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QUALITY COMPETITION AND ENTRY DETERRENCE: WHEN TO LAUNCH AN EXTRA BRAND

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Quality competition and entry deterrence: When to launch an extra brand.

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Abstract

In this paper, we study the rational for an incumbent to launch a second brand when facing potential entry in a market with quality differentiated products and a fringe producer. Depending on market size, costs for a second brand and a potential entrant's setup cost the incumbent might use a second brand both when deterring and when accommodating entry. The analysis generates predictions about the equilibrium degree of product differentiation, the presence of a multiproduct incumbent, and the determinants of successful entry.

Keywords: Multiproduct firms, quality competition, vertical product differentiation, entry accommodation, entry deterrence.

JEL Classifications: L13, D43, M31

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1. Introduction

How should an airline such as Lufthansa react to the threat of entry posed by low cost carriers? Should it reduce its premium quality? Or should it introduce a second medium or low quality brand in order to deter entry? And how should it act if it cannot deter entry? Should it nevertheless produce a second brand, when it accommodates entry? We address these questions in a model of vertical product differentiation, in which we allow an incumbent to offer multiple quality differentiated products. Taking into account the possibility of different entry costs of the entrant and the incumbent's second brand, we examine how the incumbent's optimal strategy depends on these cost parameters. In our setup of sequential entry, we also account for the existence of a low quality fringe product and producer, respectively. In the case of airlines this fringe product might be travel by railways, which for instance in continental Europe offer a low quality substitute on medium ranges. Different from Bonnisseau and Lahmandi-Ayed, (2006), we show that in our setup of one active and one passive incumbent, the high quality incumbent might well launch a second brand. Therefore, no specific assumptions about the shape of costs of quality (see the discussion in Cheng et al., 2011, Cheng and Peng, 2013) are necessary in our framework to obtain a multiproduct result. Our analysis shows that incumbents might employ different strategies for entry deterrence. If the fixed costs to launch a second brand are high, it might well be optimal to reduce the high quality below the maximum quality level in order to deter entry. This holds as long as such a move would not enable the entrant to become the premium quality provider. However, if entry has to be accommodated, it is always optimal for the incumbent to launch a second, medium quality brand in order to leave only a low quality niche for the entrant and to protect the premium brand.

Our paper contributes to the research on the behavior of established firms in the light of potential entry. The focus is on the implications of various types of asymmetry between incumbents and entrants. Such asymmetries might arise due to absolute cost advantage, information advantages etc., (see Gilbert, 1989 for an overview of barriers to entry). The impact of potential competition for markets where firms engage in product differentiation has also been studied intensively. Contrary to our paper, the majority of the research analyzes the behavior of a single incumbent facing the potential entry of other firms. Omori and Yarrow (1982), for instance, analyze a monopolistic situation where the incumbent can introduce additional products and show how product diversification in addition to limit pricing can be used to deter entry. Bonanno (1987) shows that product specification instead of product proliferation can be used as a deterring device. The limitation of analyzing a single incumbent and thereby neglecting the importance of the interplay between actual and potential competition was addressed by Donnenfeld and Weber (1995). They study how the interplay of competition among incumbents and the magnitude of potential entrants's setup costs determines the spectrum of products in equilibrium. They show that incumbents can use limit qualities as a deterring device.

More recently, the question of multiproduct quality competition and the optimal product range of entrants and incumbents has been examined thoroughly (see Johnson and Myatt, 2003 and 2006). However, these papers employ a more symmetric setup with simultaneous entry, in which entry deterrence is less of an issue. Related to our paper is the result that incumbents never offer lower qualities than entrants even if they supply multiple brands.

Our approach is closest related to a paper of Donnenfeld and Weber (1995) who analyze the potential entry in a vertical differentiated market. They find that potential entry is blockaded if fixed cost of entry are sufficiently high. For moderately high levels of entry costs the unique equilibrium is one of entry deterrence. If the level of entry costs is relatively low there is a unique equilibrium where both incumbents accommodate entry and engage in maximal product differentiation. For medium levels of cost of entry both types of equilibria can arise.

Contrary to their approach we treat the low quality incumbent as a rather static player with respect to quality. As mentioned above he can only choose between producing the lowest quality and leaving the market. Given setup cost being sunk cost and positive revenues it is a dominant strategy to stay in the market. However the high quality incumbent is given the opportunity to introduce an extra brand, the determinants of which are the focus of this paper. Next to fixed cost of entry for firm E and fixed cost of an extra brand for the high quality incumbent we introduce a third parameter, the market size. Market size measures the spread between the lowest and highest (potential) qualities.

In a nutshell, our paper combines the interplay of actual and potential competition between two incumbents and a potential entry. Particular features are a fringe incumbent limited to the choice of lowest quality and the possibility of product proliferation by the high quality incumbent.

The paper is organized as follows. The model is presented in section 2, section 3 discusses the results and concludes.

2. The model

Our model will be based on a textbook model for differentiated products by Tirole (1988). The model is very similar to the one of Donnenfeld and Weber (1992), but the utility specification of the Tirole model is mathematically more convenient for our purposes. We consider an industry with two incumbents, 1 and 2 and a potential entrant, E. The firms operate in a market with differentiated products. Products differ in one dimension referred to as their quality measured by a closed interval $\left[\underline{q},\bar{q}\right]$.

For analytical simplicity we assume that one of the incumbents –the low quality incumbent (*LQI*)- is limited with respect to quality competition in the sense that he is restricted to the choice of the lowest quality or to leave the market. Contrary to this the so called high quality incumbent (*HQI*) not only can freely choose a quality on the interval, but he can also decide at a later stage whether to introduce an additional brand. Once the incumbents made their choices, firm E decides in the third stage whether to enter and -given an extra brand by the HQI- in which segment. We will refer to the segment below the medium quality as the low quality segment (*LQS*) and to the segment above the medium quality as the high quality segment (*HQS*). In the fourth stage price competition takes place. Firm E enters the market if and only if it can guarantee itself positive profits.

Stage1	The LQI decides whether to stay in the market with quality $\underline{q}$ or leave the market, the HQI decides whether to stay in the market with quality $q \in [\underline{q}, \bar{q}]$ or leave the market. Choices are made simultaneously.
Stage2	The HQI decides whether to introduce a extra brand located on $[\underline{q}, \bar{q}]$.
Stage3	The choices are observed and the potential entrant decides whether to enter the market and selects its quality.
Stage4	Price competition takes place.

Table 1 Sequence of moves.

Firms are homogenous with respect to their production cost (constant marginal cost c) with two exceptions: whereas the incumbents' fixed cost for production capacity for one brand are sunk costs, the entrant has to bear fixed cost F^E only if she enters the market. However, if the HQI decides to introduce an extra brand it has to invest F^{EB} into capacities for production variety beyond the sunk investment. All other costs than F^E and F^{EB} are assumed to be zero.

On the consumer side there is heterogeneity with respect to preference for quality θ which is assumed to be uniformly distributed across the population of consumers between $\underline{\theta} \geq 0$ and $\bar{\theta} = 1 + \underline{\theta}$. Preferences are represented by a utility function $u(\theta, q, p) = \theta q - p$, where q stands for the quality of the consumed product and p for the price of the product.

We will make the following two assumptions, being qualitatively equivalent to those made in Tirole (1988).

Assumption 1 $\quad \bar{\theta} \geq 8\underline{\theta}$

This assumption is concerned with a sufficient heterogeneity among consumer preferences. The second assumption guarantees that in the price equilibrium the market is "covered", i.e. each consumer is willing to buy a product.

Assumption 2 $\quad c + \frac{\bar{\theta} - 2\underline{\theta}}{3}(\bar{q} - \underline{q}) \leq \underline{\theta}\underline{q}$

For now we will assume F^{EB} to be zero and leave the general case for the discussion. There are various constellations that need to be considered, ranging from the textbook case with two incumbents and no entrance to the case of 3 brands for the incumbents and one brand for the entrant. The equilibrium prices and profits are all referred to Appendix A. Before we continue we have to clarify some notation. Let $\pi_i(q_1, q_2, q_3), i \in \{HQI, LQI, E\}$ denote the profit of the HQI, the LQI or the entrant E for any triple of qualities $q_1 < q_2 < q_3$ and $\pi_i(q_1, q_E, q_2, q_3), i \in \{HQI, LQI, E\}$ be the profit of the HQI, the LQI or the entrant E for any tuple of qualities $q_1 < q_E < q_2 < q_3$, i.e. $\pi_i(q_1, q_E, q_2, q_3), (\pi_i(q_1, q_2, q_E, q_3))$ denotes the profit if E entered the LQS (HQS). Let $q^*_{E,LQS}(q_1, q_2, q_3)(q^*_{E,HQS}(q_1, q_2, q_3))$ denote the optimal location of the entrant's brand in the LQS and the HQS respectively. Finally, the optimal location for E's brand if located between the two incumbent's qualities will be denoted by $q^*_E(q_1, q_2)$.

It is worthwhile to mention, that in our model an extra brand is launched for strategic reasons only, because $\pi_{HQI}(q_1, q_2, q_3)$ increases in q_2, i.e. in the absence of potential entry the HQI would never find it profitable to launch an extra brand. In other words, a second brand is launched for either entry deterrence or to accommodate entry in a more profitable way. The precise conditions for either of the two purposes are what is analyzed in the following sections.

2.1. Optimal accommodation

In the following we will state an insight concerning the profits of the firms that will be helpful for deriving the optimal accommodation and deterrence behavior of the HQI. All proofs are referred to Appendix B.

Lemma 1 $\forall q_1 < q_2 < q_3 : \pi_{HQI}(q_1, q_2, q^*_{E,HQS}, q_3) < \pi_{HQI}(q_1, q^*_E, q_3) < \pi_{HQI}(q_1, q^*_{E,LQS}, q_2, q_3)$.

Lemma 1 states that the HQI earns c.p. higher profits if no extra brand is launched in comparison to a situation with an extra brand in the market and entrance in the HQS by E. However launching an extra brand and entrance in the LQS is more profitable than entrance in the absence of an extra brand. This has a direct consequence with respect to the accommodation behavior. The second part of the inequality in Lemma 1 implies that the HQI will never accommodate entrance without the launch of an extra brand. To see this, note that the inequality stated in Lemma 1 holds for all quality levels of the extra brand. By choosing a level sufficiently close the top brand entrance will be more profitable in the LQS for E but will still induce larger profits than accommodation without an extra brand. The first part of the inequality in Lemma 1 tells us that entrance will never be accommodated in the HQS.

Proposition 1 below describes the optimal accommodation behavior for the high quality incumbent.

Proposition 1 -entry accommodation-

(1) Entry will never be accommodated with one brand.
(2) Entry will only be accommodated in the LQS.
(3) If entry is accommodated the HQI will locate its premium brand at the upper bound of the quality interval.
(4) The extra brand will be located such that the potential profits for an entrant are equalized in the two segments, above and below the medium brand.

As mentioned above (1) and (2) are direct consequences of Lemma 1, (3) is due to the fact that c.p. profits for all firms increase when the quality for the top brand is increased. Finally, (4) stems from the fact, that if Firm E would earn higher profits in the HQS and therefore enter this segment, the HQI can by increasing the quality of the extra brand decrease the profits for E in the HQS. If the quality of the extra brand is increased by that much that E prefers to enter the LQS, profits for the HQI jump above $\pi_{HQI}(q_1, q^*_E, q_3)$, which was a upper bound for the profits of what the HQI would have earned given entrance in the HQS. Hence entry accommodation is characterized by potential profits for an entrant that are higher in the LQS. However, it turns out that $\pi_{HQI}(q_1, q^*_{E,LQS}, q_2, q_3)$ is monotonously decreasing in q_2. Hence, the HQI will not increase the quality for the extra brand beyond the level where profits for the entrant in both segments are equalized.

2.2. Optimal interior deterrence

Before we turn to the optimal deterrence behavior of the HQI we will state another helpful observation.

Lemma 2 $\forall q_1 < q_E < q_2 < q_3 : \pi_E\left(q_1, q_E^*, q_2\right) = \pi_E\left(q_1, q_{E,LQS}^*, q_2, q_3\right)$.

That is if entry occurs in the LQS, the presence of an extra brand has no impact on the profits of the entrant. In other words, if the two neighboring qualities are produced by different firms, only the levels of those two neighboring qualities determine the entrant's profit. This is a common feature of Salop-type of models (Salop 1979).

As a consequence to deter entry with one brand or an extra brand the same q_2 needs to be chosen. This has an immediate consequence for the optimal deterrence strategy described in the next proposition. Proposition 2 is concerned with deterring interior entrance only, i.e. we will at a later stage have to control for the incentive of the entrant to locate at the boundary of the quality spectrum.

Proposition 2 –interior entry deterrence-

(1) Optimal deterrence makes use of an extra brand.
(2) The extra brand will be placed as close as possible to the top brand under the restriction that the potential entrant earns his fixed cost of entry in the low quality segment in which case we assume that (s)he refrains from entry.
(3) The top brand will be located as close as possible to the boundary of the quality spectrum such that the entrant earns at most his fixed cost of entry in the HQS.

To deter intermediate entry or entry in the low quality segment respectively the high quality incumbent chooses the same q_2 (see Lemma 2). Furthermore for any triple $q_1 < q_2 < q_3$ $\pi_{HQI}\left(q_1, q_2\right) - \pi_{HQI}\left(q_1, q_2, q_3\right) = -\frac{1}{4}\left(q_3 - q_2\right)\bar{\theta}^2 < 0$. This holds in particular for the level of q_2 that deters interior entry, i.e. it always pays to launch an extra brand instead of just lowering the quality of the top brand to the deterring level. The second claim stems from the fact that in the absence of potential entry the high quality incumbent does not find it profitable to launch an extra brand. In other words $\pi_{HQI}\left(q_1, q_2, q_3\right)$ increases in q_2. The third claim follows from the fact that $\pi_{HQI}\left(q_1, q_2, q_3\right)$ increases in q_3 and again that the profit of the entrant is independent from q_3 in the case of entry in the LQS. If the fixed cost of entry for E fall below the threshold that is determined by equal profits in both segments and a top brand located at $\bar{q}$ then the HQI might consider to shrink effective market size, i.e. the difference in quality between the highest and lowest quality in the market, by placing q_3 below $\bar{q}$. In this case for entry deterrence to be effective we have to control for the profits E could earn by entering the market with quality $\bar{q}$. The following observation will help us to incorporate the option of top level entry by E into our previous results with respect to interior deterrence.

Lemma 3 Let $q_l < a < b < \bar{q}$ such that $\pi_E\left(q_l, q^*_{E,LQS}, a, b\right) = \pi_E\left(q_l, a, q^*_{E,HQS}, b\right) \equiv K$, then: $\pi_E\left(q_l, a, b, q_E = \bar{q}\right) < K \Rightarrow \pi_{HQI}\left(q_l, a, b\right) < \pi_{HQI}\left(q_l, q^*_E, \bar{q}\right)$.

Lemma 3 shows that as long as the incentive constraint for the entrant not to enter at the top quality is not binding, the incumbent rather deters entry than to accommodate. In other words deterrence with two brands with a top quality below the maximum quality is not restricted by the decline in profits for the HQI due to an increasing number of brands and therefore stronger price competition, but by the incentive constraint for the entrant not to enter above the top brand of the incumbents which would induce a significant decrease in profits for the HQI.

Once the incentive constraint for the entrant not to enter at the top quality binds, the HQI engages in maximal product differentiation and accommodates entry in the LQS. This accommodation is realized by the medium brand located such that the entrant is indifferent between the two segments in which case we assume that the lower segment is chosen. Now we can state the main result of the paper that identifies the Nash equilibrium realized as a function of the setup cost for firm E.

Theorem There exist unique values $0 < F^L < F^M < F^H$ such that: For $F^E \geq F^H$ the unique Nash equilibrium is *Blockaded Entry* with maximal product differentiation, for $F^M \leq F^E < F^H$ the unique Nash equilibrium is *Entry Deterrence* with brand proliferation and maximal product differentiation, for $F^M \leq F^E < F^H$ the unique Nash equilibrium is *Entry Deterrence* with brand proliferation and non-maximal product differentiation, for $0 \leq F^E < F^L$ the unique Nash equilibrium is *Entry Accommodation* in the low quality segment and maximal product differentiation.

F^L, F^M, F^H are given by:

$$F^H = \pi_E\left(\underline{q}, q^*_E, \bar{q}\right),\ F^M = \pi_E\left(\underline{q}, q^*_{E,LQS}, a, \bar{q}\right) = \pi_E\left(\underline{q}, a, q^*_{E,HQS}, \bar{q}\right)$$
$$F^L = \pi_E\left(\underline{q}, q^*_{E,LQS}, a, b\right) = \pi_E\left(\underline{q}, a, q^*_{E,HQS}, b\right) = \pi_E\left(\underline{q}, a, b, q_E = \bar{q}\right).$$

Note that all the thresholds in the theorem are increasing functions in the size of the quality spectrum $\bar{q} - \underline{q}$. This is quite intuitive since the thresholds refer to the potential profits for the entrant which increase if the quality spectrum increases. An increasing quality spectrum reduces price competition in equilibrium and induces a higher equilibrium quality for the entrant and thereby larger revenues.

3. Discussion and Conclusion

We would have obtained qualitatively similar results for the type of utility functions analyzed in Jaskold Gabszewicz and Thisse (1979), however our specification is mathematically more convenient. We further conjecture that our results can be generalized analog to Donnenfeld and Weber (1995) for a class of models where profits functions share some features like homogeneity of degree zero and so forth (see Assumptions (1)-(9) in Donnenfeld and Weber (1995)).

In the paper we assumed that the high quality incumbent bears no fixed cost for launching an extra brand, i.e. $F^{EB} = 0$. We will in following discuss the introduction of strictly positive cost for an extra brand. Although positive cost for an extra brand being plausible it appears rather unlikely that the incumbent faces higher cost than the entrant, i.e. we restrict on $F^{EB} \leq F^{E}$. It turns out that there is one qualitative change in results. The relation of costs between accommodation and deterrence is unaltered though, i.e. as long as it is feasible for the incumbent to deter entry (s)he will do so. However, the decision with respect to optimal deterrence strategy is influenced by the introduction of positive cost for an extra brand. In particular as soon as entry becomes profitable given maximal product differentiation it is no longer optimal to deter with two brands. Because for slightly lower fixed cost for E, entry can be deterred by lowering the top quality and fixed cost for the extra brand can be saved. Once the restriction for the entrant not to enter the market with maximal quality turns binding, it will be optimal again to deter with an extra brand covering the maximal quality as well. Furthermore, the optimal accommodation strategy is unchanged since the higher profits in case of accommodation in the low quality segment and thus keeping the entrant away from the premium segment more than compensates for the additional costs of an extra brand. In summary, the introduction of costs of an extra brand for the high quality incumbent introduces a threshold $F^{M} < \tilde{F} < F^{H}$ such that for F^{EB} above $\tilde{F}$ the high quality incumbent will deter with one brand by non-maximal product differentiation. For values below $\tilde{F}$ the high quality incumbent will deter with brand proliferation and maximal product differentiation. Apart from that all other conditions stated in our theorem remain unchanged. Hence our results are not driven by negligible fixed cost for an extra brand by the high quality incumbent and remain unchanged for $F^{EB} \leq F^{E}$.

Starting from a market with differentiated products we analyzed a model where the high quality incumbent was given the opportunity to launch an extra brand. An option that is not profitable in the absence of potential entry, but which might be exercised by the incumbent for strategic reasons if entry by a new player is present. The strategic value of an extra brand stems from its potential to deter entry or to manipulate entry in a way that is favorable for the high quality incumbent. In our model in equilibrium the incumbent will never accommodate entry without brand proliferation. The extra brand is used to induce the entrant to enter the low quality segment. By that the incumbent can secure high profits from its top brand. For low cost of entry the incumbent cannot deter entry. Under this condition and for fixed cost for the extra brand that do not exceed those of the entrant entry accommodation with brand proliferation and maximal product differentiation constitutes the unique Nash equilibrium of the game. Once fixed cost of entry reach a certain threshold it pays off for the incumbent to shrink interior quality spectrum, i.e. to refrain from maximal product differentiation. Under the condition that the entrant does not prefer to introduce a brand with maximal quality, the incumbent will prefer to deter entry in this fashion than to accommodate entry in the low quality segment. Further increasing set up cost will enable the incumbent to achieve both, deterrence and maximal product differentiation. For fixed cost of entry close to what an entrant could maximally earn given no brand proliferation and maximal product differentiation be the incumbents it pays off for the incumbent to deter entry by again shrinking interior quality spectrum with a top quality below maximal quality and thereby saving the fixed cost for a launch of an extra brand. Finally, for even higher set up cost the high quality incumbent need not to fear entry and will thus engage in maximal product differentiation and no brand proliferation. Noteworthy for markets with a larger maximal difference in quality it is c.p. more likely to observe entry accommodation than entry deterrence.

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Appendix A[2]

1. Price equilibria, profits

1.1. Two qualities by incumbents, no entry, i.e. $q_1 < q_2$

$$p_1 = c + \frac{1}{3}(q_2 - q_1)(\overline{\theta} - 2\underline{\theta}) \qquad p_2 = c + \frac{1}{3}(q_2 - q_1)(2\overline{\theta} - \underline{\theta})$$

$$\pi_{LQI}(q_1, q_2) = \frac{1}{9}(q_2 - q_1)(\overline{\theta} - 2\underline{\theta})^2 \qquad \pi_{HQI}(q_1, q_2) = \frac{1}{9}(q_2 - q_1)(2\overline{\theta} - \underline{\theta})^2$$

1.2. Two qualities by incumbents and intermediate quality by entrant, i.e. $q_1 < q_E < q_2$

$$p_1 = c + \frac{(q_E - q_1)((q_2 - q_E)\overline{\theta} - (4q_2 - q_E - 3q_1)\underline{\theta})}{6(q_2 - q_1)} \qquad p_E = c + \frac{(q_E - q_1)(q_2 - q_E)(\overline{\theta} - \underline{\theta})}{3(q_2 - q_1)} \qquad p_2 = c + \frac{(q_2 - q_E)((3q_2 + q_E - 4q_1)\overline{\theta} - (q_E - q_1)\underline{\theta})}{6(q_2 - q_1)}$$

$$\pi_{LQI}(q_1, q_E, q_2) = \frac{(q_E - q_1)((q_2 - q_E)\overline{\theta} - (4q_2 - q_E - 3q_1)\underline{\theta})^2}{36(q_2 - q_1)^2} \qquad \pi_E(q_1, q_E, q_2) = \frac{(q_E - q_1)(q_2 - q_E)(\overline{\theta} - \underline{\theta})^2}{9(q_2 - q_1)}$$

$$\pi_{HQI}(q_1, q_E, q_2) = \frac{(q_2 - q_E)((3q_2 + q_E - 4q_1)\overline{\theta} - (q_E - q_1)\underline{\theta})^2}{36(q_2 - q_1)^2}$$

1.3. Extra brand by high quality incumbent, $q_1 < q_2 < q_3$

$$p_1 = c + \frac{1}{3}(q_2 - q_1)(\overline{\theta} - 2\underline{\theta}) \qquad p_2 = c + \frac{1}{3}(q_2 - q_1)(2\overline{\theta} - \underline{\theta}) \qquad p_3 = c + \frac{1}{6}((3q_3 + q_2 - 4q_1)\overline{\theta} - (q_2 - q_1)\underline{\theta})$$

$$\pi_{LQI}(q_1, q_2, q_3) = \frac{1}{9}(q_2 - q_1)(\overline{\theta} - 2\underline{\theta})^2 \qquad \pi_{HQI}(q_1, q_2, q_3) == \frac{1}{36}((9q_3 + 7q_2 - 16q_1)\overline{\theta}^2 - 16(q_2 - q_1)\overline{\theta}\underline{\theta} + 4(q_2 - q_1)\underline{\theta}^2)$$

1.4. Extra brand by high quality incumbent and entry in the low quality segment, $q_1 < q_E < q_2 < q_3$

[2] Results were derived with the help of the software Mathematica® version 9.0, the file is available upon request.

$$p_1 = c + \frac{\left(q_E - q_1\right)\left(\left(q_2 - q_E\right)\overline{\theta} - \left(4q_2 - q_E - 3q_1\right)\underline{\theta}\right)}{6\left(q_2 - q_1\right)} \qquad p_E = c + \frac{\left(q_E - q_1\right)\left(q_2 - q_E\right)\left(\overline{\theta} - \underline{\theta}\right)}{3\left(q_2 - q_1\right)} \qquad p_2 = c + \frac{\left(q_2 - q_E\right)\left(\left(3q_2 + q_E - 4q_1\right)\overline{\theta} - \left(q_E - q_1\right)\underline{\theta}\right)}{6\left(q_2 - q_1\right)}$$

$$\pi_{LQI}\left(q_1, q_E, q_2, q_3\right) = \frac{\left(q_E - q_1\right)\left(\left(q_2 - q_E\right)\overline{\theta} - \left(4q_2 - q_E - 3q_1\right)\underline{\theta}\right)^2}{36\left(q_2 - q_1\right)^2} \qquad \pi_E\left(q_1, q_E, q_2, q_3\right) = \frac{\left(q_E - q_1\right)\left(q_2 - q_E\right)\left(\overline{\theta} - \underline{\theta}\right)^2}{9\left(q_2 - q_1\right)}$$

$$\pi_{HQI}\left(q_1, q_E, q_2, q_3\right) = \frac{\begin{array}{c}\left(q_1^2\left(9q_3 + 7q_2 - 16q_E\right) + q_2^2\left(9q_3 - 3q_E\right) - 5q_2q_E^2 - q_E^3 + 2q_1\left(-3q_2\left(q_2 + 3q_3\right) + 8q_2q_E + 4q_E^2\right)\right)\overline{\theta}^2 \\ -2\left(q_2 - q_E\right)\left(q_E - q_1\right)\left(3q_2 + q_E - 4q_1\right)\overline{\theta}\underline{\theta} + \left(q_E - q_1\right)^2\left(q_2 - q_E\right)\underline{\theta}^2\end{array}}{36\left(q_2 - q_1\right)^2}$$

1.5. Extra brand by high quality incumbent and entry in the high quality segment, $q_1 < q_2 < q_E < q_3$

$$p_1 = c + \frac{\left(q_2 - q_1\right)\left(-\left(q_E - q_2\right)\left(q_3 - q_E\right)\overline{\theta} - 2\left(3q_1\left(q_3 - q_2\right) - q_E\left(4q_3 - q_E\right) + q_2\left(q_3 + 2q_E\right)\right)\underline{\theta}\right)}{3\left(q_2^2 + 4q_1\left(q_3 - q_2\right) + 2q_2q_E - q_E\left(4q_3 - q_E\right)\right)}$$

$$p_2 = c - \frac{\left(q_2 - q_1\right)\left(q_E - q_2\right)\left(2\left(q_3 - q_E\right)\overline{\theta} - \left(4q_3 - q_E - 3q_2\right)\underline{\theta}\right)}{3\left(q_2^2 + 4q_1\left(q_3 - q_2\right) + 2q_2q_E - q_E\left(4q_3 - q_E\right)\right)}$$

$$p_E = c - \frac{\left(q_E - q_2\right)\left(q_3 - q_E\right)\left(\left(3q_E + q_2 - 4q_1\right)\overline{\theta} - 2\left(q_2 - q_1\right)\underline{\theta}\right)}{3\left(q_2^2 + 4q_1\left(q_3 - q_2\right) + 2q_2q_E - q_E\left(4q_3 - q_E\right)\right)}$$

$$p_3 = c - \frac{\left(q_3 - q_E\right)\left(2\left(4q_1q_2 - q_2^2 - 3q_1q_3 + \left(3q_E - 2q_2 - q_1\right)q_E\right)\overline{\theta} - \left(q_2 - q_1\right)\left(q_E - q_2\right)\underline{\theta}\right)}{3\left(q_2^2 + 4q_1\left(q_3 - q_2\right) + 2q_2q_E - q_E\left(4q_3 - q_E\right)\right)}$$

$$\pi_{LQI}\left(q_1,q_2,q_E,q_3\right)=\left(q_2-q_1\right)\frac{\left(-\left(q_E-q_2\right)\left(q_3-q_E\right)\overline{\theta}-2\left(3q_1\left(q_3-q_2\right)-q_E\left(4q_3-q_E\right)+q_2\left(q_3+2q_E\right)\right)\underline{\theta}\right)^2}{\left(3\left(4\left(q_3-q_2\right)\left(q_E-q_1\right)-\left(q_E-q_2\right)^2\right)\right)^2}$$

$$\pi_{E}\left(q_1,q_2,q_E,q_3\right)=\left(q_3-q_2\right)\left(q_E-q_2\right)\left(q_3-q_E\right)\left(\frac{\left(3q_E+q_2-4q_1\right)\overline{\theta}-2\left(q_2-q_1\right)\underline{\theta}}{3\left(4\left(q_3-q_2\right)\left(q_E-q_1\right)-\left(q_E-q_2\right)^2\right)}\right)^2$$

$\pi_{HQI}\left(q_1,q_2,q_E,q_3\right)=\pi_{HQI}^{LQ}\left(q_1,q_2,q_E,q_3\right)+\pi_{HQI}^{HQ}\left(q_1,q_2,q_E,q_3\right)$, where LQ (HQ) denotes low quality brand (high quality brand).

$$\pi_{HQI}^{LQ}\left(q_1,q_2,q_E,q_3\right)=\left(q_2-q_1\right)\left(q_E-q_1\right)\left(q_E-q_2\right)\left(\frac{2\left(q_3-q_E\right)\overline{\theta}-\left(4q_3-q_E-3q_2\right)\underline{\theta}}{3\left(4\left(q_3-q_2\right)\left(q_E-q_1\right)-\left(q_E-q_2\right)^2\right)}\right)^2$$

and

$$\pi_{HQI}^{HQ}\left(q_1,q_2,q_E,q_3\right)=\left(q_3-q_E\right)\left(\frac{\left(6\left(q_3-q_2\right)\left(q_E-q_1\right)+2\left(q_E-q_2\right)\left(q_2-q_1\right)\right)\overline{\theta}-\left(q_2-q_1\right)\left(q_E-q_2\right)\underline{\theta}}{3\left(4\left(q_3-q_2\right)\left(q_E-q_1\right)-\left(q_E-q_2\right)^2\right)}\right)^2$$

1.6. Extra brand by high quality incumbent and entry at the top quality, $q_1<q_2<q_3<q_E$

$$p_1=c+\frac{2\left(q_2-q_1\right)\left(\left(q_E-q_3\right)\overline{\theta}-\left(4q_E-q_3-3q_1\right)\underline{\theta}\right)}{3\left(4q_E+q_2-q_3-4q_1\right)}\qquad p_2=c+\frac{\left(q_2-q_1\right)\left(4\left(q_E-q_3\right)\overline{\theta}-\left(4q_E-q_3-3q_1\right)\underline{\theta}\right)}{3\left(4q_E+q_2-q_3-4q_1\right)}$$

$$p_3=c+\frac{\left(q_E-q_3\right)\left(\left(3q_3+q_2-4q_1\right)\overline{\theta}-4\left(q_2-q_1\right)\underline{\theta}\right)}{3\left(4q_E+q_2-q_3-4q_1\right)}\qquad p_E=c+\frac{2\left(q_E-q_3\right)\left(\left(3q_3+q_2-4q_1\right)\overline{\theta}-\left(q_2-q_1\right)\underline{\theta}\right)}{3\left(4q_E+q_2-q_3-4q_1\right)}$$

$$\pi_{LQI}(q_1,q_2,q_3,q_E)=\frac{4(q_2-q_1)\left((q_E-q_3)\overline{\theta}-(4q_E-q_3-3q_1)\underline{\theta}\right)^2}{9(4q_E+q_2-q_3-4q_1)^2}$$

$$\pi_E(q_1,q_2,q_3,q_E)=\frac{4(q_E-q_3)\left((9q_E+2q_2-3q_3-8q_1)\overline{\theta}+(q_2-q_1)\underline{\theta}\right)\left((3q_3+q_2-4q_1)\overline{\theta}-(q_2-q_1)\underline{\theta}\right)}{9(4q_E+q_2-q_3-4q_1)^2}$$

$$\pi_{HQI}(q_1,q_2,q_3,q_E)=-\frac{1}{9(4q_E+q_2-q_3-4q_1)^2}[(q_E-q_3)\left(16q_1^2+q_2^2-2q_2q_3-9q_3^2+8q_2q_E+18q_E^2-4q_1(2q_2-5q_3+11q_E)\right)\overline{\theta}^2$$
$$+(q_2-q_1)(q_E-q_3)(-28q_1+q_2+19q_3+8q_E)\overline{\theta}\underline{\theta}-(q_2-q_1)\left(q_1(9q_2+17q_3-26q_E)+(q_3-4q_E)^2+q_2(-11q_3+2q_E)\right)\underline{\theta}^2]$$

1.7. Profits of four independent incumbents, $q_1<q_2<q_3<q_4$

$$\pi_1(q_1,q_2,q_3,q_4)=(q_2-q_1)\left(\frac{\left((q_3-q_2)(q_4-q_3)\overline{\theta}-2\left(q_3(4q_4-q_3)-3q_1(q_4-q_2)-q_2(q_4+2q_3)\right)\underline{\theta}\right)}{3\left(4(q_4-q_2)(q_3-q_1)-(q_3-q_2)^2\right)}\right)^2$$

$$\pi_2(q_1,q_2,q_3,q_4)=(q_2-q_1)(q_3-q_1)(q_3-q_2)\left(\frac{2(q_4-q_3)\overline{\theta}-(4q_4-3q_2-q_3)\underline{\theta}}{3\left(4(q_4-q_2)(q_3-q_1)-(q_3-q_2)^2\right)}\right)^2$$

$$\pi_3(q_1,q_2,q_3,q_4)=(q_4-q_2)(q_E-q_2)(q_4-q_3)\left(\frac{(3q_3+q_2-4q_1)\overline{\theta}-2(q_2-q_1)\underline{\theta}}{3\left(4(q_4-q_2)(q_3-q_1)-(q_3-q_2)^2\right)}\right)^2$$

$$\pi_4(q_1,q_2,q_3,q_4)=(q_4-q_3)\left(\frac{(8q_2q_1-2q_2^2-6q_1q_4-2q_1q_3-4q_2q_3+6q_4q_3)\overline{\theta}-(q_3-q_2)(q_2-q_1)\underline{\theta}}{3\left(4(q_4-q_2)(q_3-q_1)-(q_3-q_2)^2\right)}\right)^2$$

NOTE: $\pi_2(q_1,q_2,q_E,q_3)=\pi_{HQI}^{LQ}(q_1,q_2,q_E,q_3)$; $\pi_4(q_1,q_2,q_E,q_3)=\pi_{HQI}^{HQ}(q_1,q_2,q_E,q_3)$

2. Optimal entrant quality

2.1. Two qualities by incumbents and intermediate quality by entrant, i.e. $q_1 < q_E < q_2$

$$q_E^*\left(q_1, q_2\right) = \frac{q_1 + q_2}{2}$$

2.2. Extra brand by high quality incumbent and entry in the low quality segment, $q_1 < q_E < q_2 < q_3$

$$q_{E,LQS}^*\left(q_1, q_2 . q_3\right) = \frac{q_1 + q_2}{2}$$

2.3. Extra brand by high quality incumbent and entry in the high quality segment, $q_1 < q_2 < q_E < q_3$

$$q_{E,HQS}^*\left(q_1, q_2, q_3\right) = \frac{1}{6\left(\left(21q_3 - 13q_2 - 8q_1\right)\overline{\theta} + 4\left(q_1 - q_2\right)\underline{\theta}\right)}$$

$$2^{2/3}\Omega^{1/3} + 6\left(3q_2^{\ 2} + q_2 q_3 + 4q_3^{\ 2} + 4q_1\left(-5q_2 + 3q_3\right)\right)\overline{\theta} + 12\left(q_1 - q_2\right)\left(q_2 + q_3\right)\underline{\theta}$$

$$+1/\Omega^{1/3} 242^{1/3}\left(q_2 - q_3\right)\left((-64q_1^{\ 3} + 23q_2^{\ 3} + 50q_2^{\ 2}q_3 + 3q_2 q_3^{\ 2} - 12q_3^{\ 3} + 4q_1^{\ 2}\left(43q_2 + 5q_3\right) + q_1\left(-119q_2^{\ 2} - 106q_2 q_3 + 33q_3^{\ 2}\right)\right)\overline{\theta}^{\,2}$$

$$+2\left(q_1 - q_2\right)\left(16q_1 - 11q_2 - 5q_3\right)\left(2q_1 + q_2 - 3q_3\right)\overline{\theta}\underline{\theta} - \left(q_1 - q_2\right)^2\left(16q_1 - 11q_2 - 5q_3\right)\underline{\theta}^2)$$

where

$$\Omega = \sqrt{\Sigma} + 432\left(q_2 - q_3\right)^2$$

$$\left(\begin{array}{l}\left(\begin{array}{l}256q_1^{\ 4} + 5q_2^{\ 4} + 79q_2^{\ 3}q_3 + 163q_2^{\ 2}q_3^{\ 2} + q_2 q_3^{\ 3} + 8q_3^{\ 4} - 8q_1^{\ 3}\left(73q_2 + 55q_3\right) + q_1^{\ 2}\left(430q_2^{\ 2} + 892q_2 q_3 + 214q_3^{\ 2}\right)256q_1^{\ 4} \\ +5q_2^{\ 4} + 79q_2^{\ 3}q_3 + 163q_2^{\ 2}q_3^{\ 2} + q_2 q_3^{\ 3} + 8q_3^{\ 4} - 8q_1^{\ 3}\left(73q_2 + 55q_3\right) + q_1^{\ 2}\left(430q_2^{\ 2} + 892q_2 q_3 + 214q_3^{\ 2}\right)\end{array}\right)\overline{\theta}^{\,3} \\ -\left(q_1 - q_2\right)\left(256q1^3 + 23q_2^{\ 3} + 28q_2^{\ 2}q_3 - 337q_2 q_3^{\ 2} + 30q_3^{\ 3} - 4q_1^{\ 2}\left(53q_2 + 139q_3\right) + q_1\left(-97q_2^{\ 2} + 618q_2 q_3 + 247q_3^{\ 2}\right)\right)\overline{\theta}^{\,2}\underline{\theta} \\ +\left(q_1 - q_2\right)^2\left(64q_1^{\ 2} - 94q_1 q_2 + 7q_2^{\ 2} - 34q_1 q_3 + 80q_2 q_3 - 23q_3^{\ 2}\right)\overline{\theta}\underline{\theta}^2 + 3\left(q_1 - q_2\right)^3\left(q_2 - q_3\right)\underline{\theta}^3 \\ \end{array}\right)$$

and

$$\Sigma = 6912(q_1-q_2)^2(q_2-q_3)^3\left((8q_1+13q_2-21q_3)\overline{\theta}+4(-q_1+q_2)\underline{\theta}\right)^2\left((4q_1-q_2-3q_3)\overline{\theta}^2-2(2q_1+q_2-3q_3)\overline{\theta}\underline{\theta}+(q_1-q_2)\underline{\theta}^2\right)$$

$$\begin{pmatrix} (4q_1-q_2-3q_3)\left(256q_1^3-68q_2^3-159q_2^2q_3-6q_2q_3^2-23q_3^3-48q_1^2(11q_2+5q_3)+3q_1(11q_2+5q_3)^2\right)\overline{\theta}^2 \\ -2\left(512q_1^4+4q_2^4+189q_2^3q_3+471q_2^2q_3^2-149q_2q_3^3-3q_3^4-32q_1^3(43q_2+21q_3)-q_1(5q_2-q_3)(q_2+7q_3)(41q_2+23q_3)+6q_1^2(177q_2^2+334q_2q_3+q_3^2)\right)\overline{\theta}\underline{\theta} \\ +(q_1-q_2)\left(256q_1^3-68q_2^3-159q_2^2q_3-6q_2q_3^2-23q_3^3-48q_1^2(11q_2+5q_3)+3q_1(11q_2+5q_3)^2\right)\underline{\theta}^2 \end{pmatrix}$$

The above term for the optimal quality $q^*_{E,HQS}$ is analytically not tractable, but we can give upper and lower bound for $q^*_{E,HQS}$:

a) $q^*_{E,HQS}(q_1,q_2,q_3) > q^*_E(q_2,q_3) = \frac{q_2+q_3}{2}$, i.e. the presence of a brand at q_1 induces the entrant to locate its HQS-brand at a higher level (closer to q_3):

$$\left.\frac{\partial \pi_E(q_1,q_2,q_E,q_3)}{\partial q_E}\right|_{q_E=q^*_E(q_2,q_3)} = \frac{4(q_3-q_2)\left((3q_3+5q_2-8q_1)\overline{\theta}-4(q_2-q_1)\underline{\theta}\right)\left((q_3-q_2)\overline{\theta}+8(q_2-q_1)\underline{\theta}\right)}{3(7q_3+9q_2-16q_1)^3} \qquad (1)$$

This term is strictly positive by Assumption 1. That is, the derivative of the entrant's profits with respect to the quality level of its own brand is positive given an entry in the HQS at the level that would be optimal in the absence of the LQI. Hence, increasing q_E beyond $q^*_E(q_2,q_3)$ increases profits. Therefore, $q^*_{E,HQS}(q_1,q_2,q_3) > q^*_E(q_2,q_3)$ must hold by uniqueness of interior extreme points.

b) $q^*_{E,HQS}(q_1,q_2,q_3) < \frac{q_2+2q_3}{3}$

$$\left.\frac{\partial\pi_E(q_1,q_2,q_E,q_3)}{\partial q_E}\right|_{q_E=\frac{q_2+2q_3}{3}} = -3\left((q_3+q_2-2q_1)\bar{\theta}-(q_2-q_1)\underline{\theta}\right)$$

$$\frac{\left((18(q_2-q_1)(q_3-q_1)+(3q_3+q_1-4q_2)(q_3-q_2))\bar{\theta}-(q_2-q_1)(13q_3-4q_2-9q_1)\underline{\theta}\right)}{4(5q_3+4q_2-9q_1)^3}<0$$

This term is strictly positive by Assumption 1. That is, the derivative of the entrant's profits with respect to the quality level of its own brand is positive given an entry in the HQS at the level that would be optimal in the absence of the LQI. Hence, increasing q_E beyond $\frac{q_2+2q_3}{3}$ decreases profits. Therefore, $q^*_{E,HQS}(q_1,q_2,q_3)<\frac{q_2+2q_3}{3}$ must hold by uniqueness of interior extreme points.

2.4. Extra brand by high quality incumbent and entry at the top quality, $q_1<q_2<q_3<q_E$

$$q^*_E(q_1,q_2,q_3)=\bar{q}$$

3. Partial derivatives

3.1. HQI

$$\frac{\partial\pi_{HQI}(q_1,q^*_E,q_2)}{\partial q_1}=-\frac{1}{288}(7\bar{\theta}-\underline{\theta})^2<0 \qquad \frac{\partial\pi_{HQI}(q_1,q^*_{E,LQS},q_2,q_3)}{\partial q_2}=-\frac{1}{288}\left(23\bar{\theta}^2+14\bar{\theta}\underline{\theta}-\underline{\theta}^2\right)<0$$

3.2. Entrant

$$\frac{\partial\pi_E(q_1,q^*_E,q_2)}{\partial q_1}=-\frac{1}{36}(\bar{\theta}-\underline{\theta})^2<0 \qquad \frac{\partial\pi_E(q_1,q^*_{E,LQS},q_2,q_3)}{\partial q_2}=\frac{1}{36}(\bar{\theta}-\underline{\theta})^2>0$$

$$\frac{\partial\pi_E(q_1,q_2,q_E,q_3)}{\partial q_2}<0:$$

$\pi_E(q_1,q_2,q_E,q_3) = f\cdot g$, where $f=(q_3-q_2)(q_E-q_2)(q_3-q_E)$ and $g=h^2=\left(\frac{(3q_E+q_2-4q_1)\bar{\theta}-2(q_2-q_1)\underline{\theta}}{3\left(4q_1(q_3-q_2)(q_E-q_1)-(q_E-q_2)^2\right)}\right)^2, h>0$. Hence,

$$\frac{\partial\pi_E(q_1,q_2,q_E,q_3)}{\partial q_2} = f'\cdot g + f\cdot g' < 0 \Leftrightarrow f'\cdot h^2 + f\cdot 2hh' < 0 \Leftrightarrow f'\cdot h + f\cdot 2h' < 0 \overset{f'=-f\left(\frac{1}{q_3-q_2}+\frac{1}{q_E-q_2}\right)}{\Leftrightarrow} 2h' < \left(\frac{1}{q_3-q_2}+\frac{1}{q_E-q_2}\right)h$$

$$\overset{h=\frac{\varphi}{\psi}}{\Leftrightarrow} 2\left(\frac{\varphi'}{\varphi}-\frac{\psi'}{\psi}\right) < \frac{1}{q_3-q_2}+\frac{1}{q_E-q_2} \Leftrightarrow 2\left(\frac{\bar{\theta}-2\underline{\theta}}{(3q_E+q_2-4q_1)\bar{\theta}-2(q_2-q_1)\underline{\theta}} - \frac{2(q_E-q_1+q_2-q_1)}{4q_1(q_3-q_2)(q_E-q_1)-(q_E-q_2)^2}\right) < \frac{1}{q_3-q_2}+\frac{1}{q_E-q_2}$$

Note that the LHS of the last inequality decreases in q_2, whereas the RHS increases in q_2. Thus, it suffices to show that the inequality holds for $q_2 \equiv q_1$. The inequality reduces to: $-\frac{1}{q_3-q_1}-\frac{1}{3(q_E-q_1)}-\frac{4}{4q_3-q_E-3q_1}-\frac{4\underline{\theta}}{3\bar{\theta}(q_E-q_1)}<0$, which is true for $q_1<q_2<q_E<q_3$.

$$\frac{\partial\pi_E(q_1,q_2,q_E,q_3)}{\partial q_3} > 0$$

$$\frac{\partial\pi_E(q_1,q_2,q_E,q_3)}{\partial q_3} = \frac{(q_E-q_2)^2\left(q2^2+4q1(-q2+q3)-2q2(q3-2qE)-qE(2q3+qE)\right)\left((3q_E+q_2-4q_1)\bar{\theta}-2(q_2-q_1)\underline{\theta}\right)^2}{-9\left(4(q_3-q_2)(q_E-q_1)-(q_E-q_2)^2\right)^3} > 0$$

Appendix B

Proof Lemma 1:

a) $\pi_{HQI}\left(q_1, q_E^*, q_3\right) < \pi_{HQI}\left(q_1, q_{E,LQS}^*, q_2, q_3\right)$

$\pi_{HQI}\left(q_1, q_{E,LQS}^*, q_2, q_3\right)$ is monotone decreasing in q_2 (see 3.1 in Appendix A). Taking the limit $q_2 \to q_3$ yields $\pi_{HQI}\left(q_1, q_{E,LQS}^*, q_2, q_3\right) = \pi_{HQI}\left(q_1, q_E^*, q_3\right)$.

Hence, $\pi_{HQI}\left(q_1, q_E^*, q_3\right) = \lim_{q_2 \to q_3} \pi_{HQI}\left(q_1, q_{E,LQS}^*, q_2, q_3\right) < \pi_{HQI}\left(q_1, q_{E,LQS}^*, q_2, q_3\right), \forall q_1 < q_2 < q_3$.

b) $\pi_{HQI}\left(q_1, q_2, q_{E,HQS}^*, q_3\right) < \pi_{HQI}\left(q_1, q_E^*, q_3\right)$

Let the superscript HQ (LQ) denote the low quality brand and the high quality brand of the incumbent, respectively (see 1.5. in Appendix A).

$$\pi_{HQI}\left(q_1, q_2, q_{E,HQS}^*, q_3\right) = \pi_{HQI}^{LQ}\left(q_1, q_2, q_{E,HQS}^*, q_3\right) + \pi_{HQI}^{HQ}\left(q_1, q_2, q_{E,HQS}^*, q_3\right) \overset{(*)}{\leq} \tag{2}$$

$$\frac{4\left(q_2-q_1\right)\left(q_3-q_1\right)\left(q_3-q_2\right)\left(\bar{\theta}-2\underline{\theta}\right)^2}{9\left(3q_3+q_2-4q_1\right)^2} + \pi_{HQI}^{HQ}\left(q_1, q_2, q_{E,HQS}^*, q_3\right) \overset{(**): q_{E,HQS}^* > q_E^*}{\leq} \tag{3}$$

$$\frac{4\left(q_2-q_1\right)\left(q_3-q_1\right)\left(q_3-q_2\right)\left(\bar{\theta}-2\underline{\theta}\right)^2}{9\left(3q_3+q_2-4q_1\right)^2} + \pi_{HQI}^{HQ}\left(q_1, q_2, q_E^*, q_3\right) \tag{4}$$

It turns out that (4) decreases in q_2 (see $(***)$). Therefore, evaluating (4) at $q_2 = q_1$ gives us the upper bound $\frac{8}{49}\left(q_3-q_1\right)\bar{\theta}^2$, which is below $\pi_{HQI}\left(q_1, q_E^*, q_3\right) = \frac{1}{288}\left(q_3-q_1\right)\left(7+6\underline{\theta}\right)^2$.

$(*)$:

$$\pi_{HQI}^{LQ}\left(q_1, q_2, q_E, q_3\right) =$$

$$\underbrace{\frac{\left(q_2-q_1\right)\left(q_E-q_1\right)\left(q_E-q_2\right)}{9\left(q_2^2+4q_1\left(q_3-q_2\right)+2q_2q_E-q_E\left(4q_3-q_E\right)\right)^2}}_{\text{increasing in } q_E} \underbrace{\left(2\left(q_3-q_E\right)\bar{\theta}-\left(4q_3-3q_2-q_E\right)\underline{\theta}\right)^2}_{\text{maximized at boundaries of } q_E} \leq \tag{5}$$

$$\frac{\left(q_2-q_1\right)\left(q_3-q_1\right)\left(q_3-q_2\right)}{\left(3q_3+q_2-4q_1\right)^2} \max\left\{\frac{4}{9}\left(\bar{\theta}-2\underline{\theta}\right)^2, \underline{\theta}^2\right\} \overset{A1}{=} \frac{\left(q_2-q_1\right)\left(q_3-q_1\right)\left(q_3-q_2\right)}{\left(3q_3+q_2-4q_1\right)^2}\frac{4}{9}\left(\bar{\theta}-2\underline{\theta}\right)^2 \tag{6}$$

$(**)$: see 2.3 a) in Appendix A.

(∗∗∗):

$\pi_{HQI}^{HQ}\left(q_1,q_2,q_E^*,q_3\right)=\frac{2\left(q_3-q_2\right)\left(\left(6q_3+8q_2-14q_1\right)\bar{\theta}-\left(q_2-q_1\right)\underline{\theta}\right)^2}{9\left(7q_3+9q_2-16q_1\right)^2}$. Thus, equation (4) becomes:

$$\left(q_3-q_2\right)\left[\frac{4\left(q_2-q_1\right)\left(q_3-q_1\right)\left(\bar{\theta}-2\underline{\theta}\right)^2}{9\left(3q_3+q_2-4q_1\right)^2}+\frac{2\left(\left(6q_3+8q_2-14q_1\right)\bar{\theta}-\left(q_2-q_1\right)\underline{\theta}\right)^2}{9\left(7q_3+9q_2-16q_1\right)^2}\right], \tag{7}$$

which decreases in q_2. QED

<u>Proof Proposition 1:</u>

(1) Suppose that entry occurs and that the high quality incumbent did not launch a second brand. Note that in this case the high quality incumbent will engage in maximal product differentiation. However this is not a best response for the incumbent since (s)he could increase profits by introducing a second brand sufficiently close to its top brand. By that either entrance is deterred or entrance occurs in the low quality segment. In the latter case profits will strictly increase by Lemma 1. In the former case, note that $\pi_{HQI}\left(q_1,q_2,q_3\right)>\pi_{HQI}\left(q_1,q_E^*,q_3\right)$. The inequality holds because $\pi_{HQI}\left(q_1,q_2,q_3\right)$ is increasing in q_2 and $\pi_{HQI}\left(q_1,q_1,q_3\right)=\frac{1}{4}\left(q_3-q_1\right)\bar{\theta}^2>\frac{1}{288}\left(q_3-q_1\right)\left(7\bar{\theta}-\underline{\theta}\right)^2=\pi_{HQI}\left(q_1,q_E^*,q_3\right)$ by Assumption 1. Therefore, if entry is deterred by a second brand profits for the HQI must be higher than in case of accommodation with no second brand.

(2) By Lemma 1 $\pi_{HQI}\left(q_1,q_2,q_{E,HQS}^*,q_3\right)<\pi_{HQI}\left(q_1,q_E^*,q_3\right)$, i.e. if entry occurs in the HQS the HQI would prefer not to be in the market with a second intermediate quality. However by (1) we know that entry will never be accommodated with one brand. Hence, if entry is accommodated in equilibrium then entry must occur in the LQS.

(3) The claim follows from $\frac{\partial \pi_{HQI}\left(q_1,q_{E,LQS}^*,q_2,q_3\right)}{\partial q_3}\underset{\text{independent of } q_3}{\overset{q_{E,LQS}^*}{=}}\frac{\partial \pi_{HQI}\left(q_1,q_E,q_2,q_3\right)}{\partial q_3}=\frac{\bar{\theta}^2}{4}>0$.

(4) Note that $\pi_{HQI}\left(q_1,q_{E,LQS}^*,q_2,q_3\right)$ decrease in q_2 (see 3.1 in Appendix A) and $\pi_E\left(q_1,q_{E,LQS}^*,q_2,q_3\right)$ increases in q_2 (see 3.2 in Appendix A). Furthermore, both $\pi_E\left(q_1,q_2,q_{E,HQS}^*,q_3\right)$ and $\pi_{HQI}\left(q_1,q_2,q_{E,HQS}^*,q_3\right)$ decrease in q_2. The former holds, because $\pi_E\left(q_1,q_2,q_E,q_3\right)$ decreases q_2 (see 3.2 in Appendix A).

This and (3) imply that the HQI will decrease q_2 and thereby increase profits until the constraint that the entrant prefers entrance in the LQS becomes binding. This is exactly the case when the profits in both segments are equal in which we assumed for E to enter the LQS.

QED

Proof Lemma 2: Note that $\pi_E(q_l, q_E, q_2) = \pi_E(q_l, q_E, q_2, q_3)$ (see Appendix A). Hence the optimal quality choices of E and reduced profits coincide. QED

Proof Proposition 2:

(1) Since $\pi_E(q_l, q_E, q_2) = \pi_E(q_l, q_E, q_2, q_3)$, to deter intermediate entry or entry in the LQS respectively, the HQI chooses the same q_2. Furthermore, $\pi_{HQI}(q_l, q_2) - \pi_{HQI}(q_l, q_2, q_3) = -\frac{1}{4}(q_3 - q_2)\bar{\theta}^2 < 0$, i.e. it always pays to launch a second brand instead of just lowering the quality of the top brand.

(2) Note that $\frac{\partial \pi_{HQI}(q_l, q_2, q_3)}{\partial q_2} = \frac{1}{36}(\bar{\theta} - 2\underline{\theta})(7\bar{\theta} - 2\underline{\theta}) > 0$, i.e. the HQIs' profits increase in the quality level of the second brand. Hence in order to deter entry the incumbent will move q_2 away from its top brand only to the extent that profits for the entrant become sufficiently low.

(3) The claim follows from $\pi_{HQI}(q_l, q_2, q_3)$ being increasing in q_3 (see 1.3 in Appendix A).

QED

Proof Lemma 3: The proof will proceed in two steps:

Step one:

$$\exists! b^{crit} : \forall q_l < a < b < q_3, b \geq b^{crit} : \pi_{HQI}(q_l, a, b) > \pi_{HQI}(q_l, q_E^*, q_3) \tag{8}$$

, i.e. irrespective of the choice of quality of the intermediate brand the profit for the HQI if no entrance occurs is higher than the profit from accommodation with maximal product differentiation if the top quality is sufficiently high (above b^{crit}).

Since $\pi_{HQI}(q_l, a, b)$ is increasing in a, b^{crit} is determined by solving:

$$\pi_{HQI}(q_l, q_l, b^{crit}) = \pi_{HQI}(q_l, q_E^*, q_3) \tag{9}$$

, which yields: $b^{crit} = \frac{23q_l + 49q_3 + 12\underline{\theta}(5q_l + 7q_3 + 3(q_l + q_3)\underline{\theta})}{72(1+\underline{\theta})^2}$. (10)

Step two:

$$\forall b \le b^{crit}:$$
$$\min_{q_1<a<b}\left\{\pi_{E,123E}\left(q_1,a,b,q_E\right)\right\} > \pi_E\left(q_1,q_E^*,q_3\right) > \max_{q_1<a<b}\left\{\pi_E\left(q_1,q_{E,LQS}^*,a,b\right),\pi_E\left(q_1,a,q_{E,HQS}^*,b\right)\right\} \quad (11)$$

In words, if $b \le b^{crit}$ the least w.r.t. to a that E can earn if (s)he enters with the top quality is more than the most w.r.t. a what (s)he could earn in LQS or HQS.

Note that:

$$\begin{aligned}&\max_{q_1<a<b}\left\{\pi_E\left(q_1,q_{E,LQS}^*,a,b\right),\pi_E\left(q_1,a,q_{E,HQS}^*,b\right)\right\}\\&=\max_{q_1<a<b}\left\{\pi_E\left(q_1,q_{E,LQS}^*,b,b\right),\pi_E\left(q_1,q_1,q_{E,HQS}^*,b\right)\right\}\end{aligned} \quad (12)$$

$$\max_{q_1<a<b}\left\{\pi_E\left(q_1,q_{E,LQS}^*,b,b\right),\pi_E\left(q_1,q_1,q_{E,HQS}^*,b\right)\right\}=\pi_E\left(q_1,q_{E,LQS}^*,b,b\right) \quad (13)$$

Equation (12) follows from the fact that $\pi_E\left(q_1,q_{E,LQS}^*,a,b\right)$ increases in a and that $\pi_E\left(q_1,a,q_{E,HQS}^*,b\right)$ decreases in a (see (4) in the proof of Proposition 2). Equation (13) follows from $\pi_E\left(q_1,q_{E,LQS}^*,b,b\right)=\pi_E\left(q_1,q_E^*,b\right)$ and the fact that profits for E are decreasing in the number of brands in the market, i.e. $\pi_E\left(q_1,q_E^*,b\right)-\pi_E\left(q_1,q_1,q_{E,HQS}^*,b\right)=\frac{1}{144}\left(b-q_1\right)\left(\bar{\theta}^2-8\bar{\theta}\underline{\theta}+4\underline{\theta}^2\right)$. This difference in payoffs is strictly positive by Assumption 1. Finally, $\pi_E\left(q_1,q_E^*,q_3\right)>\pi_E\left(q_1,q_E^*,b\right)$ follows from $\pi_E\left(q_1,q_E^*,b\right)$ being increasing in b, $\frac{\partial\pi_E\left(q_1,q_E^*,b\right)}{\partial b}=\frac{1}{36}\left(\bar{\theta}-\underline{\theta}\right)>0$. Hence we established the second inequality in equation (11).

Regarding the first inequality in equation (11), note that $\pi_{E,123E}\left(q_1,a,b,q_E\right)$ is monotonically decreasing or increasing in a, since

$$\frac{\partial\pi_{E,123E}\left(q_1,a,b,q_E\right)}{\partial a}=\frac{4\overbrace{\left(b-q_3\right)}^{<0}\overbrace{\left(b-q_3+3\left(q_3-q_1\right)\underline{\theta}\right)}^{\text{sign independent of a}}\overbrace{\left(a-3b-4q_1+6q_3+3\left(a-b-2q_1+2q_3\right)\underline{\theta}\right)}^{>0}}{\underbrace{9\left(a-b-4q_1+4q_3\right)^3}_{>0}}$$

, hence $\min_{q_1<a<b}\left\{\pi_{E,123E}\left(q_1,a,b,q_E\right)\right\}=\min\left\{\pi_{E,123E}\left(q_1,q_1,b,q_E\right),\pi_{E,123E}\left(q_1,b,b,q_E\right)\right\}$.

What is left to show is:

$$\min\left\{\pi_{E,123E}\left(q_1,q_1,b,q_E\right),\pi_{E,123E}\left(q_1,b,b,q_E\right)\right\}>\pi_E\left(q_1,q_E^*,q_3\right). \quad (14)$$

Equations (15) and (16) establish this relation.

$$\pi_{E,123E}\left(q_1,q_1,b^{crit},q_E\right)-\pi_E\left(q_1,q_E^*,q_3\right)=\left(q_3-q_1\right)\varphi\left(\theta\right)>0 \quad (15)$$

$, b > b^{crit}$ where

$$\omega(\theta) = \frac{(23+12\underline{\theta}(5+3\underline{\theta}))(265+12\underline{\theta}(61+3\underline{\theta}(19+6\underline{\theta})))(599+12\underline{\theta}(155+3\underline{\theta}(53+18\underline{\theta})))}{13436928(1+\underline{\theta})^6} - \frac{1}{36} > 0, \forall \underline{\theta} > 0$$

$$\varphi(\theta) = \frac{(1+\underline{\theta})^2 (23+12\underline{\theta}(5+3\underline{\theta}))(167+12\underline{\theta}(29+15\underline{\theta}))}{4(239+12\underline{\theta}(41+21\underline{\theta}))^2} - \frac{1}{36} > 0, \forall \underline{\theta} > 0 .$$

and

$$\pi_{E,123E}\left(q_1, q_1, b^{crit}, q_E\right) - \pi_E\left(q_1, q_E^*, q_3\right) = \left(q_3 - q_1\right)\omega(\theta) > 0 \tag{16}$$

, where , which establishes the first inequality in equation (11).

By step two $\pi_E\left(q_1, a, b, q_E = \overline{q}\right) > K = \pi_E\left(q_1, q^*_{E,LQS}, a, b\right) = \pi_E\left(q_1, a, q^*_{E,HQS}, b\right), \forall b \leq b^{crit}$. Hence, for $\pi_E\left(q_1, a, b, q_E = \overline{q}\right) < K$ to be satisfied we need $b > b^{crit}$ which implies by step one $\pi_{HQI}\left(q_1, a, b\right) > \pi_{HQI}\left(q_1, q_E^*, \overline{q}\right)$ QED

Proof Theorem:

Blockaded Entry Let $F^H \equiv \pi_E\left(\underline{q}, q_E^*, \overline{q}\right)$. If $F \geq F^H$, E cannot earn strictly positive profits by entering the market, hence entry is blockaded. In that case the HQI will maximize profits by maximal product differentiation. Hence, blockaded entry is the unique Nash equilibrium.

Entry Deterrence (with maximal product differentiation) Let $\underline{q} < a < \overline{q}$ such that $\pi_E\left(\underline{q}, q^*_{E,LQS}, a, \overline{q}\right) = \pi_E\left(\underline{q}, a, q^*_{E,HQS}, \overline{q}\right) \equiv F^M$. For $F^M \leq F < F^H$, E cannot profitable enter the market given the optimal interior deterrence strategy (Proposition 2), i.e. $q_3^* = \overline{q}$ and $q_2^* : \pi_E\left(\underline{q}, q^*_{E,LQS}, q_2^*, \overline{q}\right) = F$. According to Proposition 2, entry is accommodated only in the LQS, i.e. in particular with two brands. Furthermore, $q_3 = \overline{q}$ by (3) of Proposition 2. To actually allow E to enter the LQS, the HQI would need to allocate its intermediate brand such that $\pi_E\left(\underline{q}, q^*_{E,LQS}, q_2, \overline{q}\right) > F$. Note that in that case, on the one hand, $q_2 > a$ as $\pi_E\left(\underline{q}, q^*_{E,LQS}, a, \overline{q}\right)$ increases in a and $F^M \leq F$. On the other hand, $\pi_E\left(\underline{q}, q_2 > a, q^*_{E,HQS}, \overline{q}\right) < F^M \leq F$. However, lowering q_2 will not only increase profits ($\pi_{HQI}\left(q_1, q^*_{E,LQS}, q_2, q_3\right)$ decreases in q_2, see (4) in the proof of Proposition 1) given entry, but will eventually induce E not to enter the market which will additionally increase profits of the HQI (see Lemma 1). Hence, entry deterrence with brand proliferation and maximal product differentiation is the unique Nash equilibrium.

Entry Deterrence (without maximal product differentiation) Let $\underline{q} < a < b < \overline{q}$ such that $\pi_E\left(\underline{q}, q^*_{E,LQS}, a, b\right) = \pi_E\left(\underline{q}, a, q^*_{E,HQS}, b\right) = \pi_E\left(\underline{q}, a, b, q_E = \overline{q}\right) \equiv F^L$. For $F^L \leq F < F^M$ deterrence with $q_3 = \overline{q}$ is no longer feasible, because E will find it profitable to enter either the LQS or the

HQS depending on the location of q_2. According to Proposition 2, the HQI will reduce the quality of its top brand only to the extent that the equalized profits in the LQS and the HQS just match the level of fixed cost of entry of E. Remember that the deterring levels for q_2 and q_3 are unique, because $\pi_E\left(\underline{q}, q^*_{E,LQS}, a, b\right) = F$ is independent of b and determines q_2, whereas the condition $\pi_E\left(\underline{q}, a, q^*_{E,HQS}, b\right) = F$ determines q_3. This strategy will indeed deter entry as long as $F^L \leq F$. For lower values of F the entrant will find it profitable to enter the market with the maximum level of quality. We need to compare the profits generated by this optimal entry deterrence with the profits from optimal entry accommodation. We know from Proposition 1 that optimal entry accommodation equalizes profits of LQS and HQS. Furthermore, for the premium brand $q_3 = \overline{q}$. We therefore have to compare $\pi_{HQI}\left(\underline{q}, a, b\right)$ with $\pi_{HQI}\left(\underline{q}, q^*_{E,LQS}, q_2^{eq}, \overline{q}\right) = \pi_{HQI}\left(\underline{q}, q_2^{eq}, q^*_{E,HQS}, \overline{q}\right)$, where q_2^{eq} denotes the level that equalizes profits in both segments given maximal product differentiation.

Again, the independence of $\pi_E\left(\underline{q}, q^*_{E,LQS}, a, b\right)$ w.r.t. b and the fact that $\pi_E\left(\underline{q}, q^*_{E,LQS}, a, b\right)$ increases in a implies $a < q_2^{eq}$.

Hence, it suffices to show that:
$\pi_{HQI}\left(\underline{q}, a, b\right) > \pi_{HQI}\left(\underline{q}, q^*_{E,LQS}, a, \overline{q}\right)\left(> \pi_{HQI}\left(\underline{q}, q^*_{E,LQS}, q_2^{eq}, \overline{q}\right) = \pi_{HQI}\left(\underline{q}, q_2^{eq}, q^*_{E,HQS}, \overline{q}\right)\right)$, the last inequality follows from $a < q_2^{eq}$ and that $\pi_{HQI}\left(\underline{q}, q^*_{E,LQS}, q_2, \overline{q}\right)$ decreases in q_2.

To show $\pi_{HQI}\left(\underline{q}, a, b\right) > \pi_{HQI}\left(\underline{q}, q^*_{E,LQS}, a, \overline{q}\right)$, we will proceed in four steps.

(1) Lower bound for a

Note that $\pi_{HQI}\left(\underline{q}, \underset{+}{a}, \underset{+}{b}\right)$ and $\dfrac{\partial \pi_{HQI}\left(\underline{q}, q^*_{E,LQS}, a, \overline{q}\right)}{\partial a} < 0$ (see 3.1 in Appendix A). Furthermore, for $\tilde{a} = \dfrac{3\underline{q} + 2b}{5}$:

$$\pi_E\left(\underline{q}, q^*_{E,LQS}, \tilde{a}, b\right) < \pi_E\left(\underline{q}, \tilde{a}, q^*_{E,HQS}, b\right) \underset{\text{of } q^*_{E,HQS}}{\overset{\text{by definition}}{<}} \pi_E\left(\underline{q}, \tilde{a}, q^*_E, b\right) \tag{17}$$

Hence, if $\pi_{HQI}\left(\underline{q}, \tilde{a}, b\right) > \pi_{HQI}\left(\underline{q}, q^*_{E,LQS}, \tilde{a}, \overline{q}\right)$, then $\pi_{HQI}\left(\underline{q}, a, b\right) > \pi_{HQI}\left(\underline{q}, q^*_{E,LQS}, a, \overline{q}\right), \forall a > \tilde{a}$.

(2) Critical value for b

Solving

$$\pi_{HQI}\left(\underline{q}, \tilde{a}, b\right) = \pi_{HQI}\left(\underline{q}, q^*_{E,LQS}, \tilde{a}, \overline{q}\right) \tag{18}$$

w.r.t. to b yields a critical level $b^{crit} = \underline{q} + \frac{180\overline{\theta}^2\left(\overline{q}-\underline{q}\right)}{259\overline{\theta}^2 - 114\overline{\theta}\underline{\theta} + 31\underline{\theta}^2}$.

So far, we have established that

$$\pi_{HQI}\left(\underline{q}, a, b\right) > \pi_{HQI}\left(\underline{q}, q^*_{E,LQS}, a, \overline{q}\right), \forall a > \tilde{a}, b > b^{crit}. \qquad (19)$$

(3) Lower bound for b

To get a lower bound for b, we make use of: $\pi_E\left(\underline{q}, q^*_{E,LQS}, a, b\right) = \pi_E\left(\underline{q}, a, b, \overline{q}\right)$ by definition of a and b. Solving this equation for b yields

$$\tilde{b} = \frac{-8\left(\overline{q}-\underline{q}\right)\left(4\overline{\theta}-\underline{\theta}\right)\sqrt{\left(1+\left(8\overline{\theta}+\underline{\theta}\right)^2\right)} + \left(4\underline{q}\left(1-64\overline{\theta}^2+8\overline{\theta}\underline{\theta}+2\underline{\theta}^2\right)+\overline{q}\left(8\left(4\overline{\theta}-\underline{\theta}\right)\left(14\overline{\theta}+\underline{\theta}\right)\right)-5\right)}{48\overline{\theta}\left(4\overline{\theta}-\underline{\theta}\right)-1}$$

(4) $\tilde{b} > b^{crit}$

As the last step, we establish that lower bound exceeds the critical level of b, what guarantees that the HQI prefers entry deterrence (see equation (19)). The claim follows from the fact that $\tilde{b} - b^{crit}$ increases in $\overline{q}$ and vanishes at $\overline{q} = \underline{q}$

Hence the unique Nash equilibrium is entry deterrence with product proliferation and non-maximal product differentiation.

Entry Accommodation Finally for $0 \le F < F^L$ entry cannot be deterred and the unique Nash equilibrium is entry accommodation in the LQS and maximal product differentiation (Proposition 1). We will proof that by contradiction. Let us assume that $\exists c, d : \underline{q} < c < d < \overline{q} : \max\left\{\pi_E\left(\underline{q}, q^*_{E,LQS}, c, d\right), \pi_E\left(\underline{q}, c, q^*_{E,HQS}, d\right), \pi_E\left(\underline{q}, c, d, q_E = \overline{q}\right)\right\} \le F < F^L$, i.e. entry deterrence is feasible for a given fixed cost of entry . Consider $\underline{q} < a < b < \overline{q}$ as in the definition of F^L. Note that for given b, $\pi_E\left(\underline{q}, q^*_{E,LQS}, a, b\right)$ increases in a and $\pi_E\left(\underline{q}, a, q^*_{E,HQS}, b\right)$ decreases in a.

Furthermore, for given a, $\pi_E\left(\underline{q}, q^*_{E,LQS}, a, b\right)$ is independent of b (see Lemma 2) and $\pi_E\left(\underline{q}, a, q^*_{E,HQS}, b\right)$ increases in b. The latter is true since $\pi_E\left(q_1, q_2, q_E, q_3\right)$ increases in q_3. To induce profits for E below F^L in the LQS and the HQS the two previous observation imply that both a and b must be reduced. As a consequence, profit for entry at the top level $\pi_E\left(\underline{q}, a, b, q_E = \overline{q}\right)$ increases, i.e. $\max\left\{\pi_E\left(\underline{q}, q^*_{E,LQS}, c, d\right), \pi_E\left(\underline{q}, c, q^*_{E,HQS}, d\right)\right\} \le F < F^L$ implies $\pi_E\left(\underline{q}, c, d, q_E = \overline{q}\right) > F^L$. Hence entry cannot be deterred. QED