Chapter H.1 – Implicit and Explicit Collusion

Competition Policy and Strategy

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English version of my 2020 lecture

Implicit and Explicit Collusion - Basics

- We always assumed that firms behave competitively
  - Market outcome depends on
    - Market structure (e.g., number of firms).
    - Production technology (e.g., *Economies of Scale*).
    - Mode of competition (Cournot, Bertrand, Stackelberg, ...).
  - Market power influenced by
    - supply-side (e.g., potential competition).
    - demand-side (e.g., buyer power, switching cost, network effects).
- In the following, we analyze cooperation between firms
  - (Illegal) explicit collusion/cartels through agreements on prices, quantities, customer allocation, etc.
    - The goal is to raise profits by, e.g., jointly raising prices
    - Results in loss of consumer surplus and welfare
  - Implicit or tacit collusion
  - (Legal) horizontal agreements (e.g., Research joint ventures).

Implicit and Explicit Collusion – Prisoner's Dilemma

- Collusion between firms inherently unstable due to firms' incentive to deviate from collusive agreement
- Example:
  - Homogeneous good with inverse demand $p(q) = a - q$.
  - Cournot-Duopoly
  - $C_i(q_i) = c$ for all $i \in \{1, 2\}$.
- Indicies: C for competition, K for collusion, M for monopoly, D for deviation/defection, F for fooled
- Each firm has two options: cooperate or deviate
  - Cooperation
    - Joint profits are maximized when total output is $q_M = \frac{a-c}{2}$, such that $\pi_M = \left(\frac{a-c}{2}\right)^2$
    - Each firm produces half of the monopoly output, $q_K = \frac{a-c}{4}$ and realizes $\pi_K = \frac{\pi_M}{2} = \frac{(a-c)^2}{8}$
  - Deviation
    - Deviating firm follows best-response function
    - If both firms deviate, competitive outcome: $\pi_C = \left(\frac{a-c}{3}\right)^2$
    - $\pi_C < \pi_K$ such that both firms have an incentive to participate in a collusive agreement

Implicit and Explicit Collusion – Prisoner's Dilemma

- Reaction function in Cournot $q_i(q_j) = \arg\max_{q_i} \ \pi_i(q_i, q_j) = \frac{a-c-q_j}{2}$.
- If firm j cooperates by setting $q_K = \frac{a-c}{4}$ and firm i deviates, we get:

$$q_D = q_i(q_K) = \frac{a-c-q_K}{2} = \frac{3(a-c)}{8} > q_K.$$

- Deviating firm i realizes profits $\pi_D = \left(\frac{3(a-c)}{8}\right)^2 > \pi_K$.
- Thus, there is an incentive to participate in a collusive agreement, but also the incentive to deviate from the agreement (instability!)
- When firm i deviates, firm j's profits decrease to $\pi_F = \frac{3(a-c)^2}{32} < \pi_C$ (firm j is fooled).
- Prisoner's Dilemma:
  - Cooperation is beneficial for both firms: $\pi_K > \pi_C$
  - Unilateral incentive to deviate: $\pi_D > \pi_K$
  - If i deviates, j is "fooled": $\pi_C > \pi_F$

Implicit and Explicit Collusion – Prisoner's Dilemma

i \ j	Cooperate	Deviate
Cooperate	1/8, 1/8	3/32, 9/64
Deviate	9/64, 3/32	1/9, 1/9

Figure 1: Payoff-matrix Prisoner's Dilemma with $a - c \equiv 1$, Cournot.

Implicit and Explicit Collusion – Prisoner's Dilemma

- Suppose both firms compete in prices (Bertrand)
- If both firms compete/deviate, $\pi_C = 0$
- If both firms cooperate/collude, each firm charges $p_M = \frac{a+c}{2}$ such that $\pi_K = \frac{\pi_M}{2}$
- Firm i can deviate from the collusive agreement by setting $p_D = p_M - \epsilon$, $\epsilon \to 0$.
- Deviating firm i realizes $\pi_D \approx \pi_M > \pi_K$.
- Firm j realizes $\pi_F = \pi_C = 0$.

Implicit and Explicit Collusion – Prisoner's Dilemma

i \ j	Cooperate	Deviate
Cooperate	1/8, 1/8	0, 1/4
Deviate	1/4, 0	0, 0

Figure 2: Pay-matrix Prisoner's Dilemma with $a - c \equiv 1$, Bertrand.

Implicit and Explicit Collusion – Repeated Prisoner's Dilemma

- Firms interact repeatedly on the market
- Interaction in multiple periods $t = 0, 1, \ldots, T$. Analysis begins in $t = 0$ ("today")
    - In every period t, firms play stage game (e.g., interaction á la Cournot or Bertrand)
    - Equilibrium of repeated game can differ from that of stage game
- Discount factor $\delta \in [0, 1]$ as a function of interest rate $r \in \mathbb{R}_+$: $\delta = \frac{1}{1+r}$.
- W.l.o.g., only consider firm i (drop index, symmetry)
- Payoffs are discounted per period profits/present value

$$V = \pi_0 + \delta\pi_1 + \delta^2\pi_2 + \cdots + \delta^{t-1}\pi_{t-1} + \delta^t\pi_t + \delta^{t+1}\pi_{t+1} + \cdots + \delta^{T-1}\pi_{T-1} + \delta^T\pi_T = \sum_{t=0}^{T} \delta^t \pi_t.$$

- Example: $\pi_t = 10\ \forall t,\ T = 4$ and $\delta = 0.9 (\Leftrightarrow r \approx 11.11\%)$:

$$V = 10 + 9 + 8.1 + 7.29 + 6.561 = 40.951.$$

- Intuition
  - Deviation profitable in every period, however, deviation can be **punished**
  - Punishment takes the form of, e.g., deviating in all subsequent periods ("Grim Trigger")
- Present value competition:

$$V_C = \sum_{t=0}^{T} \delta^t \pi_C = \frac{1-\delta^{T+1}}{1-\delta}\pi_C.$$

- Present value collusion:

$$V_K = \sum_{t=0}^{T} \delta^t \pi_K = \frac{1-\delta^{T+1}}{1-\delta}\pi_K.$$

- Present value deviation (in $t = 0$):

$$V_D = \pi_D + \sum_{t=1}^{T} \delta^t \pi_C = \pi_D + \frac{\delta-\delta^{T+1}}{1-\delta}\pi_C.$$

Implicit and Explicit Collusion – Repeated Prisoner's Dilemma

- Subgame perfect equilibrium
  - Nash equilibrium in every subgame (stage game)
  - Must not contain threats that are not credible
- Consider a finite game, $T < \infty$
  - In $t = T$: No future period such that deviation always profitable. Both firms deviate in T.
  - In $t = T - 1$: firm i knows that j deviates in period T. Both firms deviate in $T - 1$.
  - In $t = T - 2$: firm i knows that j deviates in period $T - 1$. Both firms deviate in $T - 2$.
  - ...
- Selten Theorem (Reinhard Selten 1930-2016):
  If a game with a single Nash equilibrium is played a finite number of times, that Nash equilibrium will arise in every subgame.
  Here: (deviate, deviate) is Nash equilibrium in every period for finite T

Implicit and Explicit Collusion – Repeated Prisoner's Dilemma

- Consider infinite game, $T \to \infty$ (Supergame).
- We have $\lim_{T\to\infty} \delta^{T+1} = 0 \; \forall \delta < 1$.
- Present value competition:
$$V_C = \sum_{t=0}^{T} \delta^t \pi_C = \frac{1}{1-\delta}\pi_C.$$
- Present value collusion:
$$V_K = \sum_{t=0}^{T} \delta^t \pi_K = \frac{1}{1-\delta}\pi_K.$$
- Present value deviation in $t = 0$ (Grim Trigger):
$$V_D = \pi_D + \sum_{t=1}^{T} \delta^t \pi_C = \pi_D + \frac{\delta}{1-\delta}\pi_C.$$
- V_C not relevant:
    - We have $V_C < V_K$ because $\pi_C < \pi_K$.
    - We have $V_C < V_D$ because $\pi_C < \pi_D$.

Implicit and Explicit Collusion – Repeated Prisoner's Dilemma

- The firms' discount factor determines whether cartel is stable or not.
- Discount factor measures δ firms' "patience": the higher δ, the higher a firm values future profits.
- Deviation promises additional profits $\pi_D - \pi_K > 0$ in the period of deviation (short-run gain).
- However, deviation is punishes such that $\pi_K - \pi_C$ is lost in every subsequent period (long-run loss).
- Collusion is said to be stable if and only if $V_K \geq V_D$, i.e.,

$$\frac{1}{1-\delta}\pi_K \geq \pi_D + \frac{\delta}{1-\delta}\pi_C.$$

- Rearranging yields:

$$\underbrace{\frac{\delta}{1-\delta}(\pi_K - \pi_C)}_{\text{long-run loss starting in } t=1} > \underbrace{\pi_D - \pi_K}_{\text{short-run gain in } t=0}.$$

Implicit and Explicit Collusion – Repeated Prisoner's Dilemma

- long-run loss increases in δ: the more firms value future profits the higher the long-run loss from deviation. If firm are myopic ($\delta = 0$), the long-run loss is zero.
- Short-run gain is independent of δ because it is realized immediately (in $t = 0$).
- There exists a $\delta \in (0, 1)$ such that $V_K = V_D$.
    - Follows from intermediate value theorem
        - Define $\Delta V \equiv V_K - V_D = \frac{\delta}{1-\delta}(\pi_K - \pi_C) - (\pi_D - \pi_K)$.
        - For $\delta \to 0$, $\Delta V = -(\pi_D - \pi_K) < 0$.
        - For $\delta \to 1^-$, $\Delta V \to +\infty > 0$.
        - ΔV is continuous in δ.
- A firm with $\delta = \delta_{\text{krit}}$ is indifferent between deviation and collusion.
- For all $\delta > \delta_{\text{krit}}$ ($\delta < \delta_{\text{krit}}$) collusion is (not) stable.
- Computation of δ_{krit}:

$$V_K - V_D = 0 \;\Leftrightarrow\; \pi_D - \pi_K = \frac{\delta}{1-\delta}(\pi_K - \pi_C) \Rightarrow \delta_{\text{krit}} \equiv \frac{\pi_D - \pi_K}{\pi_D - \pi_C}.$$

Implicit and Explicit Collusion – Repeated Prisoner's Dilemma

- Example 1: symmetric Cournot-duopoly, no costs in production, linear demand $q(p) = 24 - p$.
- Subgame/stage game (Slides 2 and 3):
  - Competition: $\pi_C = 64$.
  - Collusion: $\pi_K = 72$.
  - Deviation: $\pi_D = 81$.
- Present value collusion: $V_K = \frac{72}{1-\delta}$.
- Present value deviation $V_D = 81 + \frac{64\delta}{1-\delta}$.
- Collusion is stable if $V_K \geq V_D$, i.e., if $\delta \geq \frac{9}{17} \equiv \delta_{\text{krit}}$.
- Equivalent approach:
  - Short-run gain: $\pi_D - \pi_K = 9$.
  - Long-run loss: $\pi_K - \pi_C = 8$ per period such that discounting yields $\frac{8\delta}{1-\delta}$.
  - Collusion is stable if long-run loss exceeds short-run gain from deviation: $9 < \frac{8\delta}{1-\delta} \;\Leftrightarrow\; \delta > \frac{9}{17} \equiv \delta_{\text{krit}}$.
- If actual discount factor δ exceeds $\frac{9}{17}$, collusion is stable.

Implicit and Explicit Collusion – Repeated Prisoner's Dilemma

- Example 2: symmetric Bertrand-duopoly, no costs in production, linear demand $q(p) = 24 - p$.
- Subgame/stage game:
  - Competition: $\pi_C = 0 \Rightarrow$ long-run loss larger than in Cournot.
  - Collusion: $\pi_K = 72$.
  - Deviation: $\pi_D = 144 \Rightarrow$ short-run gain larger than in Cournot.
- Present value collusion: $V_K = \frac{72}{1-\delta}$.
- Present value deviation: $V_D = 144$.
- Collusion is stable if $V_K \geq V_D$, i.e., if $\delta \geq \frac{1}{2} \equiv \delta_{\text{krit}}$.
- If actual discount factor δ exceeds $\frac{1}{2}$, collusion is stable.
- Here: collusion more stable in Bertrand than in Cournot because critical discount factor in Cournot ($\frac{9}{17}$) exceeds critical discount factor in Bertrand ($\frac{1}{2}$).

Implicit and Explicit Collusion – Repeated Prisoner's Dilemma

- In our consideration, the threat of punishment was sometimes sufficient to stabilize collusion.
- Firms did not have to communicate.
- This mechanism is called tacit collusion.
- Ivaldi et al. (2003)[1]:
    - Market outcome where firms realize supra-competitive profits
    - Firms agree tacitly/implicitly (i.e., without explicit communication) to set prices above (quantities below, etc.) competitive levels.
    - Requires repeated interaction
    - Firms have to be sufficiently patent
    - Agreement requires sufficiently severe and certain punishments

[1] *The Economics of Tacit Collusion*, IDEI Toulouse Final Report for DG Competition, European Commission

Implicit and Explicit Collusion – Repeated Prisoner's Dilemma

- Explicit collusion or cartel:
  - Same goal as tacit collusion, but firms communicate
  - Agreements on prices, rebates or innovation, market sharing, customer allocation ...
  - 101 TFEU, Sherman Act Sect. 1 applies §1 GWB, per se infringement
  - Cartels can involve side payments (example: citric acid cartel, see Harrington and Skrzypacz (2011, AER))
- The exact mechanisms how communication actually works are unclear
  - Solve coordination problems
    - Every price above marginal cost is equilibrium in Supergame
    - Which price should we set?
  - Detect deviation with more than 2 firms and uncertainty (see esp. the recent work of Awaya and Krishna).
  - Popular object of study in experiments (see esp. the work of Fonseca and Normann).

Implicit and Explicit Collusion – Law

Art. 101(1) TFEU

The following shall be prohibited as incompatible with the internal market: all agreements between undertakings, decisions by associations of undertakings and concerted practices which may affect trade between Member States and which have as their object or effect the prevention, restriction or distortion of competition within the internal market, and in particular those which:

a) *directly or indirectly fix purchase or selling prices or any other trading conditions;*
b) *limit or control production, markets, technical development, or investment;*
c) *share markets or sources of supply; [...]*

Implicit and Explicit Collusion – Law

Art. 101(3) TFEU

The provisions of paragraph 1 may, however, be declared inapplicable in the case of:

- *any agreement or category of agreements between undertakings,*
- *any decision or category of decisions by associations of undertakings,*
- *any concerted practice or category of concerted practices,*

which contributes to **improving the production or distribution of goods or to promoting technical or economic progress, while allowing consumers a fair share of the resulting benefit**, *and which does not:*

a) *impose on the undertakings concerned restrictions which are not indispensable to the attainment of these objectives;*
b) *afford such undertakings the possibility of eliminating competition in respect of a substantial part of the products in question.*

Implicit and Explicit Collusion – Law

§1 GWB

- As 101 TFEU
- Agreements between undertakings, decisions by associations and concerted practices which impede, restrict or distort competition (object or effect)
- Exact wording in German:

  Vereinbarungen zwischen Unternehmen, Beschlüsse von Unternehmensvereinigungen und aufeinander abgestimmte Verhaltensweisen, die eine Verhinderung, Einschränkung oder Verfälschung des Wettbewerbs bezwecken oder bewirken, sind verboten.

Implicit and Explicit Collusion – Competition Authorities

Decisions by EC (s. `https://competition-policy.ec.europa.eu/system/files/2022-11/cartels_cases_statistics.pdf`, rounded).

Period	# Decisions	# Undetakings	Fines EUR
1990–1994	10	185	344 282 550
1995–1999	9	45	270 963 500
2000–2004	29	156	3 157 348 710
2005–2009	33	199	7 863 307 787
2010–2014	31	180	7 598 728 479
2015–2019	26	107	8 187 380 159
2020–2022	15	42	2 222 928 000
Total	153	914	29 745 465 612

Year	Undetaking	Case	Fines EUR
2016	Daimler	Trucks	1 008 776 000
2017	Scania	Trucks	880 523 000
2016	DAF	Trucks	752 679 000
2008	Saint Gobain	Carglass	715 000 000

Implicit and Explicit Collusion – Competition Authorities

- Communication between cartel members generates evidence (e-Mails, witnesses, protocols, etc.); "Smoking Gun"
- Competition authorities can detect evidence using audits or complaints from consumers
- Another tool is leniency, where firms can report cartel and/or cooperate to reduce their fines
- Formally:
    - Detection probability ρ.
    - Fines F.
- Cf. Aubert, C., Rey, P. und Kovacic, W. E. (2006). The impact of leniency and whistle-blowing programs on cartels. *International Journal of Industrial Organization*, Vol. 24, Issue 6, pp. 1241–1266.
- Model where collusion is fined

Implicit and Explicit Collusion – Competition Authorities

- Per period profits with expected fines
  - In periods when cartel is active, $\pi_K - \rho F$
  - When deviating, $\pi_D - \rho F$
- Present values
  - Collusion: $V_K = \frac{1}{1-\delta}(\pi_K - \rho F)$.
  - Deviation: $V_D = \pi_D - \rho F + \frac{\delta}{1-\delta}\pi_C$.
- Critical discount factor now reads

$$\delta_{\text{krit}}^{\text{CA}} \equiv \frac{\pi_D - \pi_K}{\pi_D - \rho F - \pi_C}.$$

- Without competition authority, we had $\delta_{\text{krit}} = \frac{\pi_D - \pi_K}{\pi_D - \pi_C}$.
- Activity of competition authority c. p. destabilizes collusion as $\delta_{\text{krit}}^{\text{CA}} > \delta_{\text{krit}}$
- Competition authority deters collusion in the range $\delta \in \left[\delta_{\text{krit}}, \delta_{\text{krit}}^{\text{CA}}\right)$
  - For all $\delta < \delta_{\text{krit}}$, collusion is unstable even w/o competition authority
  - For all $\delta \geq \delta_{\text{krit}}^{\text{CA}}$, collusion is stable despite competition authority
  - Recall that tie-breaking rule was that collusion arises whenever $\delta \geq \delta_{\text{krit}}$

Implicit and Explicit Collusion – Competition Authorities

- Leniency program
  - Installed in EU in 2002 (http://europa.eu/rapid/press-release_IP-02-247_en.htm?locale=en)
  - Allows firms to self-report cartel activity
  - Self-reporting leads to reduction in fines; even full immunity is possible (https://competition-policy.ec.europa.eu/cartels/leniency_en)
  - Evidence that leniency improved cartel deterrence (Miller, 2009 AER)
- Caveats
  - Potentially reduces deterrent effect of fines (see following slides)
  - Damage claims can reduce firms' incentive to apply for leniency
- Formally:
  - Deviation and leniency-application reduces fines to $f < F$ such that $V_D = \pi_D - f + \frac{\delta}{1-\delta}\pi_C$.
  - With $V_K = \frac{1}{1-\delta}(\pi_K - \rho F)$, the critical discount factor reads

$$\delta_{\text{krit}}^{LP} = \frac{\pi_D - f - (\pi_K - \rho F)}{\pi_D - f - \pi_C}.$$

- Leniency program improves deterrence iff $\delta_{\text{krit}}^{LP} > \delta_{\text{krit}}^{\text{CA}}$. This is the case when $f < \rho F$.

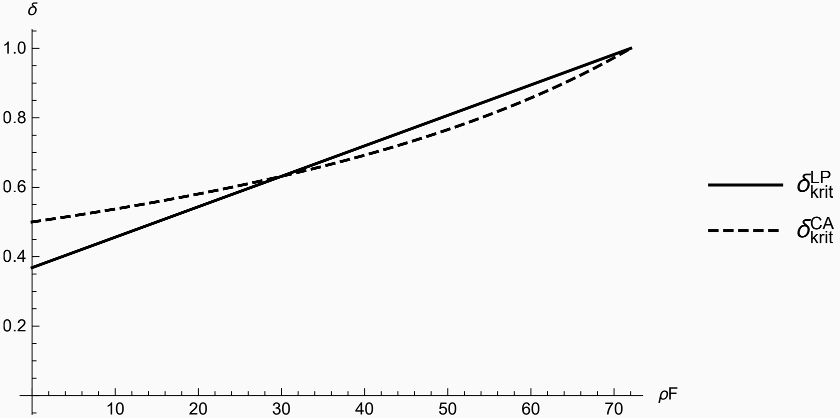

Figure 3: Discount factor $\delta_{\text{krit}}^{CA}$ and $\delta_{\text{krit}}^{LP}$ with $\pi_D = 144$, $\pi_K = 72$ and $\pi_C = 0$ for $f = 30$.

Implicit and Explicit Collusion – Facilitating Factors

Factors that facilitate collusion (cf Motta 2004: 142–150).

- Incentive Constraint: Short-run gain from deviation lower than long-run loss. Collusion is stabilized if
  - Short-run gain $\pi_D - \pi_K$ decreases.
  - Per-Period loss after deviation $\pi_K - \pi_C$ increases.
  - Firms become more patient (higher δ).
- Facilitating factors:
  - Concentration
    - The more firms, the smaller the share of monopoly profits in cartel, $\pi_K = \frac{\pi_M}{n}$.
    - Profits from deviation relative large in relation to punishment
  - Entry barriers
    - Collusion raises industry profits, which facilitates entry
    - Entrant destabilizes cartel by decreasing cartel profits (integrate entrant into agreement?)
  - Links between companies
    - Contacts between employees/management
    - Cross-ownership can stabilize collusion because harm of fooled firms is partially internalized

Implicit and Explicit Collusion – Facilitating Factors

- Frequency of orders
  - With infrequent/large order, incentive to deviate is higher
  - Frequent orders makes punishment easier
- Buyer Power
  - Powerful buyers (esp. in B2B) can threaten to
    - buy from cartel-outsider/entrant
    - produce own product
    - Bargaining environment important: disagreement profits/outside options
  - Combine multiple orders (see above)
  - Auction design
- Evolution of demand
  - Uncertainty
    - I. i. d. shocks (unforeseen): incentive to deviate when positive shock occurs (same as with frequency of orders)
    - Cyclical demand: collusion more stable in times of increasing demand, because gain from deviation today is small in relation to future loss
    - Collusion less stable the more shocks occur, e.g., because deviation is harder to detect

Implicit and Explicit Collusion – Facilitating Factors

- Homogeneity
  - Ambiguous effect: the more differentiated the products, the lower ...
    - ... profits from deviation ⇒ stabilizes collusion.
    - ... the effectiveness of punishment ⇒ destabilizes collusion.
- Symmetry and capacities
  - Firms with high capacities c. p. have a higher incentive to deviate if capacities are not fully utilized at cartel prices
  - Firms with lower capacities have limited ability to punish
  - Overall, effects of over-capacities are unclear
- Multi-market contact
  - Deviation in one market leads to punishment in multiple markets
  - deviation in multiple markets increases incentive to deviate

Implicit and Explicit Collusion – Facilitating Factors

- Caveat of facilitating factors (cf Garrod and Olczak, 2018 IJIO)
  - Facilitating factors do not distinguish between tacit and explicit collusion
  - Explicit collusion is illegal
  - If market conditions facilitate (tacit) collusion, then why do we see explicit collusion?
- What is the relevant counterfactual scenario?

Implicit and Explicit Collusion – Damage Claims and Pass-On

- Collusion raises prices and harms consumers
- Consumers can claim damages from cartelists (in case of explicit collusion!)
- Challenge: claimant has to show that damage occurs and quantify the damage
- Basic idea
  - Identify (percentage) overcharge $\kappa = \frac{p_K - p_C}{p_C}$
  - Competitive price p_C computed in counterfactual scenario
    - Econometric methods: Time-series/Panel analysis or difference-in-difference
    - Theory-based: simulation
  - Damnum emergens: lost profits
  - Lucrum cessans: quantity/volume effects
  - interest

Implicit and Explicit Collusion – Damage Claims and Pass-On

- Collusion often on intermediate products (graphite electrodes, rails, elevators, trucks, carglass, methionine, animal feeding phosphates, lysine, zinc, steel, ...)
- Depending on type of good and mode of competition, buyer passes on a part of the overcharge to consumers $\Rightarrow$ Pass-On
- Passing-On Defence
  - Cartelists can argue that direct buyer passed on overcharge to indirect buyer(s)
  - Thus, claims of direct buyer only partially valid or not valid at all
- Example:
  - Collusion affects variable input of direct buyers, increasing their marginal costs from c to c'
  - Direct buyers in perfect competition and demand is perfectly inelastic (vertical demand function)
  - 100% pass-on as prices downstream increase from c to c'
  - No quantity effects as demand is inelastic
  - Perfect pass-on in the long run, even if collusion affects fixed input

- Present value competition, start date $t = 0$

$$V_C = \pi_C + \delta\pi_C + \delta^2\pi_C + \cdots + \delta^T\pi_C$$

- Multiply with δ

$$\delta V_C = \delta\pi_C + \delta^2\pi_C + \cdots + \delta^T\pi_C + \delta^{T+1}\pi_C$$

- Using $V_C - \delta V_C$, we get

$$(1-\delta)V_C = (1-\delta^{T+1})\pi_C \;\Leftrightarrow\; V_C = \frac{1-\delta^{T+1}}{1-\delta}\pi_C$$

Implicit and Explicit Collusion – Appendix: Finite and Infinite Geometric Series

- Present value competition, start date $t = 1$

$$V'_C = \delta\pi_C + \delta^2\pi_C + \delta^3\pi_C + \cdots + \delta^T\pi_C$$

- Multiply with δ

$$\delta V'_C = \delta^2\pi_C + \delta^3\pi_C + \cdots + \delta^T\pi_C + \delta^{T+1}\pi_C$$

- Using $V_C - \delta V_C$, we get

$$(1-\delta)V'_C = (\delta - \delta^{T+1})\pi_C \;\Leftrightarrow\; V'_C = \frac{\delta - \delta^{T+1}}{1-\delta}\pi_C$$

Implicit and Explicit Collusion – Appendix: Critical Discount Factor w/o CA

- For $\delta = \delta_{\text{krit}}$, we have $V_K = V_D$ such that

$$\begin{aligned} \frac{1}{1-\delta}\pi_K &= \pi_D + \frac{\delta}{1-\delta}\pi_C \\ \Leftrightarrow \qquad \pi_K &= \pi_D - \delta\pi_D + \delta\pi_C \\ \Leftrightarrow \delta(\pi_D - \pi_C) &= \pi_D - \pi_K \\ \Leftrightarrow \qquad \delta &= \frac{\pi_D - \pi_K}{\pi_D - \pi_C} \equiv \delta_{\text{krit}}. \end{aligned}$$