R&D cooperation with unit-elastic demand

Georg Götz

This draft: September 2005.

Abstract: This paper shows that R&D cooperation leads to the monopoly outcome in terms of price and quantity if demand is unit-elastic. If the demand function exhibits an upper bound for the willingness to pay, R&D cooperation is inferior to a scenario in which firms cooperate both in their R&D and their output decision.

Keywords: R&D cooperation, spillovers, cartelization

JEL-Classification No.: L13, O31

Department of Economics, University of Vienna, BWZ - Bruenner Str. 72, A-1210 Vienna, Austria, phone/fax: +43 1 4277 -374 66/-374 98, email: georg.goetz@univie.ac.at, http://homepage.univie.ac.at/georg.goetz/
1. Introduction

The question of whether R&D cooperation might have adverse consequences for product market competition is a rather disputed issue. There are concerns that firms which cooperate in one field might well cooperate in another one as well (see Martin 2002). Rather than entering this discussion, I show that under certain conditions R&D cooperation directly leads to the monopoly outcome in terms of price and quantity. If demand is unit-elastic, a Cournot duopoly, in which firms cooperate in R&D, leads to a price that approaches infinity and R&D expenditures that go to zero. The duopoly differs from the monopoly case only in terms of profits, which are lower in the duopoly case. If R&D spillovers are perfect, the cooperative and the non-cooperative choice of R&D expenditures coincide. If the demand function exhibits an upper bound for the willingness to pay, duopoly prices and quantities under R&D cooperation are exactly that of the monopoly. However, R&D expenditures are inefficiently low. I show that a scenario, in which firms cooperate in R&D and compete in the quantity setting stage of the game, is inferior to full cartelization.

A related paper is Kline (2000). He shows that R&D cooperation might lead to lower R&D expenditures than non-cooperation if certain conditions on demand and costs are satisfied. The case of a unit-elastic demand analyzed in this paper is a special case of the conditions stated by Kline. My analysis complements Kline's results in that it calculates actual levels of R&D expenditures whereas Kline makes only comparisons at the margin.
2. The model

I consider a homogeneous good duopoly, in which firms set quantities. In order to make the points clearly, I employ the functional forms introduced by Dasgupta and Stiglitz (1980). Inverse demand reads

\[ p = \sigma (q_1 + q_2)^{-\frac{1}{\varepsilon}} \]  

(1)

Here, \( q_1 \) and \( q_2 \) denote the outputs of the two firms, \( p \) denotes the product price. \( \sigma \) is a positive parameter that measures market size. \( \varepsilon \) denotes the constant price elasticity of demand.

Firm \( i \)'s constant, marginal costs \( c_i \) depend on \( x_i \), the firm's R&D expenditures in the following way:

\[ c_i(x_i) = \beta (x_i + \theta x_j)^{-\alpha} \quad i, j = 1, 2; i \neq j. \]  

(2)

I assume a two-stage game in which firms simultaneously choose R&D expenditures in the first stage. In the second stage firms simultaneously choose outputs. In the case of R&D cooperation firms choose R&D expenditures of both firms cooperatively in order to maximize joint profits. The setup follows Kamien, Mueller, and Zang (1992) by assuming input spillovers and in the definition of R&D cooperation.

Second stage quantities for given levels of R&D expenditures derive from the profit function

\[ \pi_i = \left( \sigma (q_1 + q_2)^{-\frac{1}{\varepsilon}} - \beta (x_i + \theta x_j)^{-\alpha} \right) q_i - x_i \quad i, j = 1, 2; i \neq j. \]  

(3)

The first order conditions with respect to quantities can be written as

\[ \frac{\partial \pi_i}{\partial q_i} = \frac{\sigma}{\varepsilon} (q_1 + q_2)^{-\frac{(1+\varepsilon)/\varepsilon}{\varepsilon}} \left( (\varepsilon - 1) q_i + \varepsilon q_j - \beta (x_i + \theta x_j)^{-\alpha} \right) = 0, \quad i, j = 1, 2; i \neq j. \]  

(4)
Closed-form solutions do not exist for arbitrary values of the demand elasticity $\varepsilon$. However, they are straightforward to calculate for the special case $\varepsilon = 1$. Equilibrium quantities read in this case

$$q_i = \frac{\alpha \left( x_i + \theta x_j \right)^\alpha \left( \theta x_i + x_j \right)^{2\alpha}}{\beta \left( \left( x_i + \theta x_j \right)^\alpha + \left( \theta x_i + x_j \right)^\alpha \right)^{\frac{1}{2}}}, \quad i, j = 1, 2; \quad i \neq j. \quad (5)$$

Note that the quantities are well defined for a demand elasticity of 1. A symmetric equilibrium of the quantity setting stage exists even for inelastic demand as long as $\varepsilon$ is greater than 1/2. The equilibrium price is also a function of R&D expenditures and reads

$$p = \beta \left( \left( x_i + \theta x_j \right)^{-\alpha} + \left( \theta x_i + x_j \right)^{-\alpha} \right), \quad i, j = 1, 2; \quad i \neq j. \quad (6)$$

The reduced profit functions as a function of R&D expenditures are

$$\pi_i^* \left( x_i, x_j \right) = \frac{\alpha \sigma \left( x_i + \theta x_j \right)^{2\alpha}}{\left( \left( x_i + \theta x_j \right)^\alpha + \left( \theta x_i + x_j \right)^\alpha \right)^{\frac{1}{2}}} - x_i, \quad i, j = 1, 2; \quad i \neq j. \quad (7)$$

### 2.1 Non-cooperative R&D

Differentiating the reduced profit function with respect to $x_i$ and solving for an equilibrium with symmetric R&D expenditures $x^N$ yields

$$x^N = \frac{\alpha \sigma (1 - \theta)}{4 (1 + \theta)}. \quad (8)$$

Effective R&D expenditures $x_i + \theta x_j$ are equal to $\alpha \sigma (1 - \theta) / 4$.

Note that individual and effective R&D expenditures are strictly decreasing in the spillover parameter $\theta$. Even more, as $\theta$ approaches one, $x^N$ approaches 0. With unit elastic demand, a perfect spillover would lead firms to invest as little as possible in R&D.
2.2 Cooperative R&D

Under R&D cooperation firms choose R&D activities cooperatively in order to maximize joint profits. In the quantity setting stage, firms play a non-cooperative game. The objective function is

\[
\max_{x_1, x_2} \pi_1^* (x_1, x_2) + \pi_2^* (x_2, x_1) = \frac{\sigma \left( (x_1 + \theta x_2)^{2\alpha} + (x_2 + \theta x_1)^{2\alpha} \right)}{\left( (x_1 + \theta x_2)^{\alpha} + (\theta x_1 + x_2)^{\alpha} \right)^2} - x_1 - x_2. \quad (9)
\]

The derivative of this expression with respect to \( x_1 \) reads

\[
\frac{\partial (\pi_1^* + \pi_2^*)}{\partial x_1} = \frac{2\sigma \alpha (1-\theta^2) x_2 \left( (x_1 + \theta x_2)^{\alpha} - (x_2 + \theta x_1)^{\alpha} \right)}{\left( (x_1 + \theta x_2)^{1-\alpha} (\theta x_1 + x_2)^{1-\alpha} \right) \left( (x_1 + \theta x_2)^{\alpha} + (\theta x_1 + x_2)^{\alpha} \right)^3} - 1. \quad (10)
\]

Again I solve for an equilibrium with symmetric R&D expenditures. If one assumes that \( x_1 = x_2 = x^C \), the first expression on the r.h.s of (10) becomes 0, and the value of the derivative is equal to \(-1\). I obtain the result that the R&D-cooperators invest in R&D as a monopolist would: i.e. as little as possible. The joint profit of the duopolists is, however, smaller than that of the monopolists. Due to competition in the second stage of the game, profit for each duopolist approaches \( \sigma/4 \) as R&D expenditures go to 0. Therefore, joint profits are just half of the monopoly profit. Note here that in the case with symmetric R&D expenditures \( x^C \), joint profits can be simplified to

\[
\pi_1^* + \pi_2^* = \frac{\sigma}{2} - 2x^C. \quad (11)
\]

3. Extension: Demand with finite maximum willingness to pay

While the above section clearly makes the relevant point – R&D cooperation might lead to the monopoly outcome in terms of final output – I now present an example that does not rely
on limit arguments. To do that, I assume that consumers have a maximum finite willingness to pay that is equal to the parameter $\sigma$. The demand curve now exhibits a flat part at a price of $\sigma$ and a kink at an aggregate output of one. This extension leaves the equilibrium of the game with R&D competition unchanged as long as

$$\sigma > \frac{2^{-\frac{1}{1+\alpha}} \beta^{\frac{1}{1+\alpha}}}{(\alpha(1-\theta))^{\alpha/(1+\alpha)}}.$$  \hspace{1cm} (12)

I assume this condition to hold. This assumption ensures that price and quantity in the equilibrium without R&D cooperation are different from the monopoly price and quantity.

Both monopoly and R&D cooperation would lead to a price of $\sigma$ and an aggregate output of one. This is a consequence of the fact that price elasticity is infinite along the flat part of the demand curve, whereas the arguments from the above section extend to the unit-elastic part of the demand curve. One obtains the result that R&D cooperation is sufficient for a duopoly, which competes in the final stage of the game, to implement the monopoly solution in terms of output and price. The joint profit, however, is smaller. To see this, I derive equilibrium R&D expenditures under R&D cooperation. Again looking for a symmetric equilibrium, I solve for $\bar{x}$, the maximum value of R&D expenditures, which is compatible with both firms producing an output of 1/2 each. Setting the quantity in (5) equal to 1/2 and imposing symmetry yields

$$\bar{x} = \frac{2^{\frac{1}{\alpha}} \beta^{\frac{1}{\alpha}}}{(1+\theta)\sigma^{\alpha/(1+\alpha)}}.$$  \hspace{1cm} (13)

For higher values of $x$, the firms would produce a total output of more than one in the second stage of the game due to lower marginal costs. Note that $\bar{x}$ is smaller than the cost minimizing value of R&D expenditures. This can be seen by differentiating $\pi_1 + \pi_2$ as defined in equation (3) with respect to $x_1$ and $x_2$, taking into account that individual outputs are 1/2. The resulting value is
greater than $\bar{x}$ for the values of $\sigma$ that satisfy (12). This yields another surprising result: R&D cooperation with non-cooperation in the quantity setting stage is inferior to full cartelization! While output and prices are identical, sole R&D cooperation leads to higher average costs.

4. Policy conclusions

There is a clear policy conclusion from the above analysis. Do not allow R&D cooperation for the development of products with inelastic demand. However, if the authority does not prevent such a cooperation, then it should even suggest a full cartelization of the participants of a cooperative agreement.

5. References


