Abstract: This paper analyses the adoption and diffusion of new technology in a market for a differentiated product with monopolistic competition. It is shown that in a noncooperative equilibrium ex-ante identical firms adopt a new technology at different dates. This equilibrium can be described by a simple distribution function. For non-identical firms, the conditions are stated under which a positive relationship between firm size and speed of adoption exists. It is demonstrated that increased competition often promotes diffusion. Diffusion is shown to occur more slowly in the noncooperative solution than in a constrained social optimum.

Keywords: Adoption, diffusion, monopolistic competition

JEL-Classification: O31

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1. Introduction

The productivity and welfare enhancing powers of a new product are only realized when the product is adopted by its potential users. With respect to technological change the adoption of new technologies ranks in importance with firms’ research and development activities (see OECD 1997). Firms usually adopt new technologies sequentially rather than simultaneously, and in most cases the vast majority of eventual users adopt well after the introduction of a new product. Empirical studies have shown that observed patterns of adoption often can be described by S-shaped or sigmoid diffusion curves.\(^1\)

This paper is a contribution to explanations of these regularities which take the adoption decision of an individual firm as the starting point.\(^2\) This decision is explained explicitly out of the profit maximizing behavior of the firm. Adoption costs are assumed to be falling over time. Diffusion, i.e. sequential rather than simultaneous adoption by individual firms, may result for two reasons.

a) Rank effects: the potential users differ with respect to the (expected) returns from adoption. Reasons are differences in firm size, R&D expenditures, market shares (see Karshenas/Stoneman 1993, 1995) or even the prior beliefs about the profitability of a new technology (see Jensen 1982).

b) Stock effects: in an industry with a priori identical firms, diffusion will occur if payoffs from adoption depend on the stock of firms already using a new technology. This was shown by Reinganum (1981a, b). She assumed a pattern of profits which emerges, for instance, in a Cournot oligopoly.

The model of monopolistic competition presented in this paper allows the integration of both effects and enables one to address Schumpeterian themes in the analysis of the adoption of a new technology. In particular, the influence of both firm size and the intensity of competition on the pace and pattern of diffusion can be examined.

Monopolistic competition is formalized here using a derivation of Dixit/Stiglitz (1977) and Spence (1976). The ‘large group’ assumption (Chamberlin 1965) underlying this market structure ensures virtually no strategic interaction among firms. Nevertheless, individual firms have monopoly power due to producing a special brand of a differentiated product. Because of

\(^{1}\) For an overview see Thirtle/Ruttan 1987

\(^{2}\) A different branch of studies on diffusion is based on the ‘epidemic’ approach, where the rate of adoption at a certain date depends on the stock of users who have already adopted. This setting yields a differential equation and results in the above described diffusion pattern. For further discussion of this approach as well as for a survey of the equilibrium models and of the empirical findings of the analysis of diffusion see Karshenas/Stoneman (1995).
these two features, models of monopolistic competition combine the simplicity of decision-theoretic models with the notion of competition and rivalry prevalent in strategic (or game-theoretic) models.\(^3\)

The model presented here uses a setting similar to Reinganum (1981a,b). The results of this paper can, therefore, be most easily evaluated by comparing them to the results of Reinganum’s so called ‘game-theoretic’ approach. The equilibrium in my model can be described by a simple distribution function. Diffusion, rather than simultaneous adoption, is the outcome of the model. This holds true even for ex-ante identical firms. My model shows that it is not the existence of strategic interactions which leads to diffusion but rather an impact of other firms’ actions on the payoffs of an individual firm. It is the result of stock effects. For the case of 'small' agents, this result confirms a principle stated by Quirmbach (1986): stock effects imply an asymmetry in payoffs of adoptions which give rise to differing rather than uniform adoption dates in both market and planner solutions.

In the equilibrium of my model, the profits of firms adopting at different dates are equalized. Rent equalization is not a feature of Reinganum’s models. In the Nash-equilibria derived by Reinganum, ‘the early adopter does better than the later one’ as Fudenberg and Tirole (1985, p. 384) show. Reinganum’s result is due to the assumption that firms must precommit to an adoption date at the outset of the game. If firms (in a duopoly) can observe and respond immediately to their rival’s actions, profits are equalized due either to pre-emptive adoption or to collusion with a 'late' uniform adoption date (see Fudenberg and Tirole 1985). My model shows that neither pre-emptive adoption nor collusion occur if the agents are 'small'. In a sense, the model derives Reinganum type equilibria for the case of no precommitment.

In contrast to Fudenberg and Tirole’s approach, in my model the number of firms can be varied without posing problems. This makes it is easy to address the question of market entry. Although I will assume a fixed number of firms most of the time, in some examples, I will explore the effect of free entry. The model of monopolistic competition can also take into account heterogeneity among firms. Particularly, I consider differences in firm size. The model predicts a positive relation between firm size and the speed of adoption. This result presumes that adoption costs are independent of firm size.

Two further properties of the model are worth mentioning. The model makes it possible to derive hypotheses for empirical studies directly from the explicit equilibrium distribution rather than from first order conditions alone. As I will show, in the case of stock effects the former method predicts signs for coefficients which are the opposites of the ones predicted by the latter method. Furthermore, increased competition is shown to often promote diffusion and

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\(^3\) For the classification see Beath et al. (1995).
to lead invariably to an earlier start of the diffusion process. Note that more competition here means a lower degree of (exogenous) product differentiation and therefore more intense price competition.

In order to address policy questions, I derive a second-best adoption pattern. I show that the adoption of a new technology takes longer in the market solution than in the socially optimum solution. Quirmbach (1986) derives the opposite result in a Cournot model with homogeneous goods.

The remainder of the paper is organized as follows: In Sections 2 and 3 the basic model is described, and the noncooperative equilibrium is derived. In these two sections the number of firms is assumed to be fixed. In Section 4 the question of entry is addressed. Section 5 is devoted to an integration of rank effects into the basic model and an examination of the implications which can be drawn from stock effects for empirical studies. Section 6 examines the influence of changes in different parameters on the equilibrium distribution, and in Section 7 the welfare analysis is presented. Section 8 concludes.

2. The basic model

I consider the evolution of an industry in continuous time. In this industry, a continuum of firms produces different varieties of a differentiated product. For the moment I assume that there is no entry, but a fixed measure $n$ of active firms. For simplicity I refer to this measure as the ‘number of firms’. The question of entry will be dealt with in Section 4.

The preference ordering of identical consumers is described by the intertemporal utility function

$$U = \int_0^\infty e^{-rt}(x_0(t) + \log C(t))dt,$$

(1)

where $x_0(t)$ is the consumption of the numeraire in time $t$ and $C(t)$ is a consumption index of the Dixit-Stiglitz type with

$$C(t) = \left(\int_0^n y(j, t)^\alpha dj\right)^{1/\alpha} \text{ and } 0 < \alpha < 1.$$ 

Here $y(j, t)$ is the amount of variety $j$ of the differentiated product which is demanded by a consumer at time $t$. Because of the quasi-linear instantaneous utility function, the demand function for the differentiated goods does not change over time. Furthermore, demand is independent of the consumers' income, as long as total discounted income of each consumer is greater than $1/r$. I assume this to be the case. I denote the number of consumers by $E$. As each consumer's spending on the differentiated product is equal to one, $E$ is equal to the total instantaneous expenditure on the differentiated product. With the above assumptions, one gets
the instantaneous aggregate demand function $Y(j, t)$ for variety $j$ at time $t$ (see, for instance, Grossman/Helpman 1991, Chapter 3):

$$Y(j, t) = \frac{p(j, t)^{(1-\alpha)}}{\int_0^\infty p(z, t)^{\alpha/(1-\alpha)} \, dz} E,$$

where $p(j, t)$ is the price of variety $j$ in time $t$.

The demand function (2) is isoelastic with the elasticity of demand $\sigma = 1 / (1 - \alpha)$. Actions of rivals which result in price changes enter the demand function through the integral in the denominator. In this paper this term is referred to as the ‘price index’, although it differs from the price index used by Dixit/Stiglitz (1977) and does not have all the properties of a conventional price index. As is well known (see Dixit/Stiglitz 1977 and Grossman/Helpman 1991, Chapter 3), this demand function gives rise to a simple mark-up pricing rule for given marginal costs $c$. For the profit maximizing price $p$ one gets

$$p = c / \alpha.$$

Firms produce with constant marginal costs $\bar{c}$. In time $t = 0$ a new technology becomes available which, once adopted, allows production with lower marginal costs $\hat{c}$. The discounted costs $X$ of purchasing the new technology and integrating it in the production process depend on the date $T$, at which production should take place at the lower marginal costs. The function $X(T)$ is assumed to be decreasing and convex in $T$ so that $X'(T) < 0$ and $X''(T) > 0$. I also assume $X(0) = \infty$ and $X(\infty) = 0$. With this adoption cost function, earlier adoption is more expensive, and eventually all firms adopt.

The adoption cost function $X(T)$ allows for two different interpretations with respect to the underlying adoption process in the model presented here. First, the adoption process could be understood as a ‘time-consuming activity’ which implies a certain adjustment path (see Reinganum 1989, p. 897). Second, adoption costs could consist only of the cost of purchasing the new capital-embodied technology. Firms are then able to choose instantaneous adoption at every moment of the game (see Fudenberg/Tirole 1985). An ongoing decrease of the price of the respective capital good may be due to technical progress.

The operating profits $\pi$ of a single firm can be determined as a function of its own and of its rivals’ behavior. Operating profits are defined as the difference between revenue and variable costs. Because of the above mentioned pricing rule, the price a firm charges depends only on whether or not the firm has adopted the new technology. The adoption decision determines the level of marginal cost. The actions of rivals enter the firm’s profit function via the price index. This term may be simplified, based on the fact that competitors will also charge prices according to the above rule. Assuming that a fraction $q$ of the rivals have already adopted, the price index may be written as follows:

$$\int_0^\infty p(z)^{\alpha/(1-\alpha)} \, dz = \alpha^{\alpha/(1-\alpha)} n \left[ q (\hat{c}^{\alpha/(1-\alpha)} - \bar{c}^{\alpha/(1-\alpha)}) + \bar{c}^{\alpha/(1-\alpha)} \right],$$

$$p = c / \alpha.$$
In the derivation of this term, the low cost firms which have already adopted the new technology are ordered in the interval \([0, q_n]\).

Using the pricing rule and the term for the price index, operating profits are

\[
\pi^{q(t)}_{\alpha}(t) = (1 - \alpha) \frac{c^{a/(a-1)} - e^{a/(a-1)}}{\{q(t)(c^{a/(a-1)} - e^{a/(a-1)}) + e^{a/(a-1)}\} n^E}
\]  

(5)

for a firm which has not yet adopted by time \(t\), where \(q(t)\) is the fraction of all firms which have adopted before \(t\). Henceforth, subscripts 1 and 0 indicate whether or not a firm has adopted. The superscripts denote the fraction of rivals who already use the new technology. The flow of profits is

\[
\pi^{q(t)}_{\alpha}(t) = (1 - \alpha) \frac{c^{a/(a-1)}}{\{q(t)(c^{a/(a-1)} - e^{a/(a-1)}) + e^{a/(a-1)}\} n^E}
\]  

(6)

for a firm which already has adopted in \(t\). The adoption behavior of the rivals enters the profit function of a single firm only by the fraction \(q\) of firms which have adopted. Because this fraction changes over time it is described as a function of \(t\).

Before deriving the noncooperative solution of the model, I want to mention some properties of the payoffs (5) and (6).

**Property P1**: The operating profit of a firm decreases as the number of firms producing with the new technology increases.

**Property P2**: The profit increase \(\pi^{q(t)}_{\alpha}(t) - \pi^{q(t)}_{\alpha}(t)\) induced by the adoption diminishes as the share of firms which already have adopted grows.

The second property is especially interesting in connection with Reinganum (1981a, b) and Quirmbach (1986). It shows that the demand functions derived from the Dixit-Stiglitz utility function imply a payoff structure similar to the one Reinganum assumed for the case of a discrete number of firms: „the increase in profit rates due to adopting \((m - 1)\)th is greater than that due to adopting \(m\)th“ (Reinganum 1981b, p. 619). According to Quirmbach (1986) this satisfies a necessary condition for diffusion. In a recent empirical paper, Stoneman and Kwon (1996) show that profit flows in actual diffusion processes exhibit property P2. They derive significant stock effects in the case of the adoption of microprocessors in the UK engineering and metalworking sector, thus underpinning the importance of developments in the output market for a firm’s decision to adopt a new technology.
3. The noncooperative equilibrium of the model

The starting point for the derivation of an equilibrium distribution \( q(t) \) is the individual firm’s choice of the optimal adoption date.\(^4\) A firm maximizes the discounted value of total profits by choosing the adoption date \( T \). Denoting the profit function as \( \Pi(T) \), the optimization problem of the firm is as follows:

\[
\max_T \Pi(T) = \int_0^T e^{-rT} \pi^0(t) \, dt + \int_T^\infty e^{-rT} \pi^1(t) \, dt - X(T)
\]  
(7)

The profit depends on the firm’s own adoption date as well as on the adoption dates of its rivals summarized by the distribution function \( q(t) \) with \( t \in [0, \infty) \). The distribution \( q(t) \) indicates how firms spread across potential adoption dates. The range of \( q(t) \) is the unit interval. If the distribution function is continuous, the profit function can be differentiated with respect to \( T \). One gets the first order condition of the profit maximization problem:

\[
\frac{d\Pi(T)}{dT} = e^{-rT} \left( \pi^0(T) - \pi^1(T) \right) - X'(T) = 0.
\]  
(8)

The first term of the derivative in (8) gives the (discounted) value of the decrease in profits induced by adopting marginally later. In the optimum this must be equal to the resulting decrease in cost. The cost decrease is the second term.

In order to get a concave profit maximization problem I make the following assumption.

**Assumption 1:**

\[ re^{-rT} \left( \pi^0_0 - \pi^0_1 \right) - X''(T) < 0 \]  
for all \( T \).

This assumption ensures that the second order condition is satisfied for a firm whose rivals never adopt. Furthermore it is assumed that a finite solution of the optimization problem exists both for a single adopting firm and for a firm choosing to adopt later than all of its rivals. In the latter case, \( q(t) = 1 \) holds.

Now I am ready to derive the equilibrium distribution. This is done in Proposition 1. Before that I make a definition and state 2 lemmas.

A distribution \( q(t) \) is a Nash equilibrium distribution if it satisfies the following conditions:

a) Given that the rival firms are distributed across the adoption dates according to \( q(t) \), the profit \( \Pi \) of a single firm must be equal for all dates for which the density is positive. This requirement is necessary because none of the ex ante identical firms would choose an adoption date for which the payoff is less than for other dates. The density cannot be positive at that date.

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\(^4\) Here a heuristic approach is used in the sense that I choose individual firms as a starting point although a single firm is of no importance at all when the industry consists of a continuum of firms. It serves to develop an economic intuition for the results derived later on.
b) For all dates where the density vanishes, profits must be less than or equal to profits at dates with a positive density. This is a usual condition for a Nash equilibrium; an equilibrium strategy must not be dominated by another strategy.

From the first order condition (8), Lemmas 1 and 2 follow (for Proofs see Appendix):

**Lemma 1:** The range for which the density can be strictly positive is restricted to the interval \([T_L, T_H]\) where \(T_L\) and \(T_H\) are defined by

\[
e^{-\pi T} \left( \pi_i^0 - \pi_0^0 \right) = -X'(T), \quad \text{and} \quad (9)
\]

\[
e^{-\pi T} \left( \pi_i^1 - \pi_0^1 \right) = -X'(T). \quad \text{resp.} \quad (10)
\]

**Lemma 2:** The density function is strictly positive in the interval \([T_L, T_H]\).\(^5\)

Now I am ready to state the main proposition of this section of the paper.

**Proposition 1:** The function

\[
q(t) = \begin{cases} 
0 & \text{for } t \leq T_L \\
- e^{-\pi} \frac{(1 - \alpha)E}{X(t)\pi} - \frac{\tilde{e}^{\alpha/(a-1)}}{(\tilde{c}^{\alpha/(a-1)} - \tilde{e}^{\alpha/(a-1)})} & \text{for } T_L \leq t \leq T_H \\
1 & \text{for } T_H \leq t 
\end{cases} \quad (11)
\]

is a Nash equilibrium distribution in adoption dates for the industry considered. It describes the unique equilibrium share of firms making use of the new technology at some time \(t\). This share is strictly increasing in the interval \([T_L, T_H]\).

**Proof:** From Lemma 1 and Lemma 2 it immediately follows that for \(q(t)\) to be an equilibrium distribution it has to satisfy the following conditions:

\[
\frac{d\Pi(T)}{dT} = 0 \quad \text{for all } T \in [T_L, T_H] \quad \text{and} \quad (12)
\]

\[
q(t) = 0 \text{ if } t < T_L \text{ and } q(t) = 1 \text{ if } t > T_H. \quad (13)
\]

From (12) the equilibrium distribution can then be determined explicitly (for the corresponding range). Making use of (5) and (6) in (8), the condition (12) becomes

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\(^5\) Note that this lemma excludes an equilibrium in which all firms choose a uniform adoption date. The reason for this non-existence is the non-differentiability of the profit function for dates there a jump in the distribution function occurs. For a further discussion see Reinganum 1981a, Götz (forthcoming) or the working paper version of this paper which is available from the author upon request.
This condition can now be solved for \( q(t) \), and one immediately gets the respective part of equation (11). Because there is only a single solution for this simple equation, the resulting distribution function must be unique.

Continuity of the distribution function, which is asserted in the above proposition by using weak inequalities, is proved by inserting \( T_L \) and \( T_H \) in (11) and taking into account the conditions (9) and (10). Together with assumption 1, the first order condition (8) ensures that \( q(t) \) is strictly increasing. This is equivalent to the density being positive over the interval \( [T_L, T_H] \).

\[
\frac{d\Pi(T)}{dT} = e^{-rT} \left( 1 - \alpha \right) \frac{\tilde{c}^{\alpha/(a-1)} - \tilde{c}^{\alpha/(a-1)}}{q(T) \left( \tilde{\tilde{c}}^{\alpha/(a-1)} - \tilde{c}^{\alpha/(a-1)} \right)} - X'(T) = 0. \tag{14}
\]

Q.E.D.

The equilibrium derived here is a flow of adoptions between \( T_L \) and \( T_H \) such that all firms make the same profits. The result provides no information about the behavior of a single firm in equilibrium. Nevertheless the proportion of firms which have already adopted at a certain time is unique. It is instructive to compare the equilibrium derived here with the results of Reinganum (1981a, 1981b) and of Fudenberg and Tirole (1985). A common feature of all of these approaches is the occurrence of diffusion. In contrast to Reinganum, however, the model presented here does not exhibit different profits for firms adopting at different dates. The main objection of Fudenberg and Tirole does not apply to my approach. Contrary to Fudenberg and Tirole, pre-emptive adoption or collusion to a 'late' uniform adoption date cannot occur in the equilibrium of my model. No firm will adopt earlier than \( T_L \), the adoption date of a firm whose rivals never adopt, or later than \( T_H \). The reason is that the agents are 'small' in the Chamberlin model. The actions of a single firm do not affect the payoffs of its rivals and therefore firms cannot act strategically. The resulting equilibrium is similar to Reinganum's with the exception of a continuum of agents and rent equalization.

4. Sunk costs, uncertainty, and entry

The above equilibrium distribution has been derived under the assumption of a fixed number of firms. In a model of monopolistic competition the question of how free entry would change the results arises. I will examine this question for two different cases. The first case is the existence of sunk entry costs, the second the combination of sunk costs with uncertainty about the new technology.

Entry into markets for differentiated products is often assumed to cause sunk costs. These costs may either consist of the R&D expenditures necessary to design a new variety of the product or of the advertising expenditures for creating a brand image. They seem to be substantial in many markets (see, for example, Romer 1990 and Sutton 1991). Suppose now that the start of the industry is in \( t_0 = 0 \). A consequence of sunk costs is that profitable entry
cannot take place at some date \( t_1 > 0 \). Firms entering at \( t_0 = 0 \) would earn strictly positive profits in this case because of the flow of operating profits between \( t_0 \) and \( t_1 \). At \( t = 0 \) firms will enter the industry as long as the discounted equilibrium profits (denoted above as \( \Pi(T) \) for a firm adopting at \( T \)) are greater than the sunk costs. In the appendix it is shown that the profits decrease when the number of firms increases. Given the sunk costs and parameters relating to the new technology, one gets a unique number of active firms.

Under the set-up just described, changes in parameters related to the new technology or policy measures aimed towards its adoption would result in changes in the number of active firms. This, however, need not be the case, if characteristics of the new technology are uncertain at the start of the industry. Suppose, for instance, that the only feature of the new technology which is uncertain, is the extent of the cost reduction. Assume further that the industry emerges at some date \( t_e < 0 \) and that the realization of the cost parameter becomes common knowledge at \( t = 0 \). Note that different realizations imply that total profits from time \( t = 0 \) onwards (that is \( \Pi(0) \)) differ. Entry cannot take place in \( t = 0 \), if the value of the profit flow between \( t_e \) and \( t = 0 \) is at least as large as the difference in the discounted profits from the smallest and the largest cost reduction. The reason is that \( \Pi(t_e) \) is, even in the worst case, not smaller than \( \Pi(0) \) in the best case regardless of the realization of the cost parameter. Of course, one can always find values for \( t_e \) so that no entry in \( t = 0 \) occurs. While expectations about future innovations or future policy measures influence the number of firms entering in \( t_e \), in many cases the realizations will have no further effect on the number of firms. This seems to be especially the case for innovations arising in mature industries. As I am interested mainly in the adoption and diffusion of specific innovations which are already well known, I will generally take the number of firms as given in the remainder of the paper. However, I will point out possible relationships between the number of firms and other parameters of the model, when this relationship matters for the questions addressed.

5. Rank and stock effects

5.1 Rank effects

In this subsection I present an extension of the basic model in order to capture rank effects. It is also shown how the introduction of some differences in firms’ characteristics solves the problem, that the basic model does not determine the adoption date of individual firms but only the aggregate diffusion pattern. A simple way to incorporate rank effects into the basic model is by adding an additional parameter \( A(j) \) into the Dixit-Stiglitz consumption index \( C(t) \) in the following way:

\[ C(t) = \sum_{j} A(j) D(t-j) \]

More detailed examples can be found in the working paper version of this paper.
\[ C(t) = \left( \int_0^t (A(j)y(j,t))^{\frac{1}{\alpha}} \right)^{\frac{1}{\alpha}}. \]

With respect to \( A(j) \), assume that it is a continuous and monotonously decreasing function of the firm index \( j \). That is, firms are ranked in terms of the parameter in such a way, that firms with higher consumer valuations have a lower index number. The parameter captures the idea that consumers value some varieties more than others, be that for reasons of product design or other product characteristics. The aggregate demand function resulting from this specification is

\[
Y(j,t) = \frac{A(j)^{\alpha/(1-\alpha)} p(j,t)^{1/(\alpha-1)} E}{\int_0^a p(z,t)^{\alpha/(\alpha-1)} A(z)^{\alpha/(1-\alpha)} dz}. \quad (15)
\]

(15) shows that higher values of \( A \) imply higher demand ceteris paribus and that firm sizes are different. The optimal pricing rule for an individual firm does not change when \( A(j) \) is introduced. Given an arbitrary adoption pattern of the rivals, the gain from adopting the new technology turns out to be

\[
\pi_i(j) - \pi_i(0) = \frac{(1-\alpha)A(j)^{\alpha/(1-\alpha)} E}{\int_0^a c(j)^{\alpha/(\alpha-1)} A(j)^{\alpha/(1-\alpha)} dj} \left( c^{\alpha/(\alpha-1)} - \overline{c}^{\alpha/(\alpha-1)} \right). \quad (16)
\]

This term is increasing in \( A(j) \), therefore the gain from adoption is always greater for larger firms, as it is assumed in the rank models.

The following proposition states the consequences of the heterogeneity introduced in this section.

**Proposition 2:** i) A firm \( j \) will adopt earlier in equilibrium than a firm \( i \) if \( A(j) > A(i) \). That is, larger firms will adopt earlier than the smaller ones.

ii) Diffusion will start earlier and take longer in the model with heterogeneous firms than in the basic model.

**Proof:** See appendix.

By assuming a specific functional form for \( A(j) \), namely

\[ A(j) = 1 + \varepsilon \left( \frac{1}{2} - \frac{j}{n} \right), \quad \text{where } \varepsilon > 0 \]

it can be seen that small differences between firms lead to unique adoption dates for individual firms. As \( \varepsilon \) converges to 0 the diffusion pattern converges to the equilibrium distribution of the basic model, but the adoption dates of individual firms are unique.

A simple example shows that the relation between firm size and date of adoption breaks down, if one abandons the indivisibility assumption with respect to the new technology. If one were to allow for a linear relationship between firm size and adoption costs, the connection between firm size and date of adoption would vanish. To see this, assume that \( X(T) \) describes
the per unit costs for adoption, and that total adoption costs are just $X(T) y_{LR}$, where $y_{LR}$ is the long run production level. The first order condition is

\begin{equation}
e^{-rT} \left( \pi_n(j) - \pi_s(j) \right) - X(T)y_{LR}(j) = 0. \end{equation}

Inspection of (15) and (16) immediately shows that the terms which differ for different firms (the $A(j)$ terms in the numerators) cancel out. Therefore, firm size does not matter under these circumstances.

While ‘most of the empirical work (...) has found a positive relation between firm size and speed of adoption’ (Karshenas and Stoneman 1995), there are exceptions (see, for instance, Colombo and Mosconi 1995). Contrary to pure rank effects models, stock effects models are able to explain diffusion also in the case where adoption costs for new technologies are not independent of firm size.

5.2 Stock effects and the hazard rate

Turning back to the basic model, I want to examine the conclusions which can be drawn from stock effects for duration models. This kind of econometric model is quite common in empirical studies of diffusion processes. Typically, the hazard rate $h$ is estimated, where $h$ is ‘defined as the probability of a firm adopting the new technology in the small time interval $t + dt$, conditional on having not adopted the technology by time $t$’ (Karshenas and Stoneman 1995, p. 285). Starting with Karshenas and Stoneman (1993), these models have only recently been extended to test for the empirical relevance of stock effects. Using a first order condition like (8) and adding a stochastic error term, Karshenas and Stoneman (1993) derive the following hypothesis: The hazard rate should be negatively related to the stock of users of the new technology, if the stock effect is to be significant.

Their conclusion draws on property P2 mentioned above. This property also holds in their model. What is important is that their hypothesis is derived from a first order condition and not from the equilibrium (distribution) itself. I now want to show that taking the equilibrium distribution leads to the opposite hypothesis: a positive relation between the hazard rate and the stock of users of the new technology is to be expected, if stock effects are important.

The hazard rate $h$ is defined as $h(t) = q(t)/(1 - q(t))$, where the dot denotes the time derivative which is equivalent to the density function here. In order to simplify calculations I use the explicit adoption cost function $X(t) = e^{-(r+\beta)t}$, where $\beta$ is a positive constant capturing the decrease in cost induced by technical progress (see Fudenberg and Tirole 1985). With this specification, one gets the density function

\begin{equation}
q(t) = \frac{\beta(1 - \alpha)E}{(r + \beta)^n} e^{\beta t}.
\end{equation}

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7 Long run here denotes the period after the diffusion process in the industry has stopped. I therefore assume for simplicity that firms can produce above long run capacity for a transition period.
As this term is increasing in $t$, and the denominator of the hazard function must be nonincreasing, the hazard rate must be an increasing function of time. Because latter dates must be associated with greater values of the stock of users $q$, empirical studies returning a positive coefficient on the stock of users would be in accordance with the basic model.

Colombo and Mosconi (1995) find a positive but not significant coefficient on the number of competitors already using the new technology. The respective coefficient in Karshenas and Stoneman (1993) is positive and significant, but having the opposite hypothesis, the authors take that result as indicating epidemic effects. The above analysis shows that a distinction between stock and epidemic effects cannot be easily drawn. Another point worth stressing is that deriving hypothesis only from first order conditions may be misleading.

6. The effects of parameter changes: Some comparative static results

In this section I turn to the question of how the diffusion pattern may vary with changes in parameters. Because of the representation of the equilibrium by an explicit function, comparative static results can be derived in a simple way. The effects of an increase in the flow of expenditures $E$, of an increase in the cost reduction and of a decrease of the interest rate $r$ are discussed in detail in the working paper version of this paper. There it is shown that all of these changes speed up diffusion, as long as the number of firms is assumed to be constant. Here I want to focus on the effects of two parameters, namely the number of firms $n$ and the parameter $\alpha$, which is related to the elasticity of substitution and the elasticity of demand. Both of these parameters allow the inclusion of the Schumpeterian themes of the relation of firm size, competitiveness, and new technology. In the Chamberlin model, changes in $n$ induce changes in firm size only, while the parameter $\alpha$ can be regarded as a measure of competitiveness. I first want to consider the effects of these two parameters separately. As changes in $\alpha$ induce large changes in operating profits, I examine the case of an endogenous number of firms afterwards. The effect of an exogenous change in $n$ is stated in the next proposition.

**Proposition 3:** An increase in the number of firms $n$ implies a decrease of $q(t)$ for every $t$ in the range in which adoption takes place. Diffusion occurs more slowly, and the share of firms using the new technology is smaller at any time than in the original situation.

**Proof:** By partially differentiating (11) the proposition is obtained immediately. Q.E.D.

---

8 Of course the simplicity is due to the explicit utility and demand function used. It seems to be essential to use special functional forms in order to derive a wide range of results. This can be seen from Reinganum (1981b) and Quirmbach (1986), who use linear functions.

9 Changes of $n$ can be the result of changes in either sunk costs or in time of market birth.
The increase in \( n \) implies a decrease in average firm sales measured by \( E/n \). As the above discussion of rank effects shows, decreasing the firm size delays adoption, if adoption costs are independent of the firm size.

The effect of the parameter \( \alpha \) on the diffusion pattern is ambiguous, if one holds fixed the number of firms \( n \). This can be seen from

\[
\frac{\partial q(t)}{\partial \alpha} = e^{-rt} \frac{E}{X'(t) n} \log(\frac{\hat{c}}{\bar{c}}) \left( \frac{\hat{c}}{\bar{c}} \right)^{\alpha/(a-1)} \left( \frac{\hat{c}}{\bar{c}} \right)^{-1} (1-\alpha)^2
\]

which is the derivative of the relevant part of the equilibrium distribution (11) with respect to \( \alpha \). The first term on the right-hand side of (18) is negative (\( X'(t) < 0 \)) and the second one is - inclusive of the minus sign - positive (note that \( \hat{c} / \bar{c} < 1! \)). On the one hand, the increase in \( \alpha \) implies a greater elasticity of demand \( 1/(1-\alpha) \), and leads, therefore, to a smaller mark-up and, ceteris paribus, to a smaller return to adoption. On the other hand, a given cost reduction now implies a larger shift of demand from the dearer to the cheaper varieties because of the increased elasticity of substitution. A given price reduction leads to a larger increase in demand; the new technology provides a larger profit gain and, therefore, induces earlier adoption.

The results of a simulation, presented in figure 1, give more information about the influence of \( \alpha \) on the diffusion pattern.

- Figure 1: The diffusion pattern as a function of the parameter \( \alpha \).

The functional forms and parameters used are \( X(T) = \int_0^T e^{-rt} \beta^b dt \), \( E = 10 \), \( \hat{c} / \bar{c} = 2 / 3 \), \( r = 1 / 10 \), \( n = 2 \), \( b = 2 \).

The diagram suggests that an increasing degree of competitiveness in an industry not only leads to an earlier start of the diffusion process, but also to an expansion of this process. The time interval between the earliest and the latest adoption widens. As the case with \( \alpha = 9/10 \)
shows, the second effect may be so strong that diffusion is completed later than in the case of less competition.

As noted above, an increase in \( \alpha \) will decrease the number of firms as the mark-up and profits go down. The associated increase in firm size induces firms ceteris paribus to adopt earlier. In order to get the total effect of the change in \( \alpha \) on the diffusion pattern, I make the number of firms endogenous. An extended version of the above simulation gives the following results: Increasing \( \alpha \) leads to a shift of the diffusion curve to the left, if the impact of \( \alpha \) on \( n \) is taken into account. Although the time elapsed between the first and the last adoption is longer in industries with a higher degree of competition, the last adopters adopt earlier than in the case with less competition. Therefore, increased competition promotes diffusion in the set-up used in the simulation.\(^{11}\)

The results derived here shed some light on Reinganum's (1981b) results for a Cournot oligopoly with homogeneous goods. Using a linear demand function, Reinganum shows that (most) firms will delay adoption, if the number of firms is increased.\(^ {12}\) The above discussion indicates that from the two effects which are associated with changes in \( n \) in the Cournot model, namely a firm size effect and a change in the market power, the former one apparently dominates the latter one. As the analysis of the effects of \( \alpha \) shows, more competition often speeds up diffusion, especially if the related changes in the number of firms are accounted for. Loosely speaking, more competition in the Chamberlin model means that it is more difficult

\(^{10}\) For \( \alpha \) equal to 0.85 and 0.9, respectively, I calculated the number of firms so that total profits are 0 assuming that the market emerges at \( t = 0 \), and that the sunk costs are equal to the total profits earned by firms when \( n = 2 \) and \( \alpha = 0.8 \). These high values for \( \alpha \) are of interest for two reasons: First, they include \( \alpha = 0.9 \), for which the time period between the first and the last adoption is particularly long (see figure 1). Second, they imply reasonable mark-ups between 11 (for \( \alpha = 0.9 \)) and 25 (for \( \alpha = 0.8 \)) percent (see Martins et al. 1996).

The values for the number of firms \( n \), the first adoption \( T_L \), and the last adoption \( T_H \) are depicted in the following table for the respective values of \( \alpha \):

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( n )</th>
<th>( T_L )</th>
<th>( T_H )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>2</td>
<td>0.5</td>
<td>1.2</td>
</tr>
<tr>
<td>0.85</td>
<td>146/100</td>
<td>0.3</td>
<td>1.1</td>
</tr>
<tr>
<td>0.9</td>
<td>95/100</td>
<td>0.2</td>
<td>1.0</td>
</tr>
</tbody>
</table>

More familiar values for the firm numbers are obtained by multiplying expenditure \( E \) and \( n \) by a common term, say 100. In this case nothing changes except \( E \) and \( n \).

\(^{11}\) Note that figure 1 would capture the total effect of the change in \( \alpha \), if we were in the case with (linear) size dependent adoption costs (see section 5). The change in \( n \) would not affect the diffusion pattern in that case.

\(^{12}\) Reinganum shows that there may be one firm which adopts earlier and some late adopters whose adoption date is unchanged. This case can only arise, however, if innovation is drastic in the sense that the equilibrium price falls below the original marginal costs as soon as \( m (\leq n) \) firms have adopted. The late adopters will temporarily cease production in this case. I do not consider this case here.
for the firms to differentiate their products. Price competition is tougher in this case. This relation also explains the surprising property, that more competition is associated with fewer firms in a free entry equilibrium.

My theoretical results are in line with empirical results. There are signs of a positive relationship between the degree of competitiveness and the speed of diffusion (see Kamien and Schwartz (1982), p. 100f and, more recently, Colombo and Mosconi (1995), who found a negative but insignificant influence of the Herfindahl concentration index on diffusion in their study). However, with respect to the effect of the market structure ambiguity remains (see Karshenas and Stoneman (1995)).

7. The welfare analysis

Now I turn to the welfare analysis. Because I want to focus on the efficiency properties of the diffusion pattern, I assume that the social planner takes the number of firms as given. I also assume that the planner is not able to influence the (monopoly) pricing behavior of the firms. The second-best optimum is derived as follows. The social planner maximizes the utility of the consumers. The objective function $V$ can be written as:

$$V = E \int_0^\infty e^{-rt} \left( x_0(t) + \log \left[ n^{(1-\alpha)/\alpha} \left( c^{(\alpha/(\alpha-1))} - d^{(\alpha/(\alpha-1))} \right) \right] \right) dt. (19)$$

$V$ is derived from (1) using (2), the pricing rule (3), and (4). The per-period expenditure of a consumer for the differentiated products is denoted as $s(t)$. The objective function $V$ is maximized by choosing $x_0(t)$, the expenditure $s(t)$ and the distribution function $q(t)$. Total expenditure must not be greater than total (discounted) income $I$ augmented by the profits of the firms in the differentiated goods industry. This constraint reads:

$$E \int_0^\infty e^{-rt}(x_0(t) + s(t))dt = I + \int_0^\infty \Pi(j) dj (20)$$

where

$$\int_0^\infty \Pi(j) dj = \int_0^\infty e^{-rt}(1-\alpha)Es(t)dt - nX(T) + n\int_0^T q(t)X'(t)dt. (21)$$

The expression for the operating profits is especially simple here. The reason is that every firm charges the same constant mark-up. Total per-period profits of the industry are, therefore, always a share of $(1-\alpha)$ of the expenditures on the differentiated goods $(Es(t))$. The last two terms on the rhs. of (21) are the adoption costs for the industry. These expressions are simplified by partial integration. The date $T$ denotes the last date at which the planner wants a firm to adopt.

The results for the Lagrange multiplier $\lambda$ associated with (20) and for $s(t)$, respectively, are

$$\lambda = 1 \quad \text{and} \quad s(t) = 1/\alpha \quad \text{for all } t.$$
In order to get the optimality conditions with respect to \( q(t) \), I define a Hamiltonian \( H \) which consists of the expressions which depend on \( q(t) \):

\[
H = e^{-\eta t} E \left[ (1 - \alpha) \log \left( \frac{c^{\alpha/a_i} - c^{-\alpha/a_i}}{c^{\alpha/a_i} + c^{-\alpha/a_i}} \right) + nq(t)X'(t) \right].
\] (22)

Note that I used \( \lambda = 1 \) here. The first order condition reads

\[
\frac{\partial H}{\partial q^S(t)} = \frac{(1 - \alpha)(c^{\alpha/a_i} - c^{-\alpha/a_i})e^{-\eta t} E}{\alpha q^S(t)(c^{\alpha/a_i} - c^{-\alpha/a_i}) + c^{\alpha/a_i}} + nX'(t) = 0.
\] (23)

Of course, the range for the second-best distribution \( q^S(t) \) is restricted to the unit-interval.

Comparing (23) with the corresponding equation for the market solution (14) shows that the expressions differ only by the factor \( \alpha \) in the denominator of (23). This gives the following result:

**Proposition 4:** The new technology is adopted later in the market solution than in the second-best solution. That is,

\[
q^S(t) > q(t) \quad \text{for all} \; t \in [T_L^S, T_H].
\]

Here \( T_L^S \) denotes the date of the first adoption in the second best solution. Several points about the second-best solution are worth mentioning.

First, the planner chooses the per period expenditure \( s(t) \) in a way that compensates for the differentiated goods firms’ monopoly pricing.

Second, the planner chooses diffusion rather than simultaneous adoption. As Quirmbach (1986) has demonstrated, this result is obtained, if the incremental benefit from the \( i \)-th adoption is greater than that from the \( i+1 \)-st. As can be seen from the first term of the Hamiltonian (23), in my model the marginal utility from the adoptions is a decreasing function of \( q \). Simultaneous adoption cannot be optimal. Because the above derivative also gives the marginal utility for the case of pricing according to marginal costs, the diffusion result holds in a first-best solution as well. In this respect the Chamberlin model differs from a homogeneous goods model. In the first-best solution in a world of homogeneous goods and constant marginal costs, only one firm would adopt the new technology and serve the whole market.

Third, the new technology spreads too slowly in the market solution, compared to a constrained social optimum. Quirmbach (1986) derives the opposite second-best welfare result for a Cournot model with linear demand functions: The new technology is, in general, adopted too rapidly in the decentralized solution. The model presented here shows that such a result is not an inevitable consequence of market structures with imperfect competition. The result also differs from Dixit and Stiglitz’ (1977) result, that the market solution is 'sort of ideal', if one considers monopolistic competition.
There is an interesting point to note with respect to the implementation of the second-best solution. If a planner were to introduce a subsidy, so that the expenditure $s(t)$ were equal to that in the second best solution, then the second-best diffusion pattern would result in the market solution without the necessity for further measures. Given the number of firms, larger expenditures would induce firms to adopt earlier. If one were to look for a third-best optimum, where the planner would take the expenditure $s(t)$ as given (i.e., as 1), equation (23) would still describe the optimal diffusion pattern. The market solution is not third best in that sense.

8. Conclusions

In this paper, a framework has been presented which allows the integration of two effects which feature prominently in the recent literature on diffusion, namely stock and rank effects. Testable hypotheses have been easily derived from the model’s equilibrium, a simple distribution function. The welfare analysis reveals that diffusion occurs too slowly in the decentralized solution compared to a constrained social optimum. While the result is derived for a particular market structure and special functional forms, it indicates that policy makers’ conventional wisdom about the introduction of a new technology (compare Stoneman and Diederens 1994, esp. p. 919) may not be so wrong. Policy makers often state that diffusion should be accelerated by the government.

An additional case for governmental action directed towards the diffusion of new technology would arise, if there were additional market imperfections like technological externalities. In the context of the adoption of a new technology, externalities could exist on the supply side of the new technology, for instance because of learning by doing, or on the demand side, for instance as network externalities. The framework developed here seems to be sufficiently flexible and simple to include both of these effects. Furthermore, a more explicit treatment of the supply side would be possible than the mere adoption cost function used here. Models with these components would cover the complex interactions of the demand and the supply side, which may arise in the diffusion of the new technology. The development of such models appears to be a promising task for future research.

Appendix

Proof of Lemma 1: First, I show that $T_L < T_H$. This follows from assumption 1 and the inequality $\pi_i^0 - \pi_i^0 > \pi_i^1 - \pi_i^0$. The inequality is a special case of property P2. The lemma is proven by inspecting (8) at the margins of the range of the distribution function. At these points $q(t)$ will be equal to 0 and 1, respectively and one gets the conditions (9) and (10). These conditions are satisfied for $T_L$ and $T_H$ respectively; at values smaller than $T_L$ or greater than $T_H$ profit must be less because of the second order condition. Q.E.D.
Proof of Lemma 2: To prove this statement it is demonstrated first that in equilibrium
\[ \Pi(T_L) = \Pi(T_H). \]

Suppose there would be a date \( \tilde{T} < T_H \) with positive density and \( q(\tilde{T}) = 1 \), that is all firms have already adopted in \( \tilde{T} \). In this case it would be true that \( \Pi(T_H) > \Pi(\tilde{T}) \), because, by assumption 1, the profit function is concave in the interval \([\tilde{T}, T_H]\) and its maximum value is reached at \( T_H \). Therefore a date \( \tilde{T} \) with the above asserted property cannot exist. The density has to be positive in an arbitrarily small interval with the right boundary point \( T_H \). Profits must be equal to \( \Pi(T_H) \) in this interval.

An analogous reasoning holds for \( T_L \). Here \( T_L < \tilde{T} \) and \( q(\tilde{T}) = 0 \) is chosen as a starting point. It is not possible that the share of firms having adopted is 0 until time \( \tilde{T} \). The profit of all dates in a small interval with the left boundary point \( T_L \) must amount to \( \Pi(T_L) \).

The equality of profits is proved by observing that the density is positive at the intervals at \( T_L \) and at \( T_H \), which only can be the case if the profits are equal in these intervals.

In a second step it is demonstrated that there exists no interval \([\tilde{T}, \hat{T}]\) with \( T_L < \tilde{T}, \hat{T} < T_H \) such that the density is 0 over that range. Again, the proof is by contradiction. Suppose that there would be such an interval. Then one could assume without loss of generality, that the density is positive on the interval \([T_L, \tilde{T}]\). Furthermore it would be true that \( q(\tilde{T}) = q(\hat{T}) \). The profit function is strictly concave on the interval \([\tilde{T}, \hat{T}]\) by assumption 1. Because the density is assumed to be positive to the left of \( \tilde{T} \) and 0 to the right, concavity implies that the profit is strictly decreasing, and it follows that \( \Pi(\tilde{T}) > \Pi(\hat{T}) \). This implies that the density would have to vanish in \( \tilde{T} \) and for all subsequent dates. The density could not become positive again because the share of firms having adopted is equal to that in \( \tilde{T} \), where the first order condition (8) was satisfied by assumption. But this statement contradicts the result derived above, that density must be positive in a neighborhood of \( T_H \). Therefore it is proved that the density must be strictly positive on the interval \([T_L, T_H]\). In this equilibrium, the proportion of the firms using the new technology must be strictly monotonically increasing. \( \text{Q.E.D.} \)

Proof that the discounted profits \( \Pi(T) \) decrease with the number of active firms (section 4):

Using the equilibrium distribution \( q(t) \) from equation (11) the discounted equilibrium profits \( \Pi(T) \) depicted in equation (7) can be determined explicitly. One gets
\[
\Pi(T) = \frac{(1 - \alpha)E}{nr}(1 - e^{-\alpha T_L} + e^{-\alpha T_H}) - \frac{\hat{c}^{\alpha/(\alpha-1)} X(T_H) - e^{\alpha/(\alpha-1)} X(T_L)}{\hat{c}^{\alpha/(\alpha-1)} - e^{\alpha/(\alpha-1)}} \quad \text{for all } T \in [T_L, T_H] \tag{24}
\]

Because of the envelope theorem, the sign of the derivative of the profit function with respect to \( n \) is determined solely by the direct effect. The indirect effect via changes in \( T_L \) and \( T_H \) cancels out. The direct effect is obviously negative. \( \text{Q.E.D.} \)
Proof of Proposition 2: Part i) can be deduced, for instance, from conditions (10) and (9) and the proof of Lemma 1. The proof that $T_L < T_H$ implies that a greater gain from adoption leads to earlier adoption. In the heterogeneity case different gains are implied by (16).

Part ii) follows from inspection of (16), (10) and (9). Note that the gain from adopting is greater for the first adopter in the heterogeneous case (that is, firm 0) than for the first adopter in the basic model. To see this note that $A(0) > A(i)$ for all $i \in (0, n]$ and therefore

$$\frac{A(0)^{a/(1-a)}}{\int_0^n c(j)^{a/(n-1)} A(j)^{a/(1-a)} dj} > \frac{1}{\int_0^n c(j)^{a/(n-1)} dj}.$$ 

The expression on the r.h.s. is the respective term for the homogeneous case. The reasoning is analogous for the last adopter. In that case the inequality is reversed. Q.E.D.

References


