## Problem Set 6 (due date: 25.11.2013)

1. Consider a monopolist who can produce a product with quality $s$. Depending on the quality level, consumer's utility is $U=\theta s-p$. The number of consumers is $N$ and $\theta$ is equally distributed in the interval $[0, N]$. The total cost function is $C(q, s)=q\left[\frac{c s^{2}}{2}\right]$, where $q$ is the quantity.
a) Determine the marginal costs with respect to quantity and to quality, respectively.
b) Derive the demand function for a product of quality $s$.
c) Determine the optimal quality level from the viewpoint of both the monopolist and a social planner for a given quantity $q$. Show that the monopolist chooses a lower quality than the socially optimal one, given that output is $q$.
d) Derive both optimum quantity and optimum quality for the monopolist and the social planner. Show that the monopolist and a social planer provide the same quality but different quantities.

Now assume that the utility function is $U=\theta+(\alpha-\theta) s-p$.
e) * Interpret the utility function and derive the demand function. Draw the demand curve and show how it changes with the quality level $s$. Interpret the demand curve.
f) *What is the relation between the quality offered by the monopolist and the social planer at a given quantity?
2. Sony is considering manufacturing two versions of a new laptop. One will meet high performance standards. The other one will only meet medium performance standards. To make the second, Sony uses cheaper material and then crimps the keyboard of the high performance machine with the result that the marginal costs of each product is identical at $\$ 500$.
Sony knows that there are two types of consumers for these new machines. "Techies" have the (indirect) utility function $V_{t}=2000(z-1)$ and "Normals" have the (indirect) utility function $V_{N}=$ $1000(z-1)$. In each case, $z$ is a measure of product quality, which can be chosen from the interval $[2,3]$. Sony also knows that there are $N_{t}$ techies and $N_{N}$ normals in the target market.
a) Interpret the utility functions! Use technology levels for this purpose.
b) Suppose that Sony can identify the two consumer types and sell to them separately. What is its profit-maximizing strategy?
c) Now suppose that Sony does not know the type of each consumer. Show how Sony's profitmaximizing strategy is determined by the relative numbers of each type of consumer.

