On entire solutions of an elliptic system modelling phase separation ${\bf r}$

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We present several results concerning existence and qualitative properties of entire solutions to

$$\begin{cases} \Delta u_i = u_i \sum_{j \neq i} u_j^2 & \text{in } \mathbb{R}^N \\ u_i > 0 & \text{in } \mathbb{R}^N \end{cases} \qquad i = 1, \dots, k,$$

and we discuss their application in the asymptotic analysis for the system

$$\begin{cases} -\Delta u_i = f_i(x, u_i) - \beta u_i \sum_{j \neq i} a_{ij} u_j^2 & \text{in } \Omega \\ u_i > 0 & \text{in } \Omega \end{cases} \qquad i = 1, \dots, k$$

in the limit of strong competition $\beta \to +\infty$.