

Allocation under Fixed Book-Price Arrangements.

Abstract: We apply a standard monopolistic competition model to analyse fixed book-price arrangements. It is shown that fixed-price arrangements support title variety. That is, an increase in book sellers' price margin produces more titles. However, increased title variety is costly and consumers' welfare goes down. Thus, fixed book prices support title variety but at the cost of redistributing welfare from consumers to book sellers. Introducing opportunity costs of reading (van der Ploeg, 2004) we show that an increase in book sellers' price margin can improve consumers' welfare.

Keywords. Book market, fixed book price, monopolistic competition.

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The market for books functions much along the lines suggested by Chamberlain (1933): There are many consumers and many titles, meaning that a single title cannot have total control over the market; nevertheless, each title is exclusive in the meaning that there are non-price differences from one title to another. Also, it is easy to put new titles on the market. Indeed, Bittlingmayer

(1992) and, more recently, van der Ploeg (2004 and 2005) suggest that the book market is a monopolistically competitive market.

Monopolistically competitive markets function inefficiently in that they pick out the wrong number of product varieties and the wrong number of copies per variety (see Spence, 1976, and Dixit and Stiglitz, 1977). This explains why anti-competitive practices and public subsidies might have positive consequences beyond the obvious redistributive effects in favour of book suppliers. That is, one cannot rule out that what the “folklore” says is correct: that books are special, and therefore, they need some kind of protection from market forces. In this paper we apply a standard model of a monopolistically competitive industry and examine fixed book-price arrangements. Fixed book-price agreements are widely used throughout Europe and they are supposed to promote title variety which seems to be an objective for cultural policy (van der Ploeg, 2005, page 16).

Clearly, the consumption of a book is rival and excludable, making the individual title a private good. This suggests, on one hand, that the book market is fairly well-functioning. As a consequence, from the welfare economics point of view, fixed book-price arrangement is misplaced. On the other hand, as demonstrated by Spence (1976), a monopolistically competitive market typically fails along three measures. With respect to books, the number of titles and the number of copies per title is typically inefficient. Also, the market selects the ideal mix of titles only in rare circumstances. That is, the title portfolio is wrong. We are concerned with the first two problems (number of titles and number of copies per title) and ask if fixed book-price agreements

increase the number of titles surviving in the market, and how the suspension of price competition changes welfare for consumers and suppliers of books.

Van der Ploeg (2004) suggests that the time needed for reading is a significant cost in comparison to paying for the book. It certainly seems obvious that books are different in comparison to other sorts of consumption goods on this point. When the opportunity cost of reading a book is the income that could have been earned by working when reading, the opportunity cost will most likely exceed the price of the book rather substantially. Of course, when consumers of books take into consideration the socially relevant opportunity cost when deciding to purchase a book, the fact that it is time consuming to read will not change the comparison between the market's outcome and the welfare-maximising allocation. However, when the consumer's calculation of opportunity cost differs from the cost that is relevant for society at large, it is relevant to take into account how this cost affects the market, and, in turn, how the opportunity cost might change the relationship between the market outcome and the welfare-maximising allocation.

There are only few empirical studies dealing with the book market. With respect to the market for paper books, Greco (2000) studies concentration in the American-consumer book industry, and Hjorth-Andersen (2000) models changes in the market for books in Denmark. However, neither of these studies concern the issue discussed here. Latcovich and Smith (2001), Peltier and Moreau (2012), and Tang, Smith and Montgomery (2010) study the emergence of internet shopping and book sales. Bittlingmayer (1992) uses German data to study pricing in the book market but does not analyse welfare issues and market interventions. Van der Ploeg (2004 and 2005) offers a

thorough view on the economics of the book market but does not analyse the problems relating to the monopolistically competitive nature of the book market. Our paper proceeds as follows. The model is introduced in Section 1. In Section 2, we analyse equilibrium, and the relationship between title variety and fixed book-price arrangements are the subjects of Section 3. In Sections 4 and 5 the welfare implications of fixed book-price arrangements are worked out. Section 6 concludes. The formal analysis is in the Appendix.

1. The Book Market

To analyse the effect of a fixed book-price arrangement we consider a model where publishers operate through bookshops. Bookshops, in turn, are responsible for the sale to consumers. Evidently, one of the most distinct characteristics of the book market is that it is impossible to find a perfect substitute for some given title even when the number of titles is huge (see, for example, Bittlingmayer, 1992, and van der Ploeg, 2004). We model this by assuming that titles cannot be arbitrarily close. This idea of no-neighbouring varieties implies that each title is a monopoly; that is, titles are not strategically interrelated. On the other hand, we expect that competition between varieties harness the monopoly profit per title. This idea rests on the assumption that the number of potential varieties is much larger than the actual number of varieties that survive. In the case of books, this assumption means that there are unpublished manuscripts that are put into production if this creates a profit. Thus, if there is profit, new titles enter and this process comes to a stop when existing titles makes a non-negative profit and a new title will give rise to a loss.

Preferences. A utility structure that allows us to describe the market in the way explained above is $u(q) = \alpha^{-1}m^\alpha$, where $q = q_1, q_2, \dots, q_n$ and $m = \sum_{i=1}^n q_i^{\theta_i}$ (see Spence, 1976, and Dixit and Stiglitz, 1977). Here, q_i is the number of copies of title i , n is the number of titles that are actually in print, and α and θ_i are parameters. The parameters satisfy $0 < \alpha < 1$ and $0 < \theta_i < 1$. The marginal utility of an increase of m is positive and decreasing for the restriction on α . This utility function captures a preference for variety as long as θ_i is bounded from above by 1.

Following Benassy (1996) it can be argued that it is more appropriate to model preferences as $u(q) = \alpha^{-1}m^\alpha, m = n^v \sum_{i=1}^n q_i^{\theta_i}$. This gives a less tight relationship between consumers' preference for variety and the monopoly power per title. It is often argued that books serve cultural purposes that people appreciate in addition to personal consumption of books. Obviously, books are private goods since they are excludable (by being sold) and non-divisible (assuming that they are not shared). However, this clearly does not exclude the possibility that consumers also value diversity of titles *per se*. One interpretation of $m = n^v \sum_{i=1}^n q_i^{\theta_i}$ is that the term n^v describes the value of existence of titles.¹ We consider the case of $v = 0$ as well as that where utility is increasing at a decreasing rate in the number of titles, i.e., setting $0 < v < 1$.

Costs. Following Bittlingmayer (1992), notice that book production is characterised by fixed costs. These include reviews and proof reading of manuscripts, cover designs and marketing activities.

¹ An alternative approach is to let private utility be $u(q_i) = \alpha^{-1}m^\alpha$, where $m = \sum_{i=1}^n q_i^{\theta_i}$ and social welfare $S(q_i) = \alpha^{-1}m^\alpha + vm$. Trivially, this implies that the market delivers too little title variety and too few copies per title.

Once the publication decision has been made, these costs are a fixed cost for the title in question even if they are discretionary expenses at the firm level. Moreover, we assume that there is a fixed cost per copy, meaning that the average cost is decreasing in the number of copies per title. The fixed marginal cost are, for example, storage and transportation costs. Overall, with respect to costs, the production costs of a given title is $c(q_i) = cq_i + F$, where F is the fixed cost, and c is the constant marginal cost that is identical across titles. We assume that one publisher publishes one title.²

Fixed book prices. We consider books sold to final customers through (intermediaries) bookshops – markets that, in Europe, are characterised by fixed book-price agreements. Suppose that a bookshop pays the publisher w_i per copy of title i and that is all the costs of getting the book. If there is price competition among bookshops, bookshops end up setting a price of w_i per copy of title i . To model as simply as possible the workings of a fixed book-price arrangement we assume that title i is sold at $p_i = w_i + \phi$, where the term ϕ is profit to the bookshop. This is arguably a very simple way to model the fixed price arrangement but it allows us to focus on the issue of whether harnessing competition in the book market is beneficial in promoting varied literary production (see Ringstad, 2004).

The social optimum. The social optimum is the welfare-maximising combination of output per title and number of titles. When we consider the symmetric case, $m = n^{1+\nu}q^\theta$, we can as well ask for

² Under the assumptions about respect to costs made here, each publisher prints one title, cf. Dixit and Stiglitz (1977, page 299).

combinations of m and q that maximise welfare. Thus, the welfare maximum is given by the solution to:

$$\text{Argmax}_{m,q} W = \alpha^{-1}m^\alpha - n(cq + F), m = n^{1+v}q^\theta \quad (1)$$

The bookshops' sales margin enters as income for bookshops and expenses for buyers of books and is why it cancels from the welfare expression. In the Appendix, the solution to the maximisation problem is found to be:

$$q^* = \theta / (1 - \theta + v) \cdot F / c \quad (2)$$

$$m^{*1/(1+v)-\alpha} = (1 - \theta + v) / F \cdot \left(\theta / (1 - \theta + v) \cdot F / c \right)^{\theta/(1+v)} \quad (3)$$

The term $1/(1 + v) - \alpha$ is positive because a second-order condition gives $\alpha < 1/(1 + v)$. The solution reflects that the optimum number of titles minimizes the cost of achieving some given value of m . Since average costs are declining it is clear that marginal-cost pricing, which is a precondition for maximizing social welfare, is infeasible since each title would give a loss due to the fixed cost. Thus, implementation of the welfare-maximising allocation presupposes that some sort of lump-sum transfer makes it possible for publishers to recoup the fixed-cost expenses.

2. Equilibrium and title variety

The equilibrium is determined by the price decisions of publishers. Prices are set so as to maximize per-title profit that is given by $\pi(q_i) = (p_i - \phi)q_i - cq_i - F$. Under the assumption that each publisher's share of the market is sufficiently small, the individual publisher acts as if her sale has

no effect on the sale of other publishers, i.e., $\partial m / \partial q_i = 0$. Under the no-neighbouring condition, and if we ignore income-effects, the publisher sees that the price that a title can sell for is given by $p_i = m^{\alpha-1} n^v \theta_i q_i^{\theta_i-1}$. The publishing-profit of title i is:

$$\pi(q_i) = (m^{\alpha-1} n^v \theta_i q_i^{\theta_i-1} - \phi) q_i - c q_i - F \quad (4)$$

This expression for profit shows that each publisher enjoys a monopoly while at the same time, her sale is affected (through the term m) by the decisions of other publishers. Under monopoly conditions, profit-maximising behaviour disturbs pricing away from equality between price and marginal cost but it enables the publisher to cover the fixed publishing costs. Moreover, the expression reveals how the number of copies of competing titles changes the profitability of one title. As is common, we shall focus on symmetric equilibrium in order to bring out most clearly the problems associated with the market's selection of the number of copies per title and the number of titles.³ In a symmetric equilibrium, we have that output per title and number of titles are (shown in the Appendix):

$$\bar{q} = \theta / (1 - \theta) \cdot F / (c + \phi), \quad (5)$$

$$\bar{m}^{1/(1+v)-\alpha} = \theta (1 - \theta) / F \left(\theta / (1 - \theta) \cdot F / (c + \phi) \right)^{\theta/(1+v)} \quad (6)$$

respectively. It is worth noting that, in deriving equations (5), we have utilised that the price per title is determined by a demand function with constant elasticity (of $1 - \theta$, numerically). From this it follows that first-order condition for a profit maximum is $(p_i - c) / p_i = 1 - \theta$, where the left-

³ It is of obvious interest to ask also, whether the market picks out the right titles or if a specific group of titles are left out systematically. This is difficult to answer and is left to a future paper.

hand side is the Lerner index. We have also utilized that entry and exit decisions drives profit to zero, i.e., we have $p - (c + \phi) - F/\bar{q} = 0$.⁴ Combining the zero-profit condition with the expression for the profit maximizing price we have result in equation (5). The value of \bar{m} in equation (6) follows from solving for \bar{m} so that the price satisfies $(p_i - c)/p_i = 1 - \theta$.

Comparison of equations (2) and (5) shows:

Proposition 1. For $\phi = v = 0$, the number of copies per title in the market allocation exceeds the socially optimal number. When $0 < v < 1$, the number of copies per title in the market allocation exceeds or falls short of the socially optimal number as $\phi \lesseqgtr \tilde{\phi}$, where $\tilde{\phi} = (1 - \theta)/vc$.

In the situation of $\phi = 0$, that is in the absence of a fixed price arrangement, the upshot of Proposition 1 is that the market is inefficient. More precisely, overall utility is given by $u(q) = \alpha^{-1}m^\alpha$ and the proposition implies, for some fixed value of m , that there are too many copies relative to the number of titles. Since the number of titles in the market is decreasing in ϕ , this might suggest that increasing ϕ from $\phi = 0$ can increase welfare. The reasoning behind such a policy is that the sales margin effectively means that publishers' marginal cost increases.

Increasing marginal costs changes the balance between the fixed cost and the variable cost so that

⁴ Follows from $w - c - F/\bar{q} = 0$ and $w = p - \phi$.

fixed costs become, relatively speaking, less important. In turn, it becomes more profitable to introduce new titles (and cut back on the number of copies of existing titles).

3. Welfare and Sales Margin

Although the result in Proposition 1 suggests that an increase of the sales margin might have a favourable effect on the balance between the number of titles and the number of copies per title, we need to compare the number of copies per title as well as the number of titles to see whether consumer welfare is affected positively or negatively by bookshops' sales margins. Rather than comparing copies per title and the number of titles we compare the number of copies per title and the values for variable m . We have:

Proposition 2. The market's allocation is inefficient as $m^* > \bar{m}$. The number of copies per titles is smaller or larger than the optimum number in accordance with $q^* \geq \bar{q}$ as $(1 - \theta)\phi \geq vc$.

The relation between the optimum numbers of copies per title is a repetition from Proposition 1. Using (3) and (6) in the Appendix it follows right away that $\bar{m} < m^*$. Combining the observations that $\bar{m} < m^*$ and, for $\phi = 0$, $\bar{q} > q^*$, we see that the market supplies too few titles. One of the central speculations about the book market is that fixed-price arrangements are beneficial because they protect variety. That is, if this thesis is correct, ϕ is ideally bounded from below at zero. To examine if this is the situation, consider (initially) the case where books are a good that is

of value only for personal consumption. This is the case of $v = 0$. Comparison of the values of \bar{q} and q^* shows that the market, in this situation, supports the socially optimal number of copies per title when bookshops' sales margin is nullified. However, the market is nevertheless inefficient since $m^* > \bar{m}$. Consider a small increase of ϕ from $\phi = 0$. The welfare effect of this is negligible when we look at the number of copies per title. Nevertheless, even if there is a favourable effect on the balance between the number of titles and the number of copies per title, it is welfare-reducing to increase ϕ . This is so because (follows from (6)) \bar{m} goes down with an increase in the sales margin. The latter is a consequence of titles being less than perfect substitutes. Overall, an increase in the sales margin reduces the number of copies per title to below the socially optimal value and pushes the value of m even further away from the optimal value. Hence, if book variety is not valued *per se*, an increase of the bookshops' sales margin is unambiguously welfare-reducing. That is, in welfare terms, it is costly to promote the number of titles.

Consider next how changes in ϕ relate to welfare when consumers value the number of titles *per se*. This is the case of $v > 0$. If, in this situation $\phi = 0$, we have $\bar{q} > q^*$ and $\bar{m} < m^*$. When the sales margin increases \bar{q} goes down. Since the number of copies per title is excessive, this is a positive effect and it dominates the adverse effect of the decline in \bar{m} welfare goes up as ϕ goes up. To see if this can occur, define $\bar{W} = \alpha^{-1}m^\alpha - (m/q^\theta)^{1/(1+v)}(cq + F)$, where $\bar{W} = \alpha^{-1}\bar{m}^\alpha - (\bar{m}/\bar{q}^\theta)^{1/(1+v)}(c\bar{q} + F)$. These combinations of q and m produce equal welfare and the iso-welfare contours are circular. If the change of q and m that follows an increase in ϕ implies a move to the interior of the iso-welfare contour, welfare improves. To rule out that welfare improves with increases of the sales margin, notice that the slope of the iso-welfare contour at

$\{\bar{m}, \bar{q}\}$ comes from evaluating $dm/dq|_W = (\partial \bar{W}(\bar{m}, \bar{q})/\partial m)^{-1} \partial \bar{W}(\bar{m}, \bar{q})/\partial q$. The market opportunities, on the other hand, are described by equations (5) and (6) and from these,

$$dm/dq|_M = \theta \bar{m}/\bar{q} \cdot (1 - (1 + v)\alpha)^{-1}.$$

In the Appendix we show that $dm/dq|_W < dm/dq|_M$. That is, an increase in ϕ moves $\{\bar{m}, \bar{q}\}$ to a point outside the iso-welfare contour, which is why welfare decreases unambiguously. We summarise this in Proposition 3 (complete proof in the Appendix).

Proposition 3. An increase in the sales margin reduces welfare.

4. A Second Look at Welfare and Sales Margin

Van der Ploeg (2004, page 7) suggests that it is proper to include, in welfare evaluations, the opportunity cost of the time spent reading. The reason is that the real cost of buying and reading a book by far exceeds the price paid for the book. This argument is based on the observation that the cost of reading a title is measured by the number of hours used, multiplied by the wage. From a social point of view, the alternative cost is the wage as taxes on wage incomes are transfers. From the personal point of view the relevant opportunity cost is income net of the tax. Let the

gross cost be δ and the private cost δ_t . Either of these measures exceeds, quite substantially, the price of a title.⁵

When the social opportunity cost is δ per title, welfare is $W = \alpha^{-1}m^\alpha - n(\delta q + F)$. The social optimum is described by (use δ for c in Section 1 of the Appendix):

$$q^* = \theta / (1 - \theta + v) \cdot F / \delta \quad (7)$$

$$m^{*1/(1+v)-\alpha} = (1 - \theta + v) / F \cdot \left(\theta / (1 - \theta + v) \cdot F / \delta \right)^{\theta/(1+v)} \quad (8)$$

Taking into account alternative costs, the buyer's total cost for a title is $p_i + \delta_t$. Proceeding like before,⁶ this means that publisher profit per title is $\pi_i = m^{\alpha-1}n^v\theta q_i^\theta - (\delta_t + \phi)q_i - F$, and the market's allocation is characterised by (use δ_t for c in Section 2 of the Appendix):

$$\bar{q} = \theta / (1 - \theta) \cdot F / (\delta_t + \phi) \quad (9)$$

Using this in the zero-profit restriction we get:

$$\bar{m}^{1/(1+v)-\alpha} = \theta \cdot (1 - \theta) / F \left(\theta / (1 - \theta) \cdot F / (\delta_t + \phi) \right)^{\theta/(1+v)} \quad (10)$$

⁵ It can be argued as in van der Ploeg (2004) that the individual buyer assesses foregone income using the going-wage rate. However, if the number of hours spent reading is non-negligible (which is why includes them), the withdrawal of labour drives the wage up. This is one more reason that the private opportunity cost falls short of the social opportunity cost.

⁶ In order to simplify we will also assume that marginal production cost is zero (see Bittlingmayer, 1992).

The first thing to notice is that including opportunity costs does not change our results when $\delta_t = \delta$. However, when the personal opportunity cost differs from the social opportunity cost, comparison of equations (7) and (9) shows that $\bar{q} > q^*$ when $\phi = 0$. Also, comparison of equations (8) and (10) shows, for the case of $\phi = 0$, that $m^* \geq \bar{m}$ as:

$$\left(1 + \frac{v}{1 - \theta}\right)^{1 - \theta/(1+v)} \left(\delta_t/\delta\right)^{\theta/(1+v)} \geq \theta. \quad (11)$$

If we ignore that variety in titles is valued *per se* (setting $v = 0$), inspection of equation (11) shows that $\bar{m} > m^*$ when $(\delta_t/\delta)^\theta < \theta$. That is, when the private opportunity cost is sufficiently low relative to the socially relevant opportunity cost, $\bar{q} > q^*$ and $\bar{m} > m^*$ when the sales margin is vanishing. In this situation, welfare improves when ϕ is increased (since \bar{q} as well as \bar{m} goes down as ϕ increases). It is straightforward to verify that the condition in equation (11) gives $\bar{m} > m^*$ when v increases if $\bar{m} > m^*$ for $v = 0$.⁷ We summarise this in Proposition 4.

Proposition 4. When the private opportunity cost is sufficiently low relative to the social opportunity cost, an increase of the sales margin from being zero increase welfare.

The explanation for the result in the proposition is, of course, that book purchase is inefficiently large because consumers undervalue the real cost of using them. An increase in the sales margin

⁷ Rewriting equation (11) to $(1 + v/(1 - \theta))^{1+v-\theta} (\delta_t/\delta)^\theta \geq \theta^{1+v}$ and taking logs, we have $\theta \ln(\delta_t/\delta) \geq (1 + v) \ln(\theta) - (1 - \theta + v) \ln(1 + v/(1 - \theta))$. We know that the inequality is “<” for $v = 0$. Because the right-hand side is decreasing in v , we have “<” as v increases.

moves the book market in the direction of being smaller, which is unambiguously beneficial in this situation.

5. Conclusion

Fixed book-price agreements clearly reduce competition in the market for books. Allowing the publishing industry to maintain high prices redistributes welfare from consumers of books to this industry. However, because the book market is a monopolistically competitive market, it might be possible that fixed book-price arrangements have some real positive effects, alongside the redistributive effects that benefit publishing.

We have examined this question using a standard model of monopolistic competition. Defining the sales margin as the difference between the selling-price of books and the price bookshops pay to book-producers, it turns out that, sometimes, a strictly positive sales margin promotes more title variety than when the sales margin is competed away. In this meaning, fixed book prices admit more title variety than does the market. A positive sales margin advances title variety when book buyers' preferences make it relatively easy to find a substitute title, and when price response to new titles is strong.

However, supporting title variety by fixed book-price deals in terms of welfare. Hence, the fixed book price lowers overall welfare while it redistributes welfare from consumers to booksellers.

Van der Ploeg (2004) suggests that the opportunity cost must be inclined toward an assessment of the book market. We have shown that this is correct when consumers set a different value on this

cost compared to the socially relevant opportunity cost. In this situation, a strictly positive sales margin is beneficial. The reason is, however, that the book market without regulation is too bulky and that the sales margin tends to slim the market.

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Appendix

1. The social optimum.

$$W = \alpha^{-1}m^\alpha - \sum_{i=1}^n (cq_i + F), m = n^v \sum_{i=1}^n q_i^\theta, \quad (\text{A.1})$$

$0 < \alpha < 1, 0 < \theta < 1$. In the symmetric case, $m = n^{1+v}q^\theta$, so that we write welfare as

$$W = \alpha^{-1}m^\alpha - (m/q^\theta)^{1/(1+v)}(cq + F), \quad (\text{A.2})$$

For any fixed value of m , the optimal allocation satisfies $q^* = \min_q (m/q^\theta)^{1/(1+v)}(cq + F)$, or

$$q^* = \theta / (1 - \theta + v) \cdot F / c. \quad (\text{A.3})$$

Writing $V^* = q^{-\theta/(1+v)}(cq + F)$, welfare is $W = \alpha^{-1}m^\alpha - m^{1/(1+v)}V^*$ and the optimum value of m is characterised by:

$$m^{*\alpha-1} - (1+v)^{-1}m^{*1/(1+v)-1}V^* = 0. \quad (\text{A.4})$$

under the assumption that the second order condition

$$(\alpha - 1)m^{*\alpha-2} - ((1+v)^{-1} - 1)(1+v)^{-1}m^{*1/(1+v)-2}V^* < 0 \quad (\text{A.5})$$

is satisfied. It is easy to see that this requires $\alpha < 1/(1+v)$. Rewriting (A.4) we have

$$m^{*1/(1+v)-\alpha} = (1+v)V^{*-1}, \text{ or}$$

$$m^{*1/(1+v)-\alpha} = (1-\theta+v)/F \cdot \left(\theta/(1-\theta+v) \cdot F/c \right)^{\theta/(1+v)} \quad (\text{A.6})$$

2. The Market Solution

The price that variety i sells for is $p_i = m^{\alpha-1}n^\alpha \theta q_i^{\theta-1}$. Hence, the profit per title is

$$\pi_i = m^{\alpha-1}n^\alpha \theta q_i^\theta - (c + \phi)q_i - F. \quad (\text{A.7})$$

Because the publication of one title (variety) does not trigger responses in the form of changed prices for other titles (varieties), profit maximisation calls for $\theta p_i - c + \phi = 0$, or:

$$p_i = (c + \phi)/\theta. \quad (\text{A.8})$$

The zero-profit condition is $p_i q_i - (c + \phi)q_i - F = 0$ and using (A.8), we have:

$$\bar{q} = \theta/(1-\theta) \cdot F/(c + \phi). \quad (\text{A.9})$$

We can use (A.8) to get:

$$n^{1-(1+v)\alpha} = \theta/(c + \phi) \cdot \theta q^{\alpha\theta-1} \quad (\text{A.10})$$

or,

$$n^{1-(1+v)\alpha} = \theta^{(1-\theta)/F} \cdot \left(\theta/(1-\theta) \cdot F/(c+\phi)q \right)^{\alpha\theta} \quad (\text{A.11})$$

Alternatively, from (A.10), we have $(c+\phi)/\theta = n^{(1+v)\alpha-1} \theta q^{\alpha\theta-1}$. Using that $n = \left(m/q^\theta \right)^{1/(1+v)}$,

we have:

$$(c+\phi)/\theta = \left(m/q^\theta \right)^{1/(1+v)((1+v)\alpha-1)} \theta q^{\alpha\theta-1} \quad (\text{A.12})$$

or:

$$\bar{m}^{1/(1+v)-\alpha} = \theta^{(1-\theta)/F} \left(\theta/(1-\theta) \cdot F/(c+\phi) \right)^{\theta/(1+v)} \quad (\text{A.13})$$

3. Proof of Proposition 2.

We have:

$$\bar{V} = 1/\theta \left((1-\theta)/\theta \cdot (c+\phi)/F \right)^{\theta/(1+v)} \cdot F/(1-\theta) \quad (\text{A.14})$$

$$V^* = \left((1-\theta+v)/\theta \cdot c/F \right)^{\theta/(1+v)} \cdot F/(1-\theta+v). \quad (\text{A.15})$$

Now, rewriting shows that $\bar{V} \geq V^*$ as

$$1/\theta \cdot (1-\theta)^{\theta/(1+v)-1} (c+\phi)^{\theta/(1+v)} \geq (1-\theta+v)^{\theta/(1+v)-1} c^{\theta/(1+v)}. \quad (\text{A.16})$$

As $1/\theta > 1$, $\theta/(1+v) > 0$ and $\theta/(1+v) - 1 < 0$, it follow that $\bar{V} > V^*$ so that $V^{*-1} > \bar{V}^{-1}$, in turn implying that $m^* > \bar{m}$.

End of proof.

4. Proof of Proposition 3.

Welfare is

$$W = \alpha^{-1}m^\alpha - \left(\frac{m}{q^\theta}\right)^{1/(1+v)} (cq + F).$$

The welfare effect of a change in ϕ can be calculated directly as $dW/d\phi = \partial W/\partial m \cdot d\bar{m}/d\phi + \partial W/\partial q \cdot d\bar{q}/d\phi$. Using the above expression for W :

$$\partial W/\partial q = -\left(\frac{m}{q^\theta}\right)^{1/(1+v)} \left(c - \frac{\theta}{(1+v)}(c + F/q)\right) \quad (\text{A.17})$$

or, using (A.9) and (A.13),

$$\partial W/\partial q = -\theta m^\alpha \cdot \frac{1 - \theta/F}{(1+v)} \left(\frac{(1 - \theta + v)}{(1+v)} \cdot c - \frac{(1 - \theta)}{(1+v)} \cdot (c + \phi)\right). \quad (\text{A.18})$$

Once more Using the above expression for W :

$$\partial W/\partial m = 1/m \cdot \left(m^\alpha - \frac{1}{(1+v)} \left(\frac{m}{q^\theta}\right)^{1+v} (cq + F)\right). \quad (\text{A.19})$$

Or, using (A.9) and (A.13),

$$\partial W/\partial m = m^\alpha \left(1 - \frac{\theta}{(1+v)} \left(\frac{(c + (1 - \theta)\phi)}{(c + \phi)}\right)\right). \quad (\text{A.20})$$

Using (A.18) and (A.19) the slope of the indifference curve is:

$$\left. \frac{dm}{dq} \right|_w = \theta^{m/q} \cdot ((1 + v)(c + \phi) - \theta(c + (1 - \theta)\phi))^{-1} (\theta(vc - (1 - \theta)\phi)). \quad (\text{A.21})$$

The combination of m and q that is feasible in the market is given by (A.9) and (A.13) and we have:

$$\left. \frac{dm}{dq} \right|_M = \theta^{m/q} \cdot (1 - (1 + v)\alpha)^{-1}. \quad (\text{A.22})$$

By comparison of (A.21) and A.22) we have $\left. \frac{dm}{dq} \right|_M > \left. \frac{dm}{dq} \right|_w$. And, since m as well as q goes down as the sales margin increases, welfare decreases.

End of proof.