Chapter B – Oligopoly: Bertrand- and Cournot-Competition

Competition Policy and Strategy – German Title: Wettbewerbspolitik und -strategie

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English version of my 2020 lecture

Oligopoly – Foundations in Game Theory

- Oligopoly: competition between few firms such that there is strategic interdependence between the firms. That is, firms take other firms' behavior into account when making their own decisions.
- Fims "play a game" in the sense of Game Theory (=formal analysis of strategic decisions).
- Definition of a game
    - *Players*: set of payers $\mathcal{N} = \{1, 2, \ldots, n\}$.
    - *Actions*: player $i \in \mathcal{N}$ picks a strategy profile $\sigma_i \in \Sigma_i$, where Σ_i contains all available strategies of player i (strategy set). The strategy space is then $\Sigma = \times_{i \in \mathcal{N}} \Sigma_i$.
    - *Payoffs*: player i's payoff as a function of its own strategy profile σ_i and the strategy profiles of the other players, σ_{-i}, with $-i := \mathcal{N} \setminus i$ indicating all players except for i. Usually, $\pi_i : \Sigma \to \mathbb{R}$.
    - *Information*: defines the *timing* of the game. Simultaneous (sequential) games are characterized by players making simultaneous (sequential) decisions. Also contains uncertainty ("nature").

Oligopoly – Foundations in Game Theory

- We usually assume that players are **rational**. This means that given the players' preferences they behave optimally ("internal consistency"). s. bspw. Sen, A. K. (1977). Rational fools: A critique of the behavioral foundations of economic theory. Philosophy & Public Affairs, 317-344.
- Rationality is *common knowledge*: Every player knows that all other players are rational; every player knows that all other players know that all players are rational, etc.
- Nash Equilibrium:
    - A strategy profile $\sigma^* = (\sigma_1^*, \sigma_2^*, \ldots, \sigma_n^*) \in \Sigma$ constitutes a Nash equilibrium when all players choose their *best responses* based on all other players' best responses.
    - Player i's best response is a strategy that maximizes i's payoff, $\sigma_i^* = \arg\max_{\sigma_i} \pi(\sigma_i, \sigma_{-i})$.
    - In a Nash equilibrium there is no incentive to deviate:

$$\pi_i(\sigma_1^*, \ldots, \sigma_i^*, \ldots, \sigma_n^*) \geq \pi_i(\sigma_1^*, \ldots, \sigma_i', \ldots, \sigma_n^*) \ \forall \sigma_i' \in \Sigma_i, i \in \mathcal{N}.$$

Oligopoly – Cournot Competition

- In Cournot Oligopoly (Antoine-Augustin Cournot, 1801-1877), the firms' instrument is **output**.
- Simultaneous game (sequential quantitiy competition: Stackelberg)
- Consider the following example:
  - Firms have identical costs, $C(q_i) = cq_i \ \forall i \in \mathcal{N}$.
  - Demand is linear $p(q_i) = a - bQ$.
  - Goods are homogeneous.
  - *One-shot game*, no uncertainty.

Oligopoly – Cournot Competition

- Total output in the market is $Q = q_i + q_{-i}$, with q_{-i} denoting the output of all firms except i.
- Firm i takes into account q_{-i} when choosing its own output:

$$\max_{q_i} \ \pi_i(q_i, q_{-i}) = (p(q_i, q_{-i}) - c)\, q_i.$$

- Optimality requires

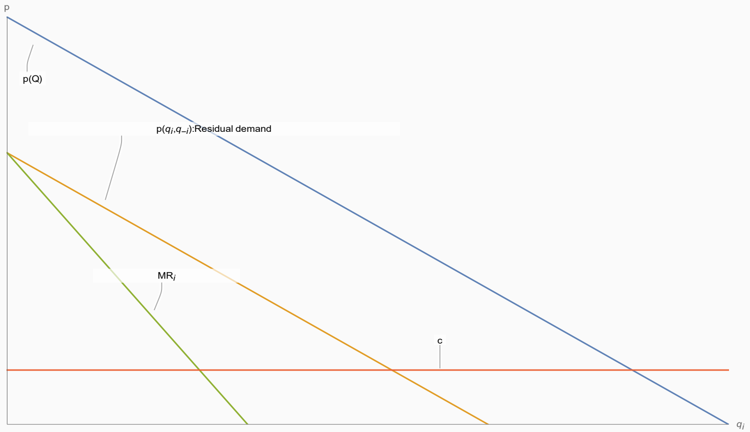
$$\underbrace{\frac{\partial p(q_i, q_{-i})}{\partial q_i} q_i + p(q_i, q_{-i})}_{\text{marginal revenue}} = c.$$

- Given that $Q = q_i + q_{-i}$, we have $p(q_i, q_{-i}) = a - b(q_i + q_{-i})$.
- FOC yields i**'s best response function**:

$$q_i(q_{-i}) = \frac{a-c}{2b} - \frac{q_{-i}}{2}.$$

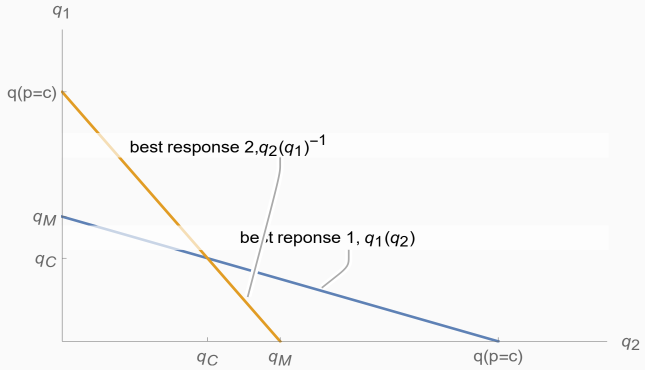
Oligopoly – Cournot Competition

Residual demand for firm i



Oligopoly – Cournot Competition

- Let $n = 2$.
- Reaction/best response functions $i = 1$: $q_1(q_2) = \frac{a-c}{2b} - \frac{q_2}{2}$.
- For $q_2 = \frac{a-c}{b}$ (perfect competition), $q_1 = 0$; for $q_2 = 0$, $q_1 = \frac{a-c}{2b}$ (monopoly)



Oligopoly – Cournot Competition

- Let $n \in \mathbb{N}$. With symmetric firms, $q_{-i} = (n-1)q_i$.
- Insert in FOC/reaction function of firm i to obtain i's equilibrium output:

$$q_C = \frac{1}{b}\frac{a-c}{n+1}.$$

- Industry output in equilibrium reads $Q_C = nq_C = \frac{n}{b}\frac{a-c}{n+1}$. The ensuing price reads:

$$p_C = c + \frac{a-c}{n+1},$$

with individual profits and (total) consumer surplus

$$\pi_C = \frac{1}{b}\left(\frac{a-c}{n+1}\right)^2, \quad CS_C = \frac{1}{2b}\left(\frac{n(a-c)}{n+1}\right)^2.$$

Oligopoly – Bertrand Competition

- With Bertrand Competition (nach Joseph L. F. Bertrand, 1822-1900), firms compete in prices.
- Simultaneous Game
- Consider the following example:
    - Firms have identical costs, $C(q_i) = cq_i \; \forall i \in \mathcal{N}$.
    - Goods are homogeneous.
    - *One-shot game*, no uncertainty.
    - No capacity constraints.
    - Let $n = 2$.

Oligopoly – Bertrand Competition

- Basic concept: consumers buy from cheapest firm.
- Without capacity constraints, firm with lowest price serves all consumers.
- It is usually assumed that with equal prices, consumers allocate symmetrically among firms.
- We can thus compute demand of firm i, $i, j \in \{1, 2\}, i \neq j$:

$$q_i(p_i, p_j) = \begin{cases} q(p_i) & p_i < p_j \\ \frac{q(p_i)}{2} & p_i = p_j \\ 0 & p_i > p_j. \end{cases}$$

Oligopoly – Bertrand Competition

- Firm i's profit as a function of (p_i, p_j)

$$\pi_i(p_i, p_j) = \begin{cases} q(p_i)(p_i - c) & p_i < p_j \\ \frac{q(p_i)}{2}(p_i - c) & p_i = p_j \\ 0 & p_i > p_j. \end{cases}$$

- No firm chooses $p_i < c$ as this would imply losses for every unit sold.
- No firm chooses $p_i > p_M$ with p_M being the monopoly price. For all $p_j > p_M$, $p_i(p_j) = p_M$ is best response.
- For all $p_j \in (c, p_M]$, i's best reponse is to undercut j such that $p_i(p_j) = p_j - \epsilon$ with arbitrarily small $\epsilon > 0$.

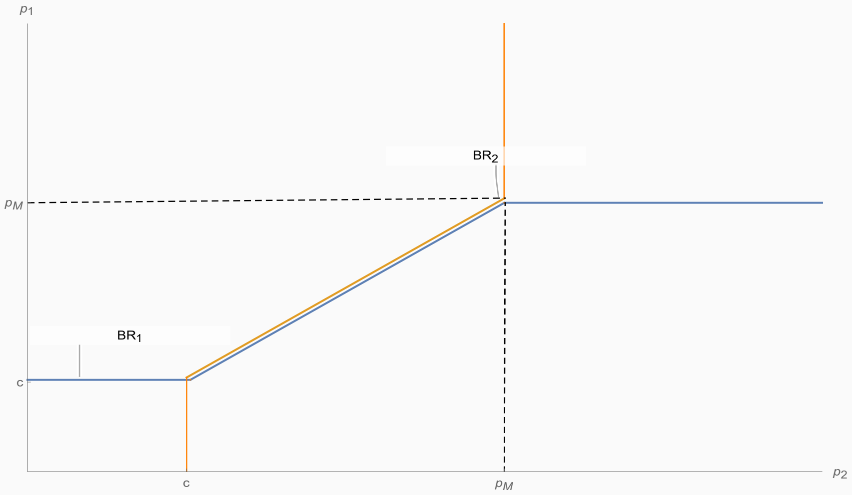
Oligopoly – Bertrand Competition

- Reaction function of firm i:

$$p_i(p_j) = \begin{cases} p_M & p_j > p_M \\ p_j - \epsilon & p_j \in (c, p_M] \\ c & p_j = c. \end{cases}$$

- Nash Equilibrium at $p_i(p_j) = p_j(p_i) = c$.
- Intuition:
  - Undercut rival until $p = c$.
  - At $p = c$, undercutting stops as it is no longer profitable.
  - Bertrand-Paradox (Economics!): With the given assumptions, two firms suffice to generate an efficient market equilibrium (perfect competition, long run, allocative efficiency, zero profits).

Oligopoly – Bertrand Competition



Oligopoly – Cournot vs. Bertrand

- Bertrand: allocative efficiency (under certain conditions!)
- Cournot: price above marginal costs ($n < \infty$) $\rightarrow$ according to Bertrand-logic, this is no equilibrium!
- Practical relevance?
- Extensions:
  - Differentiated goods: i is monopolist for its own variant (Hotelling, vertical product differentiation; IO-Course on Economics2Go!)
  - Repeated games, *tacit collusion*
  - Capacity constraints (Bertrand-Edgeworth; Kreps and Sheinkman 1983, Bell).
  - Uncertainty about costs, demand, etc.

Annex – Derivation Cournot Equilibrium With Linear Demand

- Assume $p(q) = a - bq$ and constant marginal costs c with $n \geq 2$ firms.
- i's profit reads $\pi_i = (a - b(q_i + q_{-i}) - c)q_i$.
- FOC: $a - 2bq_i - bq_{-i} - c = 0$
- In symmetric equilibrium: $q_{-i} = (n-1)q_i$.
- Insert to get $a - 2bq_i - b((n-1)q_i) - c = 0 \;\Leftrightarrow\; q_i = \frac{a-c}{b(n+1)}$.
- Industry ouput: $nq_i = \frac{n(a-c)}{b(n+1)}$.
- Equilibrium price: $p = a - b\frac{n(a-c)}{b(n+1)} = \frac{a+nc}{n+1} + c - c = c + \frac{a-c}{n+1}$.
- Individual firm's proifts: $\pi_C = (c + \frac{a-c}{n+1} - c)\frac{a-c}{b(n+1)} = \frac{1}{b}\left(\frac{a-c}{n+1}\right)^2$.
- Consumer surplus in equilibrium: $CS = \frac{1}{2}\left[a - \left(c + \frac{a-c}{n+1}\right)\right]\frac{n(a-c)}{b(n+1)} = \frac{1}{2b}\left(\frac{n(a-c)}{n+1}\right)^2$.