Solution branches of nonlinear Schrödinger equations entering the essential spectrum

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In this talk we consider the stationary nonlinear Schrödinger equation

$$-\Delta u - \lambda u = Q(x)|u|^{p-2}u \qquad \text{in } \mathbb{R}^N, \quad N \geq 3,$$

where Q is a bounded and nonnegative weight function with compact support, $2 < p < \frac{2N}{N-2}$ and λ is a real parameter. While, as a consequence of Sobolev's inequality, the value $\lambda = 0$ is not an L^s-bifurcation point for $s > \frac{2N}{N-2}$, we will show that, for a range of subcritical exponents p, there exist solution branches which can be continued to values $\lambda > 0$ thereby entering the essential spectrum of $-\Delta$. For $\lambda > 0$, these solutions are not in $H^1(\mathbb{R}^N)$, but they remain in $L^q(\mathbb{R}^N)$ for $q > \frac{2N}{N-1}$ and the branches are continuous with respect to the L^q-norm. These results were obtained in collaboration with Tobias Weth.