}

-25. + 0.623835 n
\text{prof2ndEntrant2ndEntrantLarge1stSmall1stAt0[f2block4thEntrant, n]} \ // \text{Expand}
-25. + 0.623835 n

\text{prof3sym[f2block4thEntrant, n] [[2]]}
0.0660519 n

\text{prof2ndEntrant3firmsDeter4thEntrant1stAt0[f2block4thEntrant, n]}
0.0811014 n

\text{prof2ndEntrant3firmsDeter4thEntrant1stAtI[f2\_\_\_\_, n\_\_, i\_] :=
\{(\text{redProfit3ofN2ndFirm[n]} /\{x[1] \rightarrow i, x[2] \rightarrow \text{Re}[\text{x2ndEntrantDeter4th[i, f2]]},
\text{react3MR} /\{x[1] \rightarrow i, x[2] \rightarrow \text{Re}[\text{x2ndEntrantDeter4th[i, f2]]}\})[[1]],

\text{Plot[prof2ndEntrant3firmsDeter4thEntrant1stAtI[f2block4thEntrant, 50, i], \{i, 0, .2\]}

- Graphics -

\text{prof2ndEntrant2ndEntrantLarge1stSmall1stAtI[f2\_\_\_, n\_\_, i\_] :=
\text{profits2FirmsOfN[i, deterCenterMR[i, f2], -1/2, n] [[2]]} /\ f[2] \rightarrow 25

\text{Plot[}
\text{prof2ndEntrant2ndEntrantLarge1stSmall1stAtI[f2block4thEntrant, 50, i], \{i, 0, .2\]}

- Graphics -
The profit difference profit of the 2nd entrant in the 2 firm case (2nd L) minus profit of the 2nd entrant in the three firm case reaches a maximum at i=0. Thus in order to induce the second entrant to use the L-tech the first entrant has to locate at 0.

(Old: The profit of the second firm is larger in the three firm case than the profit of the first entrant in the equilibrium with three firms. The reason is that the first entrant locates at 0 rather than inside the market area. From this it follows
that an area exists where the first entrant is large below the area where the first entrant is small and the second entrant
is large!!)

\[
\text{profAsym}[f2block4thEntrant, nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrant1stAtQ[f2block4thEntrant]]
\]

3.73578

\[
(\text{profits2FirmsOfN}[0, \text{deterCenterMR}[0, f2block4thEntrant], -1/2, n][[1]] /.
 n -> nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrant1stAtQ[f2block4thEntrant]
\]

5.79268

\[
\text{prof3sym}[f2block4thEntrant, nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrant1stAtQ[f2block4thEntrant]]
\]

\{1.14955, 3.04256, 2.15708\}

\[
\text{prof2ndEntrant3firmsDeter4thEntrant1stAtI}[f2block4thEntrant, nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrant1stAtQ[f2block4thEntrant], 0]
\]

3.73578

\[
nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrant1stAtQ[f2block4thEntrant]
\]

46.0631130857056342406618

5.79 / 3.74

1.54813

The difference between the two three firm cases is quite large!

\[
\text{profAsym}[f2noDeter3rdEntrant, nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrant1stAtQ[f2noDeter3rdEntrant]]
\]

3.68219

\[
(\text{profits2FirmsOfN}[0, \text{deterCenterMR}[0, f2noDeter3rdEntrant], -1/2, n][[1]] /.
 f[1] -> f2noDeter3rdEntrant n) /.
 n -> nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrant1stAtQ[f2noDeter3rdEntrant]
\]

6.42642

The first entrant will induce the second entrant to use the large scale technology as soon as that is possible!

\[
(\text{profits2FirmsOfN}[0, \text{deterCenterMR}[0, f2block4thEntrant], -1/2, n] /.
 n -> nOf3smallFirms[f2block4thEntrant]
\]

\{5.63639, 2.96047\}

\[Spatial\_publication.nb\] 64
I plot fig3MR for the domain where it applies.
Show[fig3MR, fig4MR, fig5MR, fig6MR]

- Graphics -

Show[fig1a, fig2a, fig3a, fig1MR, fig2MR, fig3MR, fig4MR, fig5MR, fig6MR, AxesOrigin -> {0.02, 40}, AxesLabel -> {FontForm["fS/N", {"Times-Italic", 12}], FontForm["N", {"Times-Italic", 12}]}]

N

fs/N

- Graphics -

fig1a = Plot[nOfSmallF2[f2],
{f2, f2IndifferentLargeAndSmallRival1stFirmSmall, f2Deter1stEntr}]

- Graphics -
f2block4thEntrant
f2noDeter3rdEntrant
0.0219904
0.0258377
f2IndifferentLargeAndSmallRival1stFirmSmall
0.0439468
nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrant
nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrant
fig3a = Graphics[Line[{{f2noDeter3rdEntrant, 10}, {f2noDeter3rdEntrant, nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrant[ f2noDeter3rdEntrant]}}]]
- Graphics -
fig4a = Graphics[Line[{{f2IndifferentLargeAndSmallRival1stFirmSmall, nOf2ndEntrantIndifferentLAndSTech1stENtrantSmall[ f2IndifferentLargeAndSmallRival1stFirmSmall]}, {f2IndifferentLargeAndSmallRival1stFirmSmall, nOf1stEntrantIndifferent2AsymSituations[ f2IndifferentLargeAndSmallRival1stFirmSmall]}}]]
- Graphics -
Show[fig1a, fig2a, fig3a, fig4a, fig1MR, fig2MR,
  fig3MR, fig4MR, fig5MR, fig6MR, AxesOrigin -> {0.02, 40},
  AxesLabel -> {FontForm["f_2/N"], {"Times-Italic", 12]}, FontForm["N",
  {"Times-Italic", 12}}], PlotRange -> {40, 75}]

Consistency check: the profit of the first entrant in the two asymmetric cases.
No problem arises when comparing the profits in the cases where the first entrant uses S-tech. The profits in both situations are proportional to n. Above I determined the value of f2 (f2IndifferentLargeAndSmallRival1stFirmSmall) where the switch takes place.
The locations in the range where the first entrant is small and the second is large.

prof2ndEntrant3firmsOfI[f2_, n_, i_] :=
   ((redProfit3ofN2ndFirm[n] /. {x[1] -> i, x[2] -> location2ndEntrant3rdAt1[i],

prof2ndEntrant2ndEntrantLarge1stSmallOfI[f2_, n_, i_] :=
   profits2FirmsOfN[i, deterCenterMR[i, f2], -1/2, n] [[2]] / . f[2] -> 25

iOf1stEntrantInAsy[f2_, n_] := i /. FindRoot[prof2ndEntrant3firmsOfI[f2, n, i] ==
   prof2ndEntrant2ndEntrantLarge1stSmallOfI[f2, n, i], {i, 0.14, 0.141}]

iOf1stEntrantInAsy[0.04, 50]
0.169692

nOf1stEntrantIndifferent2AsymSituations[0.04]
63.301
The constraint \textit{iOf2ndCannotDeter} becomes binding for small values of \( n \) already.

\begin{verbatim}
iOf2ndCannotDeter[0.04]
iOf1stEntrantInAsy[0.04, nOf1stEntrantIndifferent2AsymSituations[0.04]]
0.145814
0.399407

location2ndEntrant3rdAt1[0.1458]
0.694765
\end{verbatim}

Locations, prices and market shares at \textit{f2noDeter3rdEntrant} at the locus \textit{nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrant} (Regions II and IV; analogous loci apply in the cases below).

\begin{verbatim}
f2noDeter3rdEntrant
nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrant[
 f2noDeter3rdEntrant]
0.0258377
45.533518829502932933696
\end{verbatim}
Profits: symmetric case with two small firms

((0.839803 - .5) .5 - 0.0258376539619856115`) 45.533518829502932

6.55973

Profits: asymmetric case first entrant small. 1st entrant

((0.99692 - .5) 0.34190929351262644 - 0.0258376539619856115`) 45.533518829502932

6.55974

Profits: asymmetric case first entrant small. 2nd entrant

((0.95644748331396947`) (1 - 0.34190929351262644)) 45.533518829502932 - 25

3.66013

Locations, prices and market shares at f2=1/16
nOfLargeF2[1/16.]
73.5612 - 1.80231 \times 10^{-14} I

deter[1/16.]
0.356196 + 3.70074 \times 10^{-17} I

deterLargeIncumbent[1/16.]
0.00328523

\([\text{demand}[1] / . \text{prices2Firms} / .
\{c[1] \rightarrow 0, c[2] \rightarrow 1/2, x[1] \rightarrow \text{deterLargeIncumbent}[1/16.], x[2] \rightarrow 1\}) / n\)[[1]]
0.584156

\([\text{prices2Firms} / .
\{c[1] \rightarrow 0, c[2] \rightarrow 1/2, x[1] \rightarrow \text{deterLargeIncumbent}[1/16.], x[2] \rightarrow 1\})[[1]]

\([\text{demand}[1] / . \text{prices2Firms} / .
\{c[1] \rightarrow 1/2, c[2] \rightarrow 1/2, x[1] \rightarrow \text{deter}[1/16.], x[2] \rightarrow 1\}) / n\)[[1]]
0.559366 + 6.16791 \times 10^{-18} I

\([\text{prices2Firms} / . \{c[1] \rightarrow 1/2, c[2] \rightarrow 1/2, x[1] \rightarrow \text{deter}[1/16.], x[2] \rightarrow 1\})[[1]]
\{p[1] \rightarrow 1.22024 - 3.34595 \times 10^{-17} I, p[2] \rightarrow 1.06736 - 4.05553 \times 10^{-17} I\}

Locations, prices and market shares at f2Deter1stEntr:

\(f2Deter1stEntr\)
0.05331106166397552288738235

\(nOfLargeF2[f2Deter1stEntr]\)
71.2218023662634510203 + 0. \times 10^{-20} I

deter[f2Deter1stEntr]
0.4223158741616795777241 + 0. \times 10^{-29} I

deterLargeIncumbent[f2Deter1stEntr]
0.061316041725369257193617

\([\text{demand}[1] / . \text{prices2Firms} / . \{c[1] \rightarrow 0, c[2] \rightarrow 1/2,
x[1] \rightarrow \text{deterLargeIncumbent}[f2Deter1stEntr], x[2] \rightarrow 1\}) / n\)[[1]]
0.598996113980920540303558

\([\text{prices2Firms} / . \{c[1] \rightarrow 0, c[2] \rightarrow 1/2,
x[1] \rightarrow \text{deterLargeIncumbent}[f2Deter1stEntr], x[2] \rightarrow 1\})[[1]]
Overinvestment.

Locations, prices and market shares at f2block4thEntrant at the locus
nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrant1stAt0 (regions IV and V)

nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrant1stAt0[f2block4thEntrant]
46.0631130857056342406618

nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrant1stAt0[f2block4thEntrant]
0.32735

The output of the large firm.

nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrant1stAt0[f2block4thEntrant]
1 - (nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrant1stAt0[f2block4thEntrant]
30.9844

Locations, prices and market shares at f2block4thEntrant
nOf1stEntrantIndifferent2AsymSituationsDeterBySmall1stentrant
nOf1stEntrantIndifferent2AsymSituationsDeterBySmall1stentrant[f2block4thEntrant]
64.1294
The output of the large firm.

```
deterLargeIncumbent[f2block4thEntrant]
```

0.310616

```
deterLargeIncumbent[f2] - deterBySmall1stentrant[f2]
```

0

```
   x[1] -> deterLargeIncumbent[f2block4thEntrant], x[2] -> 1}) / n) [[1]]
```

0.67265

```
(((prices2Firms) /. {c[1] -> 0, c[2] -> 1/2,
   x[1] -> deterLargeIncumbent[f2block4thEntrant], x[2] -> 1})) [[1]]
```

\{p[1] \rightarrow 0.927428, p[2] \rightarrow 0.951339\}

This quantity could be cheaper produced with the small scale technology. The respective fixed costs are

```
nOf1stEntrantIndifferent2AsymSituationsDeterBySmall1stentrant[f2block4thEntrant]
```

```
   x[1] -> deterLargeIncumbent[f2block4thEntrant], x[2] -> 1}) / n) [[1]]
```

43.1367

\[ p_1 \rightarrow 0.927428, p_2 \rightarrow 0.951339 \]

Now I derive the values for the case where the first entrant uses the large scale technology.

Depending on the parameter values the second might use either S-tech or L-tech.

The first thing I determine is the locus for which the constraint for the deterrence of a large second entrant becomes binding. Given \( f_2 \), for smaller values of \( n \), the condition for the deterrence of a third, small entrant is binding.
Next I calculate the first entrant's location which prevents the second entrant from using L-tech. \textit{iDeterLargeEntrant} is the solution.

\begin{verbatim}
Solve[ ((profits2Firms[i, 1, 0][[2]] - 25) - (profits2Firms[i, 1, 1/2][[2]] - f2 n)) == 0, i] \end{verbatim}

\begin{verbatim}
{{i \to 0.598043}, {i \to 1.62196}}
\end{verbatim}
The locus dividing the asymmetric case (with the first entrant being large) and the case with two large firms derives from the condition \( \text{iDeterLargeEntrant} = \frac{1}{2} \). \( \text{nOfLargeEntry} \) gives the maximum values of \( n \) (given \( f_2 \)) for which the first entrant is able to prevent the second entrant from using L-tech.

\[
\text{Solve}[\text{iDeterLargeEntrant}[f_2] == 1/2, n]
\]

\[
\{[n \rightarrow \frac{225}{1 + 9 f_2}]\}
\]

\[
\text{nOfLargeEntry}[f_2_] = n /. \text{Solve}[\text{iDeterLargeEntrant}[f_2] == 1/2, n][[1]]
\]

\[
\frac{225}{1 + 9 f_2}
\]

\[
\text{fig6a} = \text{Plot}[\text{nOfLargeEntry}[f_2], \{f_2, f_2\text{block4thEntrant}, 1/16\}]
\]

- Graphics -

Prices and demand for these locations.

\[
\text{prices2Firms} /. \{c[1] \rightarrow 0, c[2] \rightarrow 1/2, x[1] \rightarrow 1/2, x[2] \rightarrow 1\}
\]

\[
\{[p[1] \rightarrow \frac{3}{4}, p[2] \rightarrow \frac{3}{4}]\}
\]

\[
(\text{demand}[1] / . \text{prices2Firms}) /. \{c[1] \rightarrow 0, c[2] \rightarrow 1/2, x[1] \rightarrow 1/2, x[2] \rightarrow 1\}
\]

\[
\{\frac{3 n}{4}\}
\]

In what follows I derive the diagrams used in the paper.
fig7a = Plot[144, {f2, 1/16, 0.068}]

fig8a = Graphics[Line[{{0.02, 30}, {0.025, 30}}]]

fig9a = Graphics[Line[{{1/16, 10}, {1/16, 144}}]]

<<Graphics`Arrow`

fig10a = Graphics[{Hue[.7], Arrow[{0.0227149, 53.}, {0.0227149, 45.7002}, HeadScaling -> Relative]}]

fig11a = Graphics[{Dashing[.005, .01], Line[{{f2Deter1stEntr, 10}, {f2Deter1stEntr, nOfLargeF2[f2Deter1stEntr]}}]}]
fig12a = Graphics[{Hue[.7], Arrow[{0.0250882, 78.6483}, {0.0227149, 45.7002}], HeadCenter -> .02, HeadLength -> .022]}

- Graphics -

figEquilibriumRanges = Show[fig1a, fig2a, fig3a, fig4a, fig5a, fig6a, fig7a, fig9a, fig10a, fig1MR, fig2MR, fig3MR, fig4MR, fig5MR, fig6MR, Graphics[{Text[FontForm["III",{"Times",10}],{0.024,37.6137}],{1,0}]}], Graphics[{Text[FontForm["II",{"Times",10}],{0.05,46.8733}],{1,0}]}], Graphics[{Text[FontForm["IV",{"Times",10}],{0.0227149,53.}],[-.2,-1]]}, Graphics[{Text[FontForm["V",{"Times",10}],{0.0309538,55.1389}],{1,0}]}], Graphics[{Text[FontForm["VI",{"Times",10}],{0.0377,102.431}],{1,0}]}], Graphics[{Text[FontForm["VII",{"Times",10}],{0.0377,157.276}],{1,0}]}], Graphics[{Text[FontForm["I",{"Times",10}],{0.065,168.673}],{-1,0}]}], AxesOrigin -> {0.021,30}, AxesLabel -> {FontForm["fr/N"],{"Times-Italic",12}}, FontForm["N"],{"Times-Italic",12}]], PlotRange -> {30,190}];

N

figEquilibriumRangesNew = Show[fig0a, fig1a, fig2a, fig3a, fig4a, fig5a, fig6a, fig7a, fig9a, fig10a, fig1MR, fig2MR, fig3MR, fig4MR, fig5MR, fig6MR, (* Graphics[Text[FontForm["III",{"Times",10}],{0.024,37.6137}],{1,0}] ], Graphics[Text[FontForm["II",{"Times",10}],{0.05,46.8733}],{1,0}] ], Graphics[Text[FontForm["IV",{"Times",10}],{0.0227149,53.}],[-.2,-1]]}, Graphics[Text[FontForm["V",{"Times",10}],{0.0309538,55.1389}],{1,0}]}], Graphics[Text[FontForm["VI",{"Times",10}],{0.0377,102.431}],{1,0}]}], Graphics[Text[FontForm["VII",{"Times",10}],{0.0377,157.276}],{1,0}]}], Graphics[Text[FontForm["I",{"Times",10}],{0.065,168.673}],{-1,0}]}], AxesOrigin -> {0.021,30}, AxesLabel -> {FontForm["fr/N"],{"Times-Italic",12}}, FontForm["N"],{"Times-Italic",12}]], PlotRange -> {30,190}];

NN

fig12a = Graphics[{Hue[.7], Arrow[{0.0250882, 78.6483}, {0.0227149, 45.7002}], HeadCenter -> .02, HeadLength -> .022]}

- Graphics -

figEquilibriumRanges = Show[fig1a, fig2a, fig3a, fig4a, fig5a, fig6a, fig7a, fig9a, fig10a, fig1MR, fig2MR, fig3MR, fig4MR, fig5MR, fig6MR, Graphics[{Text[FontForm["III",{"Times",10}],{0.024,37.6137}],{1,0}]}], Graphics[{Text[FontForm["II",{"Times",10}],{0.05,46.8733}],{1,0}]}], Graphics[{Text[FontForm["IV",{"Times",10}],{0.0227149,53.}],[-.2,-1]]}, Graphics[{Text[FontForm["V",{"Times",10}],{0.0309538,55.1389}],{1,0}]}], Graphics[{Text[FontForm["VI",{"Times",10}],{0.0377,102.431}],{1,0}]}], Graphics[{Text[FontForm["VII",{"Times",10}],{0.0377,157.276}],{1,0}]}], Graphics[{Text[FontForm["I",{"Times",10}],{0.065,168.673}],{-1,0}]}], AxesOrigin -> {0.021,30}, AxesLabel -> {FontForm["fr/N"],{"Times-Italic",12}}, FontForm["N"],{"Times-Italic",12}]], PlotRange -> {30,190}];
Augmenting the diagramm with curves with constant fixed costs \( f_S \).

```math
fig1c = Plot[1/f2, {f2, 0.02, 0.065}]
fig2c = Plot[2/f2, {f2, 0.02, 0.065}]
fig3c = Plot[6/f2, {f2, 0.02, 0.065}]
fig4c = Plot[9/f2, {f2, 0.02, 0.065}]
```

General::spell : Possible spelling error: new
symbol name "fig1c" is similar to existing symbols \( \text{fig1, fig1a} \).

General::spell : Possible spelling error: new
symbol name "fig2c" is similar to existing symbols \( \text{fig2, fig2a} \).
Possible spelling error: new symbol name "fig3c" is similar to existing symbol "fig3a".

Possible spelling error: new symbol name "fig4c" is similar to existing symbol "fig4a".
figVariousFixedCosts = Show[fig1c, fig2c, fig3c, fig4c, figEquilibriumRanges,
    Graphics[Text[FontForm["fS = 1", {"Times-Italic", 10}], {0.0651512, 16.0} , {-1, 0}]],
    Graphics[Text[FontForm["fS = 2", {"Times-Italic", 10}], {0.0651512, 31.0} , {-1, 0}]], Graphics[
    Text[FontForm["fS = 6", {"Times-Italic", 10}], {0.0651512, 93.0} , {-1, 0}]], Graphics[Text[FontForm["fS = 9", {"Times-Italic", 10}], {0.0651512, 137} , {-1, 0}]],
    AxesOrigin -> {0.02, 10}, PlotRange -> {10, 190},
    AxesLabel -> {StyleForm["fS/N", FontFamily -> "Times-Italic", FontSize -> 12],
    StyleForm["N", FontFamily -> "Times-Italic", FontSize -> 12]}];

- Graphics -
I have to check whether further entry of large and small firms is blockaded at the combinations I derived. While it is clear that a large firm can be deterred up to $N \approx 1000$, it is open what the situation looks like for a small third entrant. To check whether entry at 0 is blockaded for the situation where one is near the values for which a large second firm is deterred by locating close to the center. It would be sufficient to show that entry is blockaded for $f_s/n = 49/1944$ and the center location of the large incumbent.

```math
\{\text{redProfit3[1] /. } \{x[1] \to 0, x[2] \to 1/2, \\
x[3] \to 1, c[1] \to 1/2, c[2] \to 0, c[3] \to 1/2\}[[1]] + f[1] == f2 n \\
\frac{n}{144} == f2 n
\}
```

Solve:\n```
\{\text{redProfit3[1] /. } \{x[1] \to 0, x[2] \to 1/2, x[3] \to 1, \\
c[1] \to 1/2, c[2] \to 0, c[3] \to 1/2\}[[1]] + f[1] == f2 n, f2[[1]]
\}```
Entry at 0 is clearly blockaded!? That seems to be wrong: Consider the case where \( N = 144 \). Entry of a small firm would be possible if \( f < 1 \) But such fixed costs are excluded by the condition that the second entrant chooses iTech rather than sTech. That is: Entry is blockaded!!

\[
\text{Solve} \left( \text{redProfit3}[2] /. \{ x[1] \to 0, x[2] \to 1/2, x[3] \to 1, c[1] \to 0, c[2] \to 1/2, c[3] \to 0 \}[[1]] + f[2] == f2 n, f2 \right)
\]

\[
\{ \{ f2 \to \frac{1}{2} \} \}
\]

\( N[\%] \)

\[
\{ \{ f2 \to 0.0138889 \} \}
\]

Entry at the the center if the two large firms locate at the edges is blockaded as well (with \( c = 1/2 \)).

\( \text{nOfLargeEntry}[1/16.] \)

144.

\( \text{nOfLargeEntry}[1/16.] 1/16 \)

9.

\( 6/144 \)

\[
\frac{1}{24}
\]

Next I show that a third small entrant would locate between the 2 incumbents for quite large values of the the first entrants location. This holds actually for \( x[1] \) about 1/2!.

\[
\text{Solve} \left( \text{redProfit3}[1] /. \{ x[1] \to 0, x[2] \to i, x[3] \to 1, c[1] \to 1/2, c[2] \to 0, c[3] \to 1/2 \}[[1]] + f[2] == \text{redProfit3}[2] /. \{ x[1] \to i, x[2] \to x2opt[i], x[3] \to 1, c[1] \to 0, c[2] \to 1/2, c[3] \to 1/2 \}[[1]] + f[2], i \right)
\]

\[
\{ \{ i \to \text{Root\[1 + 9 \#1 - 223 \#1^2 + 817 \#1^3 - 1114 \#1^4 + 629 \#1^5 - 128 \#1^6 + 9 \#1^7 + \#1^8 & 3\]} \},
\{ \{ i \to \text{Root\[1 + 9 \#1 - 223 \#1^2 + 817 \#1^3 - 1114 \#1^4 + 629 \#1^5 - 128 \#1^6 + 9 \#1^7 + \#1^8 & 4\]} \},
\{ \{ i \to \text{Root\[1 + 9 \#1 - 223 \#1^2 + 817 \#1^3 - 1114 \#1^4 + 629 \#1^5 - 128 \#1^6 + 9 \#1^7 + \#1^8 & 5\]} \},
\{ \{ i \to \text{Root\[1 + 9 \#1 - 223 \#1^2 + 817 \#1^3 - 1114 \#1^4 + 629 \#1^5 - 128 \#1^6 + 9 \#1^7 + \#1^8 & 6\]} \}
\]

\( N[\%] \)

\[
\{ \{ i \to 0.123968 \}, \{ i \to 0.501271 \},
\{ i \to 0.990778 - 0.0914023 \}, \{ i \to 0.990778 + 0.0914023 \} \}
\]

The first term gives a wrong solution. For very close locations the calculation of profits go wrong. The second term gives the solution. As it is greater than 1/2 it is never a restriction. The result is a little bit strange; the third entrant would locate quite close to the second.
The equilibria for $c=1/8$ as a function of the fixed costs and of the market size.

Now I go through the same stages as in the case of $1/2$. I skip the monopoly using the small scale technology.

**A monopolist using the large scale technology**

```
profits2Firms[1/2, 1, 1/8] // Expand
{25 n/64 - f[1], 9 n/64 - f[2]}
9/64.
```

0.140625

Small entrants are blockaded for $f2 \geq 9/64$, large entrants for $n \leq 144$.

Region of deterred entry

```
Solve[(25/n - f2) == 1/8, n]
```

\[
\left\{ \left\{ n \rightarrow \frac{200}{1 + 8 f2} \right\} \right\}
\]

```
fig0a = Plot[200/\(1 + 8 f2\), \{f2, 9/64, .15\}, PlotStyle -> Dashing[\{.001, .005\}]]
```

The case with two active firms

The code in this case is nearly the same as in the case $c=1/2$. Therefore I will only give comments in case of differences.

Important: The functions for the symmetric cases (all firms have the same marginal costs) need not be changed as profits depend only on the differences in costs but not on their level. Therefore I can still use them, and do not state them again.

```
Solve[deter[f2] == 0, f2]
```

\[
\left\{ \left\{ f2 \rightarrow \frac{1}{8} \right\} \right\}
\]
In the case of two small firms, maximum product differentiation is possible for \( f_2 \geq 1/8 \).

\[
x_{2\text{opt}}[i_] = x[2] /.
\]

\[
\text{Solve}[[\text{focLoc}[2][[1]]]/.\{x[1] \rightarrow i, x[3] \rightarrow 1, c[1] \rightarrow 0, c[2] \rightarrow 1/8, c[3] \rightarrow 1/8\}] == 0, x[2][[2]]]
\]

\[
\frac{-3 - 8 \, i + 3 \, i^2 + 2 \sqrt{3 - 4 \, i + i^2} - i \sqrt{3 - 4 \, i + i^2}}{4 \, (-3 + i)}
\]

The evaluation of the next input takes a while. To save that time, the result stated in the next but one line can be used as an input as well.

\[
d\text{eterLargeIncumbent}[f2_] =
\]

\[
i/\text{Solve}[[\text{redProfit3}[2]/.\{x[1] \rightarrow i, x[2] \rightarrow x_{2\text{opt}}[i], x[3] \rightarrow 1,
\]
\]

\[
\]

\[
\text{deterLargeIncumbent}[f2_] =
\]

\[
\text{Root}[-12167 - 25344 \, f2 + 1327104 \, f2^2 + 50784 \, f2^3 + 283392 \, f2^4 + 391152 \, f2^5 + 2654208 \, f2^6 + 3860864 \, f2^7 + 28992 \, f2^8 - 165888 \, f2^9 + 6144 \, f2^{10} + 10752 \, f2^{11} + 512 \, f2^{12} + 512 \, f2^{13} + 512 \, f2^{14} + 512 \, f2^{15} + 512 \, f2^{16} + 64 \, f2^{17} + 1, 1]
\]

\[
\text{deterLargeIncumbent}[f2_] =
\]

\[
\text{Root}[-12167 - 25344 \, f2 + 1327104 \, f2^2 + 50784 \, f2^3 + 283392 \, f2^4 + 391152 \, f2^5 + 2654208 \, f2^6 + 3860864 \, f2^7 + 28992 \, f2^8 - 165888 \, f2^9 + 6144 \, f2^{10} + 10752 \, f2^{11} + 512 \, f2^{12} + 512 \, f2^{13} + 512 \, f2^{14} + 512 \, f2^{15} + 512 \, f2^{16} + 64 \, f2^{17} + 1, 1]
\]

\[
\text{Plot}[\text{deterLargeIncumbent}[f2], \{f2, 0, 3/4]\]
\]

Contrary to the case of \( c=1/2 \), entry of a small firm at the center is blocked in the case of 1 large and 1 small firm (for \( f_2 \neq f\text{OfSmallEntrantBlockaded} \)).
nOfSmallF2[f2_] := n /. InterpolateRoot[
    (profAsym[f2, n]) == profSymSmallF[f2, n], {n, 100., 100.01}, AccuracyGoal -> 14]

nOfSmallF2[0.02]
104.8101766488533055123

Additional problem for c=1/8. Entry of a third small entrant is blockaded for f2>0.10577 (=fOfSmallEntrantBlockaded)
in the case of 1 large and 1 small firms. The profit of the large firm in this case is equal to

\[
\text{profitsFirms[0, 1, 1/8][[1]] = } \frac{1}{18} ((625 n) / 64 - 18 f[1])
\]

nOfSmallEntrantBlockaded1largeFirm[f2_] = 
    n /. Solve[(profitsFirms[0, 1, 1/8][[1]] / f[1] -> 25) == profSymLargeF[f2, n][[1]]]

\[
\frac{25}{1152} + f2 - \left((-2 + 2^{1/3} - 2 I 2^{1/3} \sqrt[3]{3} + 8 (-2 - 243 f2 + 9 \sqrt[3]{\sqrt[3]{2}} \sqrt[3]{4 + 243 f2})^{1/3} - (4 - 486 f2 + 18 \sqrt[3]{\sqrt[3]{2}} \sqrt[3]{4 + 243 f2}^{2/3})^2 I \sqrt[3]{3} (-2 - 243 f2 + 9 \sqrt[3]{\sqrt[3]{2}} \sqrt[3]{4 + 243 f2})^{1/3} + I (-4 - 486 f2 + 18 \sqrt[3]{\sqrt[3]{2}} \sqrt[3]{4 + 243 f2}^{2/3}) + I \sqrt[3]{3} (-4 - 486 f2 + 18 \sqrt[3]{\sqrt[3]{2}} \sqrt[3]{4 + 243 f2}^{2/3})^{3/2} + 3888 (-2 - 243 f2 + 9 \sqrt[3]{\sqrt[3]{2}} \sqrt[3]{4 + 243 f2})^{3/2} / 2 \right)
\]

In the case of 2 small firms entry is blockaded for f2>1/8!

NEW! Problem whether constraint for second firm using L-tech is binding. I need the definition from below.

Here I derive the region of excessive entry deterrence.

iDeterLargeEntrant[f2_] = i /. 
    Solve[((profitsFirms[i, 1, 0][[2]] - 25) - (profitsFirms[i, 1, 1/8][[2]] - f2 n)) == 0, i][[1]]; // Simplify

\[
2 + 36 f2 + \frac{900 - \frac{1}{2} \sqrt[3]{4096 (-450 + n + 18 f2 n)^2 - 64 n (-28800 + (47 + 1152 f2) n))}{n}
\]

iDeterLargeEntrant[1/8] /. n -> 144.
0.23047
-0.540569

Calculation of value of n at which iDeterLargeEntrant is no longer binding.

\[
f2SecondEntrantMustMoveToDeter3rdEntrant
\]

0.036001292934144601140
FindRoot[iDeterLargeEntrant[f2SecondEntrantMustMoveToDeter3rdEntrant] ==
deterLargeIncumbent[f2SecondEntrantMustMoveToDeter3rdEntrant],
{n, 200}, AccuracyGoal -> 15]
{n -> 357.009}

N[%, 12]
{n -> 357.008651328}

Plot[deterLargeIncumbent[.14], {n, 130, 160}]

nOfLargeEntrantBlockaded[f2_] =
n /. Solve[(((profits2Firms[deterLargeIncumbent[f2], 1, 0][2] - 25) -
(profits2Firms[deterLargeIncumbent[f2], 1, 1/8][2] - f2 n)) == 0, n][[1]]

25/ (f2 - 47/ (1152 (-1 + Root[(-12167 - 25344 f2 + 1327104 f2^2 + 50784 #1 + 283392 f2 #1 - 2654208
f2^2 #1 - 83352 #1^2 - 617472 f2 #1^2 + 1327104 f2^2 #1^2 + 68096 #1^3 + 506880 f2 #1^3 -
28992 #1^4 - 165888 f2 #1^4 + 6144 #1^5 + 18432 f2 #1^5 - 512 #1^6 &, 1])) +
Root[(-12167 - 25344 f2 + 1327104 f2^2 + 50784 #1 + 283392 f2 #1 - 2654208 f2^2 #1 -
83352 #1^2 - 617472 f2 #1^2 + 1327104 f2^2 #1^2 + 68096 #1^3 + 506880 f2 #1^3 -
28992 #1^4 - 165888 f2 #1^4 + 6144 #1^5 + 18432 f2 #1^5 - 512 #1^6 &, 1]) -
(18 (-1 + Root[(-12167 - 25344 f2 + 1327104 f2^2 + 50784 #1 + 283392 f2 #1 - 2654208 f2^2 #1 -
83352 #1^2 - 617472 f2 #1^2 + 1327104 f2^2 #1^2 + 68096 #1^3 + 506880 f2 #1^3 -
28992 #1^4 - 165888 f2 #1^4 + 6144 #1^5 + 18432 f2 #1^5 - 512 #1^6 &, 1])) +
Root[(-12167 - 25344 f2 + 1327104 f2^2 + 50784 #1 + 283392 f2 #1 - 2654208 f2^2 #1 -
83352 #1^2 - 617472 f2 #1^2 + 1327104 f2^2 #1^2 + 68096 #1^3 + 506880 f2 #1^3 -
28992 #1^4 - 165888 f2 #1^4 + 6144 #1^5 + 18432 f2 #1^5 - 512 #1^6 &, 1]) -
(72 (-1 + Root[(-12167 - 25344 f2 + 1327104 f2^2 + 50784 #1 + 283392 f2 #1 - 2654208 f2^2 #1 -
83352 #1^2 - 617472 f2 #1^2 + 1327104 f2^2 #1^2 + 68096 #1^3 + 506880 f2 #1^3 -
28992 #1^4 - 165888 f2 #1^4 + 6144 #1^5 + 18432 f2 #1^5 - 512 #1^6 &, 1])))

nOfLargeEntryTest[f2_] = n /. Solve[iDeterLargeEntrant[f2] == 0, n]

{28800/47 + 1152 f2}

nOfLargeEntryTest[.1]

{177.559}
fig6a = Plot[nOfLargeEntryTest[f2],
            {f2, fOfSmallEntrantBlockaded, 9/64}, PlotStyle -> Dashing[{.001, .005}]]

- Graphics -

fOfSmallEntrantBlockaded // N

0.105774

Show[fig3a, fig4a, %]

- Graphics -
\text{fig7a = Plot[nOfLargeEntrantBlockaded[f2],}
\{f2, .036, fOfSmallEntrantBlockaded\}, \text{PlotStyle -> Dashing[\{.001, .005\}]}

\text{Plot[nOfLargeEntryTest[f2], \{f2, 1/10, .15\}]}

\text{Show[\%, \%\%]}
Show[%, fig1b]

- Graphics -

\[D\]

\[\text{nOfLargeEntrantBlockaded[f2SecondEntrantMustMoveToDeter3rdEntrant]}\]

\[357.0086513275698045\]

\[iIndifferentLargeAndSmallRival\]

\[-1 - \frac{3}{2\sqrt{2}} + \sqrt{3 + \frac{3}{\sqrt{2}}}\]

\[\text{Solve[iIndifferentLargeAndSmallRival == deterLargeIncumbent[f2], f2]}\]

\[\{\{f2 \rightarrow \frac{1}{1536} \left(576 + 597\sqrt{2} - 564\sqrt{3 + \frac{3}{\sqrt{2}}} - 48\sqrt{2\left(3 + \frac{3}{\sqrt{2}}\right)} - \sqrt{3\left(479270 + 360320\sqrt{2} - 369472\right)} + 107568\left(3 + \frac{3}{\sqrt{2}}\right) + 18048\sqrt{2\left(3 + \frac{3}{\sqrt{2}}\right)} - 259288\sqrt{2\left(3 + \frac{3}{\sqrt{2}}\right)}\}\}\},\]

\[\{f2 \rightarrow \frac{1}{1536} \left(576 + 597\sqrt{2} - 564\sqrt{3 + \frac{3}{\sqrt{2}}} - 48\sqrt{2\left(3 + \frac{3}{\sqrt{2}}\right)} + 107568\left(3 + \frac{3}{\sqrt{2}}\right) + 18048\sqrt{2\left(3 + \frac{3}{\sqrt{2}}\right)} - 259288\sqrt{2\left(3 + \frac{3}{\sqrt{2}}\right)}\}\}\}

Maximum total costs in the 2 small firms case (f2=9/64, n=nOfSmallEntrantBlockaded2SmallFirms)

\[1369 / 64. + 136 / 16\]

\[27.625\]

Total costs with equal market shares.
totalCosts[f2_, n_] = f2 n + n/16

\[ \frac{n}{16} + f2 n \]

nBothFirmsUnderinvest[f2_] = n /. Solve[totalCosts[f2, n] == 25, n][[1]]

\[ \frac{400}{1 + 16 f2} \]

Plot[nBothFirmsUnderinvest[f2], {f2, 1/8, 9/64}]

- Graphics -

Show[%, fig3a, fig4a]

- Graphics -

Solve[totalCosts[f2, 260.] == 25, f2]

\{\{f2 \to 0.0336538\}\}

The value of f2 indicates that both firms overinvest should be possible.

iTest = i /.

Solve[((profits2Firms[i, 1, 0][[2]] - 25) - (profits2Firms[i, 1, 1/8][[2]] - 19)) == 0, i][[1]] // Simplify

\[ 2 + \frac{-216 - \frac{1}{2} \sqrt{746496 - 6912 n + 17 n^2}}{n} \]

profits2Firms[i, 1, 1/8]

\{\{-\frac{216 + 2 i + i^2}{18 (-1 + i)} f[1], -\frac{216 - 4 i + i^2}{18 (-1 + i)} f[2]\}\}
\[ f[2] \]
\[ f[2] \]
\[ \text{demand}[2] \]
\[
\]
\[ \text{Plot[iTest, \{n, 140, 170\}]} \]

- Graphics -

\[ \text{Plot[Evaluate[\{profits2Firms[iDeterLargeEntrant[f2], 1, 1/8][2] - f2 n\} /. \{f[2] -> 0, n -> nOfLargeEntryTest[f2]\}], \{f2, 1/8, 9/64\}]} \]

- Graphics -

\[ \text{Plot[iDeterLargeEntrant[9/64] /. \{f[2] -> 0\}, \{n, 120, 160\}]} \]

- Graphics -
Underinvestment by firm 2 in the asymmetric region. Reason: Small technology reduces competition, increases prices and therefore profits.

Underinvestment by both firms cannot be an equilibrium outcome because the operating profits are the same in both cases. But that means firms care for the fixed costs only. They put the variable costs on to the consumers. That is, my solution would be right. We have an innovation preventing cartel.

That argument does not hold for large values of n, that is in the region with 2 L firms. The reason is that it would be profitable for the second firm to use the L-technology any way.

Does underinvestment occur for c=1/2, f2 and the respective value of n.

With this low fixed costs underinvestment could apply only for the first entrant as the seconds market share is smaller than 1/2. Variable costs are therefore smaller than 20.

Checking the market shares.

```
a[2, 3] = 1
```
Underinvestment occurs in the case $c=1/2$ only for the first entrant. The extent is quite small, less than 1 percent of total costs could be saved.

```
prices2Firms


deter[1/16.]

0.356196 + 3.70074 \times 10^{-17} I

(demand[1] / . prices2Firms) / {c[_] \rightarrow 0, x[1] \rightarrow deter[1/16.], x[2] \rightarrow 1}

\{(0.559366 + 6.16791 \times 10^{-18} I) n\}

% / . n \rightarrow 73.561947442069435`

{41.1476 + 4.53719 \times 10^{-16} I}

% .5 + 1/16 73.561947442069435`

{25.1714 + 2.26859 \times 10^{-16} I}

a[2, 3] =.

Entry not blockaded at $f2=49/1944$ and deterLargeIncumbent[49/1944]. That is, the first entrant is not able to deter the third small entrant alone!

```
profits2Firms[0, 1, 0][[1]] // Expand

n / 2 - f[1]

nOfSmallEntrantBlockaded2SmallFirms[f2_] =

n /. Solve[{(profits2Firms[0, 1, 1/8][[1]] / . f[1] \rightarrow 25) == n / 2 - f2 n, n}[[1]]

28800
49 + 1152 f2

Entry not blocked at $f2=49/1944$ and deterLargeIncumbent[49/1944]. That is, the first entrant is not able to deter the third small entrant alone!

```
xOptAt0Asy[i_, k_] =

c[2] \rightarrow 0, c[3] \rightarrow 1/8}[[1]], i} == 0, i}[[4]]];

xOptAt0Asy[.4, .8]

0.0146822
I determine the locations analogous to deterleft and detercenter. For values of $f_2$ smaller than $f_2$SecondEntrantMustMoveToDeter3rdEntrant, the second entrant must move to the center. I determine the locations analogous to deterleft and detercenter.

```mathematica
InterpolateRoot[
  {redProfit3[1] /. {x[1] -> xOptAt0Asy[deterLargeIncumbent[f2], 1], x[2] ->
   {f2, .04, 0.041}, AccuracyGoal -> 24, ShowProgress -> True]
```

\[ f_2 \text{SecondEntrantMustMoveToDeter3rdEntrant} = 0.03600129341446011396375182511923856229 \]

For values of $f_2$ smaller than $f_2$SecondEntrantMustMoveToDeter3rdEntrant, the second entrant must move to the center. I determine the locations analogous to deterleft and detercenter.

```mathematica
x2optAsy[i, k_] = x[2] /. Solve[
  x[2]][[2]] // Simplify

function2Asy[i_, k_] :=

numDeterCenterAsy[i_, f2_] :=
  k /. FindRoot[function2Asy[i, k] == f2, {k, 85/100, 95/100}][[1]]

numDeterCenterAsy[.4, .03]

0.909314

numDeterLeftAsy[f2_] :=
  i /. FindRoot[(redProfit3[1] /. {x[1] -> xOptAt0Asy[i, numDeterCenterAsy[i, f2]],

numDeterLeftAsy[3/100]

0.419577
 profAsymAsy[f2_, n_] := profits2FirmsOfN[numDeterLeftAsy[f2], numDeterCenterAsy[numDeterLeftAsy[f2], f2], 1/8, n][[1]]/. f[1] -> 25

nOfSecondEntrantMustMoveToDeter3rdEntrant[f2_] :=
   n /. Solve([(profAsymAsy[f2, n]) == profSymSmallF[f2, n][[1]]]

nOfSecondEntrantMustMoveToDeter3rdEntrant[49/1944]

136.458


{19.1233, 19.1233}

   {x[1] -> numDeterLeftAsy[f2], x[2] -> numDeterCenterAsy[numDeterLeftAsy[f2], f2],

function3Asy[0.02]

0.0133527

Entry of a third entrant at 1 is blockaded.

fig1a = Plot[nOfSmallF2[f2],
   {f2, f2SecondEntrantMustMoveToDeter3rdEntrant, f2Deter1stEntr}]
fig2a = Plot[nOfLargeF2[f2], {f2, f2Deter1stEntr, fOfSmallEntrantBlockaded}]
fig3a = Plot[nOfSmallEntrantBlockaded1largeFirm[f2],
   {f2, fOfSmallEntrantBlockaded, 1/8}, PlotPoints -> 50]
fig4a = Plot[nOfSmallEntrantBlockaded2SmallFirms[f2],
   {f2, 1/8, 9/64}, PlotPoints -> 50]
fig5a = Plot[nOfSecondEntrantMustMoveToDeter3rdEntrant[f2],
   {f2, 0.02, f2SecondEntrantMustMoveToDeter3rdEntrant}]

- Graphics -
The case where the first entrant is small and the second entrant is large!

Location of the potential entrant if the first entrant is small and the second entrant is large!

\[ x_{2opt1stSmall[i_\_]} = x[2]/. \]
\[ \text{Solve}\{(\text{focLoc}[2][[1]] /\_{\{x[1] \rightarrow i, x[3] \rightarrow 1, c[1] \rightarrow 1/8, c[2] \rightarrow 1/8, c[3] \rightarrow 0\}} == 0, x[2]][[3]]\]
\[ -9 + i^2 - 2 \sqrt{3 - 4 i + i^2} + i \sqrt{3 - 4 i + i^2} \]
\[ 4 (-3 + i) \]

deterBySmall1stentrant[f2_] = 

Root[{-12167 - 25344 f2 + 1327104 f2^2 + 50784 #1 + 283392 f2 #1 - 
  2654208 f2^2 #1 - 83352 #1^2 - 617472 f2 #1^2 + 1327104 f2^2 #1^2 + 68096 #1^3 + 
  506880 f2 #1^3 - 28992 #1^4 - 165888 f2 #1^4 + 6144 #1^5 + 18432 f2 #1^5 - 512 #1^6 &, 1]

deterBySmall1stentrant[.05]
  0.330617

Plot[Evaluate[profits2Firms[deterBySmall1stentrant[f2], 1, -1/8][[1]] /. 
  {f[1] -> f2, n -> 1}], {f2, 0, 1/16}]

- Graphics -

Plot[profSymSmallF[f2, 1], {f2, 0.01, 1/16}]

- Graphics -
 profSymSmallF seems to be the right alternative.

The graph shows that iOf2ndCannotDeter is always binding.

\[
x2opt1stSmallBothMove[i_, k_] = x[2] /.
\]

\[
\frac{1}{4 (-2 + i - k)} \left( -2 i + i^2 - 6 k + 2 i k - 3 k^2 + \sqrt{-2 i + i^2 + 2 k - 2 i k + k^2} - i \sqrt{-2 i + i^2 + 2 k - 2 i k + k^2} + k \sqrt{-2 i + i^2 + 2 k - 2 i k + k^2} \right)
\]
The analytical solution method does not work. I use the numerical version.

\[
\text{deterCenterMR}[i_\_ , f2_\_] := \\
\text{FindRoot}[\text{function2MR}[i_, k_] := f2, \{k, 0.5\}]
\]

\[
\text{deterCenterMR}[.11, .04] = 0.703718
\]

Clear[\text{iOfIndifferentLargeAndSmallRival1stFirmSmall}\]

\[
\text{iOfIndifferentLargeAndSmallRival1stFirmSmall}[f2_] := \\
\text{iOfIndifferentLargeAndSmallRival1stFirmSmall}[f2] = \\
\text{i} /. \text{FindRoot}[\text{\{profits2Firms}[i, \text{deterCenterMR}[i, f2], -1/8][1]) /. \\
\text{\{f[1] \rightarrow f2, n \rightarrow 1\}} \rightarrow \text{\{profSymSmallF[f2, 1], \{i, .1, 0.11\}\}
\]

\[
\text{iOfIndifferentLargeAndSmallRival1stFirmSmall}[0.03] = 0.197937
\]

\[
\text{Plot}[\text{\{-deterCenter[\text{deterLeft}[f2], f2], \\
\text{iOfIndifferentLargeAndSmallRival1stFirmSmall}[f2], \\
\text{iOfIndifferentLargeAndSmallRival1stFirmSmall}[f2], \\
\text{\{f2, 0.02, 0.05\}, PlotRange \rightarrow \{0, 1\}, PlotStyle \rightarrow \{\text{Hue}[.8]\}\}
\]

\[
\text{\& Graphics::gprim : \text{Unknown Graphics primitive Null encountered.}}
\]

\[
\text{\& Graphics -}
\]

\[
\text{FindRoot[}
\text{iOf2ndCannotDeter[f2] - iOfIndifferentLargeAndSmallRival1stFirmSmall[f2] \rightarrow 0, \\
\text{\{f2, 0.035, 0.04\}\]
\]

\[
f2\text{IndifferentLargeAndSmallRival1stFirmSmall} = f2/. %
\]

\[
0.0339363
\]
\[ \text{profits2FirmsOfN[iOf2ndCannotDeter[f2],}
\]
\[ \text{deterCenterMR[Chop[iOf2ndCannotDeter[f2]], f2], -1/8, n][][]/ f[1] -> f2 n} \]
\[ \text{nOf1stEntrantIndifferent2AsymSituations[f2_] :=}
\]
\[ \text{n / . Solve[profits2FirmsOfN[iOf2ndCannotDeter[f2], f2, n] == profAsymAsy[f2, n], n]} \]
\[ \text{nOf1stEntrantIndifferent2AsymSituations[0.034]} \]
\[
\begin{align*}
0.121814
\end{align*}
\]
\[ \text{fig1MR = Plot[nOf1stEntrantIndifferent2AsymSituations[f2],}
\]
\[ \{f2, f2block4thEntrant, f2IndifferentLargeAndSmallRival1stFirmSmall}\}
\]
\[ \text{Show[fig5a, fig1MR]} \]
\[
\begin{align*}
\text{- Graphics -}
\end{align*}
\]
Equilibrium locations in the above case.
\[
\begin{align*}
\{\text{iOf2ndCannotDeter[f2block4thEntrant]},
\text{deterCenterMR[Chop[iOf2ndCannotDeter[f2block4thEntrant]], f2block4thEntrant]}\}
\{\text{iOf2ndCannotDeter[f2IndifferentLargeAndSmallRival1stFirmSmall], deterCenterMR[}
\text{Chop[iOf2ndCannotDeter[f2IndifferentLargeAndSmallRival1stFirmSmall]], f2IndifferentLargeAndSmallRival1stFirmSmall]}\}
\end{align*}
\]
\[
\begin{align*}
\{0.387694, 0.820647\}
\end{align*}
\]
\[
\begin{align*}
\{0.220192, 0.763961\}
\end{align*}
\]
The next diagrams show that \textit{iOf2ndCannotDeter} is binding. The second entrant would choose the S-tech if that were sufficient to deter a third entrant.

\begin{verbatim}
Plot[profits2Firms[iOf2ndCannotDeter[f2],
    deterCenterMR[iOf2ndCannotDeter[f2], f2], -1/8][2] /. 
    {f2, 0.02, 0.035}, PlotStyle -> Hue[.8]]
\end{verbatim}

\begin{verbatim}
Plot[profits2Firms[iOf2ndCannotDeter[f2],
    deterCenter[iOf2ndCannotDeter[f2], f2], 0][2] /. 
    {f2, 0.02, 0.035}]
\end{verbatim}

\begin{verbatim}
Show[%, %]
\end{verbatim}
Entry at the left edge is not blockaded for small values of \( f_2 \). I determine the entry deterring location next.

\[
x_{\text{OptAt0Asy1stSmall}}[j_, k_] =
\]

\[
x_{\text{OptAt0Asy1stSmall}}[.4, .8]
\]

\[
0.0777162 - 2.08167 \times 10^{-17} \text{i}
\]

\[
\text{numDeterLeftAsy1stSmall}[f_2_] := 1 /.
\text{FindRoot[[redProfit3[1] / . \{x[1] -> x_{\text{OptAt0Asy1stSmall}}[i, \text{numDeterCenterAsy}[i, f_2]],
    c[3] -> 0, f[1] -> 0, n -> 1\}]][1]] = f_2, \{i, 4/10, 5/10\}][[1]]
\]

\[
\text{numDeterLeftAsy1stSmall}[3/100]
\]

\[
0.328246
\]

\[
\text{FindRoot[\{\text{iOf2ndCannotDeter}[f_2] - \text{numDeterLeftAsy1stSmall}[f_2] == 0, \{f_2, 0.025, 0.03\}]}
\]

\[
\{f_2 \rightarrow 0.0269112\}
\]

\[
f_{20f1stSmall2ndLargeEntryAt0} = f_2 /. \%
\]

\[
0.0269112
\]

For \( f_2 \leq f_{20f1stSmall2ndLargeEntryAt0} \), the first entrant must deter entry at 0.
Plot[numDeterLeftAsylstSmall[f2],
   {f2, 0.02, f2IndifferentLargeAndSmallRival1stFirmSmall}]

Plot[{1 - deterCenter[deterLeft[f2], f2],
   iOfIndifferentLargeAndSmallRival1stFirmSmall[f2]},
   {f2, 0.02, 0.05}, PlotRange -> {0, 1}, PlotStyle -> {, Hue[.8]}]

Show[%,%]


- Graphics -

Show[%,%]


- Graphics -
Entry at the right edge is still blockaded.

The locations:

\[
\begin{align*}
{x[1]} & \rightarrow \text{numDeterLeftAsymSmall}[f2\text{block4thEntrant}], \\
{x[2]} & \rightarrow \text{deterCenterMR}[\text{Chop}[\text{numDeterLeftAsymSmall}[f2\text{block4thEntrant}]], f2\text{block4thEntrant}], \\
{x[3]} & \rightarrow 1, \\
{c[1]} & \rightarrow 1/8, \\
{c[2]} & \rightarrow 0, \\
{c[3]} & \rightarrow 1/8, \\
{n} & \rightarrow 1, \\
{f[3]} & \rightarrow 0
\end{align*}
\]

\[
\begin{align*}
{x[1]} & \rightarrow 0.290776, \\
{x[2]} & \rightarrow 0.723729
\end{align*}
\]

\[
(x[2] - x[1]) / . \%
\]

0.432952
\[ x[1] \rightarrow \text{numDeterLeftAsy1stSmall}[f2Of1stSmall2ndLargeEntryAt0], \]
\[ x[2] \rightarrow \text{deterCenterMR}[\text{Chop}[\text{numDeterLeftAsy1stSmall}[f2Of1stSmall2ndLargeEntryAt0]], \]
\[ f2Of1stSmall2ndLargeEntryAt0] \]

\[ \{x[1] \rightarrow 0.31471, x[2] \rightarrow 0.795671\} \]

The market shares in this case.

\[ ((\text{demand}[1] / . \text{prices2Firms}) / . \{c[1] \rightarrow 1/8, c[2] \rightarrow 0, x[1] \rightarrow \text{numDeterLeftAsy1stSmall}[f2block4thEntrant], \]  
\[ x[2] \rightarrow \text{deterCenterMR}[\text{Chop}[\text{numDeterLeftAsy1stSmall}[f2block4thEntrant]], f2block4thEntrant]\}) / n[[1]] \]

0.454298

\[ ((\text{prices2Firms}) / . \{c[1] \rightarrow 1/8, c[2] \rightarrow 0, \]  
\[ x[1] \rightarrow \text{numDeterLeftAsy1stSmall}[f2block4thEntrant], x[2] \rightarrow \text{deterCenterMR}[\]  
\[ \text{Chop}[\text{numDeterLeftAsy1stSmall}[f2block4thEntrant]], f2block4thEntrant]\}) \]

\[ \{p[1] \rightarrow 0.518379, p[2] \rightarrow 0.472526\} \]

\[ ((\text{demand}[1] / . \text{prices2Firms}) / . \{c[1] \rightarrow 1/8, c[2] \rightarrow 0, \]  
\[ x[1] \rightarrow \text{numDeterLeftAsy1stSmall}[f2Of1stSmall2ndLargeEntryAt0], x[2] \rightarrow \]  
\[ \text{deterCenterMR}[\text{Chop}[\text{numDeterLeftAsy1stSmall}[f2Of1stSmall2ndLargeEntryAt0]], \]  
\[ f2Of1stSmall2ndLargeEntryAt0]) / n[[1]] \]

0.475081

\[ ((\text{prices2Firms}) / . \{c[1] \rightarrow 1/8, c[2] \rightarrow 0, \]  
\[ x[1] \rightarrow \text{numDeterLeftAsy1stSmall}[f2Of1stSmall2ndLargeEntryAt0], x[2] \rightarrow \]  
\[ \text{deterCenterMR}[\text{Chop}[\text{numDeterLeftAsy1stSmall}[f2Of1stSmall2ndLargeEntryAt0]], \]  
\[ f2Of1stSmall2ndLargeEntryAt0]) \]

\[ \{p[1] \rightarrow 0.581991, p[2] \rightarrow 0.504932\} \]

\[ a[2, 3] = 1 \]

\[ ((\text{demand}[2] / . \text{prices2Firms}) / . \{c[1] \rightarrow 1/8, c[2] \rightarrow 0, \]  
\[ x[1] \rightarrow \text{numDeterLeftAsy1stSmall}[f2Of1stSmall2ndLargeEntryAt0], x[2] \rightarrow \]  
\[ \text{deterCenterMR}[\text{Chop}[\text{numDeterLeftAsy1stSmall}[f2Of1stSmall2ndLargeEntryAt0]], \]  
\[ f2Of1stSmall2ndLargeEntryAt0]) / n[[1]] \]

\[ a[ \]
\[ 2, \]
\[ 3] = . \]

1

0.524919

The small firm reaches a market share of 47.5 % although charges a price which is 16% higher than that of the large firm. The difference in price is smaller than the difference in marginal costs!

\[ \text{demand}[1] / . \]
\[ \{p[1] \rightarrow 0.581991130727700056`, p[2] \rightarrow 0.504931823421535597`, x[1] \rightarrow 0, x[2] \rightarrow 1\} \]

0.46147 n

The locations in region VI (at the same locus)
\{\text{numDeterLeftAsy}[f2\text{block4thEntrant}], \\
\text{numDeterCenterAsy}[\text{numDeterLeftAsy}[f2\text{block4thEntrant}], f2\text{block4thEntrant}]\}

\{0.390114, 0.823066\}

\%[[2]] - \%[[1]]

0.432952

(((\text{demand[1]} / . \text{prices2Firms}) / . \{c[1] -> 0, c[2] -> 1/8, \\
x[1] -> \text{numDeterLeftAsy}[f2\text{block4thEntrant}], x[2] -> \text{numDeterCenterAsy}[ \text{numDeterLeftAsy}[f2\text{block4thEntrant}], f2\text{block4thEntrant}]) / n) [[1]]

0.583649

x[2] -> \text{numDeterCenterAsy}[\text{numDeterLeftAsy}[f2\text{block4thEntrant}], f2\text{block4thEntrant}]\}

\{\{p[1] \to 0.505385, p[2] \to 0.48552\}\}

The locus between the cases with small firms and the case where the second entrant is large.

\text{prof2ndEntrant2ndEntrantLarge1stSmall}[f2_, n_] := 
\text{profits2FirmsOfN[\text{iOfIndifferentLargeAndSmallRival1stFirmSmall}[f2], \\
\text{deterCenterMR}[\text{Chop[\text{iOfIndifferentLargeAndSmallRival1stFirmSmall}[f2]]}, f2], \\
-1/8, n] [[2]] / . f[2] \to 25

\text{nOf2ndEntrantIndifferentLAndSTech1stEntrantSmall}[f2_] := n /. \text{Solve[}
\text{prof2ndEntrant2ndEntrantLarge1stSmall}[f2, n] - \text{prof2ndEntrant3firms}[f2, n] == 0, n]

\text{nOf2ndEntrantIndifferentLAndSTech1stEntrantSmall}[0.034389685913451693`]

\{90.626\}

\{x[1] \to \text{iOfIndifferentLargeAndSmallRival1stFirmSmall}[.03], \\
x[2] \to \text{location2ndEntrant3rdAtl[} \\
\text{iOfIndifferentLargeAndSmallRival1stFirmSmall}[.03]], x[3] \to 1\}

\{x[1] \to 0.197937, x[2] \to 0.710562, x[3] \to 1\}
For the locations which are relevant in the case of $c=1/8$, the problems with respect to the optimal location of the forth entrant mentioned above become important. I use the two different solutions for the respective intervals where they apply. These intervals are determined in the plots below.

```math
```

General::spell1 : Possible spelling error: new symbol name "x2opt4firmsA" is similar to existing symbol "x2opt4firms".

Root[
-24 i^2 + 16 i^3 + 48 i^2 j - 12 i^3 j + 24 j^2 - 46 i j^2 - 28 i^2 j^2 + 8 i^3 j^2 - 6 j^3 + 32 i j^3 + i^2 j^3 - 7 i j^4 + 48 i #1 - 56 i^2 #1 - 20 i^3 #1 - 48 j #1 - 4 i j #1 + 20 i^2 j #1 - 4 i^3 j #1 + 24 j^2 #1 + 60 i j^2 #1 - 15 i^2 j^2 #1 - 36 j^3 #1 - 8 i^3 j^3 #1 + 9 j^4 #1 + 18 i^3 #1^2 + 96 i j^2 #1^2 + 12 i^3 #1^2 + 18 j #1^2 - 108 i j #1^2 + 15 i^2 j #1^2 - 12 j^2 #1^2 + 12 i j^2 #1^2 + 9 j^3 #1^2 - 12 #1^3 - 64 i #1^3 - 57 j^2 #1^3 + 72 j #1^3 + 16 i j #1^3 - 15 j^2 #1^3 + 51 i #1^4 - 21 j #1^4 - 6 #1^5 & 1]
```
Plot[{{x2opt4firms[iOfIndifferentLargeAndSmallRival1stFirmSmall[f2], location2ndEntrant3rdAt1[iOfIndifferentLargeAndSmallRival1stFirmSmall[f2]], x2opt4firmsA[iOfIndifferentLargeAndSmallRival1stFirmSmall[f2], location2ndEntrant3rdAt1[iOfIndifferentLargeAndSmallRival1stFirmSmall[f2]]}, {f2, 0.0285, .029}]

Plot::plnr : x2opt4firms[iOfIndifferentLargeAndSmallRival1stFirmSmall[f2], <<24>>[<<1>>]] is not a machine-size real number at f2 = 0.0287912708055911394`.

Plot::plnr : x2opt4firms[iOfIndifferentLargeAndSmallRival1stFirmSmall[f2], <<24>>[<<1>>]] is not a machine-size real number at f2 = 0.0287815453540027945`.

Plot::plnr : x2opt4firms[iOfIndifferentLargeAndSmallRival1stFirmSmall[f2], <<24>>[<<1>>]] is not a machine-size real number at f2 = 0.0287802437962091985`.

General::stop : Further output of Plot::plnr will be suppressed during this calculation.

- Graphics -
Plot[{x2opt4firms[iOfIndifferentLargeAndSmallRival1stFirmSmall[f2],
location2ndEntrant3rdAt1[iOfIndifferentLargeAndSmallRival1stFirmSmall[f2]],
x2opt4firmsA[iOfIndifferentLargeAndSmallRival1stFirmSmall[f2],
location2ndEntrant3rdAt1[iOfIndifferentLargeAndSmallRival1stFirmSmall[f2]]},
{f2, 0.028779, .028780}, PlotPoints -> 40]

Plot::plnr: x2opt4firms[iOfIndifferentLargeAndSmallRival1stFirmSmall[f2], <<24>>[<<1>>]]
not a machine-size real number at f2 = 0.02879896816526143.`.

Plot::plnr: x2opt4firms[iOfIndifferentLargeAndSmallRival1stFirmSmall[f2], <<24>>[<<1>>]]
not a machine-size real number at f2 = 0.02879884854608336`.

Plot::plnr: x2opt4firms[iOfIndifferentLargeAndSmallRival1stFirmSmall[f2], <<24>>[<<1>>]]
not a machine-size real number at f2 = 0.0287978781663343383`.

General::stop: Further output of Plot::plnr will be suppressed during this calculation.

- Graphics -

InputForm[%]

Graphics[{{Line[{{0.02877902496400328, 0.4790272802241584},
{0.02877902496400328, 0.4790273677552867},
{0.02877902496400328, 0.479027463215573},
{0.02877902496400328, 0.4790275528689206},
{0.02877902496400328, 0.4790276390881881},
{0.02877902496400328, 0.4790277308637555},
{0.02877902496400328, 0.4790278192052444},
{0.02877902496400328, 0.4790279131030207},
{0.02877902496400328, 0.4790280035667211},
{0.02877902496400328, 0.479028095963532},
{0.02877902496400328, 0.4790281831822639},
{0.02877902496400328, 0.479028272334107},
{0.02877902496400328, 0.4790283580518909},
{0.02877902496400328, 0.4790284493259437},
{0.02877902496400328, 0.4790285371659374},
{0.02877902496400328, 0.4790286305621883},
{0.02877902496400328, 0.4790287205243849},
{0.02877902496400328, 0.4790288070525322},
{0.02877902496400328, 0.4790288991369264},
{0.02877902496400328, 0.4790289877872743},
{0.02877902496400328, 0.4790290819338597},
{0.02877902496400328, 0.4790291727664007},
{0.02877902496400328, 0.4790292601049014},
{0.02877902496400328, 0.4790293529996309},
{0.02877902496400328, 0.4790294424603252},
{0.02877902496400328, 0.4790295284869878},
{0.02877902496400328, 0.47902963403994},
{0.02877902496400328, 0.4790297082187231},
{0.02877902496400328, 0.4790298019237856},
{0.02877902496400328, 0.4790298921948227}},{0.02877902496400328, 0.4790299994935107},
{0.02877902496400328, 0.4790300957008938},
{0.02877902496400328, 0.4790301930763062},
{0.02877902496400328, 0.4790302899819385},
{0.02877902496400328, 0.4790303867864007},
{0.02877902496400328, 0.479030483391978},
{0.02877902496400328, 0.479030580099099},
{0.02877902496400328, 0.479030676808836},
{0.02877902496400328, 0.47903077339761},
{0.02877902496400328, 0.47903087009667},
{0.02877902496400328, 0.4790309667943594},
{0.02877902496400328, 0.479031063492352},
{0.02877902496400328, 0.4790311601914532}]}
\[ x_{2\text{opt4firmsCombined}}[f2_] := \text{Which}[f2 < 0.02877987567, x_{2\text{opt4firms}}[iOfIndifferentLargeAndSmallRival1stFirmSmall][f2], \text{location2ndEntrant3rdAt1}[iOfIndifferentLargeAndSmallRival1stFirmSmall][f2]], f2 < 0.0287798766, 0.479030352, \text{True}, x_{2\text{opt4firms}}[iOfIndifferentLargeAndSmallRival1stFirmSmall][f2], \text{location2ndEntrant3rdAt1}[iOfIndifferentLargeAndSmallRival1stFirmSmall][f2]]] \]

\[ x_{2\text{opt4firmsCombined}}[0.02] \]
0.443892

\[ x_{2\text{opt4firmsCombined}}[0.03] \]
0.483241

\[
\{\text{redProfit4}[2] /. \{x[1] \to iOfIndifferentLargeAndSmallRival1stFirmSmall[.025], x[2] \to x_{2\text{opt4firms}}[iOfIndifferentLargeAndSmallRival1stFirmSmall][.025], \text{location2ndEntrant3rdAt1}[iOfIndifferentLargeAndSmallRival1stFirmSmall][.025]], x[3] \to \text{location2ndEntrant3rdAt1}[iOfIndifferentLargeAndSmallRival1stFirmSmall][.025]], x[4] \to 1, n \to 1, f[2] \to .025)\}
\]

\{0.0040933\}

\[
\{\text{redProfit4}[2] /. \{x[1] \to iOfIndifferentLargeAndSmallRival1stFirmSmall[.035], x[2] \to x_{2\text{opt4firmsCombined}}[0.035], x[3] \to \text{location2ndEntrant3rdAt1}[iOfIndifferentLargeAndSmallRival1stFirmSmall][.035]], x[4] \to 1, n \to 1, f[2] \to .035)\}
\]

\{-0.0084515\}

\[ iOfIndifferentLargeAndSmallRival1stFirmSmall[.035] \]
\[ \text{location2ndEntrant3rdAt1}[iOfIndifferentLargeAndSmallRival1stFirmSmall][.035] \]
\[ x_{2\text{opt4firms}}[iOfIndifferentLargeAndSmallRival1stFirmSmall][.035], \text{location2ndEntrant3rdAt1}[iOfIndifferentLargeAndSmallRival1stFirmSmall][.035]] \]

0.225844

0.719087

\(-1.50174 + 0.427564 \text{I}\)
\begin{verbatim}

- Graphics -

FindRoot[

\{f2 \rightarrow 0.0281945\}

f2of2ndEntrantMustDeter4thEntrant = f2 /. %

0.0281945


\{0.186935, x[2] \rightarrow 0.476962, x[3] \rightarrow 0.707214\}

For x2ndEntrantDeter4th similar problems arise as with x2opt4firms. Again, I distinguish different ranges.

x2opt4firms[0.136, .6]
x2opt4firmsA[0.136, .6]

-1.59396 + 0.181115 I

0.394828

Clear[x2ndEntrantDeter4th]

x2ndEntrantDeter4th[i_, f2_] :=
\end{verbatim}
Both of the following definitions work.

```math
nOf2ndEntrantIndifferentLargeAndSmallTech1stEntrantSmallDeter4thEntrant[f2_] :=
  n /. Solve[prof2ndEntrant2ndEntrantLarge1stSmall[f2, n] -
    prof2ndEntrant3firmsDeter4thEntrant[f2, n] == 0, n]

(* nOf2ndEntrantIndifferentLargeAndSmallTech1stEntrantSmallDeter4thEntrant[f2_] :=
  n/.InterpolateRoot[prof2ndEntrant2ndEntrantLarge1stSmall[f2,n] -
    prof2ndEntrant3firmsDeter4thEntrant[f2,n]==0,{n,40,40.01}] *)

nOf2ndEntrantIndifferentLargeAndSmallTech1stEntrantSmallDeter4thEntrant[0.021]
{103.509}

fig4MR = Plot[nOf2ndEntrantIndifferentLargeAndSmallTech1stEntrantSmallDeter4thEntrant[f2],
  {f2, f2block4thEntrant, f2of2ndEntrantMustDeter4thEntrant}]
```

FindRoot::frsec : Secant method failed to converge to the prescribed accuracy after 30 iterations.

FindRoot::frsec : Secant method failed to converge to the prescribed accuracy after 30 iterations.

FindRoot::frsec : Secant method failed to converge to the prescribed accuracy after 30 iterations.

General::stop : Further output of FindRoot::frsec will be suppressed during this calculation.
Secant method failed to converge to the prescribed accuracy after 30 iterations.

FindRoot::frsec :
Secant method failed to converge to the prescribed accuracy after 30 iterations.

FindRoot::frsec :
Secant method failed to converge to the prescribed accuracy after 30 iterations.

FindRoot::frsec :
Secant method failed to converge to the prescribed accuracy after 30 iterations.

Show[fig3MR, fig4MR]
Plot[x2ndEntrantDeter4th[iOfIndifferentLargeAndSmallRival1stFirmSmall[f2], f2],
{f2, 0.022, 0.029}]

FindRoot::frsec :
Secant method failed to converge to the prescribed accuracy after 30 iterations.

FindRoot::frsec :
Secant method failed to converge to the prescribed accuracy after 30 iterations.

FindRoot::frsec :
Secant method failed to converge to the prescribed accuracy after 30 iterations.

General::stop : Further output of FindRoot::frsec will be suppressed during this calculation.

- Graphics -

x2ndEntrantDeter4th[iOfIndifferentLargeAndSmallRival1stFirmSmall[0.026], 0.026]  
nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrant[0.026]  
x2opt4firms[iOfIndifferentLargeAndSmallRival1stFirmSmall[0.026],  
x2ndEntrantDeter4th[iOfIndifferentLargeAndSmallRival1stFirmSmall[0.026], 0.026]]  
x2opt4firmsA[iOfIndifferentLargeAndSmallRival1stFirmSmall[0.026],  
x2ndEntrantDeter4th[iOfIndifferentLargeAndSmallRival1stFirmSmall[0.026], 0.026]]  
0.671378  
98.18321666151848144637  
-1.61017 + 0.118104 I  
0.450878
\textbf{x2ndEntrantDeter4th[iOfIndifferentLargeAndSmallRival1stFirmSmall][0.027], 0.027]}
\textbf{no2ndEntrantIndifferentAndSTech1stEntrantSmallDeter4thEntrant[0.027]}
\textbf{x2opt4firms[iOfIndifferentLargeAndSmallRival1stFirmSmall][0.027],}
\textbf{x2ndEntrantDeter4th[iOfIndifferentLargeAndSmallRival1stFirmSmall][0.027], 0.027]}
\textbf{x2opt4firmsA[iOfIndifferentLargeAndSmallRival1stFirmSmall][0.027],}
\textbf{x2ndEntrantDeter4th[iOfIndifferentLargeAndSmallRival1stFirmSmall][0.027], 0.027]}

FindRoot::frsec :
Secant method failed to converge to the prescribed accuracy after 30 iterations.
0.687975

FindRoot::frsec :
Secant method failed to converge to the prescribed accuracy after 30 iterations.

FindRoot::frsec :
Secant method failed to converge to the prescribed accuracy after 30 iterations.
97.247935873870538791784

FindRoot::frsec :
Secant method failed to converge to the prescribed accuracy after 30 iterations.
0.463

FindRoot::frsec :
Secant method failed to converge to the prescribed accuracy after 30 iterations.
-1.66427

\textbf{Plot[x2opt4firmsA[iOfIndifferentLargeAndSmallRival1stFirmSmall][f2],}
\textbf{x2ndEntrantDeter4th[iOfIndifferentLargeAndSmallRival1stFirmSmall][f2], f2]},
\textbf{(f2, 0.026, 0.028), PlotRange -> All]}
\textbf{Plot[x2opt4firms[iOfIndifferentLargeAndSmallRival1stFirmSmall][f2],}
\textbf{x2ndEntrantDeter4th[iOfIndifferentLargeAndSmallRival1stFirmSmall][f2], f2]},
\textbf{(f2, 0.026, 0.028), PlotRange -> All]}

FindRoot::frsec :
Secant method failed to converge to the prescribed accuracy after 30 iterations.

FindRoot::frsec :
Secant method failed to converge to the prescribed accuracy after 30 iterations.

FindRoot::frsec :
Secant method failed to converge to the prescribed accuracy after 30 iterations.

General::stop : Further output of FindRoot::frsec will be suppressed during this calculation.
Coordinates from the above diagrams.

\{(0.02688212150063569, 0.4615889903473329),
(0.02688446462047588, -0.623963292998633),
(0.02688212150063569, 0.4615889903473329),
(0.02688446462047588, -0.623963292998633),
(0.02688478954564132, 0.461619281033843),
(0.02688478954564132, 0.461619281033843)\}
fig4MR = Plot[nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrant[f2], {f2, f2block4thEntrant, .026882}]

- Graphics -


nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrantNew[0.028]

FindRoot::precw : The precision of the argument function (<<1>> - 0) is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function (<<1>> - 0) is less than WorkingPrecision (35).

96.350148742816913518165 + 0. × 10−33 I
\begin{verbatim}
fig5MR = Plot[nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrantNew[f2], {f2, .026882, f2of2ndEntrantMustDeter4thEntrant}]

FindRoot::precw: The precision of the argument function 
   \( \text{redProfit4} \) \( \hat{=} \) \( \text{8} \) \( \text{x} \) \( \hat{=} \) 0.178575, \( \text{i} \) \( \hat{=} \) 4, \( \text{f} \) \( \hat{=} \) 0.026882 \] \] is less than WorkingPrecision (35).

FindRoot::precw: The precision of the argument function 
   \( \text{redProfit4} \) \( \hat{=} \) \( \text{8} \) \( \text{x} \) \( \hat{=} \) 0.178575, \( \text{i} \) \( \hat{=} \) 4, \( \text{f} \) \( \hat{=} \) 0.026882 \] \] is less than WorkingPrecision (35).

FindRoot::precw: The precision of the argument function 
   \( \text{redProfit4} \) \( \hat{=} \) \( \text{8} \) \( \text{x} \) \( \hat{=} \) 0.17892, \( \text{i} \) \( \hat{=} \) 4, \( \text{f} \) \( \hat{=} \) 0.0269352 \] \] is less than WorkingPrecision (35).

General::stop: Further output of FindRoot::precw will be suppressed during this calculation.

\end{verbatim}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig5MR.png}
\caption{Plot of the number of second entrant indifferent between large and small tech first entrant and small deter 4th entrant.}
\end{figure}

\begin{verbatim}
x2opt4firmsCombinedNew[f2_, j_] := If[
   Head[x2opt4firms[iOfIndifferentLargeAndSmallRival1stFirmSmall[f2], j]] == Complex, 
   x2opt4firmsA[iOfIndifferentLargeAndSmallRival1stFirmSmall[f2], j], 
   Null, x2opt4firms[iOfIndifferentLargeAndSmallRival1stFirmSmall[f2], j]]
x2opt4firmsCombinedNew[.028, .7]

0.472386
\end{verbatim}
Plot[x2opt4firmsCombinedNew[.028, j], {j, .6, .8}]

    ((react3MR /. {x[1] -> iOfIndifferentLargeAndSmallRival1stFirmSmall[.028],
        x[2] -> j}) /. x[3] -> x[4])[1], n -> 1,
    f[2] -> .028})[[1]], {j, 0.5, .75}, PlotRange -> {-.05, .05}]

- Graphics -
fig3MR = Plot[nOf2ndEntrantIndifferent1AndSTech1stENtrantSmall[f2],
{f2, f2Of2ndEntrantMustDeter4thEntrant,
 f2IndifferentLargeAndSmallRival1stFirmSmall}, PlotStyle -> {Hue[.8]}]

Show[fig3MR, fig4MR, fig5MR]

iOfIndifferentLargeAndSmallRival1stFirmSmall[f2NoDeter3rdEntrant] =
 x2ndEntrantDeter4th[
 iOfIndifferentLargeAndSmallRival1stFirmSmall[f2NoDeter3rdEntrant],
 f2NoDeter3rdEntrant]

0.171686

0.66864

{redProfit3[3] /. 
 {x[1] -> iOfIndifferentLargeAndSmallRival1stFirmSmall[f2NoDeter3rdEntrant],
 x[2] -> deterCenterMR[iOfIndifferentLargeAndSmallRival1stFirmSmall[
 f2NoDeter3rdEntrant], f2NoDeter3rdEntrant],

0.0194826
Entry at the right edge is blockaded as long as \( f_2 \geq f_2\text{noDeter3rdEntrant} \). The constraint will however become binding as soon as \( f_2 \) gets smaller than \( f_2\text{noDeter3rdEntrant} \). The first entrant would be willing to locate at 0 in this case as is shown below.

For \( f_2 \leq f_2\text{noDeter3rdEntrant} \), the alternatives for the first entrant are one large or two small rivals. As is shown next, the first entrant would be willing to locate at 0 to induce the second firm to use L-tech.

\[
\text{iOfIndifferentLargeAndSmallRival1stFirmSmall3Firms} := \text{FindRoot[}
\text{(profits2Firms[i, deterCenterMR[i, f2], -1/8][[1]]) /. \{f[1] \to f2, n \to 1\} ==}
\text{(prof3sym[f2, 1][[2]]), \{i, .1, 0.11\}}]\n\]

\[
\text{iOfIndifferentLargeAndSmallRival1stFirmSmall[0.02]}\]
\[
\text{iOfIndifferentLargeAndSmallRival1stFirmSmall3Firms[0.02]}
\]

0.128509

\[-0.0744227\]

\[
\text{iOfIndifferentLargeAndSmallRival1stFirmSmall3Firms[f2\text{noDeter3rdEntrant}]}
\]

\[-0.185309\]

The parameter values which apply in this case require both firms to move (see above). The maximum profit the second entrant can realize if she uses the large scale technology and both firms must move is profAsymAsy. I derived that above and can use it here. The next diagramms show that the first entrant will prefer this situation to the case with three (small) firms.

\[
\text{Plot[}
\text{profits2FirmsOfN[numDeterLeftAsy[f2], numDeterCenterAsy[numDeterLeftAsy[f2], f2],}
\text{1/8, 1][[2]] / . f[2] \to f2, \{f2, 0.02, 0.03\}]}
\]

- Graphics -
Plot[prof3sym[f2, n][[2]] /. n -> 1, {f2, 0.02, 0.03}]

Show[%, %]

prof2ndEntrant3firmsDeter4thEntrant1stAtINew[f2_, n_, i_] :=

x2ndEntrantDeter4th[0, f2block4thEntrant]

FindRoot::frsec : The secant method failed to converge to the prescribed accuracy after 30 iterations.

0.499322

x2ndEntrantDeter4thNew[0, f2block4thEntrant]

    is less than WorkingPrecision (35).

0.48636431899353247964823716374453822
prof2ndEntrant3firmsDeter4thEntrant1stAtINew[f2block4thEntrant, n, 0]

FindRoot::precw : The precision of the argument
  is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument
  is less than WorkingPrecision (35).

0.0811009 n

prof2ndEntrant3firmsDeter4thEntrant1stAtINew[f2block4thEntrant, n, 
1 - numDeterCenterAsy[numDeterLeftAsy[f2block4thEntrant], f2block4thEntrant]]

FindRoot::precw : The precision of the argument function {

FindRoot::frsec :
Secant method failed to converge to the prescribed accuracy after 30 iterations.

FindRoot::precw : The precision of the argument function {

FindRoot::frsec :
Secant method failed to converge to the prescribed accuracy after 30 iterations.

0.0603363 n
numDeterCenterAsy[numDeterLeftAsy[f2block4thEntrant], f2block4thEntrant]
x2ndEntrantDeter4th[
  1 - numDeterCenterAsy[numDeterLeftAsy[f2block4thEntrant], f2block4thEntrant],
  f2block4thEntrant]
x2ndEntrantDeter4thNew[
  1 - numDeterCenterAsy[numDeterLeftAsy[f2noDeter3rdEntrant], f2noDeter3rdEntrant],
  f2noDeter3rdEntrant]

0.823066
0.625651

FindRoot::precw: The precision of the argument function \((\text{redProfit4}[2] /. (x[1] \to 0.176934, \llcorner 5 \lrcorner ))[1] - 0)\) is less than WorkingPrecision (35).

FindRoot::frsec:
Secant method failed to converge to the prescribed accuracy after 30 iterations.
0.669647

FindRoot::frsec:
Secant method failed to converge to the prescribed accuracy after 30 iterations.
0.633727

FindRoot::precw: The precision of the argument function \((\text{redProfit4}[2] /. (x[1] \to 0.124288, \llcorner 5 \lrcorner ))[1] - 0)\) is less than WorkingPrecision (35).

0.6317089973223385585224333623214028

iOfIndifferentLargeAndSmallRival1stFirmSmall[f2block4thEntrant]
iOfIndifferentLargeAndSmallRival1stFirmSmall[f2noDeter3rdEntrant]
numDeterCenterAsy[numDeterLeftAsy[f2block4thEntrant], f2block4thEntrant]
numDeterCenterAsy[numDeterLeftAsy[f2noDeter3rdEntrant], f2noDeter3rdEntrant]

0.144231
0.171686
0.823066
0.875712

At f2noDeter3rdEntrant a jump in the location takes place, although not a large one. For the range of \(i\) which is relevant here, x2ndEntrantDeter4th works. I use it in the next definition

\[
\text{prof2ndEntrant3firmsDeter4thEntrant1stAtI}[f2_, n_, i_] :=
\{(\text{redProfit3ofN2ndFirm}[n] /. (x[1] \to i, x[2] \to x2ndEntrantDeter4th[i, f2]),
  (\text{react3MR} /. (x[1] \to i, x[2] \to x2ndEntrantDeter4th[i, f2]))[[1]],
  c[1] \to 1/2, c[2] \to 1/2, c[3] \to 1/2)[[1]] + f[2] - f2 n)\]
prof2ndEntrant3firmsDeter4thEntrant1stAtI[f2block4thEntrant, n, 
  1 - numDeterCenterAsy[numDeterLeftAsy[f2block4thEntrant], f2block4thEntrant]]

prof2ndEntrant3firmsDeter4thEntrant1stAtI[f2block4thEntrant, n, 
  1 - numDeterCenterAsy[numDeterLeftAsy[f2noDeter3rdEntrant], f2noDeter3rdEntrant]]

0.0577488 n

0.0643977 n

Plot[1 - numDeterCenterAsy[numDeterLeftAsy[f2], f2], 
   {f2, f2block4thEntrant, f2noDeter3rdEntrant}]

- Graphics -

Plot[prof2ndEntrant3firmsDeter4thEntrant1stAtI[f2, 100, 
   1 - numDeterCenterAsy[numDeterLeftAsy[f2], f2]], {f2, f2block4thEntrant, 0.02415}]

FindRoot::frsec : Secant method failed to converge to the prescribed accuracy after 30 iterations.

FindRoot::frsec : Secant method failed to converge to the prescribed accuracy after 30 iterations.

- Graphics -
InputForm[%]

Graphics[{{Line[{{0.02199044637696176, 5.774876217922559},
{0.02207805288425768, 5.788981909806018},
{0.022135954456134, 5.80433081272495},
{0.02226332612573155, 5.818710945412776},
{0.02234961982202536, 5.832507027225911},
{0.02244147467496574, 5.847155723553489},
{0.02252989254408177, 5.861220510890955},
{0.02262387156984438, 5.876131316400783},
{0.02271441361178264, 5.890547815351251},
{0.0228015186698656, 5.904329307369263},
{0.02289418488465704, 5.918959284267867},
{0.02298341411559319, 5.933016235007597},
{0.02306920636270499, 5.946585365351289},
{0.02316055976646336, 5.960900055333362},
{0.02324847618639739, 5.974738388566108},
{0.02334195376297799, 5.989315329406313},
{0.02343199435573425, 6.003329268987515},
{0.02351859796466616, 6.016783341669546},
{0.02361076273024464, 6.031074597345932},
{0.02369949051199879, 6.044802839255487},
{0.0237937794503995, 6.059372155884643},
{0.02388463140497588, 6.073379282164248},
{0.0239720463757279, 6.086831529955713},
{0.0240650225031265, 6.1011478709436},
{0.02414999991001859, 6.11414409214694}]},
{PlotRange -> Automatic, AspectRatio -> GoldenRatio^(-1),
DisplayFunction :> $DisplayFunction, ColorOutput -> Automatic,
Axes -> Automatic, AxesOrigin -> Automatic, PlotLabel -> None,
AxesLabel -> None, Ticks -> Automatic, GridLines -> None,
Prolog -> {}, Epilog -> {}, AxesStyle -> Automatic,
Background -> Automatic, DefaultColor -> Automatic,
DefaultFont :> $DefaultFont, RotateLabel -> True, Frame -> False,
FrameStyle -> Automatic, FrameTicks -> Automatic,
FrameLabel -> None, PlotRegion -> Automatic,
ImageSize -> Automatic, TextStyle :> TextStyle,
FormatType :> $FormatType}]
prof2ndEntrant3firmsDeter4thEntrant1stAtINew[0.02412, 100, 1 - numDeterCenterAsy[numDeterLeftAsy[0.02412], 0.02412]]

FindRoot::precw :
The precision of the argument function \( \langle x \rangle - 0 \) is less than WorkingPrecision (35).

FindRoot::frsec :
Secant method failed to converge to the prescribed accuracy after 30 iterations.

FindRoot::precw :
The precision of the argument function \( \langle x \rangle - 0 \) is less than WorkingPrecision (35).

FindRoot::frsec :
Secant method failed to converge to the prescribed accuracy after 30 iterations.

FindRoot::precw :
The precision of the argument function \( \langle x \rangle - 0 \) is less than WorkingPrecision (35).

6.11085

6.10954

FindRoot::precw :
The precision of the argument function \( \langle x \rangle - 0 \) is less than WorkingPrecision (35).

FindRoot::precw :
The precision of the argument function \( \langle x \rangle - 0 \) is less than WorkingPrecision (35).

6.11108

FindRoot::frsec :
Secant method failed to converge to the prescribed accuracy after 30 iterations.

FindRoot::frsec :
Secant method failed to converge to the prescribed accuracy after 30 iterations.

6.11085

Plot[prof2ndEntrant3firmsDeter4thEntrant1stAtINew[f2, 100, 1 - numDeterCenterAsy[numDeterLeftAsy[f2], f2]], {f2, 0.02412, 0.02413}]

FindRoot::precw : The precision of the argument function \( \langle x \rangle - 0 \) is less than WorkingPrecision (35).

FindRoot::frsec :
Secant method failed to converge to the prescribed accuracy after 30 iterations.

FindRoot::precw : The precision of the argument function \( \langle x \rangle - 0 \) is less than WorkingPrecision (35).

FindRoot::frsec :
Secant method failed to converge to the prescribed accuracy after 30 iterations.

FindRoot::precw : The precision of the argument function \( \langle x \rangle - 0 \) is less than WorkingPrecision (35).

General::stop : Further output of FindRoot::precw will be suppressed during this calculation.

FindRoot::frsec :
Secant method failed to converge to the prescribed accuracy after 30 iterations.
General::stop: Further output of FindRoot::frsec will be suppressed during this calculation.

FindRoot::frsec:
Secant method failed to converge to the prescribed accuracy after 30 iterations.

FindRoot::frsec:
Secant method failed to converge to the prescribed accuracy after 30 iterations.

FindRoot::frsec:
Secant method failed to converge to the prescribed accuracy after 30 iterations.

General::stop: Further output of FindRoot::frsec will be suppressed during this calculation.
InputForm[\%150]

Graphics[{{Line[{{0.030000000062499999, 0.2719074678792008},
{0.03127213199789059, 0.25486684502407638},
{0.03189539049507806, 0.2466376225461056},
{0.03249477611656241, 0.23879239602598989},
{0.0331327886623461, 0.230515573364814},
{0.03374692873242169, 0.2226131357649406},
{0.03439969572679665, 0.2142953818790146},
{0.03502858984546846, 0.2063518113885999},
{0.03563361108843714, 0.1987732344796074},
{0.036272594557027, 0.1907776935634133},
{0.03689703494726512, 0.1831425221272698},
{0.03749293756312441, 0.1758592760900931},
{0.03812746730328057, 0.1681649858355984},
{0.03873612416773359, 0.1608184617511004},
{0.03939740815648349, 0.153068661935025},
{0.04001281926953025, 0.145662468930589},
{0.04061435750687389, 0.13859221719339811},
{0.04125452286851438, 0.13112417660703351},
{0.04187081535445175, 0.1239883491916122},
{0.04252573496468599, 0.1164619011733398},
{0.04315678169921709, 0.1092596251671767},
{0.04376395555504507, 0.1023849203118942},
{0.04440975654146991, 0.09512417813801581},
{0.044999999349999999, 0.08853368156437653}}}],
{PlotRange -> Automatic, AspectRatio -> GoldenRatio^(-1),
DisplayFunction :> $DisplayFunction, ColorOutput -> Automatic,
Axes -> Automatic, AxesOrigin -> Automatic, PlotLabel -> None,
Prolog -> {}, Epilog -> {}, AxesStyle -> Automatic,
Background -> Automatic, DefaultColor -> Automatic,
DefaultFont :> $DefaultFont, RotateLabel -> True, Frame -> False,
FrameStyle -> Automatic, FrameTicks -> Automatic,
FrameLabel -> None, PlotRegion -> Automatic,
ImageSize -> Automatic, TextStyle :> $TextStyle,
FormatType :> $FormatType]}

InputForm[\%151]
Plot prof2ndEntrant3firmsDeter4thEntrant1stAtINew[ 
  f2, 100, 1 - numDeterCenterAsy[numDeterLeftAsy[f2], f2]], 
{f2, 0.024128, f2noDeter3rdEntrant}]
Plot prof2ndEntrant3firmsDeter4thEntrant1stAtINew[ 
  f2, 100, 1 - numDeterCenterAsy[numDeterLeftAsy[f2], f2]], 
{f2, f2block4thEntrant, 0.024128}]

FindRoot::precw : The precision of the argument function 
FindRoot::frsec : Secant method failed to converge to the prescribed accuracy after 30 iterations.
FindRoot::precw : The precision of the argument function 
FindRoot::frsec : Secant method failed to converge to the prescribed accuracy after 30 iterations.
FindRoot::precw : The precision of the argument function 
General::stop : Further output of FindRoot::precw will be suppressed during this calculation.

- Graphics -
Show[%, %%]

- Graphics -

prof2ndEntrant3firmsDeter4thEntrant[f2block4thEntrant, n]
prof2ndEntrant3firmsDeter4thEntrant[f2noDeter3rdEntrant, n]

0.0618501 n

0.0573358 n

nOf2ndEntrantIndifferentLAndSTech1stEntrantSmallDeter4thEntrantBothMoveInAsy[f2_] :=
n /. Solve[profAsymAsy[f2, n] - prof2ndEntrant3firmsDeter4thEntrant1stAtI[
f2, n, 1 - numDeterCenterAsy[numDeterLeftAsy[f2], f2]] == 0, n]

nOf2ndEntrantIndifferentLAndSTech1stEntrantSmallDeter4thEntrantBothMoveInAsyNew[
f2_] :=
n /. Solve[profAsymAsy[f2, n] - prof2ndEntrant3firmsDeter4thEntrant1stAtINew[f2, n, 1 - numDeterCenterAsy[numDeterLeftAsy[f2], f2]] == 0, n]
nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrantBothMoveInAsy[0.021]
nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrantBothMoveInAsy[f2noDeter3rdEntrant]
nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrantBothMoveInAsyNew[0.021]
nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrantBothMoveInAsyNew[f2noDeter3rdEntrant]

{108.666}
FindRoot::frsec : Secant method failed to converge to the prescribed accuracy after 30 iterations.
FindRoot::frsec : Secant method failed to converge to the prescribed accuracy after 30 iterations.
{94.2988}
FindRoot::precw : The precision of the argument function (<<1>> - 0) is less than WorkingPrecision (35).
FindRoot::precw : The precision of the argument function (<<1>> - 0) is less than WorkingPrecision (35).
{108.666 + 1.47093 \times 10^{-34} I}
FindRoot::precw : The precision of the argument function {
(redProfit4[2] /. (x[1] \to 0.124288, <<5>>) [1] - 0) is less than WorkingPrecision (35).
FindRoot::precw : The precision of the argument function {
(redProfit4[2] /. (x[1] \to 0.124288, <<5>>) [1] - 0) is less than WorkingPrecision (35).

fig6MR = Plot[
nOf2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrantBothMoveInAsy[f2],
{f2, f2block4thEntrant, 0.0241282}]

- Graphics -
fig7MR = Plot[
  nOf2ndEntrantIndifferentAndSTech1stEntrantSmallDeter4thEntrantBothMoveInAsyNew[
f2], {f2, 0.0241283, f2noDeter3rdEntrant}]


General::stop : Further output of FindRoot::precw will be suppressed during this calculation.

Show[fig6MR, fig7MR]

?prof3sym

Global`prof3sym

prof3sym[f2_, n_] = 
{0.04694647607807409*n - f2*n, 0.0880423477549795*n - f2*n, 0.06881923855036964*n - f2*n}

nOf3smallFirms[f2_] := n /. Solve[{profAsymAsy[f2, n] == prof3sym[f2, n][[2]], n}][[1]]

nOf3smallFirms[f2noDeter3rdEntrant] = 93.7368
\begin{verbatim}
fig8MR = Plot[nOf3smallFirms[f2],
    {f2, f2block4thEntrant, f2noDeter3rdEntrant}, PlotStyle -> Dashing[{.1, .01}]]

- Graphics -

Show[fig6MR, fig7MR, fig8MR]

- Graphics -

FindRoot[nOf3smallFirms[f2] ==
nOf2ndEntrantIndifferentAndSTech1stEntrantSmallDeter4thEntrantBothMoveInAsyNew[
    f2][1]], {f2, 0.025, .0251}]

FindRoot::precw : The precision of the argument
    is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument
    is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument
    is less than WorkingPrecision (35).

General::stop : Further output of FindRoot::precw will be suppressed during this calculation.

{f2 \rightarrow 0.0252445}

f2Of1stLargeBelow1stSmall = f2 /. %
0.0252445
\end{verbatim}
fig8MR = 
Plot[nOf3smallFirms[f2], {f2, f2Ofo1stLargeBelow1stSmall, f2noDeter3rdEntrant}]  

- Graphics -  

Show[fig6MR, fig7MR, fig8MR]  

- Graphics -  

{1 - numDeterCenterAsy[numDeterLeftAsy[f2noDeter3rdEntrant], f2noDeter3rdEntrant],  
1 - numDeterLeftAsy[f2noDeter3rdEntrant]}  
{1 - numDeterCenterAsy[numDeterLeftAsy[f2block4thEntrant], f2block4thEntrant],  
1 - numDeterLeftAsy[f2block4thEntrant]}  
{0.124288, 0.595093}  
{0.176934, 0.609886}  

i0fIndifferentLargeAndSmallRival1stFirmSmall[f2noDeter3rdEntrant]  
0.171686  

The profits in some borderline cases.  

profAsymAsy[f2block4thEntrant,  
nOf2ndEntrantIndifferentLAndSTech1stENTrantSMallDeter4thEntrantBothMoveInAsy[  
f2block4thEntrant]]  
{6.08603}
profits2FirmsOfN[numDeterLeftAsylstSmall[f2block4thEntrant],
deterCenterMR[Chop[numDeterLeftAsylstSmall[f2block4thEntrant]],
f2block4thEntrant, -1/8, n0f1stEntrantIndifferent2AsymSituationsEntryAt0[

{21.6302}

prof1stEntrant2ndEntrantLarge1stSmallEntryAt0[f2block4thEntrant,
n0f1stEntrantIndifferent2AsymSituationsEntryAt0[f2block4thEntrant]]

{{28.3409}}

fig1MR = Plot[n0f1stEntrantIndifferent2AsymSituations[f2],
{f2, f2Of1stSmall2ndLargeEntryAt0, f2IndifferentLargeAndSmallRivalistFirmSmall}]

- Graphics -

fig4MR = Plot[n0f2ndEntrantIndifferentLAndSTech1stENtrantSmallDeter4thEntrant[f2],
{f2, f2noDeter3rdEntrant, .026882}]

- Graphics -
Show[fig3MR, fig4MR, fig5MR, fig6MR, fig7MR, fig8MR]

\[
\text{fig5a} = \text{Plot}[\text{nOfSecondEntrantMustMoveToDeter3rdEntrant}[f2],
\{f2, f2IndifferentLargeAndSmallRival1stFirmSmall,
\text{f2SecondEntrantMustMoveToDeter3rdEntrant}\}]
\]
Calculations for the loci between the asymmetric case and the case with two large firms

\( iDeterLargeEntrant \) gives the first entrant’s location such that the second is indifferent between the two technologies.

\[
\text{Solve}\left[ \left( (\text{profits2Firms}[i, 1, 0][[2]] - 25) - (\text{profits2Firms}[i, 1, \, 1/8][[2]] - f2n) \right) = 0, i \right] \\
\left\{ \left\{ i \rightarrow \frac{1}{32n} \left( -28800 + 64n + 1152f2n - \sqrt{(28800 - 64n - 1152f2n)^2 - 64n (-28800 + 47n + 1152f2n)} \right) \right\}, \left\{ i \rightarrow \frac{1}{32n} \left( -28800 + 64n + 1152f2n + \sqrt{(28800 - 64n - 1152f2n)^2 - 64n (-28800 + 47n + 1152f2n)} \right) \right\} \right\} \\
N[/]. \{ f2 \rightarrow 1/8, n \rightarrow 154 \} \\
\left\{ \left\{ i \rightarrow 0.23047 \right\}, \left\{ i \rightarrow 1.08122 \right\} \right\}
\]
The incumbent prefers the large rival. The entry of a third entrant either small or large is blockaded. The next expressions give the location from which on the incumbent prefers the large rival.

\[
\mathbb{N}[\text{iIndifferentLargeAndSmallRival}] = \frac{1}{2} \cdot \text{Simplify}
\]

\[
\text{Solve}[\text{profits2Firms}[0, 1, 0][[1]] - \text{profits2Firms}[i, 1, 1/8][[1]] == 0, i][[1]]
\]

\[
\{i \rightarrow -1 + \frac{3}{2} \sqrt{2}, i \rightarrow -1 - \frac{3}{2} \sqrt{2}\}
\]

\[
\text{nOfIndifferentLargeAndSmallRival}[f2_] = \text{Solve}[\text{iDeterLargeEntrant}[f2] == \text{iIndifferentLargeAndSmallRival}[n]][[1]]
\]

\[
\mathbb{N}[\frac{14400}{-32 + 51 \sqrt{2} - 76 \sqrt{3 + \frac{1}{\sqrt{2}}} + 48 \sqrt{2 \left(3 + \frac{1}{\sqrt{2}}\right)} + 576 f2}]
\]

The interesting thing about the above number is that it is not equal to 144. The reason seems to be that the first entrant jumps at n=144 and f2 = 9/64. Contrary to the case of c=1/2 where the incumbent moves continuously from 1/2 towards 0 (wrong: At the borderline case the first entrant always locates at 1/2. That means that the profit is greater in the case where a large rival can be deterred.), in the case of c=1/8 the first entrant must jump to iIndifferentLargeAndSmallRival (−0.202). Otherwise, she would prefer a large rival. In this case the profit accruing from the L-Tech is greater than that from the s-Tech. In order to deter L-Tech the market size must be smaller (than 144). The interesting consequence of this argument is: The first entrant enable the second entrant to choose the L-tech for n=144 or even smaller although she could deter a large entrant. The reason is that a small entrant cannot be deterred (for f2<9/64). But from the diagram below it follows that the locations which would deter a large entrant (i.e i=1/2 in the case of n=144) and allow entry of a small entrant, yield a smaller profit for the first entrant than the case with two large firms.
Above I have determined the solution for the case where entry of a third large entrant is blocked. But from n=200 onwards this is no longer true as we know from the benchmark case. I calculate the location for which the first entrant is indifferent between a large and a small rival. I use the function deter defined above. Takes very long! Therefore I state the result explicitly below.

\[ \text{Solve}\left[ \text{profits2Firms}[\text{deter}[f2],1,0][[1]]-\text{profits2Firms}[i,1,1/8][[1]]=0,i \right] \]

■ The result of the above problem is the next expression. Very long! Therefore not printed!

■ Carrying on.

\[ \text{Length}[\%] \]
\[ 4 \]
\[ \%% / . \; f2 \rightarrow 0.10000000000000000000000001 \]
\[ \{ \{ i \rightarrow 4.293370647968 + 0. \times 10^{-18} \}, \{ i \rightarrow -0.993731354820 + 0. \times 10^{-18} \}, \{ i \rightarrow 0.28918488364 + 0. \times 10^{-18} \}, \{ i \rightarrow 0.99791711915 + 0. \times 10^{-18} \} \} \]

\[ i\text{IndifferentLargeAndSmallRivalWhenEntryNotBlockaded}[f2_] = i /. \%%[[3]]; \]
\[ i\text{IndifferentLargeAndSmallRivalWhenEntryNotBlockaded}[1/8.00000000000000000000000001] \]
\[ 0.202373 + 0. \times 10^{-16} I \]
\[ i\text{IndifferentLargeAndSmallRivalWhenEntryNotBlockaded}[25/200]; \]
\[ N[\%] \]
\[ 0.202373 - 5.30139 \times 10^{-10} I \]
\[ N[\%%,30] \]
\[ 0.20237326667389333690759260275 + 0. \times 10^{-28} I \]

Note that the function deter now depends on \( n \) rather than on \( f2 \), because it is a large entrant who must be deterred.

\[ (* \text{nOfIndifferentLargeAndSmallRivalWhenEntryNotBlockaded}[f2_] := \] \[ \text{nOfIndifferentLargeAndSmallRivalWhenEntryNotBlockaded}[f2] = \] \[ \text{n/.FindRoot}[\text{iDeterLargeEntrant}[f2] = \] \[ \text{iIndifferentLargeAndSmallRivalWhenEntryNotBlockaded}[25/n],\{n,200\}] *) \]

\[ \text{Clear}[\text{nOfIndifferentLargeAndSmallRivalWhenEntryNotBlockaded}] \]
\[ \text{nOfIndifferentLargeAndSmallRivalWhenEntryNotBlockaded}[f2_] := \] \[ \text{nOfIndifferentLargeAndSmallRivalWhenEntryNotBlockaded}[f2] = \] \[ \text{n/.InterpolateRoot}[\text{iDeterLargeEntrant}[f2] = \] \[ \text{iIndifferentLargeAndSmallRivalWhenEntryNotBlockaded}[25/n],\] \[ \{n,150,150.1\},\text{AccuracyGoal} \rightarrow 16,\text{WorkingPrecision} \rightarrow 26] \]

The following expression does not converge for the above values. It converges for starting values smaller than 150.

\[ \text{Timing}[\text{nOfIndifferentLargeAndSmallRivalWhenEntryNotBlockaded}[1/8]] \]
\[ \{17.63 \text{Second}, 152.38880472112699509435148292 + 0. \times 10^{-28} I \} \]
Timing[nOfIndifferentLargeAndSmallRivalWhenEntryNotBlockaded[0.061]]
{27.52 Second, 257.057458630351595193+ 0. \times 10^{-46} I}

FindRoot[nOfIndifferentLargeAndSmallRival[f2] == 200, {f2, 0.07}]
{f2 \to 0.0872324}

f2OfEntryOfLargeEntrantNotBlockaded2LargeFirms = f2 /. %
0.0872324

nOfIndifferentLargeAndSmallRival[0.0872323711193232931]
200.

iIndifferentLargeAndSmallRivalWhenEntryNotBlockaded[25/200.0000125000000012]
0.202373+ 0. \times 10^{-16} I

Plot[Chop[iIndifferentLargeAndSmallRivalWhenEntryNotBlockaded[25.000000000000000000/n]], \{n, 200, 500\}]
Plot::plnr : Chop[iIndifferentLargeAndSmallRivalWhenEntryNotBlockaded\[\frac{\text{25.000000000000000000}}{n}\]]
is not a machine-size real number at \(n = 200.0000125000000012\).
Plot::plnr : Chop[iIndifferentLargeAndSmallRivalWhenEntryNotBlockaded\[\frac{\text{25.000000000000000000}}{n}\]]
is not a machine-size real number at \(n = 212.170097471874763\).
Plot::plnr : Chop[iIndifferentLargeAndSmallRivalWhenEntryNotBlockaded\[\frac{\text{25.000000000000000000}}{n}\]]
is not a machine-size real number at \(n = 225.442639957812129\).

General::stop : Further output of Plot::plnr will be suppressed during this calculation.

\[ - \text{Graphics} - \]

N[iIndifferentLargeAndSmallRivalWhenEntryNotBlockaded[25/200], 20]
0.2023732666738933699+ 0. \times 10^{-63} I
Now I determine the location which deter a third small entrant locating between to large incumbents. The third entrant will locate according to \( x_{2optSym} \)

\[
\text{Solve}
\begin{align*}
\text{redProfit3} & / . \{x[1] \to i, x[2] \to x_{2optSym}[i], x[3] \to 1, c[1] \to 0, c[2] \to 1/8, \ c[3] \to 0, n \to 1, f[2] \to 0\}[[1]] - f2 == 0 , i\};
\end{align*}
\]
\[
% /. f2 \to 0.035
\]
\[
\{\{i \to 0.325588\}, \{i \to 0.942309\}, \{i \to 3.36605 - 0.909549 i\}, \{i \to 3.36605 + 0.909549 i\}\}
\]

\[\text{deter3rdSmallEntrant2LargeFirms[f2_]} = i /. \]
\[
\text{Solve[}
\left\{\text{redProfit3} / . \{x[1] \to i, x[2] \to x_{2optSym}[i], x[3] \to 1, c[1] \to 0, c[2] \to 1/8, c[3] \to 0, n \to 1, f[2] \to 0\}\right\}[[1]] - f2 == 0 , i[[1]] \right\} \]
\texttt{figLoc1 = Plot[deter3rdSmallEntrant2LargeFirms[f2],}
\texttt{\{f2, 0.03, 0.09\}, PlotStyle -> Dashing[{0.01, 0.02}]]}

- Graphics -

\texttt{figLoc2 = Plot[deter[25/nOfIndifferentLargeAndSmallRivalWhenEntryNotBlockaded[f2]],}
\texttt{\{f2, 0.04, 0.09\}]]

- Graphics -
The diagram shows that it is always the condition for deterring the small entrant rather than the condition for the large third entrant which is binding (for $f_2 < 0.075$) in the case of two large firms. Entry of a small entrant might be possible for small values of $f_2$, while entry of a large entrant should be blockaded.

The next profit function shows that entry of a small entrant is blockaded when the two large firms must start to deter the third large entrant, that is at $f_2 = 0.0872323711193232931$ and $n = 200$.

```
-0.0853631127535475

FindRoot[nofIndifferentLargeAndSmallRivalWhenEntryNotBlockaded[f2];
deter[25/ nofIndifferentLargeAndSmallRivalWhenEntryNotBlockaded[f2]] ==
deter3rdSmallEntrant2LargeFirms[f2], {f2, 0.075, 0.077}]
{f2 -> 0.0764745}
```

```
f2Small3rdEntrantNotBlockaded2LargeFirms = f2 /. %
0.0764745
```

```
iIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantNotBlockadedQ[f2_] := i /. NSolve[{(profits2Firms[deter3rdSmallEntrant2LargeFirms[f2], 1, 0][[1]] -
profits2Firms[i, 1, 1/8][[1]]) /. (n -> 1, f[1] -> 0)) == 0, i]`
iIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantNotBlockadedQ0.05
{-4.26439, -1.09142, 0.358021, 0.997789}
```

```
iIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantNotBlockadedQ[f2_] :=
i /. NSolve[{(profits2Firms[deter3rdSmallEntrant2LargeFirms[f2], 1, 0][[1]] -
profits2Firms[i, 1, 1/8][[1]]) /. (n -> 1, f[1] -> 0)) == 0, i][[3]]
```
Note that \( \text{IndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantNotBlockaded} \) is the location in the 1 L, 1 S- firm case.

\[
\text{nOfIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantNotBlockaded}(f_2) = \text{nOfIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantNotBlockaded}[f_2] = \text{n} / \text{InterpolateRoot[iDeterLargeEntrant[f2] == iIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantNotBlockaded[f2]}, \{\text{n, 190, 190.1}, \text{AccuracyGoal} \to 16}\]

\[
\text{nOfIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantNotBlockaded}[0.07]
\]

234.0329822827364734029

The one condition which is important is \( \text{nOfLargeEntry} \). It states that the first entrant locates in a way so that the second entrant prefers the s-Tech over the l-tech. The second is the deterrence of the third entrant. I try the case where the first entrant locates such that the second entrant uses the L-technology. The firms must also locate in a way that both large and small scale entrants are deterred.

From the benchmark case, we know how two firms must locate in order to deter a third entrant. The trade-off for the first entrant is: 2 large firms, deterring the 3rd large entrant. (Is the entry of a small firm blockaded at the respective locations?).

I large, one small firm located such that the small firm actually prefers the s-Tech over the L-tech and such that a third small entrant at 0 is deterred.

The slope of the respective condition in the \((f_2, n)\) space should be positive!

Other conditions should not be binding. See the plots.

The next constraint which becomes binding is entry deterrence of a small entrant at the left edge in the case of a large and a small incumbent. The location of the potential entrant would be \( x_{\text{OptAt0Asy}} \)

\[
\text{deter3rdSmallEntrantAt0}[f_2] := i / \text{FindRoot}[\{\text{redProfit3}[1] /. \{x[1] \to x_{\text{OptAt0Asy}}[i, 1], x[2] \to i, x[3] \to 1, c[1] \to 1/8, c[2] \to 0, c[3] \to 1/8, n \to 1, f[1] \to 0\}\}, \{i, .5\}, \text{AccuracyGoal} \to 25, \text{WorkingPrecision} \to 35]
\]

\text{deter3rdSmallEntrantAt0}[25/ 576.]

FindRoot::precw : The precision of the argument function \( \{(\text{redProfit3}[1] /. \{x[1] \to x_{\text{OptAt0Asy}}[i, 1], x[2] \to i, x[3] \to 1, c[1] \to 1/8, c[2] \to 0, c[3] \to 1/8, n \to 1, f[1] \to 0\}\}, 11 = 0.0434028 - 0 \) is less than WorkingPrecision (35).

0.49701378768862786979021656590804905
figLoc3 = Plot[Det3rdSmallEntrantAt0[f2], {f2, 0.035, 0.04}]

FindRoot::precw : The precision of the argument function {
{redProfit3[1] /. \{x[1] \to xOptAt0Asy[i, 1], x[2] \to i, x[3] \to 1, c[1] \to \frac{1}{8}, c[2] \to 0,
   c[3] \to \frac{1}{8}, n \to 1, f[1] \to 0\}}[1] - 0.035 - 0) is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function {
{redProfit3[1] /. \{x[1] \to xOptAt0Asy[i, 1], x[2] \to i, x[3] \to 1, c[1] \to \frac{1}{8}, c[2] \to 0, c[3] \to \frac{1}{8},
   n \to 1, f[1] \to 0\}}[1] - 0.0352028 - 0) is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function {
{redProfit3[1] /. \{x[1] \to xOptAt0Asy[i, 1], x[2] \to i, x[3] \to 1, c[1] \to \frac{1}{8}, c[2] \to 0, c[3] \to \frac{1}{8},
   n \to 1, f[1] \to 0\}}[1] - 0.035424 - 0) is less than WorkingPrecision (35).

General::stop : Further output of FindRoot::precw will be suppressed during this calculation.

- Graphics -

figLoc4 =
Plot[iIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantNotBlockaded[f2],
{f2, 0.035, 0.04}]

- Graphics -
Show[%, %%]

- Graphics -

FindRoot[xOptAt0Asy[deter3rdSmallEntrantAt0[f2], 1] == 0, {f2, 0.025, 0.05}]

FindRoot::precw : The precision of the argument function \((\text{redProfit3}[1]/\text{ll}[[1]]-0.025)-0\) is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function \((\text{redProfit3}[1]/\text{ll}[[1]]-0.05)-0\) is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function \((\text{redProfit3}[1]/\text{ll}[[1]]-0.0364175)-0\) is less than WorkingPrecision (35).

General::stop: Further output of FindRoot::precw will be suppressed during this calculation.

\(f2 \rightarrow 0.0352555\)

The nonnegativity constraint for the location is not binding in the range where the function applies as we will see below.

FindRoot[iIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantNotBlockaded[f2] ==
deter3rdSmallEntrantAt0[f2], {f2, 0.03, .035}]

FindRoot::precw : The precision of the argument function \((\text{redProfit3}[1]/\text{ll}[[1]]-0.03)-0\) is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function \((\text{redProfit3}[1]/\text{ll}[[1]]-0.035)-0\) is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function \((\text{redProfit3}[1]/\text{ll}[[1]]-0.03)-0\) is less than WorkingPrecision (35).

General::stop: Further output of FindRoot::precw will be suppressed during this calculation.

\(f2 \rightarrow 0.0361273\)

\(f2\text{Small3rdEntrantAt0NotBlockaded1Large1SmallFirm} = 0.0361273283579467374\)

\(0.0361273283579467374\)

xOptAt0Asy[
deter3rdSmallEntrantAt0[f2Small3rdEntrantAt0NotBlockaded1Large1SmallFirm], 1]

FindRoot::precw : The precision of the argument function \((\text{redProfit3}[1]/\text{ll}[[1]]-0.0361273283579467374)-0\) is less than WorkingPrecision (35).

\(0.004811329935294184465539243547245\)
This graph should show that the iDerLargeEntrant condition is slack for \( f_2 < f_2 \text{Small3rdEntrantAt0NotBlockaded1Large1SmallFirm} \) under the assumption the \( n \) is smaller than the value of \( n \) for \( f_2 \text{Small3rdEntrantAt0NotBlockaded1Large1SmallFirm} \). This assumption would, however, hold only if the condition for the first entrant (\( \text{iIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantNotBlockaded} \)) would be binding. This seems not to be the case, see below. The solution seems to be the case where \( n \) must still increase. Thus the both conditions could coincide.

The condition \( \text{iIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantNotBlockaded} \) is not binding in this case. The first entrant prefers to have a small rival and to deter a third entrant. \( n_{\text{Small3rdEntrantNotBlockaded1Large1SmallFirm}} \) gives the maximum value of \( n \) such that both aims can be reached at the same time.

The solution is the following:

\[
n_{\text{Small3rdEntrantNotBlockaded1Large1SmallFirm}[f_2]} := n /. \text{Solve}[\text{deter3rdSmallEntrantAt0[f2]} == \text{iDeterLargeEntrant[f2]}, n][[1]]
\]

\[
n_{\text{Small3rdEntrantNotBlockaded1Large1SmallFirm}[0.035]}
\]

FindRoot::precw : The precision of the argument function 
\((\text{redProfit3[1]}/.\ll1\gg)[1] - 0.035\) - 0 is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function 
\((\text{redProfit3[1]}/.\ll1\gg)[1] - 0.0352028\) - 0 is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function 
\((\text{redProfit3[1]}/.\ll1\gg)[1] - 0.035424\) - 0 is less than WorkingPrecision (35).

General::stop : Further output of FindRoot::precw will be suppressed during this calculation.

\[
\begin{align*}
\text{Plot}[ & \{(\text{profits2Firms[deter3rdSmallEntrant2LargeFirms[f2], 1, 0]}[[1]] - \\
& \text{profits2Firms[deter3rdSmallEntrantAt0[f2], 1, 1/8]}[[1]]) / \\
& \{f[1] -> 0, n -> 300\}, \{f2, 0.035, 0.04\}\}
\end{align*}
\]

The condition \( \text{iIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantNotBlockaded} \) is not binding in this case. The first entrant prefers to have a small rival and to deter a third entrant. \( n_{\text{Small3rdEntrantNotBlockaded1Large1SmallFirm}} \) gives the maximum value of \( n \) such that both aims can be reached at the same time.

The solution is the following:

\[
n_{\text{Small3rdEntrantNotBlockaded1Large1SmallFirm}[f_2]} := n /. \text{Solve}[\text{deter3rdSmallEntrantAt0[f2]} == \text{iDeterLargeEntrant[f2]}, n][[1]]
\]

\[
n_{\text{Small3rdEntrantNotBlockaded1Large1SmallFirm}[0.035]}
\]

FindRoot::precw : The precision of the argument function \((\ll1\gg) - 0\) is less than WorkingPrecision (35).

361.467
The plot can be found in the Plots section. The next condition to take into account is the deterrence of a small entrant in the center in the 1 large firm, 1 small firm case. It becomes binding for \( f_2 < f_2^{\text{SecondEntrantMustMoveToDeter3rdEntrant}} \). I calculated this value above when deriving the locus between the case of 1L, 1S firm and the 2 small firms case. I can use the derivations from that section.

\[
f_2^{\text{SecondEntrantMustMoveToDeter3rdEntrant}} = 0.036001292934144601140
\]

This condition is then used in the determination of the number of firms when one large firm moves. The next condition to take into account is the deterrence of a small entrant in the center in the 1 large firm, 1 small firm case. It becomes binding for \( f_2 < f_2^{\text{SecondEntrantMustMoveToDeter3rdEntrant}} \). I calculated this value above when deriving the locus between the case of 1L, 1S firm and the 2 small firms case. I can use the derivations from that section.

\[
deter3rdSmallEntrantAt0[f_2^{\text{SecondEntrantMustMoveToDeter3rdEntrant}}] = 0.438825
\]

This condition is then used in the determination of the number of firms when one large firm moves.
The small difference is due to the numeric approach. Now I check which constraint is binding next.
Plot[(profits2Firms[deter3rdSmallEntrant2LargeFirms[f2], 1, 0][[1]] -
     profits2Firms[numDeterLeftAsy[f2], numDeterCenterAsy[numDeterLeftAsy[f2], f2],
     1/8][[1]]) / . {f[1] -> 0, n -> 350, {f2, 0.04, 0.02}}

- Graphics -

FindRoot[
   (profits2Firms[deter3rdSmallEntrant2LargeFirms[f2], 1, 0][[1]] - profits2Firms[
     numDeterLeftAsy[f2], numDeterCenterAsy[numDeterLeftAsy[f2], f2], 1/8][[1]]) / .
   {f[1] -> 0, n -> 1}, {f2, 0.04, 0.02}]

{f2 -> 0.0358684}

f2OfIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantsAt0AndAtCenterNotBlockaded
  = f2 /. %

0.0358684

numDeterLeftAsy[
  f2OfIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantsAt0AndAtCenterNotBlockaded]

0.43842
nOf1Large1SmallFirmWhen2ndEntrantMustMove[
  f2OfIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantsAt0AndAtCenterNotBlockaded]

FindRoot::frnum : Function (-25. + 0.0794719 n) is not a length 1 list of numbers at i = 0.4.

ReplaceAll::reps : 
(FindRoot[<<1>>, {i, 0.4, 0.41}]) is neither a list of replacement rules nor
  a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum : Function (-25. + 0.0794719 n) is not a length 1 list of numbers at i = 0.4.

ReplaceAll::reps : 
(FindRoot[<<1>>, {i, 0.4, 0.41}]) is neither a list of replacement rules nor
  a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum : Function (-25. + 0.0794719 n) is not a length 1 list of numbers at i = 0.4.

General::stop : Further output of FindRoot::frnum will be suppressed during this calculation.

ReplaceAll::reps : Further output of FindRoot::frnum will be suppressed during this calculation.

355.98642507920791293

deter3rdSmallEntrant2LargeFirms[
  f2OfIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantsAt0AndAtCenterNotBlockaded]

0.318833

iIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantsAt0AndAtCenterNotBlockaded[
  f2_] :=
  i /. FindRoot[{(profits2Firms[i, 1, 0][[1]] - profits2Firms[numDeterLeftAsy[f2],
      numDeterCenterAsy[numDeterLeftAsy[f2], f2], 1/8][[1]]) /.
    {n -> 1, f[1] -> 0}) == 0, {i, 0.3, 0.4}]

iIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantsAt0AndAtCenterNotBlockaded[
  f2OfIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantsAt0AndAtCenterNotBlockaded]

0.318833
figLoc1 = Plot[deter3rdSmallEntrant2LargeFirms[f2], {f2, 0.02, 0.04}]

- Graphics -

figLoc5 = Plot[
    iIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantsAt0AndAtCenterNotBlockaded[
    f2], {f2, 0.02, 0.04}, PlotStyle -> Dashing[{0.01, 0.02}]]

- Graphics -

Show[%, %%]

- Graphics -


\[
\left\{ \frac{\left( \frac{i-k}{i} - \frac{1}{2} \right) \left( -2 i + i^2 + 2 \left( \frac{1}{i} + \frac{k}{2} \right) - i \left( \frac{1}{i} + \frac{k}{2} \right) - i k + \left( \frac{1}{i} + \frac{k}{2} \right) k \right)^2}{18 \left( 1 - k \right) \left( \frac{1}{i} - \frac{k}{2} \right)^2} - f[2] \right\}
\]
deterCenterBothMove[i_, f2_] = 

{-1 + 1 - 1/2 \sqrt{1 + (48 2^{1/3} (1 + 24 f2)) / (27648 + 995328 f2 + 8957952 f2^2 + 
  \sqrt{165112971264 f2^2 + 7264970735616 f2^3 + 80244904034304 f2^4})^3 / 3) + 
  1 \over 12 2^{1/3} \left( \left( \left( 27648 + 995328 f2 + 8957952 f2^2 + 
  \sqrt{165112971264 f2^2 + 7264970735616 f2^3 + 80244904034304 f2^4} \right)^3 / 3 \right) + 
  4 (-1 + i)^2 - 4 i + 2 i^2 - 3 (1 - 4 i + 2 i^2) \right) - 
  \frac{1}{2} \sqrt{-1 - (48 2^{1/3} (1 + 24 f2)) / (27648 + 995328 f2 + 8957952 f2^2 + 
  \sqrt{165112971264 f2^2 + 7264970735616 f2^3 + 80244904034304 f2^4})^3 / 3) + 
  1 \over 12 2^{1/3} \left( \left( \left( 27648 + 995328 f2 + 8957952 f2^2 + 
  \sqrt{165112971264 f2^2 + 7264970735616 f2^3 + 80244904034304 f2^4} \right)^3 / 3 \right) + 
  4 (-1 + i)^2 - 4 i + 2 i^2 - 3 (1 - 4 i + 2 i^2) \right) - 
  \frac{1}{2} \sqrt{-1 + (48 2^{1/3} (1 + 24 f2)) / (27648 + 995328 f2 + 8957952 f2^2 + 
  \sqrt{165112971264 f2^2 + 7264970735616 f2^3 + 80244904034304 f2^4})^3 / 3) + 
  1 \over 12 2^{1/3} \left( \left( \left( 27648 + 995328 f2 + 8957952 f2^2 + 
  \sqrt{165112971264 f2^2 + 7264970735616 f2^3 + 80244904034304 f2^4} \right)^3 / 3 \right) + 
  4 (-1 + i)^2 - 4 i + 2 i^2 - 3 (1 - 4 i + 2 i^2) \right) - 
  \frac{1}{2} \sqrt{-1 - (48 2^{1/3} (1 + 24 f2)) / (27648 + 995328 f2 + 8957952 f2^2 + 
  \sqrt{165112971264 f2^2 + 7264970735616 f2^3 + 80244904034304 f2^4})^3 / 3) + 
  1 \over 12 2^{1/3} \left( \left( \left( 27648 + 995328 f2 + 8957952 f2^2 + 
  \sqrt{165112971264 f2^2 + 7264970735616 f2^3 + 80244904034304 f2^4} \right)^3 / 3 \right) + 
  4 (-1 + i)^2 - 4 i + 2 i^2 - 3 (1 - 4 i + 2 i^2) \right) - 
  \frac{1}{2} \sqrt{-1 + (48 2^{1/3} (1 + 24 f2)) / (27648 + 995328 f2 + 8957952 f2^2 + 
  \sqrt{165112971264 f2^2 + 7264970735616 f2^3 + 80244904034304 f2^4})^3 / 3) + 
  1 \over 12 2^{1/3} \left( \left( \left( 27648 + 995328 f2 + 8957952 f2^2 + 
  \sqrt{165112971264 f2^2 + 7264970735616 f2^3 + 80244904034304 f2^4} \right)^3 / 3 \right) + 
  4 (-1 + i)^2 - 4 i + 2 i^2 - 3 (1 - 4 i + 2 i^2) \right) - 
  \frac{1}{2} \sqrt{-1 - (48 2^{1/3} (1 + 24 f2)) / (27648 + 995328 f2 + 8957952 f2^2 + 
  \sqrt{165112971264 f2^2 + 7264970735616 f2^3 + 80244904034304 f2^4})^3 / 3) + 
  1 \over 12 2^{1/3} \left( \left( \left( 27648 + 995328 f2 + 8957952 f2^2 + 
  \sqrt{165112971264 f2^2 + 7264970735616 f2^3 + 80244904034304 f2^4} \right)^3 / 3 \right) + 
  4 (-1 + i)^2 - 4 i + 2 i^2 - 3 (1 - 4 i + 2 i^2) \right) -
\[
\frac{1}{2} \sqrt{\left(1 + \left(48 \frac{2^{1/3}}{2} \left(1 + 24 \frac{2}{2}\right)\right) \left(\frac{\sqrt{165112971264 \frac{f^2}{2}} + 7264970735616 \frac{f^3}{2} + 80244904034304 \frac{f^4}{2}\right)^{1/3}\right)} + 
\frac{1}{12 \ 2^{1/3}} \left(\frac{\sqrt{165112971264 \frac{f^2}{2}} + 7264970735616 \frac{f^3}{2} + 80244904034304 \frac{f^4}{2}\right)^{1/3}\right) + 
4 \left(-1 + i\right)^2 - 4 i + 2 i^2 - 3 \left(-1 + 4 i + 2 i^2\right)
\]
deterCenterBothMove[deter3rdSmallEntrant2LargeFirms[
   f2OfIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantsAt0AndAtCenterNotBlocked, 
   f2OfIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantsAt0AndAtCenterNotBlocked] 
] 
{-2.05017 - 0.919656 I, -2.05017 + 0.919656 I, 0.375665, 1.}

deterCenterBothMove[i_, f2_] = 
   -1 + i + \frac{1}{2} \sqrt{1 + \left(48 \frac{2}{3} (1 + 24 f2)\right) \sqrt{16512971264 f2^2 + 7264970735616 f2^3 + 80244904034304 f2^4}}\right)^{(1/3)} + 
   \frac{1}{12 \frac{2}{3} \left(16512971264 f2^2 + 7264970735616 f2^3 + 80244904034304 f2^4\right)^{1/3}} + 
   4 (-1 + 1)^2 - 4 i + 2 i^2 - 3 (1 - 4 i + 2 i^2)\right) + 
   \frac{1}{2} \sqrt{\left[-1 - \left(48 \frac{2}{3} (1 + 24 f2)\right) \sqrt{16512971264 f2^2 + 7264970735616 f2^3 + 80244904034304 f2^4}}\right)^{(1/3)} - 
   \frac{1}{12 \frac{2}{3} \left(16512971264 f2^2 + 7264970735616 f2^3 + 80244904034304 f2^4\right)^{(1/3)}} + 
   8 (-1 + 1)^2 + 4 i - 2 i^2 - 3 (1 - 4 i + 2 i^2) + 
   (64 (-1 + i)^2 + 48 (-1 + i) (1 - 4 i + 2 i^2) + 16 (1 + 36 f2 + 3 i - 6 i^2 + 2 i^3))/\right) 
   \left(4 \sqrt{1 + \left(48 \frac{2}{3} (1 + 24 f2)\right) \sqrt{16512971264 f2^2 + 7264970735616 f2^3 + 80244904034304 f2^4}}\right)^{(1/3)} + 
   \frac{1}{12 \frac{2}{3} \left(16512971264 f2^2 + 7264970735616 f2^3 + 80244904034304 f2^4\right)^{(1/3)}} + 
   4 (-1 + 1)^2 - 4 i + 2 i^2 - 3 (1 - 4 i + 2 i^2)\right)\right)\right)

deterCenterBothMove[.3, .03]
0.934117

deterCenterBothMove[.32, .036]

deterCenterBothMove[.26, .022]
1.00218
0.823602

iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[f2_] := 
iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[f2] = 
i /. FindRoot[{(profits2Firms[i, deterCenterBothMove[i, f2], 0][[1]] - 
   profits2Firms[numDeterLeftAsy[f2], numDeterCenterAsy[numDeterLeftAsy[f2], 
   f2], 1/8][[1]])} /. {n -> 1, f[1] -> 0} == 0, {i, 0.3, 0.4}]}
\[ i \text{IndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival}[.035] \]
0.315245

\[ i \text{IndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival}[ f2 \text{OfIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantsAt0AndAtCenterNotBlockaded} ] \]
0.318833

\[
\text{prof2ndFirmLargeBothMove}[i_-, n_-, f2_] =
\text{profits2Firms}[i, \text{deterCenterBothMove}[i, f2], 0][[2]] / . f[2] \to 0
\]

\[
\frac{1}{2} \sqrt{\left| \left(-4 i + i^2 + 4 \left| -1 + i \right|^2 + \frac{1}{2} \sqrt{\left| 1 + (48 \frac{2^{1/3}}{2} (1 + 24 f2)) \right| \left(27648 + 995328 f2 + 8957952 f2^2 + \sqrt{165112971264 f2^2 + 7264970735616 f2^3 + 80244904034304 f2^4} \right)^{1/3}} \right| - 4 (-1 + i)^2 - 4 i + 2 i^2 - 3 (1 - 4 i + 2 i^2) + (64 (-1 + i)^3 - 48 (-1 + i) (1 - 4 i + 2 i^2) + 8 (-1 + i)^2 - 4 i - 2 i^2 - 3 (1 - 4 i + 2 i^2) + (27648 + 995328 f2 + 8957952 f2^2 + \sqrt{165112971264 f2^2 + 7264970735616 f2^3 + 80244904034304 f2^4} \right)^{1/3}} \right| -
\]
\[
\left(-1 + i + \frac{1}{2} \sqrt{\left(1 + \left(48 \frac{2^{1/3}}{1 + 24 f^2}\right) \right) \left(27648 + 995328 f^2 + 8957952 f^2^2 + \sqrt{165112971264 f^2^2 + 7264970735616 f^2^3 + 80244904034304 f^2^4} \right)} \right) ^{(1/3)} + \frac{1}{12 \ 2^{1/3}} \left(27648 + 995328 f^2 + 8957952 f^2^2 + \sqrt{165112971264 f^2^2 + 7264970735616 f^2^3 + 80244904034304 f^2^4} \right) ^{(1/3)} + 4 \ (-1 + i)^2 - 41 + 2 i^2 - 3 \ (1 - 4 i + 2 i^2) +
\]
\[
\frac{1}{2} \sqrt{\left(-1 - \left(48 \frac{2^{1/3}}{1 + 24 f^2}\right) \right) \left(27648 + 995328 f^2 + 8957952 f^2^2 + \sqrt{165112971264 f^2^2 + 7264970735616 f^2^3 + 80244904034304 f^2^4} \right)} \left(27648 + 995328 f^2 + 8957952 f^2^2 + \sqrt{165112971264 f^2^2 + 7264970735616 f^2^3 + 80244904034304 f^2^4} \right) ^{(1/3)} + 4 \ (-1 + i)^2 - 41 + 2 i^2 - 3 \ (1 - 4 i + 2 i^2) -
\]
\[
\left(-1 - i + \frac{1}{2} \sqrt{\left(1 + \left(48 \frac{2^{1/3}}{1 + 24 f^2}\right) \right) \left(27648 + 995328 f^2 + 8957952 f^2^2 + \sqrt{165112971264 f^2^2 + 7264970735616 f^2^3 + 80244904034304 f^2^4} \right)} \right) ^{(1/3)} + \frac{1}{12 \ 2^{1/3}} \left(27648 + 995328 f^2 + 8957952 f^2^2 + \sqrt{165112971264 f^2^2 + 7264970735616 f^2^3 + 80244904034304 f^2^4} \right) ^{(1/3)} + 4 \ (-1 + i)^2 - 41 + 2 i^2 - 3 \ (1 - 4 i + 2 i^2) +
\]
iDeterLargeEntrantWhen2ndEntrantMustMove1stWantsLargeRival[f2_, n_] :=
  i /. FindRoot[prof2ndFirmLargeBothMove[i, n, f2] -
    25 - (profAsymAsy2ndEntrant[i, f2, n] - f2 n) == 0, {i, .4, .41}]
iDeterLargeEntrantWhen2ndEntrantMustMove1stWantsLargeRival[.03, 250]
  0.316808
Plot[iDeterLargeEntrantWhen2ndEntrantMustMove1stWantsLargeRival[f2, 250],
  {f2, .022, 0.036}]
FindRoot::frsec :
  Secant method failed to converge to the prescribed accuracy after 15 iterations.
  - Graphics -  
Plot[iDeterLargeEntrantWhen2ndEntrantMustMove1stWantsLargeRival[f2, 230],
  {f2, .022, 0.036}]
  - Graphics -
iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival increases much faster with f2 when
iDeterLargeEntrantWhen2ndEntrantMustMove1stWantsLargeRival. Therefore n will increase with increasing f2.

nOf1Large1SmallFirmWhen1stEntrantPrefers2LargeFirmsNew[f2_] :=
nOf1Large1SmallFirmWhen1stEntrantPrefers2LargeFirmsNew[f2] =
n /. InterpolateRoot[iDeterLargeEntrantWhen2ndEntrantMustMove1stWantsLargeRival[
  f2, n] - iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[f2] == 0,
  {n, 170, 171}, MaxIterations -> 40, AccuracyGoal -> 16]
nOf1Large1SmallFirmWhen1stEntrantPrefers2LargeFirmsNew[0.035]

FindRoot::frnum : Function (-25. + 0.0939748 n) is not a length 1 list of numbers at i = 0.4.

ReplaceAll::reps :
(FindRoot[<<1>> == 0, {i, 0.4, 0.41}]) is neither a list of replacement rules
nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum : Function (-25. + 0.0939748 n) is not a length 1 list of numbers at i = 0.4.

ReplaceAll::reps :
(FindRoot[<<1>> == 0, {i, 0.4, 0.41}]) is neither a list of replacement rules
nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum : Function (-25. + 0.0939748 n) is not a length 1 list of numbers at i = 0.4.

General::stop : Further output of FindRoot::frnum will be suppressed during this calculation.

ReplaceAll::reps :
(FindRoot[<<1>> == 0, {i, 0.4, 0.41}]) is neither a list of replacement rules
nor a valid dispatch table, and so cannot be used for replacing.

General::stop : Further output of ReplaceAll::reps will be suppressed during this calculation.

247.42119796017039975

nOf1Large1SmallFirmWhen1stEntrantPrefers2LargeFirmsNew[
f2OfIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantsAt0AndAtCenterNotBlockad\ned]

FindRoot::frnum : Function (-25. + 0.0942317 n) is not a length 1 list of numbers at i = 0.4.

ReplaceAll::reps :
(FindRoot[<<1>>]) is neither a list of replacement rules
nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum : Function (-25. + 0.0942317 n) is not a length 1 list of numbers at i = 0.4.

ReplaceAll::reps :
(FindRoot[<<1>>]) is neither a list of replacement rules
nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum : Function (-25. + 0.0942317 n) is not a length 1 list of numbers at i = 0.4.

General::stop : Further output of FindRoot::frnum will be suppressed during this calculation.

ReplaceAll::reps :
(FindRoot[<<1>>]) is neither a list of replacement rules
nor a valid dispatch table, and so cannot be used for replacing.

General::stop : Further output of ReplaceAll::reps will be suppressed during this calculation.

247.67009192610755389568
The loci in the area with two large firms: 1st entrant deters alone

First: Locus which separates deterrence of small and of large entrant

```math
nOf2LargeFirmsDeterLargeAboveDeterSmall[f2_] :=
  n /. FindRoot[deter3rdSmallEntrant2LargeFirms[f2] == deter[25/n], {n, 200, 210}]
nOf2LargeFirmsDeterLargeAboveDeterSmall[.036]
```

366.769
fig8a = Plot[nOf2LargeFirmsDeterLargeAboveDeterSmall[f2],
{f2, .03, f2Small3rdEntrantNotBlockaded2LargeFirms},
PlotStyle -> Dashing[.001, .005]]

- Graphics -

Second: Locus for which 1st entrant deters alone, above area where she wants large rival

deterCenterBothMove[
  iDeterLargeEntrantWhen2ndEntrantMustMove1stWantsLargeRival[.03, 260], 0.03]
0.994522

deterCenterBothMove[
  iDeterLargeEntrantWhen2ndEntrantMustMove1stWantsLargeRival[.03, 263], 0.03]
1.00718

FindRoot[deterCenterBothMove[
  iDeterLargeEntrantWhen2ndEntrantMustMove1stWantsLargeRival[.03, n], .03] == 1,
{n, 260, 263}, AccuracyGoal -> 12]

FindRoot::frsec :
  Secant method failed to converge to the prescribed accuracy after 15 iterations.
FindRoot::frsec :
  Secant method failed to converge to the prescribed accuracy after 15 iterations.
FindRoot::frsec :
  Secant method failed to converge to the prescribed accuracy after 15 iterations.
General::stop : Further output of FindRoot::frsec will be suppressed during this calculation.
{n -> 98.6663 + 8.03254 × 10^-9 I}

test[f2_] = i /. Solve[deterCenterBothMove[i, f2] == 1, i][[1]];  
test[.03]
0.365883
\text{nOf2LargeFirmsDeterLargeAboveDeterSmallf2Small[f2\_]} := n /. \text{FindRoot[}
\quad \text{test[f2\_] - iDeterLargeEntrantWhen2ndEntrantMustMove1stWantsLargeRival[f2, n] == 0,}
\quad \{n, 280, 281\}, \text{AccuracyGoal \to 12]}
\text{nOf2LargeFirmsDeterLargeAboveDeterSmallf2Small[.03]}
\text{FindRoot::frsec : Secant method failed to converge to the prescribed accuracy after 15 iterations.}
\text{FindRoot::frsec : Secant method failed to converge to the prescribed accuracy after 15 iterations.}
\text{FindRoot::frsec : Secant method failed to converge to the prescribed accuracy after 15 iterations.}
\text{General::stop : Further output of FindRoot::frsec will be suppressed during this calculation.}
98.6663 + 1.0665 \times 10^{-13} \text{I}
\text{nOf2LargeFirmsDeterLargeAboveDeterSmallf2Small[f2\_]} := n /. \text{InterpolateRoot[}
\quad \text{test[f2\_] - iDeterLargeEntrantWhen2ndEntrantMustMove1stWantsLargeRival[f2, n] == 0,}
\quad \{n, 280, 281\}, \text{AccuracyGoal \to 12]}
\text{nOf2LargeFirmsDeterLargeAboveDeterSmallf2Small[.03]}
\text{FindRoot::frnum : Function \{-25. + 0.0927521 n\} is not a length 1 list of numbers at i = 0.4\}.\text{.}
\text{ReplaceAll::reps : \{<<1\>>\} is neither a list of replacement}
\text{\text{\quad rules nor a valid dispatch table, and so cannot be used for replacing.}}
\text{FindRoot::frnum : Function \{-25. + 0.0927521 n\} is not a length 1 list of numbers at i = 0.4\}.\text{.}
\text{ReplaceAll::reps : \{<<1\>>\} is neither a list of replacement}
\text{\text{\quad rules nor a valid dispatch table, and so cannot be used for replacing.}}
\text{FindRoot::frnum : Function \{-25. + 0.0927521 n\} is not a length 1 list of numbers at i = 0.4\}.\text{.}
\text{General::stop : Further output of FindRoot::frnum will be suppressed during this calculation.}
\text{ReplaceAll::reps : \{<<1\>>\} is neither a list of replacement}
\text{\text{\quad rules nor a valid dispatch table, and so cannot be used for replacing.}}
\text{General::stop :}
\text{\quad Further output of ReplaceAll::reps will be suppressed during this calculation.}
261.2929799125260293041
fig9a = Plot[nOf2LargeFirmsDeterLargeAboveDeterSmallf2Small[f2],
   {f2, f2Small3rdEntrantAt0NotBlockaded2LargeFirms, .036},
   PlotStyle -> Dashing[.001, .005]]

FindRoot::frnum: Function (-25. + 0.0922604 n) is not a length 1 list of numbers at i = 0.4.

ReplaceAll::reps:
   (FindRoot[prof2ndFirmLargeBothMove[i, n, 0.0270924] - 25 - (<<1>> - <<1>>) == 0, <<1>>])
   is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum: Function (-25. + 0.0922604 n) is not a length 1 list of numbers at i = 0.4.

ReplaceAll::reps:
   (FindRoot[prof2ndFirmLargeBothMove[i, n, 0.0270924] - 25 - (<<1>> - <<1>>) == 0, <<1>>])
   is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum: Function (-25. + 0.0922604 n) is not a length 1 list of numbers at i = 0.4.

General::stop: Further output of FindRoot::frnum will be suppressed during this calculation.

ReplaceAll::reps:
   (FindRoot[prof2ndFirmLargeBothMove[i, n, 0.0270924] - 25 - (<<1>> - <<1>>) == 0, <<1>>])
   is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

General::stop: Further output of ReplaceAll::reps will be suppressed during this calculation.
- Graphics -

**Old and wrong! But includes derivation of \( f_{2Small3rdEntrantAt0NotBlockaded2LargeFirms} \) and derivation of fig10a**

The following definition is equivalent to \( nOf2LargeFirmsDeterLargeAboveDeterSmallf2Small \). Do not evaluate as this would redefine the newer and right definition made above.

\[
(*) \quad nOf1Large1SmallFirmWhen1stEntrantPrefers2LargeFirmsNew[f2_] := 
nOf1Large1SmallFirmWhen1stEntrantPrefers2LargeFirmsNew[f2] = 
\text{InterpolateRoot}[\text{deterLargeEntrantWhen2ndEntrantMustMove}[f2,n] - 
\text{deter3rdSmallEntrant2LargeFirms}[f2]==0, 
{n,170,171},\text{MaxIterations}\rightarrow40,\text{AccuracyGoal}\rightarrow16]
\]

\[
nOf1Large1SmallFirmWhen1stEntrantPrefers2LargeFirmsNew[0.035]
\]

247.42119796017039975
nOf2LargeFirmsDeterLargeAboveDeterSmallf2Smallf2Small

FindRoot::frnum: Function (-25.+0.0939748 n) is not a length 1 list of numbers at i = 0.4.

ReplaceAll::reps: {FindRoot[<<1>> == 0, {i, 0.4, 0.41}]} is neither a list of replacement rules
nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum: Function (-25.+0.0939748 n) is not a length 1 list of numbers at i = 0.4.

ReplaceAll::reps: {FindRoot[<<1>> == 0, {i, 0.4, 0.41}]} is neither a list of replacement rules
nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum: Function (-25.+0.0939748 n) is not a length 1 list of numbers at i = 0.4.

General::stop: Further output of FindRoot::frnum will be suppressed during this calculation.

ReplaceAll::reps: {FindRoot[<<1>> == 0, {i, 0.4, 0.41}]} is neither a list of replacement rules
nor a valid dispatch table, and so cannot be used for replacing.

General::stop: Further output of ReplaceAll::reps will be suppressed during this calculation.

249.603150561270078053

Table[prof2ndFirmLarge[deter3rdSmallEntrant2LargeFirms[f2],
  nOf1Large1SmallFirmWhen1stEntrantPrefers2LargeFirmsNew[0.035]]
  - 25 -
  (profAsymAsy2ndEntrant[deter3rdSmallEntrant2LargeFirms[f2], f2, 
  nOf1Large1SmallFirmWhen1stEntrantPrefers2LargeFirmsNew[0.035]] -
  f2 nOf1Large1SmallFirmWhen1stEntrantPrefers2LargeFirmsNew[0.035]),
  {f2, 0.0347, 0.0353, 0.0001}]
  {-0.2853, -0.263051, -0.240799, -0.218543, -0.196284, -0.174021, -0.151755}

Check indicates that this definition gives the result. It shows for how long the first entrant can induce the second to choose the large scale technology.

The next question is that concerning the constraint which will become binding next. 2 possibilities:
1. Entry of a third entrant at 0 in the case of 2 large firms. I have calculated that above. The constraint for the small firm becomes binding for f2 < f2Small3rdEntrantAt0NotBlockaded2LargeFirms (0.027). As the values for n are much smaller than 400, no problem exists with respect to a large entrant.
2. Entry of a third entrant cannot be deterred any longer in the case of 1 large and 1 small firm. By construction of numDeterLeftAsy, numDeterCenterAsy entry could happen only at the location 1. As calculated below, this constraint becomes relevant only for f2 < 0.016. This is outside of the range I consider.

From the above argumentation it follows that the next situation to consider is one in which the second entrant is large and must move to the center.

The location of the third entrant is the symmetric one in this case! That is (i+k)/2

xOptAt0AsyIncumbentsLarge[j_, k_] =
The above value is wrong as the nonnegativity constraint is binding => see below

```
xOptAt0AsyIncumbentsLarge[
  iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[f2block4thEntrant],
  deterCenterBothMove[iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[
    f2block4thEntrant], f2block4thEntrant]]
-0.0893983

{x[1] -> xOptAt0AsyIncumbentsLarge[
  iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[f2block4thEntrant],
  deterCenterBothMove[iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[
    f2block4thEntrant], f2block4thEntrant]],
  x[2] -> iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[f2block4thEntrant],
  x[3] -> deterCenterBothMove[
    iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[f2block4thEntrant],
    f2block4thEntrant]]
{x[1] -> -0.0893983, x[2] -> 0.253007, x[3] -> 0.81652}
```

```
(redProfit3[1] /. {x[1] -> xOptAt0AsyIncumbentsLarge[
  iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[f2block4thEntrant],
  deterCenterBothMove[iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[
    f2block4thEntrant], f2block4thEntrant]],
  x[2] -> iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[f2block4thEntrant],
  x[3] -> deterCenterBothMove[
    iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[f2block4thEntrant],
    f2block4thEntrant]],
  x[2] -> iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[f2block4thEntrant],
  x[3] -> deterCenterBothMove[
    iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[f2block4thEntrant],
    f2block4thEntrant]],
-0.0122616
```
Plot[{redProfit3[1] /. \{x[1] -> xOptAt0AsyIncumbentsLarge[
  iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[f2],
  deterCenterBothMove[
    iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[f2], f2],
  x[2] -> iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[f2],
  x[3] -> deterCenterBothMove[
    iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[f2], f2],

- Graphics -

Plot[{redProfit3[1] /. \{x[1] -> 0,
  x[2] -> iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[f2],
  x[3] -> deterCenterBothMove[
    iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[f2], f2],

- Graphics -

(* InterpolateRoot[{redProfit3[1] /. \{x[1] -> xOptAt0AsyIncumbentsLarge[
  iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[f2],
  deterCenterBothMove[
    iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[f2], f2],
  x[2] -> iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[f2],
  x[3] -> deterCenterBothMove[
    iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[f2], f2],
  \{f2, 0.02, 0.04\}, ShowProgress -> True] *)

a
deter3rdSmallEntrant2LargeFirms[f2Small3rdEntrantAt0NotBlockaded2LargeFirms]

0.389961

FindRoot[
   f[1] -> 0})[[1]] - f2 == 0, {f2, 0.02, .04}]

{f2 \[\rightarrow\] 0.0270924}

xOptAt0AsyIncumbentsLarge[
   deter3rdSmallEntrant2LargeFirms[f2Small3rdEntrantAt0NotBlockaded2LargeFirms], 1]

-0.0267635

f2Small3rdEntrantAt0NotBlockaded2LargeFirms = 0.0270924454317045748`

0.0270924

function2AsyLarge[i_, k_ ] :=

numDeterCenterAsyLarge[i_, f2_] := k /.
   FindRoot[{function2AsyLarge[i, k] == f2, {k, 85/100}, WorkingPrecision -> 35}][[1]]

numDeterLeftAsyLarge[f2_] :=
   i /.
   f2, {i, 4/10, 5/10}, WorkingPrecision -> 35][[1]]

xOptAt0AsyIncumbentsLarge[numDeterLeftAsyLarge[.022],
   numDeterCenterAsyLarge[numDeterLeftAsyLarge[.022], .022]]

-0.0289468
Plot[{numDeterLeftAsyLarge[f2],
    numDeterCenterAsyLarge[numDeterLeftAsyLarge[f2], f2]}, {f2, 0.024, 0.027}]


FindRoot::precw: The precision of the argument function (function2AsyLarge[0.4, k] - 0.0240000001250000000433805880106774566) is less than WorkingPrecision (35).

FindRoot::precw: The precision of the argument function (function2AsyLarge[0.5, k] - 0.0240000001250000000433805880106774566) is less than WorkingPrecision (35).

General::stop: Further output of FindRoot::precw will be suppressed during this calculation.

- Graphics -

Plot[xOptAt0AsyIncumbentsLarge[numDeterLeftAsyLarge[f2],
    numDeterCenterAsyLarge[numDeterLeftAsyLarge[f2], f2]], {f2, 0.022, 0.027}]

- Graphics -

The nonnegativity constraint for the location is always binding!

prof2ndFirmLargeEntryAt0AndAtCenterNotBlockaded[i___, n1___, f2___] :=
profits2Firms[i, numDeterCenterAsyLarge[i, f2], 0][[2]] /. {f[2] -> 0, n -> n1}

iOfDeterLargeEntrantWhenEntryOfSmallEntrantsAt0AndAtCenterNotBlockadedInAllCases[ f2___, n___] := i /. FindRoot[prof2ndFirmLargeEntryAt0AndAtCenterNotBlockadedInAllCases[ i, n, f2] -
25 - (profAsymAsy2ndEntrant[i, f2, n] - f2 n) == 0, {i, 33/100, 35/100}]
nOfEntryOfSmallEntrantsAt0AndAtCenterNotBlockadedInAllCases[f2_] :=
nOfEntryOfSmallEntrantsAt0AndAtCenterNotBlockadedInAllCases[f2] =
  n /. InterpolateRoot[
    iOfDeterLargeEntrantWhenEntryOfSmallEntrantsAt0AndAtCenterNotBlockadedInAllCases[
      f2, n] - numDeterLeftAsyLarge[f2] == 0, {n, 270, 280},
    MaxIterations -> 40, AccuracyGoal -> 16, WorkingPrecision -> 22]

nOfEntryOfSmallEntrantsAt0AndAtCenterNotBlockadedInAllCasesTest[f2_] :=
nOfEntryOfSmallEntrantsAt0AndAtCenterNotBlockadedInAllCases[f2] =
  n /. InterpolateRoot[iOfDeterLargeEntrantWhen2ndEntrantMustMove1stWantsLargeRival[
    f2, n] - numDeterLeftAsyLarge[f2] == 0, {n, 270, 280},
    MaxIterations -> 40, AccuracyGoal -> 16, WorkingPrecision -> 22]

nOfEntryOfSmallEntrantsAt0AndAtCenterNotBlockadedInAllCases[f2Small3rdEntrantAt0NotBlockaded2LargeFirms] :=
FindRoot::precw: The precision of the argument function (function2AsyLarge[0.33, k] - 0.0270924) is less than WorkingPrecision (35).

FindRoot::precw: The precision of the argument function (function2AsyLarge[0.33, k] - 0.0270924) is less than WorkingPrecision (35).

FindRoot::frnum: Function 8 - 25. + 0.0985002 n < is not a length 1 list of numbers at i = 0.330000000000000026.

ReplaceAll::reps: FindRoot[Ai1j, 9i, 33cccc cccc cc100, 35cccc cccc cc100] is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.


General::stop: Further output of FindRoot::precw will be suppressed during this calculation.

FindRoot::frnum: Function 8 - 25. + 0.0985002 n is not a length 1 list of numbers at i = 0.330000000000000026.

ReplaceAll::reps: [FindRoot[<<1>>, {i, 33/100, 35/100}]] is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum: Function 8 - 25. + 0.0985002 n is not a length 1 list of numbers at i = 0.330000000000000026.

General::stop: Further output of FindRoot::frnum will be suppressed during this calculation.

ReplaceAll::reps: [FindRoot[<<1>>, {i, 33/100, 35/100}]] is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

General::stop: Further output of ReplaceAll::reps will be suppressed during this calculation.

268.56640111864983132367
nOfEntryOfSmallEntrantsAt0AndAtCenterNotBlockadedInAllCasesTest[f2Small3rdEntrantAt0NotBlockaded2LargeFirms]

FindRoot::frnum: Function \(-25. \times 0.0922604 n\) is not a length 1 list of numbers at \(i = 0.4\).

ReplaceAll::reps:

\[\text{FindRoot}[\{0.4, 0.41\}]\] is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::precw: The precision of the argument function \{redProfit3[1] / (x[1] \rightarrow 0, x[2] \rightarrow i, \{i, n \rightarrow 1\})[1] - 0.0270924\} is less than WorkingPrecision (35).

FindRoot::precw: The precision of the argument function \{function2AsyLarge[0.4, k] - 0.0270924543170457458769462277814454\} is less than WorkingPrecision (35).

FindRoot::precw: The precision of the argument function \{function2AsyLarge[0.5, k] - 0.0270924543170457458769462277814454\} is less than WorkingPrecision (35).

General::stop: Further output of FindRoot::precw will be suppressed during this calculation.

FindRoot::frnum: Function \(-25. \times 0.0922604 n\) is not a length 1 list of numbers at \(i = 0.4\).

ReplaceAll::reps:

\[\text{FindRoot}[\{0.4, 0.41\}]\] is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum: Function \(-25. \times 0.0922604 n\) is not a length 1 list of numbers at \(i = 0.4\).

General::stop: Further output of FindRoot::frnum will be suppressed during this calculation.

ReplaceAll::reps:

\[\text{FindRoot}[\{0.4, 0.41\}]\] is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

General::stop: Further output of ReplaceAll::reps will be suppressed during this calculation.

268.56640051078474479490
nOfEntryOfSmallEntrantsAt0AndAtCenterNotBlockadedInAllCases[f2block4thEntrant]

FindRoot::precw : The precision of the argument function 
  function2AsyLarge[0.33, k] - 0.0219904 is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function 
  function2AsyLarge[0.33, k] - 0.0219904 is less than WorkingPrecision (35).

FindRoot::frnum : 
  Function (-25. + 0.0984255 n) is not a length 1 list of numbers at i = 0.33000000000000026`.

ReplaceAll::reps : 
  FindRoot[<<1>>, {i, 33/100, 35/100}] is neither a list of replacement rules nor 
  a valid dispatch table, and so cannot be used for replacing.

FindRoot::precw : The precision of the argument function 
  is less than WorkingPrecision (35).

General::stop : Further output of FindRoot::precw will be suppressed during this calculation.

FindRoot::frnum : 
  Function (-25. + 0.0984255 n) is not a length 1 list of numbers at i = 0.33000000000000026`.

ReplaceAll::reps : 
  FindRoot[<<1>>, {i, 33/100, 35/100}] is neither a list of replacement rules nor 
  a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum : 
  Function (-25. + 0.0984255 n) is not a length 1 list of numbers at i = 0.33000000000000026`.

General::stop : Further output of FindRoot::frnum will be suppressed during this calculation.

ReplaceAll::reps : 
  FindRoot[<<1>>, {i, 33/100, 35/100}] is neither a list of replacement rules nor 
  a valid dispatch table, and so cannot be used for replacing.

General::stop : 
  Further output of ReplaceAll::reps will be suppressed during this calculation.

264.04482194256149840445
nOfEntryOfSmallEntrantsAt0AndAtCenterNotBlockadedInAllCasesTest[f2block4thEntrant]

FindRoot::frnum : Function \((-25. + 0.0918662 n)\) is not a length 1 list of numbers at \(i = 0.4\).

ReplaceAll::reps : ([FindRoot[<<1>> == 0, \{i, 0.4, 0.41\}]}) is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::precw : The precision of the argument function \((8 \rightarrow f2block4thEntrant)\) is less than WorkingPrecision \((35)\).

FindRoot::precw : The precision of the argument function \((8 \rightarrow f2block4thEntrant)\) is less than WorkingPrecision \((35)\).

General::stop : Further output of FindRoot::precw will be suppressed during this calculation.

FindRoot::frnum : Function \((-25. + 0.0918662 n)\) is not a length 1 list of numbers at \(i = 0.4\).

ReplaceAll::reps : ([FindRoot[<<1>> == 0, \{i, 0.4, 0.41\}]}) is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum : Function \((-25. + 0.0918662 n)\) is not a length 1 list of numbers at \(i = 0.4\).

General::stop : Further output of FindRoot::frnum will be suppressed during this calculation.

ReplaceAll::reps : ([FindRoot[<<1>> == 0, \{i, 0.4, 0.41\}]}) is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum : Function \((-25. + 0.0918662 n)\) is not a length 1 list of numbers at \(i = 0.4\).

General::stop : Further output of FindRoot::frnum will be suppressed during this calculation.

264.04482182625200947

nOf1Large1SmallFirmWhen1stEntrantPrefers2LargeFirmsNew[f2Small3rdEntrantAt0NotBlockaded2LargeFirms]

FindRoot::frnum : Function \((-25. + 0.0903718 n)\) is not a length 1 list of numbers at \(i = 0.4\).

ReplaceAll::reps : ([FindRoot[<<1>> == 0, \{i, 0.4, 0.41\}]}) is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum : Function \((-25. + 0.0903718 n)\) is not a length 1 list of numbers at \(i = 0.4\).

ReplaceAll::reps : ([FindRoot[<<1>> == 0, \{i, 0.4, 0.41\}]}) is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum : Function \((-25. + 0.0903718 n)\) is not a length 1 list of numbers at \(i = 0.4\).

General::stop : Further output of FindRoot::frnum will be suppressed during this calculation.

ReplaceAll::reps : ([FindRoot[<<1>> == 0, \{i, 0.4, 0.41\}]}) is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

General::stop : Further output of ReplaceAll::reps will be suppressed during this calculation.

264.04482182625200947
n0f2LargeFirmsDeterLargeAboveDeterSmallf2Small[
f2Small3rdEntrantAt0NotBlockaded2LargeFirms]

FindRoot::frnum: Function -25. + 0.0922604n
\[n\] is not a length 1 list of numbers at i = 0.4.`.

ReplaceAll::reps :
(FindRoot[<<1\>> == 0, {i, 0.4, 0.41}]) is neither a list of replacement rules
nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum: Function -25. + 0.0922604n
\[n\] is not a length 1 list of numbers at i = 0.4.`.

ReplaceAll::reps :
(FindRoot[<<1\>> == 0, {i, 0.4, 0.41}]) is neither a list of replacement rules
nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum: Function -25. + 0.0922604n
\[n\] is not a length 1 list of numbers at i = 0.4.`.

General::stop: Further output of FindRoot::frnum will be suppressed during this calculation.

ReplaceAll::reps :
(FindRoot[<<1\>> == 0, {i, 0.4, 0.41}]) is neither a list of replacement rules
nor a valid dispatch table, and so cannot be used for replacing.

General::stop: Further output of ReplaceAll::reps will be suppressed during this calculation.

268.566400510781849797
fig10a = Plot[nOfEntryOfSmallEntrantsAt0AndAtCenterNotBlockadedInAllCases[f2],
{f2, f2block4thEntrant, f2Small3rdEntrantAt0NotBlockaded2LargeFirms},
PlotStyle -> Dashing[{{.001, .005}}]]

FindRoot::precw : The precision of the argument function (function2AsyLarge[0.33, k] - 0.0219904) is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function (function2AsyLarge[0.33, k] - 0.0219904) is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function (function2AsyLarge[0.33, k] - 0.0219904) is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function (function2AsyLarge[0.33, k] - 0.0219904) is less than WorkingPrecision (35).

FindRoot::frsec : Secant method failed to converge to the prescribed accuracy after 15 iterations.
numDeterLeftAsyLarge[f2Small13rdEntrantAt0NotBlockaded2LargeFirms]

FindRoot::precw : The precision of the argument function (redProfit3[1] /. \(x[1] \to 0, x[2] \to i, \ll i \gg, f[1] \to 0, n \to 1\)) \[1 - 0.0270924\] is less than WorkingPrecision \(35\).

FindRoot::precw : The precision of the argument function (function2AsyLarge[0.4, k] - 0.0270924543170457458769462277814454) is less than WorkingPrecision \(35\).

FindRoot::precw : The precision of the argument function (function2AsyLarge[0.5, k] - 0.0270924543170457458769462277814454) is less than WorkingPrecision \(35\).

General::stop : Further output of FindRoot::precw will be suppressed during this calculation.

0.39054815442704904890324207328468367

iDeterLargeEntrantWhen2ndEntrantMustMove1stWantsLargeRival[
f2Small13rdEntrantAt0NotBlockaded2LargeFirms,
nOfEntryOfSmallEntrantsAt0AndAtCenterNotBlockadedInAllCases[
f2Small13rdEntrantAt0NotBlockaded2LargeFirms]]

0.390548
Plot[iDetLargeEntrantWhen2ndEntrantMustMove1stWantsLargeRival[.025, n],
{n, nOf1Large1SmallFirmWhen1stEntrantPrefers2LargeFirmsNew[.025],
nOfEntryOfSmallEntrantsAt0AndAtCenterNotBlockadedInAllCases[.025]]]

FindRoot::precw : The precision of the argument function 
    function2AsyLarge[0.33, k] - 0.025 is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function 
    function2AsyLarge[0.33, k] - 0.025 is less than WorkingPrecision (35).

FindRoot::frnum : Function (-25. + 0.983878 n) is not a length 1 list of numbers at i = 0.330000000000000026.`.

ReplaceAll::reps : (FindRoot[<<1>>]) is neither a list of replacement
    rules nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::precw :
    The precision of the argument function (<<1>>) is less than WorkingPrecision (35).

General::stop : Further output of FindRoot::precw will be suppressed during this calculation.

FindRoot::frnum :
    Function (-25. + 0.983878 n) is not a length 1 list of numbers at i = 0.330000000000000026.`.

ReplaceAll::reps : (FindRoot[<<1>>]) is neither a list of replacement
    rules nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum :
    Function (-25. + 0.983878 n) is not a length 1 list of numbers at i = 0.330000000000000026.`.

General::stop : Further output of FindRoot::frnum will be suppressed during this calculation.

ReplaceAll::reps : (FindRoot[<<1>>]) is neither a list of replacement
    rules nor a valid dispatch table, and so cannot be used for replacing.

General::stop :
    Further output of ReplaceAll::reps will be suppressed during this calculation.
numDeterLeftAsyLarge[.025]

FindRoot::precw : The precision of the argument function \((\ll 1)\) is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function \(\{\text{function2AsyLarge}[0.4, k] - 0.025\}\) is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function \(\{\text{function2AsyLarge}[0.5, k] - 0.025\}\) is less than WorkingPrecision (35).

General::stop : Further output of FindRoot::precw will be suppressed during this calculation.

0.38226998513497566022681450787986056

\[\text{iDeterLargeEntrantWhen2ndEntrantMustMove1stWantsLargeRival}[.025, \text{nOfEntryOfSmallEntrantsAt0AndAtCenterNotBlockadedInAllCases}[.025]]\]

0.38227

- Plots -

9 / 64.

0.140625

\[\text{f2OfEntryOfLargeEntrantNotBlockaded2LargeFirms}\]

0.0872324

\[\text{fig1b} = \text{Plot}[\text{nOfIndifferentLargeAndSmallRival}[f2], \{f2, f2OfEntryOfLargeEntrantNotBlockaded2LargeFirms, 9 / 64]\]
fig2b = Plot[nOfIndifferentLargeAndSmallRivalWhenEntryNotBlockaded[f2],
{f2, f2Small3rdEntrantNotBlockaded2LargeFirms,
 f2OfIndifferentLargeAndSmallRivalWhenEntryNotBlockaded2LargeFirms}, PlotStyle -> {Hue[.6]}]

General::spell : Possible spelling error: new symbol name "fig2b" is similar to existing symbols (fig2, fig2a, fig2c).

Graphics

{f2, f2Small3rdEntrantAt0NotBlockaded1Large1SmallFirm,
f2Small3rdEntrantNotBlockaded2LargeFirms}
{f2, 0.0361273283579467374, 0.0764745}

{f2, f2SecondEntrantMustMoveToDeter3rdEntrant,
f2Small3rdEntrantAt0NotBlockaded1Large1SmallFirm}
{f2, 0.036001292934144601140, 0.0361273283579467374}

{f2,
 f2OfIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantsAt0AndAtCenterNotBlockaded,
f2SecondEntrantMustMoveToDeter3rdEntrant}
{f2, 0.0358684, 0.036001292934144601140}

{f2, f2Small3rdEntrantAt0NotBlockaded2LargeFirms,
f2OfIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantsAt0AndAtCenterNotBlockaded}
{f2, 0.0270924, 0.0358684}

{f2, f2block4thEntrant, f2Small3rdEntrantAt0NotBlockaded2LargeFirms}
{f2, 0.0219904, 0.0270924}
fig3b =
Plot[nOfIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantNotBlockaded[f2],
{f2, f2Small3rdEntrantAt0NotBlockaded1Large1SmallFirm, f2Small3rdEntrantNotBlockaded2LargeFirms}]

General::spell : Possible spelling error: new symbol name "fig3b" is similar to existing symbols (fig3a, fig3c).

fig4b = Plot[nOfSmall3rdEntrantNotBlockaded1Large1SmallFirm[f2],
{f2, f2SecondEntrantMustMoveToDeter3rdEntrant, f2Small3rdEntrantAt0NotBlockaded1Large1SmallFirm},
PlotStyle -> AbsoluteDashing[{2, 2}]]

fig5b = Plot[nOf1Large1SmallFirmWhen2ndEntrantMustMove[f2], {f2,
f2OfIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantsAt0AndAtCenterNotBlocked,
f2SecondEntrantMustMoveToDeter3rdEntrant}]

fig6b = Plot[nOf1Large1SmallFirmWhen1stEntrantPrefers2LargeFirmsNew[f2],
{f2, f2block4thEntrant,
f2OfIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantsAt0AndAtCenterNotBlockaded}]

General::spell : Possible spelling error: new symbol name "fig4b" is similar to existing symbols (fig4a, fig4c).

FindRoot::precw : The precision of the argument function ((<<1>> - 0) is less than WorkingPrecision (35).
FindRoot::precw : The precision of the argument function ((<<1>> - 0) is less than WorkingPrecision (35).
FindRoot::precw : The precision of the argument function ((<<1>> - 0) is less than WorkingPrecision (35).
General::stop : Further output of FindRoot::precw will be suppressed during this calculation.
Possible spelling error: new symbol name "fig5b" is similar to existing symbol "fig5a".

Possible spelling error: new symbol name "fig6b" is similar to existing symbol "fig6a".

Further output of FindRoot::frnum will be suppressed during this calculation.

Further output of ReplaceAll::reps will be suppressed during this calculation.
FindRoot::frnum: Function \(-25. + 0.0918662 n\) is not a length 1 list of numbers at \(i = 0.4\).

ReplaceAll::reps: \{FindRoot[prof2ndFirmLargeBothMove[i, n, 0.0219904] - 25 - \{<<1>> - <<1>>\} == 0, <<1>>\}\}
is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum: Function \(-25. + 0.0918662 n\) is not a length 1 list of numbers at \(i = 0.4\).

General::stop: Further output of FindRoot::frnum will be suppressed during this calculation.

ReplaceAll::reps: \{FindRoot[prof2ndFirmLargeBothMove[i, n, 0.0219904] - 25 - \{<<1>> - <<1>>\} == 0, <<1>>\}\} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

General::stop: Further output of ReplaceAll::reps will be suppressed during this calculation.

\[\text{fig7b} = \text{Plot}[nOfEntryOfSmallEntrantsAt0AndAtCenterNotBlockadedInAllCases[f2], \{f2, f2block4thEntrant, f2Small3rdEntrantAt0NotBlockaded2LargeFirms\}]\]

General::spell1: Possible spelling error: new symbol name "fig7b" is similar to existing symbol "fig7a".
Show[fig1b, fig2b, fig3b]
\[ \text{fig1c = Plot}\left[\frac{20.25}{f_2}, \{f_2, f_{2\text{block}4\text{thEntrant}}, 0.15\}, \text{PlotStyle} \to \text{GrayLevel}[0.5]\right]\]
\[ \text{fig2c = Plot}\left[\frac{19}{f_2}, \{f_2, f_{2\text{block}4\text{thEntrant}}, 0.15\}, \text{PlotStyle} \to \text{GrayLevel}[0.5]\right]\]
\[ \text{fig3c = Plot}\left[\frac{4.3}{f_2}, \{f_2, f_{2\text{block}4\text{thEntrant}}, 0.15\}, \text{PlotStyle} \to \text{GrayLevel}[0.5]\right]\]
\[ \text{fig4c = Plot}\left[\frac{10}{f_2}, \{f_2, f_{2\text{block}4\text{thEntrant}}, 0.15\}, \text{PlotStyle} \to \text{GrayLevel}[0.5]\right]\]
\[ \text{fig5c = Plot}\left[\frac{2.415}{f_2}, \{f_2, f_{2\text{block}4\text{thEntrant}}, 0.15\}, \text{PlotStyle} \to \text{GrayLevel}[0.5]\right]\]
fig6c = Plot[144, {f2, 9/64, 0.15}]
fig7c = Graphics[Line[
  {(f2OfIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantsAt0AndAtCenterNotBlockaded, nOf1Large1SmallFirmWhen1stEntrantPrefers2LargeFirmsNew[f2OfIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantsAt0AndAtCenterNotBlockaded],
   (f2OfIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantsAt0AndAtCenterNotBlockaded, nOf1Large1SmallFirmWhen2ndEntrantMustMove[f2OfIndifferentLargeAndSmallRivalWhenEntryOfSmallEntrantsAt0AndAtCenterNotBlockaded])}]]
fig8c = Graphics[Line[{(9/64, 50), (9/64, 144)}]]
fig9c = Graphics[Line[{(f2noDeter3rdEntrant, 50), (f2noDeter3rdEntrant, nOf2ndEntrantIndifferentLAndSTech1stEntrantSmallDeter4thEntrant[f2noDeter3rdEntrant][[1]]})}]
fig10c = Graphics[Line[{(f2IndifferentLargeAndSmallRival1stFirmSmall, nOf2ndEntrantIndifferentLAndSTech1stEntrantSmallDeter4thEntrant[f2IndifferentLargeAndSmallRival1stFirmSmall],
   (f2IndifferentLargeAndSmallRival1stFirmSmall, nOf1stEntrantIndifferent2AsymSituations[f2IndifferentLargeAndSmallRival1stFirmSmall][[1]]})}]


fig11c = Graphics[{Hue[.7], Arrow[{0.029, 85.3475}, {f2noDeter3rdEntrant, 94}, HeadScaling -> Relative]}]

General::spell: Possible spelling error: new symbol name "fig11c" is similar to existing symbols (fig11a, fig1c).

- Graphics -
figEquRangesOneEighth = Show[fig1a, fig2a, fig3a, fig4a, fig5a, fig1MR, fig2MR, fig3MR, fig4MR, fig5MR, fig7MR, fig8MR, fig1b, fig2b, fig3b, fig4b, fig5b, fig6b, fig6c, fig7c, fig8c, fig9c, fig10c, fig11c, Graphics[Text[FontForm["III", {"Times", 10}], {0.025, 83.3475}, {1, 0}]], Graphics[Text[FontForm["II", {"Times", 10}], {0.075, 83.3475}, {1, 0}]], Graphics[Text[FontForm["I", {"Times", 10}], {0.149, 83.3475}, {1, 0}]], Graphics[Text[FontForm["IV", {"Times", 10}], {0.032, 83.3475}, {1, 0}]], Graphics[Text[FontForm["V", {"Times", 10}], {0.026, 133.809}, {1, 0}]], Graphics[Text[FontForm["VI", {"Times", 10}], {0.0512559, 218.832}, {1, 0}]], Graphics[Text[FontForm["VII", {"Times", 10}], {0.0811496, 287.633}, {1, 0}]], PlotRange -> {{0.02, 0.15}, {60, 380}}, AxesOrigin -> {0.02, 60}, AxesLabel -> {FontForm["fs/N", {"Times-Italic", 12}], FontForm["N", {"Times-Italic", 12}]}, N}
figEquRangesOneEighthNew = Show[fig0a, fig1a, fig2a, fig3a, fig4a, fig5a, fig6a, fig7a, fig8a, fig9a, fig1MR, fig2MR, fig3MR, fig4MR, fig5MR, fig6MR, fig7MR, fig8MR, fig1b, fig2b, fig3b, fig4b, fig5b, fig6b, fig7c, fig8c, fig9c, fig10c, fig11c, Graphics[Text[FontForm["III", {"Times", 10}], {0.025, 83.3475}, {1, 0}]], Graphics[Text[FontForm["II", {"Times", 10}], {0.075, 83.3475}, {1, 0}]], Graphics[Text[FontForm["I", {"Times", 10}], {0.149, 83.3475}, {1, 0}]], Graphics[Text[FontForm["IV", {"Times", 10}], {0.032, 83.3475}, {1, 0}]], Graphics[Text[FontForm["V", {"Times", 10}], {0.026, 133.809}, {1, 0}]], Graphics[Text[FontForm["VI", {"Times", 10}], {0.0512559, 218.832}, {1, 0}]], Graphics[Text[FontForm["VII", {"Times", 10}], {0.0811496, 287.633}, {1, 0}]], PlotRange -> {{0.02, 0.15}, {60, 380}}, AxesOrigin -> {0.02, 60}, AxesLabel -> {FontForm["fS/N", {"Times-Italic", 12}], FontForm["N", {"Times-Italic", 12}]}, AxesLabel -> {FontForm["fS/N", {"Times-Italic", 12}], FontForm["N", {"Times-Italic", 12}]]]
fig1d = Graphics[{-Dashing[.001, .005],
    Line[{{1/8, 50}, {1/8, nOfSmallEntrantBlockaded2SmallFirms[1/8]}}]]

fig2d = Graphics[{-Dashing[.001, .005],
    Line[{{f2Deter1stEntr, 50}, {f2Deter1stEntr, nOfLargeF2[f2Deter1stEntr]}}]]

fig3d = Graphics[{-Dashing[.001, .005],
    Line[{{f2SecondEntrantMustMoveToDeter3rdEntrant, nOfSecondEntrantMustMoveToDeter3rdEntrant[f2SecondEntrantMustMoveToDeter3rdEntrant],
        {f2SecondEntrantMustMoveToDeter3rdEntrant, nOf1Large1SmallFirmWhen2ndEntrantMustMove[f2SecondEntrantMustMoveToDeter3rdEntrant]}}]]}

fig4d = Graphics[{-Dashing[.001, .005],
    Line[{{f2OfEntryOfLargeEntrantNotBlockaded2LargeFirms, 200}, {.15, 200}}]]

fig5d = Graphics[{-Dashing[.001, .005],
    Line[{{f2Small3rdEntrantAt0NotBlockaded2LargeFirms, nOf2LargeFirmsDeterLargeAboveDeterSmallf2Small[f2Small3rdEntrantAt0NotBlockaded2LargeFirms],
        {f2Small3rdEntrantAt0NotBlockaded2LargeFirms, 400}}]]

- Graphics -
- Graphics -
- Graphics -
- Graphics -

FindRoot::frnum: Function \(-25. + 0.0922604 n\) is not a length 1 list of numbers at \(i = 0.4\).

ReplaceAll::reps : {FindRoot[prof2ndFirmLargeBothMove[i, n, 0.0270924] - 25 - \{<<1>> - <<1>>\} == 0, \{<<1>>\}] is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum: Function \(-25. + 0.0922604 n\) is not a length 1 list of numbers at \(i = 0.4\).

ReplaceAll::reps : {FindRoot[prof2ndFirmLargeBothMove[i, n, 0.0270924] - 25 - \{<<1>> - <<1>>\} == 0, \{<<1>>\}] is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum: Function \(-25. + 0.0922604 n\) is not a length 1 list of numbers at \(i = 0.4\).

General::stop : Further output of FindRoot::frnum will be suppressed during this calculation.

ReplaceAll::reps : {FindRoot[prof2ndFirmLargeBothMove[i, n, 0.0270924] - 25 - \{<<1>> - <<1>>\} == 0, \{<<1>>\}] is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

General::stop : Further output of ReplaceAll::reps will be suppressed during this calculation.

- Graphics -
fig6d = Graphics[{Dashing[{.001, .005}],
Line[{{fOfSmallEntrantBlockaded, nOfLargeF2[fOfSmallEntrantBlockaded]},
{nOfLargeEntrantBlockaded[fOfSmallEntrantBlockaded]}]}

General::spell : Possible spelling error: new symbol
name "fig6d" is similar to existing symbols (fig6a, fig6b, fig6c).
- Graphics -

fig7d = Graphics[{Hue[.7], Arrow[{{0.0848916, 194}, {0.0895041, 194},
HeadScaling -> Relative}]]

General::spell : Possible spelling error: new symbol
name "fig7d" is similar to existing symbols (fig7a, fig7b, fig7c).
- Graphics -

nOfLargeF2[fOfSmallEntrantBlockaded] // N
149.659 - 7.46001 \times 10^{-14} I

f2Deter1stEntr
0.05331106166397552288738235

f2Small3rdEntrantAt0NotBlockaded2LargeFirms
0.0270924

f2Of1stLargeBelow1stSmall
0.0252445

\{0.0254411, 255.862\}
figEquRangesOneEighthNew = 
Show[fig0a, fig1a, fig2a, fig3a, fig4a, fig5a, fig6a, fig7a, fig8a, fig9a, 
fig10a, fig1MR, fig2MR, fig3MR, fig4MR, fig5MR, fig6MR, fig7MR, fig8MR, fig1b, 
fig2b, fig3b, fig4b, fig5b, fig6b, fig7c, fig8c, fig9c, fig10c, 
fig11c, fig1d, fig2d, fig3d, fig4d, fig5d, fig6d, fig7d, 
Graphics[Text[FontForm["III", {"Times", 10}], {0.025, 83.3475}]
, {1, 0}]],
Graphics[Text[FontForm["II,a", {"Times", 10}], {0.13, 83.3475}]
, {-1, 0}]],
Graphics[Text[FontForm["II,b", {"Times", 10}], {0.085, 83.3475}]
, {-1, 0}]],
Graphics[Text[FontForm["II,c", {"Times", 10}], {0.04, 83.3475}]
, {0, 0}]],
Graphics[Text[FontForm["I,b", {"Times", 10}], {0.14332, 83.3475}]
, {1, 0}]],
Graphics[Text[FontForm["I,a", {"Times", 10}], {0.143317, 124.203}]
, {-1, 0}]],
Graphics[Text[FontForm["V,d", {"Times", 10}], {0.032, 83.3475}]
, {.4, 0}]],
Graphics[Text[FontForm["IV", {"Times", 10}], {0.025, 133.809}]
, {-1, 0}]],
Graphics[Text[FontForm["V,a", {"Times", 10}], {0.106417, 161.966}]
, {-1, .5}]],
Graphics[Text[FontForm["V,e", {"Times", 10}], {0.0848916, 194}]
, {1, 0}]],
Graphics[Text[FontForm["V,b", {"Times", 10}], {0.0512559, 194}]
, {-1, 0}]],
Graphics[Text[FontForm["V,c", {"Times", 10}], {0.025, 194}]
, {-1, 0}]],
Graphics[Text[FontForm["VI,a", {"Times", 10}], {0.130, 178.296}]
, {-1, 0}]],
Graphics[Text[FontForm["VI,b", {"Times", 10}], {0.0811496, 300}]
, {1, 0}]],
Graphics[Text[FontForm["VI,c", {"Times", 10}], {0.03, 300}]
, {.50, 0}]],
Graphics[Text[FontForm["VI,d", {"Times", 10}], {0.025, 300}]
, {0.4, 0}]],
Graphics[Text[FontForm["VI,e", {"Times", 10}], {0.025, 255.862}]
, {0.4, 0.6}]], PlotRange -> {{0.02, 0.15}, {60, 380}}, AxesOrigin -> {0.02, 60},
AxesLabel -> {FontForm["fs/N", {"Times-Italic", 12}], FontForm["N",
{"Times-Italic", 12}]}
]
Show[figEquRangesOneEighth, fig1c, fig2c, fig3c, fig4c, fig5c,
Graphics[Text[FontForm["fS\[Equal\]20.25", {"Times-Italic", 10}]
, {0.0632566, 329.684}]
, {1, 0})],
Graphics[Text[FontForm["fS\[Equal\]19", {"Times-Italic", 10}]
, {0.0572566, 329.684}]
, {1, 0})],
Graphics[
Text[FontForm["fS\[Equal\]10", {"Times-Italic", 10}]
, {0.0953817, 111.039}]
, {1, 0})],
Graphics[
Text[FontForm["fS\[Equal\]4.3", {"Times-Italic", 10}]
, {0.0548, 83.3611}]
, {1, 0})],
Graphics[
Text[FontForm["fS\[Equal\]2.415", {"Times-Italic", 10}]
, {0.0312968, 83}]
, {1, 0})],
PlotRange -> {{0.02, 0.15}, {60, 380}}, AxesOrigin -> {0.02, 60},
AxesLabel -> {FontForm["fS/N", {"Times-Italic", 12}], FontForm["N",
{"Times-Italic", 12}]}]
Region $V_e$ in detail.
Show[fig6MR, fig7MR, fig8MR, fig5c, PlotRange -> {{.025, .026}, {93.5, 96.5}}]
\[ f_2 = 9/64. \]

1/8.

\[ N[\text{fOfSmallEntrantBlockaded}] \]

\[ f_2 \text{Deter1stEntr} \]

\[ f_2 \text{SecondEntrantMustMoveToDeter3rdEntrant} \]

\[ f_2 \text{IndifferentLargeAndSmallRivals1stFirmSmall} \]

\[ f_2 \text{of2ndEntrantMustDeter4thEntrant} \]

\[ f_2 \text{of1stSmall2ndLargeEntryAt0} \]

\[ f_2 \text{noDeter3rdEntrant} \]

\[ f_2 \text{of1stLargeBelow1stSmall} \]

\[ f_2 \text{block4thEntrant} \]

0.140625

0.125

0.105774

0.05331106166397552288738235

0.036001292934144601140

0.0339363

0.0281945

0.0269112

0.0258377

0.0252445

0.0219904

Over- and underinvestment in case with two firms using identical technologies

Underinvestment

\[ a[2, 3] = 1 \]

1

Maximum total costs in the 2 small firms case (\( f_2=9/64 \), \( n=\text{nOfSmallEntrantBlockaded2SmallFirms} \))

\[ 1369/64 + 136/16 \]

27.625

Total costs with equal market shares.

\[ \text{totalCosts}[f_2_, n_] = f_2 n + n / 16 \]

\[ \frac{n}{16} + f_2 n \]
nBothFirmsUnderinvest[f2_] = \[\frac{n}{f2}\]. Solve[totalCosts[f2, n] == 25, n][[1]]

\[\text{fig1Under = Plot}[nBothFirmsUnderinvest[f2], \{f2, 1/8, 9/64\}, \text{PlotStyle} -> \text{Hue}[.8]]\]

- Graphics -

Show[%, fig3a, fig4a]

- Graphics -

marketShareUnderinvest[f2_, i_] := 
  (((demand[i] /. prices2Firms) /. \{c[\_] -> 0, x[1] -> deter[f2], x[2] -> 1\}) / n)[[1]]

marketShareUnderinvest[1/8., 1]
marketShareUnderinvest[1/8., 2]
\[0.5 - 2.46716 \times 10^{-17} \text{I}\]
\[0.5 + 2.46716 \times 10^{-17} \text{I}\]

marketShareUnderinvest[.12, 1]
marketShareUnderinvest[.12, 2]
\[0.504046 + 0. \text{I}\]
\[0.495954 + 0. \text{I}\]

totalCostsUnderinvest[f2_, n_, i_] := f2 n + marketShareUnderinvest[f2, i] n/8
The second firm has a smaller market share in general. Thus for underinvestment of both firms the second entrant’s technology choice is the binding constraint.

\[
\text{totalCostsUnderinvest}[1/10., 130, 1] \\
\text{totalCostsUnderinvest}[1/10., 130, 2] \\
21.4702 - 5.01142 \times 10^{-17} I \\
20.7798 + 5.01142 \times 10^{-17} I
\]

\[
\text{nBothFirmsUnderinvestGen}[f2_] = \\
n /. \text{Solve}[\text{totalCostsUnderinvest}[f2, n, 2] == 25, n][[1]]; \\
\text{fig2Under} = \text{Plot}[\text{nBothFirmsUnderinvestGen}[f2], \{f2, 1/10, 1/8\}]
\]

- Graphics -

\[
\text{Show}[\text{fig1Under}, \text{fig2Under}, \text{fig3}, \text{fig4}]
\]

- Graphics -
Plot[nBothFirmsUnderinvestGen[f2], {f2, 1/10, 9/64}]

- Graphics -

Show[fig1Under, %, PlotRange -> {{.12, .13}, {130, 140}}]

- Graphics -

FindRoot[
  nBothFirmsUnderinvestGen[f2] == nOfSmallEntrantBlockaded1largeFirm[f2], {f2, 1/10}]

{f2 -> 0.106391 + 1.06568 x 10^{-17} i}

f2Underinvest = f2 /. Chop[%]
0.106391

fig2Under =
  Plot[nBothFirmsUnderinvestGen[f2], {f2, f2Underinvest, 1/8}, PlotStyle -> Hue[.9]]
Demand and prices above and below the locus dividing the regions II,a and V,a.

\[
\left(\frac{\text{demand}[1]}{\text{prices2Firms}}\right) \cdot \{c[1] \to 0, c[2] \to 1/8, x[1] \to 0, x[2] \to 1\} / \text{n}[[1]]
\]

\[
\left\{\begin{array}{l}
p[1] \to \frac{9}{8}, p[2] \to \frac{9}{8}\end{array}\right\}
\]

\[N[]\]

\[
0.520833
\]

\[
\left(\frac{\text{prices2Firms}}{\cdot \{c[1] \to 1/8, c[2] \to 1/8, x[1] \to 0, x[2] \to 1\}}\right) [[1]]
\]

\[
\left\{\begin{array}{l}
p[1] \to \frac{25}{24}, p[2] \to \frac{13}{12}\end{array}\right\}
\]

\[N[]\]

\[
\{p[1.] \to 1.04167, p[2.] \to 1.08333\}
\]

**The overinvestment case: Region V, d**

\[\text{Solve}\{	ext{totalCosts[f2, 260.] == 25, f2}\}\]

\[
\{\{f2 \to 0.0336538\}\}
\]

The value of f2 indicates that both firms overinvest should be possible.

Checking the market shares.

\[\text{prices2Firms}\]

\[
\left\{\begin{array}{l}
\]
numDeterLeftAsyLarge[f2block4thEntrant]
numDeterCenterAsyLarge[numDeterLeftAsyLarge[f2block4thEntrant], f2block4thEntrant]

0.3696
0.933113

(demand[1] /. prices2Firms) /. {c[_] -> 0,
x[1] -> numDeterLeftAsyLarge[f2block4thEntrant], x[2] -> numDeterCenterAsyLarge[
numDeterLeftAsyLarge[f2block4thEntrant], f2block4thEntrant])

{0.550452 n}

marketShare[f2_, i_] :=
((demand[i] /. prices2Firms) /. {c[_] -> 0, x[1] -> numDeterLeftAsyLarge[f2],
x[2] -> numDeterCenterAsyLarge[numDeterLeftAsyLarge[f2], f2]])[[1]]/n

marketShare[f2block4thEntrant, 2]

is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function (function2AsyLarge[0.4, k] -
0.021990446286980363382879133382630243) is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function (function2AsyLarge[0.5, k] -
0.021990446286980363382879133382630243) is less than WorkingPrecision (35).

General::stop : Further output of FindRoot::precw will be suppressed during this calculation.

0.4495484519584867698605376269884649

totalCostsGen[f2_, n_, i_] := f2 n + marketShare[f2, i] n/8

totalCostsGen[f2block4thEntrant,
numDeterLeftAsyLarge[f2block4thEntrant], 1]

FindRoot::precw : The precision of the argument function {redProfit3[1] /. \[Infty]\[DirectedEdge]\[Infty][1] - 0.0219904
is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function (function2AsyLarge[0.4, k] -
0.021990446286980363382879133382630243) is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function (function2AsyLarge[0.5, k] -
0.021990446286980363382879133382630243) is less than WorkingPrecision (35).

General::stop : Further output of FindRoot::precw will be suppressed during this calculation.

23.9744
totalCostsGen[f2block4thEntrant, nOfEntryOfSmallEntrantsAt0AndAtCenterNotBlockadedInAllCases[f2block4thEntrant], 2]

FindRoot::precw : The precision of the argument function (0.0219904) is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function (0.02199046286980363382879133382630243) is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function (0.02199046286980363382879133382630243) is less than WorkingPrecision (35).

General::stop : Further output of FindRoot::precw will be suppressed during this calculation.

20.6441

Both firms overinvest!!

f2block4thEntrant
nOfEntryOfSmallEntrantsAt0AndAtCenterNotBlockadedInAllCases[f2block4thEntrant]

5.80647

nOfEntryOfSmallEntrantsAt0AndAtCenterNotBlockadedInAllCases[f2block4thEntrant]

264.04523207351216387518

marketShare[f2block4thEntrant, 2]

nOfEntryOfSmallEntrantsAt0AndAtCenterNotBlockadedInAllCases[f2block4thEntrant]

118.701

nBothFirmsOverinvest[f2_] := n /. Solve[totalCostsGen[f2, n, 1] == 25, n][[1]]

Plot[nBothFirmsOverinvest[f2],

{f2, f2block4thEntrant, f2Small3rdEntrantAt0NotBlockaded2LargeFirms}]

FindRoot::precw : The precision of the argument function (0.0219904) is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function (0.02199046499563661569842665244323143) is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function (0.02199046499563661569842665244323143) is less than WorkingPrecision (35).

General::stop : Further output of FindRoot::precw will be suppressed during this calculation.
The region where both firms overinvest is not too small.

```
FindRoot[nBothFirmsOverinvest[f2] ==
  nOfEntryOfSmallEntrantsAt0AndAtCenterNotBlockedInAllCases[f2], {f2, .024, .025}]
```

FindRoot::precw: The precision of the argument function (redProfit3[i] /. <1>)[1] - 0.024) is less than WorkingPrecision (35).

FindRoot::precw: The precision of the argument function (function2AsyLarge[0.4, k] - 0.0240000000000000000499600361081320443) is less than WorkingPrecision (35).

FindRoot::precw: The precision of the argument function (function2AsyLarge[0.5, k] - 0.0240000000000000000499600361081320443) is less than WorkingPrecision (35).

General::stop : Further output of FindRoot::precw will be suppressed during this calculation.

FindRoot::frnum : Function (-25.*0.0983731n) is not a length 1 list of numbers at i = 0.3300000000000000026`.

ReplaceAll::reps : {FindRoot[<<1>> == 0, [i, 33/100, 35/100]]} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum : Function (-25.*0.0983731n) is not a length 1 list of numbers at i = 0.3300000000000000026`.

ReplaceAll::reps : {FindRoot[<<1>> == 0, [i, 33/100, 35/100]]} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum : Function (-25.*0.0983731n) is not a length 1 list of numbers at i = 0.3300000000000000026`.

General::stop : Further output of ReplaceAll::reps will be suppressed during this calculation.

ReplaceAll::reps : {FindRoot[<<1>> == 0, [i, 33/100, 35/100]]} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

General::stop : Further output of ReplaceAll::reps will be suppressed during this calculation.

\{f2 \rightarrow 0.0242186\}
f2Overinvest = f2 /. %
0.0242186

figOverinvest = Plot[nBothFirmsOverinvest[f2],
{f2, f2block4thEntrant, f2Overinvest}, PlotStyle -> Hue[.9]]

FindRoot::precw : The precision of the argument function (redProfit3[1] /. \!\<\<\<1\>)[1] - 0.0219904) is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function (function2AsyLarge[0.4, k] - 0.021990463798216655239320011787640) is less than WorkingPrecision (35).

FindRoot::precw : The precision of the argument function (function2AsyLarge[0.5, k] - 0.021990463798216655239320011787640) is less than WorkingPrecision (35).

General::stop : Further output of FindRoot::precw will be suppressed during this calculation.

- Graphics -

- The overinvestment case: Region VI,e

Solve[totalCosts[f2, 260.] == 25, f2]
{{f2 -> 0.0336538}}

The value of f2 indicates that both firms overinvest should be possible.

Checking the market shares.

prices2Firms


(demand[1] /. prices2Firms) /. \
{c[\_] -> 0, x[1] -> iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[ \
f2block4thEntrant, n], x[2] -> \
deterCenterBothMove[iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[ \
f2block4thEntrant], f2block4thEntrant]}

\{0.511588 n\}
\[ x[1] \rightarrow \text{iDeterLargeEntrantWhen2ndEntrantMustMove1stWantsLargeRival}[ \\
\text{f2block4thEntrant, 250}], x[2] \rightarrow \\
\text{deterCenterBothMove}[\text{iDeterLargeEntrantWhen2ndEntrantMustMove1stWantsLargeRival}[ \\
\text{f2block4thEntrant, 250}], \text{f2block4thEntrant}] \]

\[ \{x[1] \rightarrow 0.313668, x[2] \rightarrow 0.87718\} \]

\[(\text{demand}[1] /. \text{prices2Firms}) /.
\{c[\_] \rightarrow 0, x[1] \rightarrow \text{iDeterLargeEntrantWhen2ndEntrantMustMove1stWantsLargeRival}[ \\
\text{f2block4thEntrant, 250}], x[2] \rightarrow \\
\text{deterCenterBothMove}[\text{iDeterLargeEntrantWhen2ndEntrantMustMove1stWantsLargeRival}[ \\
\text{f2block4thEntrant, 250}], \text{f2block4thEntrant}]\}\]

\[\{0.531808 n\}\]

\[\text{demandOfN}[i\_, n\_] := (a[i, i + 1] - a[i, i - 1]) n\]

\[\text{demandOfN}[1, n]
\]

\[n \left( \frac{-p[1] + p[2]}{2 (-x[1] + x[2])} + \frac{1}{2} (x[1] + x[2]) \right) \]

\[\text{marketShareSmallN}[f2\_, j\_, n\_] := \\
(((\text{demandOfN}[j, n] / n) /. \text{prices2Firms}) /. \{c[\_] \rightarrow 0, x[1] \rightarrow \\
\text{iDeterLargeEntrantWhen2ndEntrantMustMove1stWantsLargeRival}[f2, n], \\
\text{x}[2] \rightarrow \text{deterCenterBothMove}[ \\
\text{iDeterLargeEntrantWhen2ndEntrantMustMove1stWantsLargeRival}[f2, n], f2])[[1]] \]

\[\text{marketShareSmallN}[\text{f2block4thEntrant}, 1, 250]\]

\[0.531808\]

\[\text{totalCostsGenSmallN}[f2\_, n\_, j\_] := f2 n + \text{marketShareSmallN}[f2, j, n] n / 8\]

\[\text{totalCostsGenSmallN}[\text{f2block4thEntrant}, \\
\text{nOf1Large1SmallFirmWhen1stEntrantPrefers2LargeFirmsNew[\text{f2block4thEntrant}], 1}]\]

\[20.2736\]

\[\text{totalCostsGenSmallN}[\text{f2block4thEntrant}, \\
\text{nOf1Large1SmallFirmWhen1stEntrantPrefers2LargeFirmsNew[\text{f2block4thEntrant}], 2}]\]

\[19.5902\]

\[\text{totalCostsGenSmallN}[\text{f2Overinvest}, \\
\text{nOfEntryOfSmallEntrantsAt0AndAtCenterNotBlockadedInAllCases[\text{f2Overinvest}], 1}]\]

\[25.\]

\[\text{f2Overinvest}\]

\[\text{nOf1Large1SmallFirmWhen1stEntrantPrefers2LargeFirmsNew[.03]}\]

\[244.83325997209676881934\]

\[\text{totalCostsGenSmallN}[0.03, \\
\text{nOf1Large1SmallFirmWhen1stEntrantPrefers2LargeFirmsNew[.03], 2}]\]

\[21.5205\]
f2Small3rdEntrantAt0NotBlockaded2LargeFirms
0.0270924

totalCostsGenSmallN[f2Small3rdEntrantAt0NotBlockaded2LargeFirms, .02, 265, 2]
17.1991

Accuracy[%]
15

nOfEntryOfSmallEntrantsAt0AndAtCenterNotBlockadedInAllCases[f2block4thEntrant]
264.044821826252009947

{x[1] -> iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[
  f2block4thEntrant], x[2] -> deterCenterBothMove[
    iIndifferentLargeAndSmallRivalWhen1stLargeWantsLargeRival[f2block4thEntrant],
    f2block4thEntrant]}%
{x[1] -> 0.253007, x[2] -> 0.81652}

x[1] -> numDeterLeftAsyLarge[f2],
x[2] -> numDeterCenterAsyLarge[numDeterLeftAsyLarge[f2], f2]}

Both firms overinvest!!

f2block4thEntrant
nOf1Large1SmallFirmWhen1stEntrantPrefers2LargeFirmsNew[f2block4thEntrant]
5.18771

nOf1Large1SmallFirmWhen1stEntrantPrefers2LargeFirmsNew[f2block4thEntrant]
235.907452690256396201548239494033385897018608518980861

marketShare[f2block4thEntrant, 2]
nOf1Large1SmallFirmWhen1stEntrantPrefers2LargeFirmsNew[f2block4thEntrant]
115.22

nBothFirmsOverinvestSmallN[f2_] :=
nBothFirmsOverinvestSmallN[f2] = n /. InterpolateRoot[
  totalCostsGenSmallN[f2, n, 1] == 25, {n, 265, 270}, AccuracyGoal -> 13][[1]]
\texttt{nBothFirmsOverinvestSmallN[f2Overinvest]}

\texttt{FindRoot::frnum \textasciitilde \texttt{Function (-25. + 0.0919584\texttt{n}) is not a length 1 list of numbers at \texttt{i = 0.4'}}.  

\texttt{ReplaceAll::reps}:
\texttt{[FindRoot[prof2ndFirmLargeBothMove[i, n, 0.0242186] - 25 - (<<\texttt{i}>> - <<\texttt{l}>>> = 0, <<\texttt{l}>>>] \text{ is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.}}}

\texttt{FindRoot::frnum \textasciitilde \texttt{Function (-25. + 0.0919584\texttt{n}) is not a length 1 list of numbers at \texttt{i = 0.4'}}.  

\texttt{ReplaceAll::reps}:
\texttt{[FindRoot[prof2ndFirmLargeBothMove[i, n, 0.0242186] - 25 - (<<\texttt{i}>> - <<\texttt{l}>>> = 0, <<\texttt{l}>>>] \text{ is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.}}}

\texttt{FindRoot::frnum \textasciitilde \texttt{Function (-25. + 0.0919584\texttt{n}) is not a length 1 list of numbers at \texttt{i = 0.4'}}.  

\texttt{General::stop \textasciitilde \texttt{Further output of \texttt{FindRoot::frnum will be suppressed during this calculation.}}}

\texttt{ReplaceAll::reps}:
\texttt{[FindRoot[prof2ndFirmLargeBothMove[i, n, 0.0242186] - 25 - (<<\texttt{i}>> - <<\texttt{l}>>> = 0, <<\texttt{l}>>>] \text{ is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.}}}

\texttt{General::stop \textasciitilde \texttt{Further output of \texttt{ReplaceAll::reps will be suppressed during this calculation.}}}

266.3901347276390152889  

\texttt{nOfEntryOfSmallEntrantsAt0AndAtCenterNotBlockadedInAllCases[f2Overinvest]}  

266.39013477780151053  

\texttt{Plot[totalCostsGenSmallN[f2Overinvest, n, 1], \{n, 250, 270\}]}  

- Graphics -
InterpolateRoot[nBothFirmsOverinvestSmallN[f2] ==
nOf1LargeSmallFirmWhen1stEntrantPrefers2LargeFirmsNew[f2],
{f2, .03312, .03313}, AccuracyGoal -> 11, ShowProgress -> True]

FindRoot::frnum : Function (0.0236931 - 1. f2) is not a length 1 list of numbers at k = 0.85.`.

ReplaceAll::reps : 
{- (-1 f2) + (0.4 - k (0.4 + 
(0.0555556 (-0.4 - i) + (-0.4 + (-k) - (-0.5 + (-0.1))) == f2}) is neither a list of replacement
rules nor a valid dispatch table, and so cannot be used for replacing.

ReplaceAll::reps : 
{- (-1 f2) + (0.4 - k (0.4 + 
(0.0555556 (-0.4 - i) + (-0.4 + (-k) - (-0.5 + (-0.1))) == f2}) is neither a list of replacement
rules nor a valid dispatch table, and so cannot be used for replacing.

ReplaceAll::reps : 
{- (-1 f2) + (0.4 - k (0.4 + 
(0.0555556 (-0.4 - i) + (-0.4 + (-k) - (-0.5 + (-0.1))) == f2}) is neither a list of replacement
rules nor a valid dispatch table, and so cannot be used for replacing.

General::stop : Further output of ReplaceAll::reps will be suppressed during this calculation.

FindRoot::frnum : Function (-25. + f2 n - 0.4 f2 n + 
0.0555556 (-0.4 - k (0.4 + 
(-0.4 - i) + (-0.4 + (-k) - (-0.5 + (-0.1))) == f2}) is not a length 1 list of numbers at i = 0.4`.

FindRoot::frnum : Function (0.0236931 - 1. f2) is not a length 1 list of numbers at k = 0.85.`.

General::stop : Further output of FindRoot::frnum will be suppressed during this calculation.

FindRoot::itraw : Raw object 0.4` cannot be used as an iterator.

FindRoot::itraw : Raw object 0.4` cannot be used as an iterator.

FindRoot::itraw : Raw object 0.4` cannot be used as an iterator.

General::stop : Further output of FindRoot::itraw will be suppressed during this calculation.

$Aborted

f2OverinvestSmallN = f2 /. %
0.0331281

f2OverinvestSmallN = 0.0331280761833980452`
0.0331281
nBothFirmsOverinvestSmallN[f2OverinvestSmallN]
nof1Large1SmallFirmWhen1stEntrantPrefers2LargeFirmsNew[f2OverinvestSmallN]

FindRoot::frnum : Function \((-25. + 0.0934655 n)\) is not a length 1 list of numbers at \(i = 0.4\). 

ReplaceAll::reps :
\{FindRoot\[prof2ndFirmLargeBothMove\[i, n, 0.0331281\] - 25 - \{<<1\[\] - <<1\[\] \} == 0, <<1\[\] \} \}

is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum : Function \((-25. + 0.0934655 n)\) is not a length 1 list of numbers at \(i = 0.4\). 

ReplaceAll::reps :
\{FindRoot\[prof2ndFirmLargeBothMove\[i, n, 0.0331281\] - 25 - \{<<1\[\] - <<1\[\] \} == 0, <<1\[\] \} \}

is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum : Function \((-25. + 0.0934655 n)\) is not a length 1 list of numbers at \(i = 0.4\). 

General::stop : Further output of FindRoot::frnum will be suppressed during this calculation.

ReplaceAll::reps :
\{FindRoot\[prof2ndFirmLargeBothMove\[i, n, 0.0331281\] - 25 - \{<<1\[\] - <<1\[\] \} == 0, <<1\[\] \} \}

is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

General::stop :
Further output of ReplaceAll::reps will be suppressed during this calculation.

246.69122095236961

FindRoot::frnum : Function \((-25. + 0.0934655 n)\) is not a length 1 list of numbers at \(i = 0.4\). 

ReplaceAll::reps :
\{FindRoot\[prof2ndFirmLargeBothMove\[i, n, 0.0331281\] - 25 - \{<<1\[\] - <<1\[\] \} == 0, <<1\[\] \} \}

is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum : Function \((-25. + 0.0934655 n)\) is not a length 1 list of numbers at \(i = 0.4\). 

ReplaceAll::reps :
\{FindRoot\[prof2ndFirmLargeBothMove\[i, n, 0.0331281\] - 25 - \{<<1\[\] - <<1\[\] \} == 0, <<1\[\] \} \}

is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum : Function \((-25. + 0.0934655 n)\) is not a length 1 list of numbers at \(i = 0.4\). 

General::stop : Further output of FindRoot::frnum will be suppressed during this calculation.

ReplaceAll::reps :
\{FindRoot\[prof2ndFirmLargeBothMove\[i, n, 0.0331281\] - 25 - \{<<1\[\] - <<1\[\] \} == 0, <<1\[\] \} \}

is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

General::stop :
Further output of ReplaceAll::reps will be suppressed during this calculation.

246.69122091151237820156
figOverInvestSmallN = Plot[nBothFirmsOverInvestSmallN[f2],
{f2, f2OverInvest, f2OverInvestSmallN}, PlotStyle -> Hue[.8]]

FindRoot::frnum: Function (-25. + 0.0919584 n) is not a length 1 list of numbers at i = 0.4.

ReplaceAll::reps:
{FindRoot[prof2ndFirmLargeBothMove[i, n, 0.0242186] - 25 - (<<1>> - <<1>>) == 0, <<1>>]}
is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum: Function (-25. + 0.0919584 n) is not a length 1 list of numbers at i = 0.4.

ReplaceAll::reps:
{FindRoot[prof2ndFirmLargeBothMove[i, n, 0.0242186] - 25 - (<<1>> - <<1>>) == 0, <<1>>]}
is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

FindRoot::frnum: Function (-25. + 0.0919584 n) is not a length 1 list of numbers at i = 0.4.

General::stop: Further output of FindRoot::frnum will be suppressed during this calculation.

ReplaceAll::reps:
{FindRoot[prof2ndFirmLargeBothMove[i, n, 0.0242186] - 25 - (<<1>> - <<1>>) == 0, <<1>>]}
is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

General::stop: Further output of ReplaceAll::reps will be suppressed during this calculation.

FindRoot::frsec:
Secant method failed to converge to the prescribed accuracy after 15 iterations.

FindRoot::frsec:
Secant method failed to converge to the prescribed accuracy after 15 iterations.

FindRoot::frsec:
Secant method failed to converge to the prescribed accuracy after 15 iterations.

General::stop: Further output of FindRoot::frsec will be suppressed during this calculation.

- Graphics -
Show[figOverinvest, figOverinvestSmallN, fig6b, fig9a, fig10a]

- Graphics -

figShad1 = Graphics[GrayLevel[.9], Polygon[{{0.0219162, 238.473}, {0.025145, 242.056}, {0.0293378, 245.639}, {0.0324864, 248.326}, {0.0291129, 256.387}, {0.0264141, 261.762}, {0.02439, 267.094}}]]

- Graphics -

figShad2 = Graphics[GrayLevel[.9], Polygon[{{0.106929, 150.}, {0.121279, 150.}, {0.125123, 148.}, {0.139217, 138.492}, {0.140498, 136.471}, {0.140498, 124.}, {0.130504, 131.347}, {0.125123, 134.409}, {0.117692, 140.533}}]]

- Graphics -
figEquRangesOneEighthNewInclFixedCosts = 
Show[figShad1, figShad2, figEquRangesOneEighthNew, fig1Under, fig2Under, 
figOverinvest, figOverinvestSmallN, fig1c, fig2c, fig3c, fig4c, fig5c, 
Graphics[Text[FontForm["UI", {"Times-Italic", 10}], {0.127189, 141}], 
{0, 0.3}]], Graphics[ 
Text[FontForm["OI", {"Times-Italic", 10}], {0.0236535, 250}], 
{-2, 0.1}], Graphics[Text[FontForm["fS=20.25", {"Times-Italic", 10}], 
(0.0632566, 329.684)], 
{-1, 0}]], Graphics[Text[FontForm["fS=19", {"Times-Italic", 10}], 
(0.0572566, 329.684)], 
{1, 0}], Graphics[Text[FontForm["fS=10", {"Times-Italic", 10}], 
(0.0953817, 111.039)], 
{-1, 0}], Graphics[Text[FontForm["fS=4.3", {"Times-Italic", 10}], 
(0.0548, 83.3611)], 
{-1, 0}], Graphics[Text[FontForm["fS=2.415", {"Times-Italic", 10}], 
(0.0312968, 83)], 
{-1.5, 1.3}], PlotRange->{{0.02, 0.15}, {60, 380}}, 
Axes->True, AxesOrigin->{0.02, 60}, 
AxesLabel->{FontForm["fS/N", {"Times-Italic", 12}], FontForm["N", 
{"Times-Italic", 12}]}, 
N,
fS/
- Derivation of the relation between prices etc. and market size (Figures 5-9 in the article)

- Checks

\[
f_{2 \text{of 1stSmall2ndLargeEntryAt0}} = 0.0269112
\]

\[
f_{2 \text{SecondEntrantMustMoveToDeter3rdEntrant}}
\]

\[
f_{2 \text{IndifferentLargeAndSmallRival1stFirmSmall}} = 0.036001292934144601140
\]

\[
f_{2 \text{SecondEntrantMustMoveToDeter3rdEntrant}}
\]

\[
f_{2 \text{IndifferentLargeAndSmallRival1stFirmSmall}}
\]

\[
f_{2c} = {f_2} / . \text{FindRoot}[\]

\[
\text{nOfSecondEntrantMustMoveToDeter3rdEntrant}[f_{2 \text{SecondEntrantMustMoveToDeter3rdEntrant}}] = 119.346
\]

\[
{4.2 / f_{2 \text{SecondEntrantMustMoveToDeter3rdEntrant}}} = 116.662
\]

\[
\text{nOfSecondEntrantMustMoveToDeter3rdEntrant}[f_{2 \text{SecondEntrantMustMoveToDeter3rdEntrant}}] = 119.346
\]

\[
\text{nOfSecondEntrantMustMoveToDeter3rdEntrant}[f_{2 \text{IndifferentLargeAndSmallRival1stFirmSmall}}] = 122.131
\]

\[
{4.2 / f_{2 \text{IndifferentLargeAndSmallRival1stFirmSmall}}} = 123.761
\]

\[
\text{nOf1stEntrantIndifferent2AsymSituations}[.032] = 132.532
\]

\[
\text{nOf1stEntrantIndifferent2AsymSituations}[] = 132.532
\]

\[
? \text{nOf1stEntrantIndifferent2AsymSituations}
\]

\[
\text{Global``nOf1stEntrantIndifferent2AsymSituations}
\]

\[
\text{nOf1stEntrantIndifferent2AsymSituations}[f_{2 \_}] := \text{n /. Solve[]}\text{prof1stEntrant2ndEntrantLarge1stSmall}[f_{2 \_}, n] = \text{profAsymAsy}[f_{2 \_}, n], n]
\]

\[
f_{2b} = {f_2} / . \text{FindRoot}[\]

\[
\text{nOf1stEntrantIndifferent2AsymSituations}[f_{2 \_}][[1]] = (4.2 / f_{2 \_}), {f_2, .03, .027}]
\]

\[
0.0328033
\]

\[
\text{nOf1stEntrantIndifferent2AsymSituations}[f_{2 \_}][[1]] = (4.2 / f_{2 \_}), {f_2, .03, .027}]
\]

\[
0.0328033
\]

\[
\text{nOf1stEntrantIndifferent2AsymSituations}[f_{2 \_}][[1]] = (4.2 / f_{2 \_}), {f_2, .03, .027}]
\]

\[
0.0328033
\]

\[
\text{nOf1stEntrantIndifferent2AsymSituationsEntryAt0} = 175.619
\]

\[
{175.619}
\]
nOf1stEntrantIndifferent2AsymSituationsEntryAt0[f2Of1stSmall2ndLargeEntryAt0]
{168.744}

f2a = f2 /. FindRoot[nOf1stEntrantIndifferent2AsymSituationsEntryAt0[f2][[1]] == (4.2/f2),
{f2, .03, .027}]
0.0238717

nOf1stEntrantIndifferent2AsymSituationsEntryAt0[f2block4thEntrant]
{180.837}

4.2/f2block4thEntrant
190.992

f2Deter1stEntr
0.05331106166397552288738235

deterCenterMR[.2, .03]
0.709317

numDeterCenterAsy[.2, .03]
0.709314

iOf1ndifferentLargeAndSmallRival1stFirmSmall[0.02616961759971874] = 
0.1738995195911052
0.1738995195911052

i0f2ndCannotDeter[.0262]
0.324872

numDeterLeftAsy1stSmall[f2Of1stSmall2ndLargeEntryAt0]
i0f2ndCannotDeter[f2Of1stSmall2ndLargeEntryAt0]
0.31471
0.31471

numDeterLeftAsy1stSmall[f2Of1stSmall2ndLargeEntryAt0+.001]
i0f2ndCannotDeter[f2Of1stSmall2ndLargeEntryAt0+.001]
0.319203
0.30062

numDeterLeftAsy1stSmall[f2Of1stSmall2ndLargeEntryAt0-.001]
i0f2ndCannotDeter[f2Of1stSmall2ndLargeEntryAt0-.001]
0.310102
0.329035
f2Of1stSmall2ndLargeEntryAt0

0.0269112

loc[f2_] := Which[f2 < f2a,
{x[1] -> numDeterLeftAsy[f2], x[2] -> numDeterCenterAsy[numDeterLeftAsy[f2], f2]},

f2 < f2Of1stSmall2ndLargeEntryAt0, {x[1] -> numDeterLeftAsy1stSmall[f2],
{x[2] -> deterCenterMR[Chop[numDeterLeftAsy1stSmall[f2]], f2]},

f2 < f2b, {x[1] -> 1Of2ndCannotDeter[f2],
{x[2] -> deterCenterMR[Chop[1Of2ndCannotDeter[f2]], f2]}, f2 < f2c,
{x[1] -> numDeterLeftAsy[f2], x[2] -> numDeterCenterAsy[numDeterLeftAsy[f2], f2]],

f2 > f2c, {x[1] -> deterLeft[f2], x[2] -> deterCenter[deterLeft[f2], f2]]}

loc[.03]

{x[1] → 0.271907, x[2] → 0.781221}

Plot[x[1] /. loc[f2], {f2, f2block4thEntrant, .05}]

- Graphics -

Plot[x[2] /. loc[f2], {f2, f2block4thEntrant, .05}]

- Graphics -
Show[%, %, PlotRange -> {0, 1}]

Plot[x[1] /. loc[4.2/n], {n, 190, 110}]

Plot[x[2] /. loc[4.2/n], {n, 190, 110}]

- Graphics -
Show[%, %, PlotRange -> {0, 1}]
Plot[x[1] /. loc[4.2/n], {n, 190, 110}, AxesLabel -> {FontForm["N", {"Times-Italic", 12}], FontForm["Locations", {"Times-Italic", 12}]}, AxesLabel -> {FontForm["N", {"Times-Italic", 12}], FontForm["Locations", {"Times-Italic", 12}]}, PlotStyle -> Dashing[{.01, .01}]]
Show[%, %%, PlotRange -> {0, 1}, AxesOrigin -> {110, 0}]
pricesNew[f2_] := (prices2Firms) /.
    numDeterCenterAsy[numDeterLeftAsy[f2], f2]}, f2 < f2Of1stSmall2ndLargeEntryAt0,
    x[2] -> deterCenterMR[Chop[numDeterLeftAsy1stSmall[f2], f2]]},
    x[2] -> deterCenterMR[Chop[iOf2ndCannotDeter[f2]], f2]},
    x[2] -> numDeterCenterAsy[numDeterLeftAsy[f2], f2]}, f2 > f2c, {c[1] -> 1/8,

pricesNew[.03]

{{p[1] -> 0.601667, p[2] -> 0.541961}}

Plot[p[1] / pricesNew[f2], {f2, f2block4thEntrant, .05}]

- Graphics -

Plot[p[1] / pricesNew[4.2/n], {n, 190, 110}]

- Graphics -
Plot[p2 /. pricesNew[4.2/n], {n, 190, 110}]

- Graphics -

Show[%, %]

- Graphics -
Plot[p1 /. pricesNew[4.2/n], {n, 190, 110}, AxesLabel -> {FontForm["N", {"Times-Italic", 12}], FontForm["Prices", {"Times-Italic", 12}]}, Plot[p2 /. pricesNew[4.2/n], {n, 190, 110}, AxesLabel -> {FontForm["N", {"Times-Italic", 12}], FontForm["Prices", {"Times-Italic", 12}]}, PlotStyle -> Dashing[{.01, .01}]]
Show[%, %%, Graphics[Line[{{110, .465}, {110, .49}}]], AxesOrigin -> {110, .475}]
marketShareFirm1[f2_] :=

marketShareFirm1[.03]

0.46795

Plot[marketShareFirm1[4.2/n], {n, 190, 110}]

- Graphics -

Plot[marketShareFirm1[4.2/n], {n, 190, 110}, AxesLabel -> 
{FontForm["N", {"Times-Italic", 12}], FontForm["Market share 1st entrant", 
{"Times-Italic", 12}]}, AxesOrigin -> {110, Automatic}]

Market share 1st entrant

- Graphics -
Prices of both firms increase at \( f_2c \), but profit of 2nd entrant falls due to large loss in market share.

```math
\text{priceIndex}[f_2_] := \text{marketShareFirm1}[f_2] \left( p[1] / . \text{pricesNew}[f_2] \right) + \\
\left( 1 - \text{marketShareFirm1}[f_2] \right) \left( p[2] / . \text{pricesNew}[f_2] \right)
```

```
priceIndex[.03] = 0.5699
```

```
pricesNew[.03]
```

```
marketShareFirm1[.03] = {{p[1] \to 0.601667, p[2] \to 0.541961}}
```

0.46795
Plot[priceIndex[4.2/n], {n, 190, 110}]

- Graphics -

Plot[priceIndex[4.2/n], {n, 190, 110}, AxesLabel ->
{FontForm["N", {"Times-Italic", 12}], FontForm["Price index", {"Times-Italic", 12}]}, AxesOrigin -> {110, Automatic}]

Price index

- Graphics -

?profits2Firms
Global`profits2Firms

profits2Firms[x1_, x2_, costdif_] :=

?profits2FirmsOfN
Global`profits2FirmsOfN

profits2FirmsOfN[x1_, x2_, costdif_, n_] =
{-(n*(-costdif + 2*x1 + x1^2 - 2*x2 - x2^2)^2 + 18*(x1 - x2)*f[1]) / (18*(x1 - x2)),
 -(n*(-costdif - 4*x1 + x1^2 + 4*x2 - x2^2)^2 + 18*(x1 - x2)*f[2]) / (18*(x1 - x2))}

costDifference[f2_] := Which[f2 < f2a, 1/8, f2 < f2b, -1/8, f2 < f2c, 1/8, f2 > f2c, 0]
costDifference[.035]
0
fixedCosts1[f2_] := Which[f2 < f2a, 25, f2 < f2b, 4.2, f2 < f2c, 25, f2 > f2c, 4.2]
fixedCosts2[f2_] := Which[f2 < f2a, 4.2, f2 < f2b, 25, f2 < f2c, 4.2, f2 > f2c, 4.2]


ReplaceAll::reps: 
{Which[<20 << 0.0238717, <<9>>] is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

ReplaceAll::reps: 
{Which[<20 << 0.0238717, <<9>>] is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

{28.6999, 15.9869}


ReplaceAll::reps: 
{Which[f2 < 0.0238717, <<8>>, {<<1>>>} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

ReplaceAll::reps: 
{Which[f2 < 0.0238717, <<8>>, {<<1>>>} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

profitsNew[.03]

{27.0279, 15.369}

Table[profitsNew[f2], {f2, .02, .03, .01}]

{{33.2379, 26.369}, {27.0279, 15.369}}}
Plot[Evaluate[profitsNew[f2]], {f2, f2block4thEntrant, .05}]

ReplaceAll::reps : (Which[f2 < 0.0238717, «8>>, {«l>>}] is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

ReplaceAll::reps : (Which[f2 < 0.0238717, «8>>, {«l>>}] is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

ReplaceAll::reps : (Which[f2 < 0.0238717, «8>>, {«l>>}] is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

General::stop : Further output of ReplaceAll::reps will be suppressed during this calculation.
Plot[Evaluate[profitsNew[4.2/n]], {n, 190, 110}]

ReplaceAll::reps : 
Which[\(\frac{-20}{n} < 0.0238717, \langle 9 \rangle\)] is neither a list of replacement rules nor
a valid dispatch table, and so cannot be used for replacing.

ReplaceAll::reps : 
Which[\(\frac{-20}{n} < 0.0238717, \langle 9 \rangle\)] is neither a list of replacement rules nor
a valid dispatch table, and so cannot be used for replacing.

ReplaceAll::reps : 
Which[\(\frac{-20}{n} < 0.0238717, \langle 9 \rangle\)] is neither a list of replacement rules nor
a valid dispatch table, and so cannot be used for replacing.

General::stop :
Further output of ReplaceAll::reps will be suppressed during this calculation.

- Graphics -

Plot[Evaluate[profitsNew[4.2/n]][[1]], {n, 190, 110}, AxesLabel ->
{FontForm["N", {"Times-Italic", 12}], FontForm["Profits",
{"Times-Italic", 12}]}, AxesOrigin -> {110, Automatic}]
Plot[Evaluate[profitsNew[4.2/n]][[2]], {n, 190, 110}, AxesLabel ->
{FontForm["N", {"Times-Italic", 12}], FontForm["Profits",
{"Times-Italic", 12}]}, PlotStyle -> Dashing[{.01, .01}]]

ReplaceAll::reps :
Which[\(\frac{-20}{n} < 0.0238717, \langle 9 \rangle\)] is neither a list of replacement rules nor
a valid dispatch table, and so cannot be used for replacing.

ReplaceAll::reps :
Which[\(\frac{-20}{n} < 0.0238717, \langle 9 \rangle\)] is neither a list of replacement rules nor
a valid dispatch table, and so cannot be used for replacing.

ReplaceAll::reps :
Which[\(\frac{-20}{n} < 0.0238717, \langle 9 \rangle\)] is neither a list of replacement rules nor
a valid dispatch table, and so cannot be used for replacing.

General::stop :
Further output of ReplaceAll::reps will be suppressed during this calculation.
ReplaceAll::reps:
{Which[\(-20/n < 0.0238717\), \(-9\)]} is neither a list of replacement rules nor
a valid dispatch table, and so cannot be used for replacing.

ReplaceAll::reps:
{Which[\(-20/n < 0.0238717\), \(-9\)]} is neither a list of replacement rules nor
a valid dispatch table, and so cannot be used for replacing.

ReplaceAll::reps:
{Which[\(-20/n < 0.0238717\), \(-9\)]} is neither a list of replacement rules nor
a valid dispatch table, and so cannot be used for replacing.

General::stop:
Further output of ReplaceAll::reps will be suppressed during this calculation.
■ Check for effect of switch of first entrant on second firm

Value of f2 such that max prof with 1 L,1s, but deterrence with 2s

\[ f2 = 0.11 \]

Locations

\{x[1] \to 0, x[2] \to 1\}

\{x[1] \to \text{deter}[f2], x[2] \to 1\}

\{x[1] \to 0.0745671 + 1.4803 \times 10^{-16} \text{I}, x[2] \to 1\}

daemand

\(((\text{demand}[1] / . \text{prices2Firms}) / . \{c[1] \to 0, c[2] \to 1/8, x[1] \to 0, x[2] \to 1\}) / n)[[1]]\]

25
48

N[\%]

0.520833

\(((\text{demand}[1] / . \text{prices2Firms}) / . \{c[1] \to 1/8, c[2] \to 1/8, x[1] \to \text{deter}[f2], x[2] \to 1\}) / n)[[1]]\]

0.512428 + 2.46716 \times 10^{-17} \text{I}

Prices

\text{prices2Firms} / . \{c[1] \to 0, c[2] \to 1/8, x[1] \to 0, x[2] \to 1\}

\{\{p[1] \to \frac{25}{24}, p[2] \to \frac{13}{12}\}\}
The second entrant gains from the switch of the first \( \Rightarrow \) positive externality!

Derivation of the market size at the switch point

\[
\begin{align*}
n & \text{OfSmallEntrantBlockaded1largeFirm}[.11] \\
& 150.123 - 5.0042 \times 10^{-14} I \\
f & 2 n \text{OfSmallEntrantBlockaded1largeFirm}[.11] \\
& 16.5135 - 5.50462 \times 10^{-15} I \\
\text{profits2FirmsOfN}[0, 1, 1/8, n \text{OfSmallEntrantBlockaded1largeFirm}[.11]] \\
& \{ \frac{1}{18} ((1466.04 - 4.88691 \times 10^{-13} I) - 18 f[1]), \frac{1}{18} ((1240.86 - 4.13628 \times 10^{-13} I) - 18 f[2]) \} \\
\text{profits2FirmsOfN}[\text{deter}[f2], 1, 0, n \text{OfSmallEntrantBlockaded1largeFirm}[.11]] \\
& \{\{0.060032 + 9.60255 \times 10^{-18} I\} \\
& \quad ((1215.36 - 6.76909 \times 10^{-13} I) - (16.6578 - 2.66454 \times 10^{-15} I) f[1]), \} \\
& \quad (0.060032 + 9.60255 \times 10^{-18} I) \\
& \quad ((1100.32 - 8.3014 \times 10^{-13} I) - (16.6578 - 2.66454 \times 10^{-15} I) f[2]) \} \\
\text{profits2FirmsOfN}[\text{deter}[f2], 1, 0, n \text{OfSmallEntrantBlockaded1largeFirm}[.11], \\
& \quad f[2] \rightarrow f 2 n \text{OfSmallEntrantBlockaded1largeFirm}[.11]] // \text{Simplify} \\
& \{56.4469 - 2.71495 \times 10^{-14} I, 52.4232 - 1.74747 \times 10^{-14} I \}
\end{align*}
\]

The second entrant gains from the switch of the first \( \Rightarrow \) positive externality!

- DO all consumers gain from entry deterrence? DOes price for all consumers go down, if a firm moves to the center? Yes!

\[
\begin{align*}
\text{prices2Firms} / \cdot c[_\_] \\
\text{priceC0} \\
& \{\{p[1] \rightarrow \\
\end{align*}
\]
prices2Firms/. \{c[1] \rightarrow 0, c[2] \rightarrow 1/2\}

\{\{p[1] \rightarrow \frac{1}{3}\left(\frac{1}{2} - 2 x[1] - x[1]^2 + 2 x[2] + x[2]^2\right),

Utility of the consumer located at 0 as a function of the location of the first entrant

\[\text{NegOfUtility} = t (x[1])^2 + p[1] \cdot \text{prices2Firms} /. \{c[1] \rightarrow 0, c[2] \rightarrow 1/2, x[2] \rightarrow 1\}\]

\{x[1]^2 + \frac{1}{3}\left(\frac{7}{2} - 2 x[1] - x[1]^2\right)\}\]

\[t = 1\]

1

\[\text{Plot}[\text{Evaluate}[\text{NegOfUtility}], \{x[1], 0, 1/2\}]\]

Utility increases clearly, because of fall in costs.