Gliederung der Vorlesung

F) Introduction

- In the majority of markets firms interact with *few competitors*
- In determining strategy each firm has to consider rival's reactions
  - *strategic interaction in prices, outputs, advertising …*
- This kind of interaction is analyzed using ***game theory***
  - assumes that "players" are rational
- Distinguish *cooperative* and *noncooperative* games
  - focus on noncooperative games
- Also consider *timing*
  - simultaneous versus sequential games

Cooperative game theory: coalitions (groups of individuals) are often the unit of analysis. Commitments and side payments allowed, outcome typically efficient, question of distribution of the pie.

Noncooperative game theory: No commitments (without extra actions), individual payoff maximization.

F) Oligopoly Theory

- No single theory
  - employ game theoretic tools that are appropriate
  - outcome depends upon *information* available
- Need a concept of ***equilibrium***
  - players (firms?) choose *strategies,* one for each player
  - combination of strategies determines *outcome*
  - outcome determines *pay-offs* (profits?)
- Equilibrium first formalized by Nash: ***No firm wants to change its current strategy given that no other firm changes its current strategy***

F) Nash Equilibrium

- Equilibrium need not be "nice"
  - firms might do better by coordinating but such coordination may not be possible (or legal)
- Some strategies can be eliminated on occasions
  - they are never good strategies *no matter what the rivals do*
- These are ***dominated strategies***
  - they are never employed and so can be eliminated
  - elimination of a dominated strategy may result in another being dominated: it also can be eliminated
- One strategy might always be chosen no matter what the rivals do: ***dominant strategy***

F) An Example

- Two airlines: Transatlantic flights, one per day
- Prices set: compete in departure times
- 70% of consumers prefer evening departure, 30% prefer morning departure
- If the airlines choose the same departure times they share the market equally
- Pay-offs to the airlines are determined by market shares
- Represent the pay-offs in a ***pay-off matrix***

A short introduction into very basic game theory.

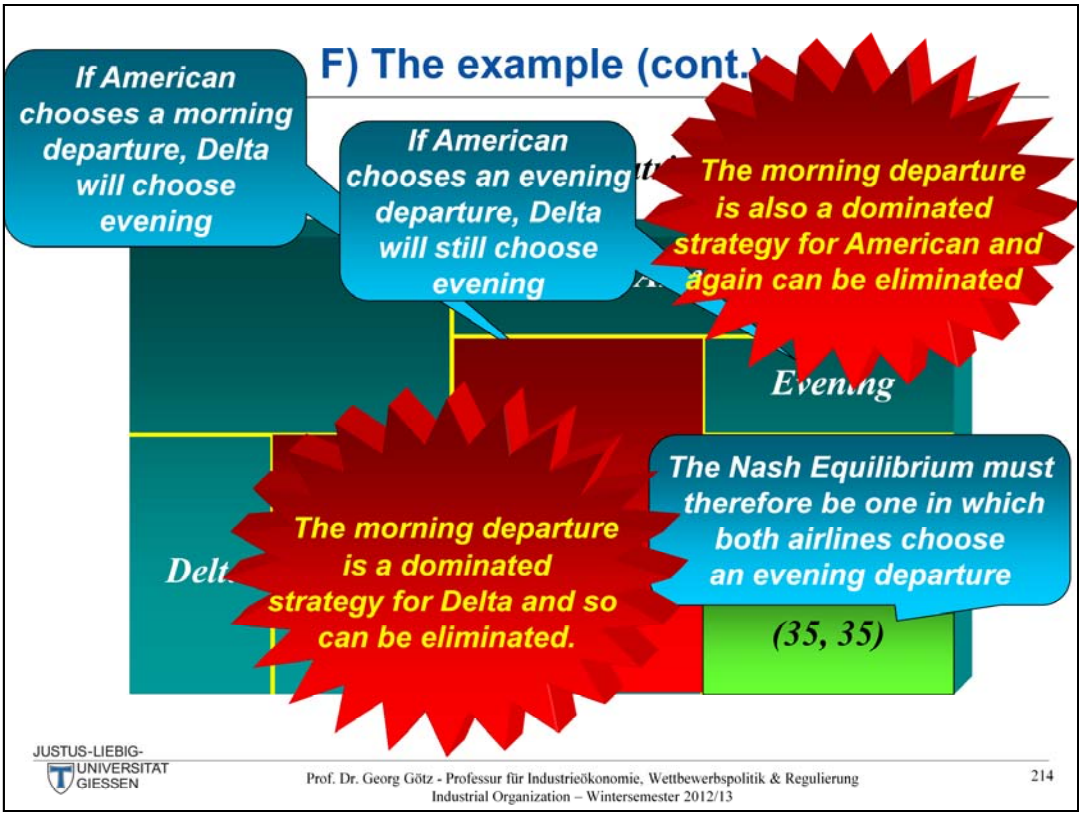
F) The example (cont.)

What is the equilibrium for this game?

The Pay-Off Matrix

		American	
		Morning	*Evening*
Delta	*Morning*	*(15, 15)*	*(30, 70)*
	Evening	*(70, 30)*	*(35, 35)*

Normal form of the game.
The left-hand number is the pay-off to Delta. The right-hand number is the pay-off to American

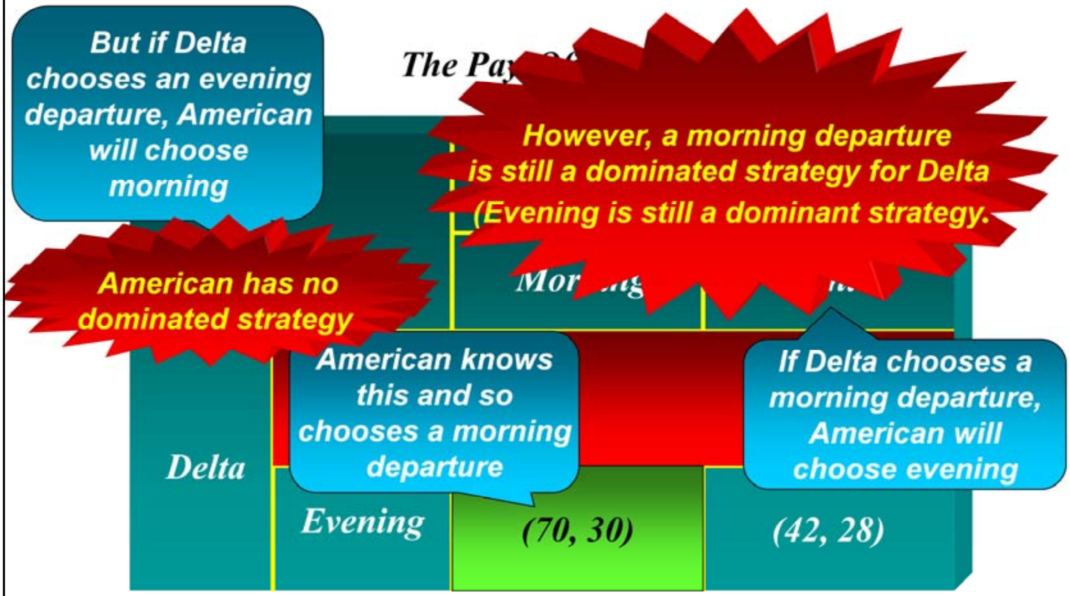


What could firms do to improve their situation: 30 % of consumers not served, only half of the market each.

F) The example (cont.)

- Now suppose that Delta has a frequent flier program
- When both airline choose the same departure times Delta gets 60% of the travelers
- This changes the pay-off matrix

F) The example (cont.)



F) Nash Equilibrium Again

- What if there are no dominated or dominant strategies?
- The ***Nash equilibrium*** concept can still help us in eliminating at least some outcomes
- Change the airline game to a pricing game:
  - 60 potential passengers with a reservation price of \$500
  - 120 additional passengers with a reservation price of \$220
  - price discrimination is not possible (perhaps for regulatory reasons or because the airlines don't know the passenger types)
  - costs are \$200 per passenger no matter when the plane leaves
  - the airlines ***must*** choose between a price of \$500 and a price of \$220
  - if equal prices are charged the passengers are evenly shared
  - Otherwise the low-price airline gets all the passengers

We don‘t allow for undercutting => restrictive simplification

Main point here is the structure of the payoff matrix!

Another example: Decision of OPEC country: Either comply with quota from cartel or produce at full capacity.

F) Nash Equilibrium (cont.)

The Pay-Off Matrix

		American	
		$P_H = \$500$	$P_L = \$220$
Delta	$P_H = \$500$	(\$9000,\$9000)	(\$0, \$3600)
	$P_L = \$220$	(\$3600, \$0)	(\$1800, \$1800)

F) Nash Equilibrium (cont.)

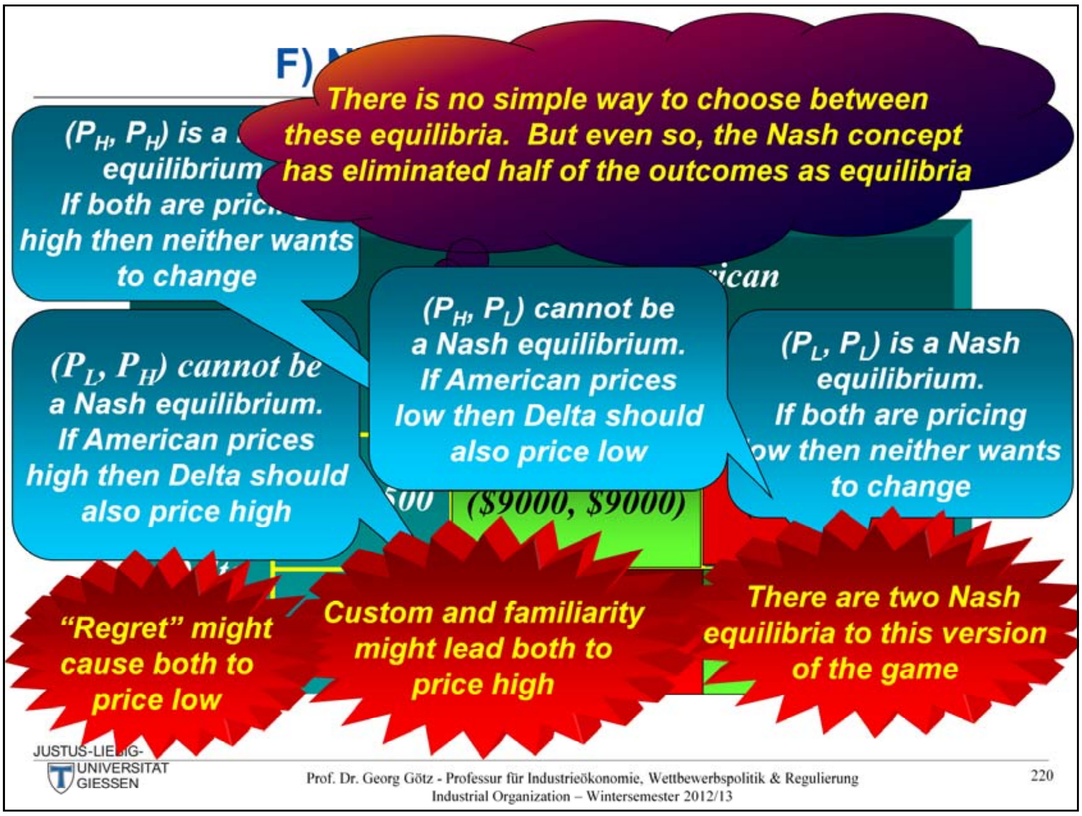
The Pay-Off Matri

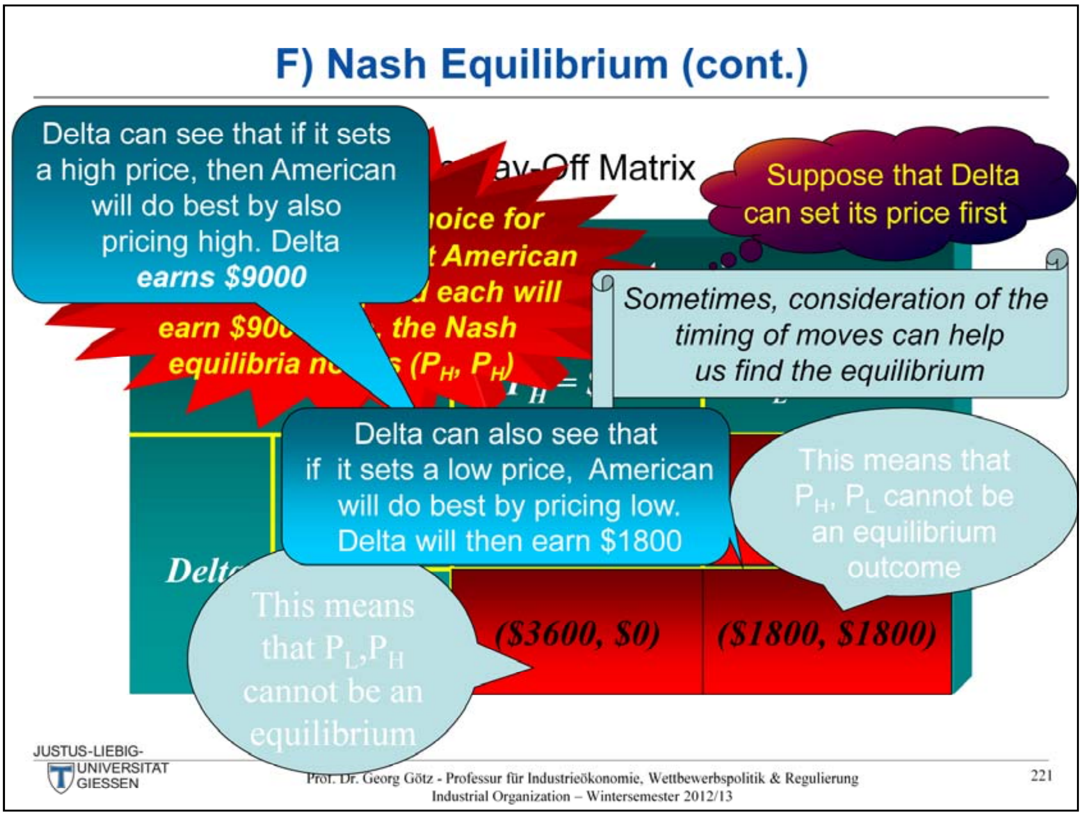
If both price high then both get 30 passengers. Profit per passenger is $300. If Delta prices low and American high then Delta gets all 180 Passengers. Profit per passenger is $20

If Delta prices high and American low then American gets all 180 Passengers. Profit per passenger is $20

If both price low they each get 90 passengers. Profit per passenger is $20

		Am	
		$P_H = \$500$	$P_L = \$220$
Delta	$P_H = \$500$		*0)*
	$P_L = \$220$	*($3600, $0)*	*($1800, $1800)*


F) N
There is no simple way to choose between these equilibria. But even so, the Nash concept has eliminated half of the outcomes as equilibria
(P_H, P_H) is a
equilibrium
If both are pric
high then neither wants to change
(P_H, P_L) cannot be a Nash equilibrium. If American prices low then Delta should also price low
(P_L, P_H) cannot be a Nash equilibrium. If American prices high then Delta should also price high
(P_L, P_L) is a Nash equilibrium. If both are pricing low then neither wants to change
($9000, $9000)
"Regret" might cause both to price low
Custom and familiarity might lead both to price high
There are two Nash equilibria to this version of the game



Elimination of an equilibrium if firms move sequentially! Here extensive form of game would be relevant! High profit equilibrium is chosen.

Equilibria in mixed strategies

- Matching pennies game (Penalty kick game)

		Goalie	
		„Left“	„Right“
Kicker	„Left“	–1, 1	1, –1
	„Right“	1, –1	–1, 1

- Equilibrium: Randomizing over actions:
  - Strategy puts positive probability on every action
  - Player indifferent between pure strategies (with positive probability), but
  - Equilibrium requires picking of particular mixed strategy

Discoordination game!

Excercise: Make game asymmetric (e.g. by natural side assumption: the kicker kicks better on his natural side, whether the keeper guesses the side correctly or not) and calculate equilibrium. Consider payoffs as expected payoffs.

More on mixed strategies with penalty kicks see in *By P.-A. CHIAPPORI, S. LEVITT, AND T. GROSECLOSE:* Testing Mixed-Strategy Equilibria When Players Are Heterogeneous: The Case of Penalty Kicks in Soccer, *THE AMERICAN ECONOMIC REVIEW SEPTEMBER 2002*

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F) Oligopoly Models

- There are three dominant oligopoly models
  - Cournot
  - Bertrand
  - Stackelberg
- They are distinguished by
  - the decision variable that firms choose
  - the timing of the underlying game
- But each embodies the Nash equilibrium concept

F1) The Cournot Model

Duopoly, homogeneous good, identical constant marginal costs c; **inverse demand function** :

$p = p(q_1 + q_2)$, where $p' < 0$.

The firms maximise

$\max \pi_1(q_1,q_2) = q_1[p(q_1 + q_2) - c]$,

$\max \pi_2(q_1,q_2) = q_2[p(q_1 + q_2) - c]$.

$\Rightarrow$ FOCs

$p(q_1 + q_2) + q_1 p'(q_1 + q_2) - c = 0$

$p(q_1 + q_2) + q_2 p'(q_1 + q_2) - c = 0$

$p' < 0$ implies $p^c > c$.

Cournot supposed that the homogeneous product was spring water. q_1 and q_2 firms' outputs

Firms maximise given an expectation of what their rivals do. Nash equilibrium: Expectations are satisfied and no incentive to deviate from optimal choice.

F1) The Cournot model: Linear Demand

Inverse demand function:
$P = A - BQ = A - B(q_1 + q_2)$

Residual demand curve: $P = (A - Bq_1) - Bq_2$

The profit-maximizing choice of output by firm 2 depends upon the output of firm 1

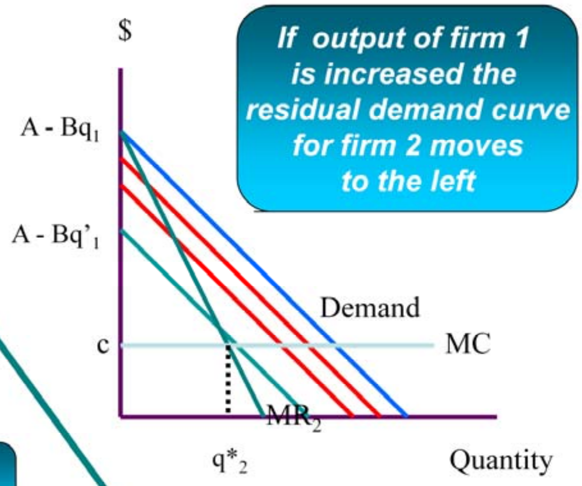
Marginal revenue for firm 2 is

$MR_2 = (A - Bq_1) - 2Bq_2$

$MR_2 = MC$

$A - Bq_1 - 2Bq_2 = c$

Solve this for output q_2



$\therefore\ q^*_2 = (A - c)/2B - q_1/2$

F1) The Cournot model (cont.)

$q^*_2 = (A - c)/2B - q_1/2$

This is the ***best response function*** for firm 2

It gives firm 2's profit-maximizing choice of output for any choice of output by firm 1

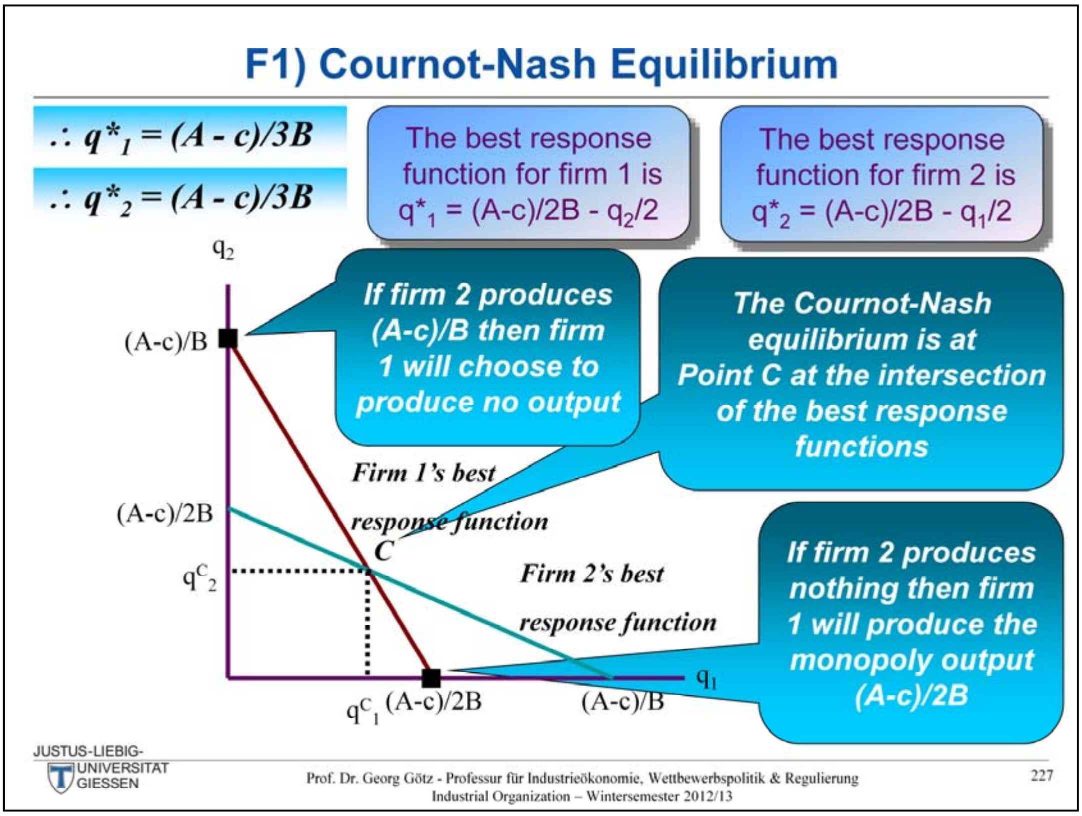
There is also a ***best response*** function for firm 1

By exactly the same argument it can be written:

$q^*_1 = (A - c)/2B - q_2/2$

Cournot-Nash equilibrium requires that both firms be on their best response functions.

Best response function is also called reaction function. Note that in the Cournot model there is neither a response nor a reaction to the rival's action since the game is simultaneous.


F1) Cournot-Nash Equilibrium
∴ q*1 = (A - c)/3B
∴ q*2 = (A - c)/3B
The best response function for firm 1 is q*1 = (A-c)/2B - q2/2
The best response function for firm 2 is q*2 = (A-c)/2B - q1/2
q2
(A-c)/B
If firm 2 produces (A-c)/B then firm 1 will choose to produce no output
The Cournot-Nash equilibrium is at Point C at the intersection of the best response functions
Firm 1's best response function
(A-c)/2B
C
qC2
Firm 2's best response function
If firm 2 produces nothing then firm 1 will produce the monopoly output (A-c)/2B
q1
qC1 (A-c)/2B
(A-c)/B
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F1) Cournot-Nash Equilibrium (cont.)

- In equilibrium each firm produces $q^C_1 = q^C_2 = (A - c)/3B$
- Total output is, therefore, $Q^* = 2(A - c)/3B$
- Recall that demand is $P = A - BQ$
- So the equilibrium price is $P^* = A - 2(A - c)/3 = (A + 2c)/3$
- Profit of firm 1 is $(P^* - c)q^C_1 = (A - c)^2/9$
- Profit of firm 2 is the same
- A monopolist would produce $Q^M = (A - c)/2B$
- **Competition between the firms causes their total output to exceed the monopoly output. Price is therefore lower than the monopoly price**
- **But output is less than the competitive output $(A - c)/B$ where price equals marginal cost and P exceeds MC**

F1) Cournot-Nash Equilibrium (cont.)

- What if there are more than two firms?
- Much the same approach.
- Say that there are N identical firms producing identical products
- Total output $Q = q_1 + q_2 + \ldots + q_N$
- Demand is $P = A - BQ = A - B(q_1 + q_2 + \ldots + q_N)$
- Consider firm 1. It's demand curve can be written:

  $P = A - B(q_2 + \ldots + q_N) - Bq_1$

  This denotes output of every firm other than firm 1

- Use a simplifying notation: $Q_{-1} = q_2 + q_3 + \ldots + q_N$
- So demand for firm 1 is $P = (A - BQ_{-1}) - Bq_1$

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General approach: N profit functions => N first order conditions => N equations in N variables (the N output levels)

Here: Symmetry assumption: All firms have identical marginal costs.

F1) The Cournot model (cont.)

$P = (A - BQ_{-1}) - Bq_1$

The profit-maximizing choice of output by firm 1 depends upon the output of the other firms

Marginal revenue for firm 1 is

$MR_1 = (A - BQ_{-1}) - 2Bq_1$

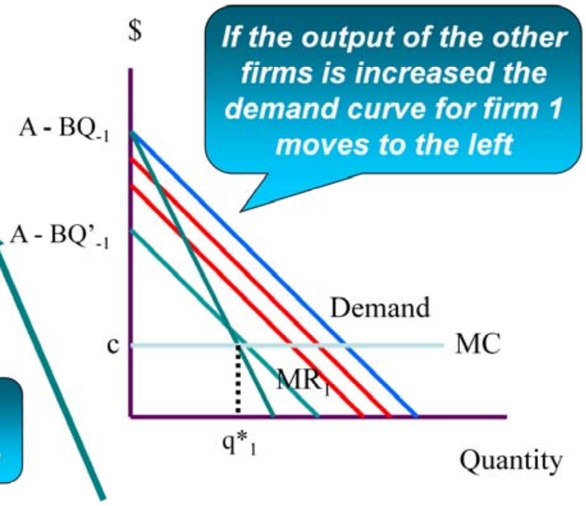
$MR_1 = MC$

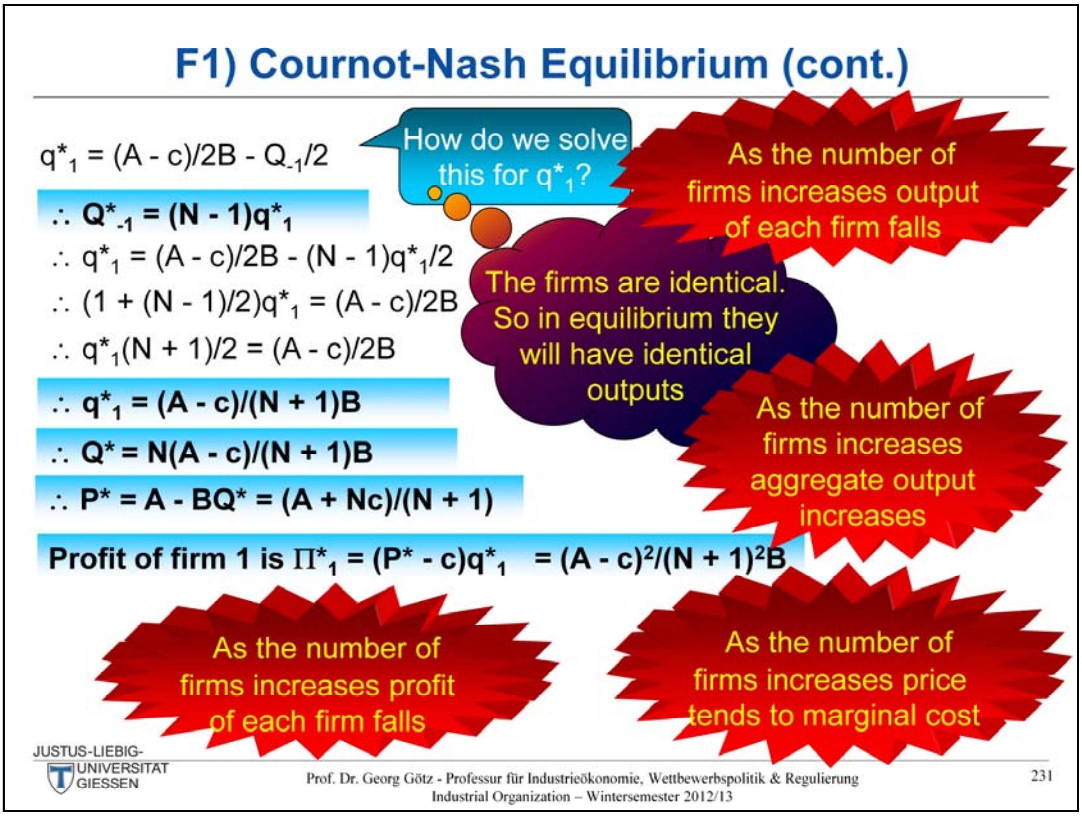
Solve this for output q_1

$A - BQ_{-1} - 2Bq_1 = c$

$\therefore q^*_1 = (A - c)/2B - Q_{-1}/2$

If the output of the other firms is increased the demand curve for firm 1 moves to the left





As the number of firms increases output of each firm falls
As the number of firms increases aggregate output increases
As the number of firms increases profit of each firm falls

F1) Cournot-Nash equilibrium (cont.)

- What if the firms do not have identical costs?
- Once again, much the same analysis can be used
- Assume that marginal costs of firm 1 are c_1 and of firm 2 are c_2.
- Demand is $P = A - BQ = A - B(q_1 + q_2)$
- We have marginal revenue for firm 1 as before
- $MR_1 = (A - Bq_2) - 2Bq_1$
- Equate to marginal cost: $(A - Bq_2) - 2Bq_1 = c_1$

Solve this for output q_1

$\therefore q^*_1 = (A - c_1)/2B - q_2/2$

$\therefore q^*_2 = (A - c_2)/2B - q_1/2$

A symmetric result holds for output of firm 2

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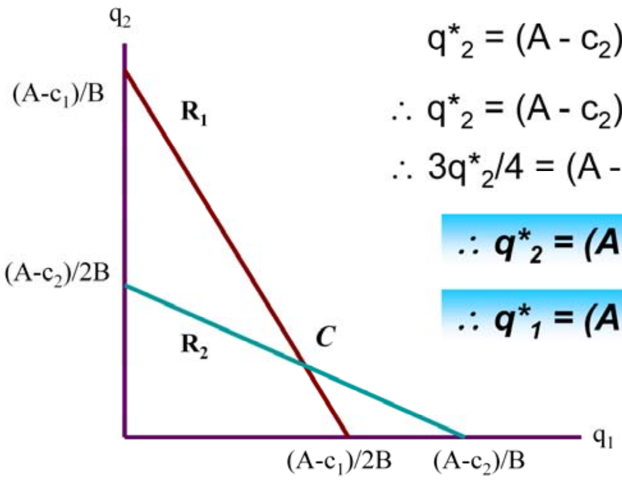
Assignment!

F1) Cournot-Nash Equilibrium



$$q^*_1 = (A - c_1)/2B - q^*_2/2$$

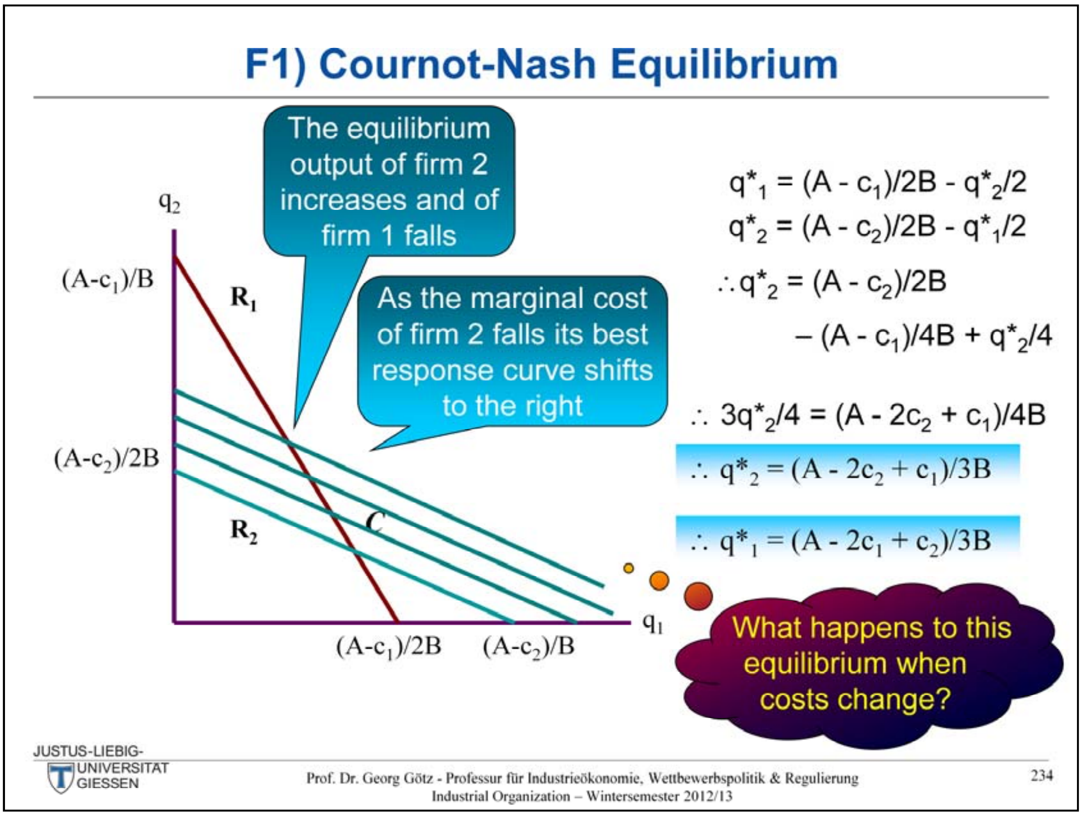
$$q^*_2 = (A - c_2)/2B - q^*_1/2$$

$$\therefore\ q^*_2 = (A - c_2)/2B - (A - c_1)/4B + q^*_2/4$$

$$\therefore\ 3q^*_2/4 = (A - 2c_2 + c_1)/4B$$

$$\therefore\ \boldsymbol{q^*_2 = (A - 2c_2 + c_1)/3B}$$

$$\therefore\ \boldsymbol{q^*_1 = (A - 2c_1 + c_2)/3B}$$



Change in costs!

F1) Cournot-Nash Equilibrium (cont.)

- In equilibrium the firms produce
  $q^C_1 = (A - 2c_1 + c_2)/3B$; $q^C_2 = (A - 2c_2 + c_1)/3B$
- Total output is, therefore, $Q^* = (2A - c_1 - c_2)/3B$
- Recall that demand is $P = A - BQ$
- So price is $P^* = A - (2A - c_1 - c_2)/3 = (A + c_1 + c_2)/3$
- Profit of firm 1 is $(P^* - c_1)q^C_1 = (A - 2c_1 + c_2)^2/9B$
- Profit of firm 2 is $(P^* - c_2)q^C_2 = (A - 2c_2 + c_1)^2/9B$
- Equilibrium output is less than the competitive level
- Output is produced inefficiently: the low-cost firm should produce all the output

F1) Concentration and Profitability

- Assume that we have N firms with different marginal costs
- We can use the N-firm analysis with a simple change
- Recall that demand for firm 1 is $P = (A - BQ_{-1}) - Bq_1$
- But then demand for firm *i* is $P = (A - BQ_{-i}) - Bq_i$
- Equate this to marginal cost c_i => FOC

$$A - BQ_{-i} - 2Bq_i = c_i$$

This can be reorganized to give the equilibrium condition:

$$A - B(Q^*_{-i} + q^*_i) - Bq^*_i - c_i = 0$$

But $Q^*_{-i} + q^*_i = Q^*$ and $A - BQ^* = P^*$

$$\therefore\ P^* - Bq^*_i - c_i = 0 \ \therefore\ P^* - c_i = Bq^*_i$$

First order condition!

F1) Concentration and profitability (cont.)

$P^* - c_i = Bq^*_i$

Divide by P* and multiply the right-hand side by Q*/Q*

$$\frac{P^* - c_i}{P^*} = \frac{BQ^*}{P^*}\ \frac{q^*_i}{Q^*}$$

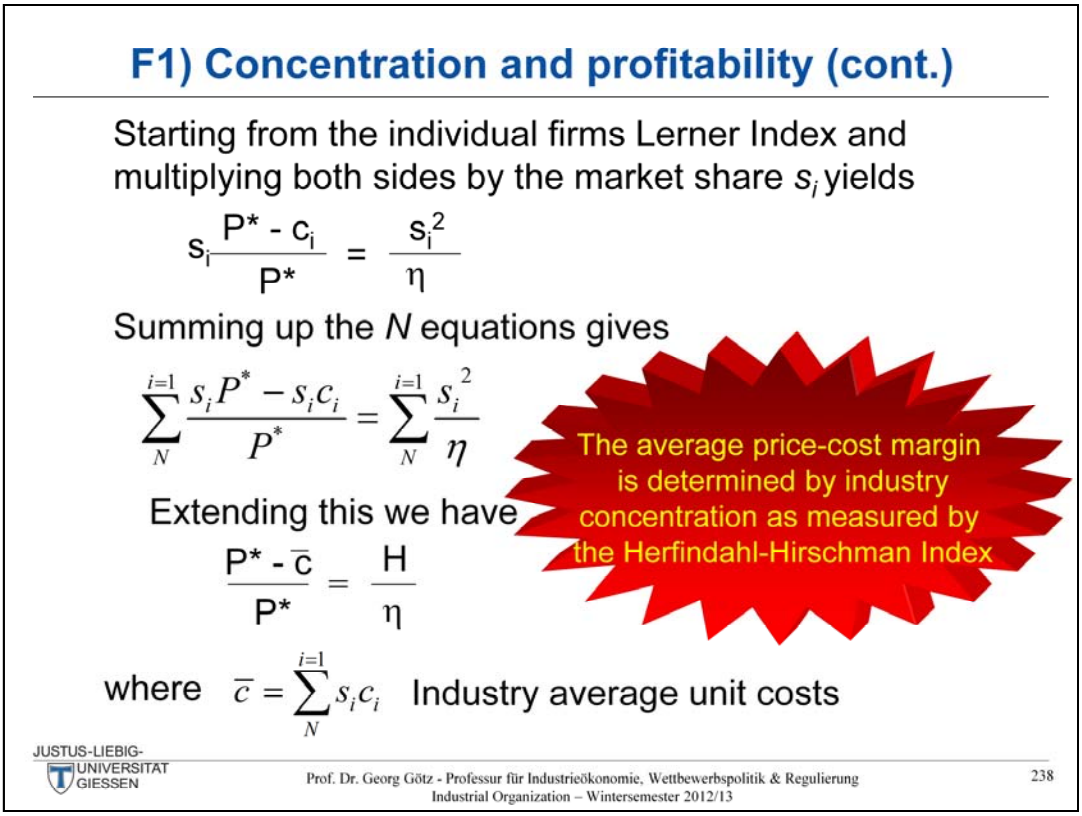
But $BQ^*/P^* = 1/\eta$ and $q^*_i/Q^* = s_i$

so: $$\frac{P^* - c_i}{P^*} = \frac{s_i}{\eta}$$

The price-cost margin for each firm is determined by its own market share and overall market demand elasticity

With general demand the Lerner Index L_i reads in the two firms case:

$$L_i = \frac{p - c_i}{p} = -\frac{q_i p'\left(q_1 + q_2\right)}{p\left(q_1 + q_2\right)} = \frac{s_i}{\eta}$$



Cournot model supports view that increases in concentration lead to higher price cost margines and therefore increases in prices. Remark: How do (average unit) costs change when concentration rises?

Average unit costs are weighted by market shares.

F2) Price Competition: Bertrand

- In the Cournot model price is set by some market clearing mechanism
- Firms seem relatively passive
- An alternative approach is to assume that firms compete in prices: this is the approach taken by Bertrand
- Leads to dramatically different results
- Take a simple example
  - two firms producing an identical product (spring water?)
  - firms choose the prices at which they sell their water
  - each firm has constant marginal cost of $10
  - market demand is Q = 100 - 2P

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Check that with this demand and these costs the monopoly price is $30 and quantity is 40 units

F2) Bertrand competition (cont.)

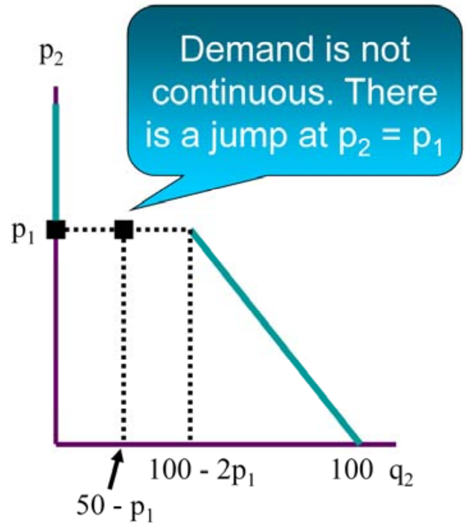
- Demand to firm 2 given p_1 (*derived demand*) is:

  $q_2 = 0$ if $p_2 > p_1$

  $q_2 = 100 - 2p_2$ if $p_2 < p_1$

  $q_2 = 50 - p_1$ if $p_2 = p_1$
  (Tie-breaker rule)

- The discontinuity in demand carries over to profit



F2) Bertrand competition: Equilibrium

Firms undercut each other as long as price is above the constant marginal cost c.

$\Rightarrow$ Unique equilibrium with both firms charging $p^B = c$.

Bertrand paradox due to the following assumptions:

a) 'Unlimited' capacities.

b) Homogeneous goods.

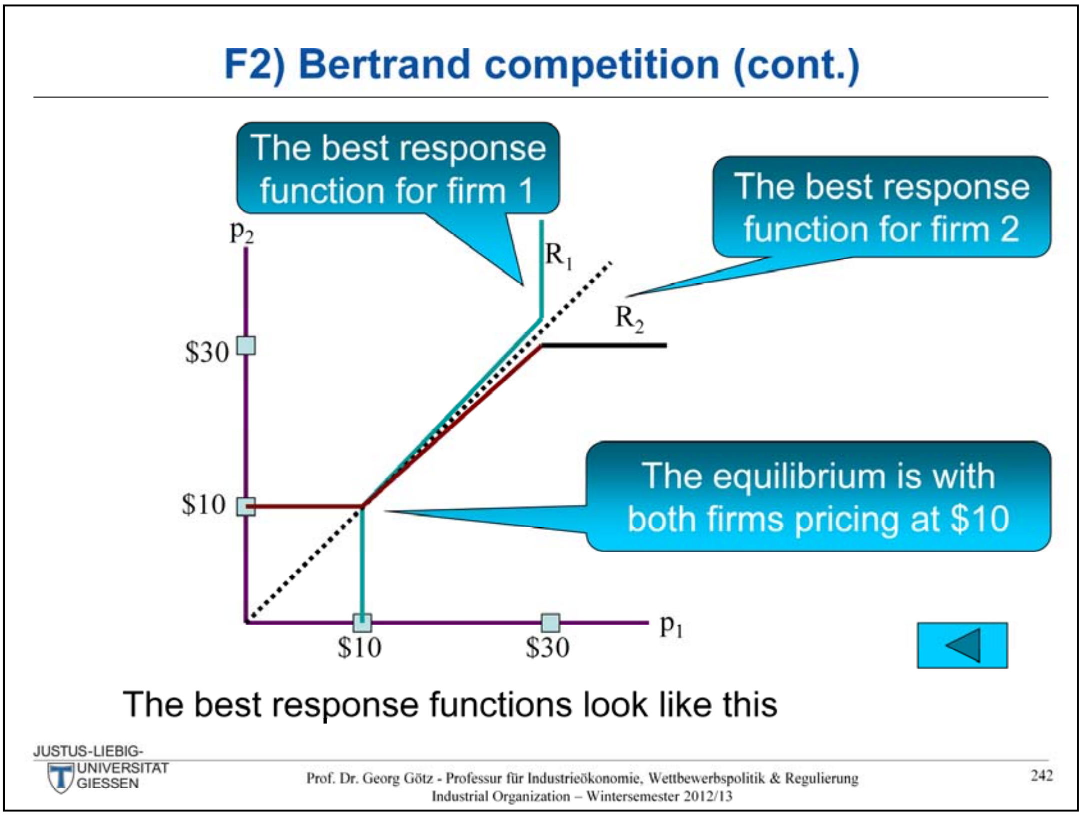
c) One shot game.

d) Identical, constant average and marginal costs.

Extensions: Two stage capacity game, differentiated products, repeated and super games, contestable markets, mixed strategy equilibria.

Tell undercut story: Important: Homogeneous products: Small difference in price leads to total demand for the low price firm.
Cutting price below cost gains the whole market but loses money on every customer. Actually with constant marginal cost each firm is indifferent about producing and not producing. But charging a different price would not be an equilibrium! The other would have an incentive to charge another price as well.

We will discuss most of the extensions furtheron.



30 is monopoly price!

F2) Bertrand competition: Different marginal costs

- Let $c_1 < c_2 \Rightarrow$ equilibrium: only firm 1 active, with c_2 as upper limit for its price, formally:
- $\max \pi(p_1, c_2) = (p_1 - c_1)x(p_1) - \lambda(p_1 - c_2)$
- Denote monopoly price as $p^m(c_1)$. Either $p_1 = p^m(c_1) < c_2$ or $p_1 = c_2$.
- **Tie-breaker rule** $x_2 = 0$**.**

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Optimization approach: Kuhn-Tucker with complementary slackness: Either λ or $p_1 - c_2$ must be zero. Just calculate the monopoly price and check whether it is below or above the rival's cost!

F2) Bertrand Equilibrium: modifications

- The Bertrand model makes clear that competition in prices is very different from competition in quantities
- Since many firms seem to set prices (and not quantities) this is a challenge to the Cournot approach
- But the Bertrand model has problems too
  - for the p = marginal-cost equilibrium to arise, both firms need enough capacity to fill all demand at price = MC
  - but when both firms set p = c they each get only half the market
  - So, at the p = mc equilibrium, there is huge excess capacity
- This calls attention to the choice of capacity
  - Note: choosing capacity is a lot like choosing output which brings us back to the Cournot model
- The intensity of price competition when products are identical that the Bertrand model reveals also gives a motivation for *Product differentiation*

Needs to be extended!

F2) Price competition with endogenous capacities

- Two firms selling homogenous products, but with endogenous capacities
- Structure of the game:

$\Rightarrow$ Firms choose capacities

$\Rightarrow$ Firms choose prices (potentially different!)

- Two stage game $\Rightarrow$ recursive solution:
  - given capacities, what are the optimal prices
  - Reduced profit function: Determine optimal capacity
- Decision on capacities and prices sequentially, but firms decisions on each of the variables simultaneously

Both firms first decide on capacities (simultaneously), then, after having observed the decision of the rival, both firms decide simultaneously on prices.

F2) Price competition with capacity constraints
The rationing rule

- Given capacity constraints and the possibility that firms charge different prices, rationing becomes important!
  - All consumers want to buy from the low price firm

  $\Rightarrow$Possibility that capacity < demand
- How does rationing look like? Who is served?

$\Rightarrow$Rationing rule

$\Rightarrow$Determines residual demand

- Two kinds: Efficient or random rationing

Rationing can in general occur by various allocation mechanisms: first come, first serve; appearance, etc.

F2) Price competition with capacity constraints
The rationing rule cont.

- Assume:
  - A continuum of consumers with unit demand and different reservation prices (uniformly distributed over [0,100]).
  - 'Number' of consumers (Density): N
  - $p_1 < p_2$: Firm 1 is the low price firm.

$\Rightarrow$ Demand function: $x(p) = N(100 - p)$

- Assume: For capacity k_1 it holds that: $k_1 < x(p_1)$

  $\Rightarrow$ Rationing

- efficient rationing rule:
  - Consumer with highest reservation prices (WTP) are served

  $\Rightarrow$ Consumers buy in order of reservation price

F2) Price competition with capacity constraints
The efficient rationing rule

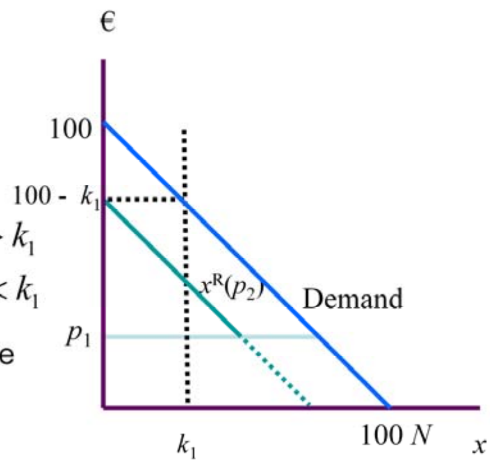
- Residual demand $x^R(p_2)$ of firm 2

$$x^R(p_2) = \begin{cases} x(p_2) - k_1 & \text{if } x(p_1) > k_1 \\ 0 & \text{if } x(p_1) < k_1 \end{cases}$$

$$x^R(p_2) = \begin{cases} N(100 - p_2) - k_1 & \text{if } x(p_1) > k_1 \\ 0 & \text{if } x(p_1) < k_1 \end{cases}$$

⇒ Only consumers with reservation price > 100 – k_1 get the product

⇒ Consumer surplus is maximized with efficient rationing!



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We look at firm 2, given firm 1’s price and capacity!

Residual demand is of course also a function of the rival’s capacity k_1 and of its price p_1.

The vertical intercept of the residual demand curve should probably read 100 – (k/N) rather than simply 100 –k. Density N is not 1!

F2) Price competition with capacity constraints
The proportional rationing rule

- Random rationing: Each consumer (with WTP > p_1) gets one unit of output with equal probability $k_1/x(p_1)$

$\Rightarrow$ Probability that an arbitrary consumer with WTP > p_2 has *not* obtained the product from firm 1: $(x(p_1)- k_1)/x(p_1)$

$\Rightarrow$ Residual demand $x^R(p_2)$ of firm 2

$$x^R(p_2)=\begin{cases} x(p_2)\dfrac{x(p_1)-k_1}{x(p_1)} & \text{if } x(p_1)>k_1 \\ 0 & \text{if } x(p_1)<k_1 \end{cases}$$

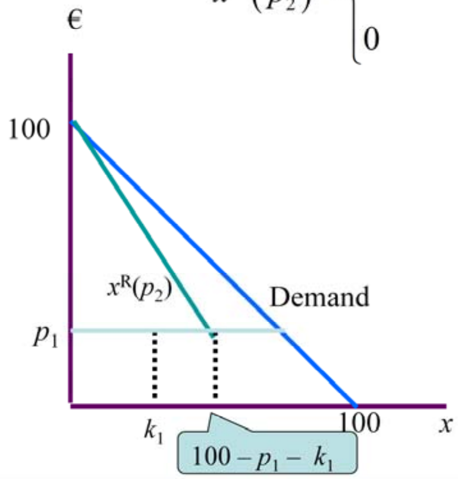
$$x^R(p_2)=\begin{cases} N(100-p_2)\dfrac{N(100-p_1)-k_1}{N(100-p_1)} & \text{if } x(p_1)>k_1 \\ 0 & \text{if } x(p_1)<k_1 \end{cases}$$

Probability of obtaining the product at price p_1: Take example $x(p_1)$ =60 and k_1 =20 => Chance of getting the product: 1/3. Chance that firm 2 gets a consumer with a high WTP (> p_2): 2/3

F2) Price competition with capacity constraints
The proportional rationing rule cont.

- Assume: $N = 1$ =>

$$x^R(p_2) = \begin{cases} (100 - p_2)\dfrac{100 - p_1 - k_1}{100 - p_1} & \text{if } x(p_1) > k_1 \\ 0 & \text{if } x(p_1) < k_1 \end{cases}$$



Explanation of the diagram: Geometry: For given price of firm 2: Residual demand divides total demand in two parts which are determined by the probabilities! (Intercept theorem (Strahlensatz)

Easiest explanation:

If price of firm 2 is such that demand is 0 (=> p=100), the residual demand is equal to total demand = 0. If price is slightly below, demand is strictly positive since there is a chance to get a consumer with a high WTP.

If price is equal to p_1, residual demand is clearly 100 - p_1 - k_1.

Since demand curve is linear we obtain the curve immediately.

F2) Price competition and choice of capacities (with the efficient rationing rule)

- Kreps & Scheinkman (1983): Efficient rationing rule and endogenous capacities

⇒ equilibrium of Bertrand-duopoly with homogeneous goods is identical to Cournot-duopoly

- Example with linear demand: $x(p) = 100 - p$
- Two stage game:

⇒ **Stage 1**: choice of capacities, $c_o \in [75,100]$ = cost per unit of capacity.

⇒ **Stage 2**: Bertrand-competition, unit costs = 0 up to capacity limit k_i, $i = 1,2$.

F2) Price competition and choice of capacities (with the efficient rationing rule)

- Solution: recursive, in several steps.
  1. Derive upper limit for capacity.
     - Note that capacity costs can never be higher than profit of a monopolist without capacity limit (= profit in second stage of the game, capacity costless)
     - Monopoly price (Note: MC =0): 50
     - (Gross) Monopoly profit: 2500
     - Capacity at most: $k_i \leq$ 2500/75 = 33.33

F2) Price competition and choice of capacities (with the efficient rationing rule)

2. Bertrand-Nash price equilibrium: Each firm sells capacity at market clearing price

$\Rightarrow$ $p^* = 100 - k_1 - k_2$

Proof: Show: Firm 1 has no incentive to sell at price $p > p^*$ if firm 2 charges $p^* = 100 - k_1 - k_2$

Residual demand of firm 1

$$x^R(p_1) = 100 - p_1 - k_2$$

Optimal price of firm 1 given quantity x_1 :

$$p_1 = 100 - x_1 - k_2$$

Note: Firm 1 would never charge a price below *p**! Furthermore, it will always charge a market clearing price given the quantity it produces. Rationing would mean selling the same quantity at a lower price!

F2) Price competition and choice of capacities (with the efficient rationing rule)

2. Bertrand-Nash price equilibrium: cont.

Profit of deviating firm 1 :

$$\Pi_1 = (100 - x_1 - k_2)x_1$$

Differentiating wrt x_1 yields:

$$\frac{\partial \Pi_1}{\partial x_1} = 100 - 2x_1 - k_2 \quad \geq 0 \text{ since } k_i \leq 33.33.$$

$\Rightarrow$decreasing x_1 below k_1 is not profitable.

$\Rightarrow$ increasing the price above p^* is not profitable

$\Rightarrow$ in stage 2 both firms make full use of capacities build up in stage 1 and charge market clearing price!

F2) Price competition and choice of capacities (with the efficient rationing rule)

3. Choice of capacities (Stage 1 of the game)
   (Taking into account optimal prices in stage 2)

⇒ max $\Pi_i = (p^* - c_0)\, k_i = (100 - k_1 - k_2 - c_0)\, k_i$

⇒ Standard Cournot optimization problem!!
(exact Cournot reduced form)

⇒ Capacity choice and subsequent price competition lead to outcome of Cournot model!

Caveat: Result not robust (rationing rule);
imperfect information about cost functions of competitors: price as signal for costs, e.g. to mislead other firms.

Gliederung der Vorlesung

G) Dynamic games: First and second movers
Stackelberg quantity leadership

- Quantity leadership: Interpret in terms of Cournot (later perhaps price leadership)
- Firms choose outputs *sequentially*
  - leader sets output first, and visibly
  - follower then sets output
- The firm moving first has a *leadership advantage*
  - can anticipate the follower's actions
  - can therefore manipulate the follower
- For this to work the leader must be able to *commit* to its choice of output
- ***Strategic commitment has value***

G1) Stackelberg quantity leadership

- Formal solution of this two stage game: Recursively
- Follower: Derive reaction function as in Cournot model. Only difference: Here the rival's (=leader's) quantity is already known

⇒ Reaction function: $q^*_{\boldsymbol{F}}$ **=** $R(q_{\boldsymbol{L}})$

- Leader: Take into account the follower's reaction

⇒ $\max_{q_L} \pi_L(q_L) = q_L \; p(q_L + R(q_{\boldsymbol{L}})) - C(q_L)$,

G) Stackelberg Equilibrium: Linear demand

- Assume that there are two firms with identical products and identical constant marginal costs c
- As in our Cournot example, let inverse demand be: $P = A - BQ = A - B(q_L + q_F)$
- From above: The follower's reaction function:

  $q^*_F = (A - c)/(2B) - q_L/2$

- The leader's optimization problem:

$\Rightarrow$ $\max \pi_L(q_L) = q_L (A - B(q_L + q^*_F)) - cq_L$

$\Rightarrow$ $\max \pi_L(q_L) = q_L (A - B(q_L + ((A - c)/(2B) - q_L/2))) - cq_L$

$\Rightarrow$ $q_L = (A - c)/(2B)$, $q_F = (A - c)/(4B)$

$\Rightarrow$ Stackelberg output (price) greater (smaller) than the respective values for Cournot model

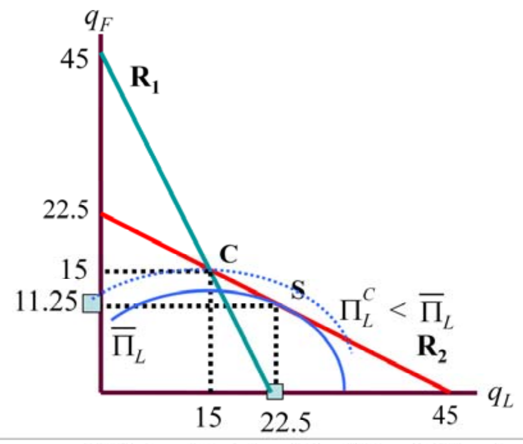
Output of Stackelberg leader = monopoly output in the case of linear demand!

Compared to Cournot: Consumers and leader gain, follower looses.

G) Stackelberg equilibrium: Graphical analysis

Isoprofit curves: combinations of q_L and q_F which give the same profit $\overline{\Pi}$ for a firm:

$\overline{\Pi}_L = (A-c)\, q_L - B\, q_L^2 - Bq_L\, q_F \Rightarrow q_F = ((A-c)\, q_L - B\, q_L^2 - \overline{\Pi}_L)/(Bq_L)$



Calculate iso-profit curves! The numbers are from an example with P = (100 – 2 Q)

Form of iso-profit curves: Horizontal (for the leader) at intersection with reaction function. Must be the case since reaction function gives the profit maximizing quantity (given the rival’s output).

Shape of the iso-profit curves: Start from horizontal part (intersection with reaction function): Increase or decrease of own quantity must lead to decrease of profit (given the rival’s output). Follows from the fact that profit is maximized at reaction function. In order to keep profit constant in case of a move away from the optimal output, the rival’s output must fall (=> increases profit).

To derive graphically the Stackelberg solution, find the iso-profit curve with the maximum profit consistent with the fact that the rival acts according to her reaction function

⇒Find the iso-profit curve which is tangent to the reaction function of the follower.

⇒Note: Profit is higher for lower iso-profit curves (Maximum when follower does not produce anything; monopoly!)

G) Stackelberg and Commitment

- It is crucial that the leader can *commit* to its output choice
  - without such commitment firm 2 should ignore any stated intent by firm 1 to produce 22.5 units
  - the only equilibrium would be the Cournot equilibrium
- So how to commit?
  - prior reputation
  - investment in additional capacity
  - place the stated output on the market
- Finally, the timing of decisions *matters*

G2) Introduction

- A firm that can restrict output by a large enough amount that the market price rises has market power
- Firms such as Microsoft (95% of PC operating systems) and Campbell's (70% of the tinned soup market) stand virtually alone as the giants in their respective industries
- Moreover, Microsoft, Campbell's and other have maintained their dominant position for many years
  - Why can't existing rivals compete away the position of such firms?
  - Why aren't new rivals lured by the profits of such dominant corporations?
- Answer: firms with monopoly power may
  - eliminate existing rivals
  - prevent entry of new firms
- Actions that eliminate existing or potential rivals is predatory conduct if they are profitable only if rivals, in fact, exit
    - e.g., R&D to reduce costs is not predatory

G2) Monopoly Power and Market Entry

- Several stylized facts about entry
  - entry is common
  - entry is generally small-scale
    - so small-scale entry is relatively easy
  - survival rate is low: >60% exit within 5 years
  - entry is highly correlated with exit
    - not consistent with entry being caused by excess profits
    - "revolving door"
    - reflects repeated attempts to penetrate markets dominated by large firms
- Not always easy to prove that this reflects predatory conduct
- But we need to understand predation it if we are to find it

G2) Predation, Predatory Pricing, and Limit Pricing

- Predatory actions come in two broad forms
  - Limit pricing: prices so low that entry is deterred
  - Predatory pricing: prices so low that existing firms are driven out
- From an economic perspective, the outcome of either action is the same—the monopolist retains control of the market
- But most legal action focuses on predatory pricing because this case has an identifiable victim—a firm that was in the market but that has left
- Can we construct a model of either limit pricing or predatory pricing? YES!
  - Stackelberg leader chooses output first
  - entrant believes that the leader is committed to this output choice
  - entrant has decreasing costs over some initial level of output

G2) Capacity Expansion and Entry Deterrence

- Central point of previous discussion
- For predation to be successful—and therefore rational—the incumbent must somehow convince the entrant that the market environment ***after*** the entrant comes in will not be a profitable one
- How can the incumbent credibly make this threat?
- One possible mechanism is to install ***capacity*** in advance of production
  - Installed capacity ***is*** a commitment to a minimum level of output
  - The lead firm can manipulate entrants through capacity choice
  - the lead firm may be able to deter entry through its capacity choice

G2) Capacity Expansion and Entry Deterrence

- An example:
  - $P = 120 - Q = 120 - (q_1 + q_2)$
  - marginal cost of *production* \$60 for incumbent and entrant
  - cost of each unit of *capacity* is \$30
  - firms also have fixed costs of F
  - incumbent chooses capacity K_1 in stage 1
  - NOTE: incumbent will always produce at least K_1 in production stage—otherwise it throws away revenue that could help cover the cost of installed capacity
  - entrant chooses capacity and output in stage 2
  - firms compete in quantities in stage 2.

State in general linear terms as in PRN.

G2) The Example (cont.)

- Consider the best response function of the entrant

  Residual demand is $P = (120 - q_1) - q_2$

  Marginal revenue is: $MR_2 = (120 - q_1) - 2q_2$

  Marginal cost is: $MC_2 = 60$

  $(120 - q_1) - 2q_2 = 60$ so $q_2 = 30 - q_1/2$

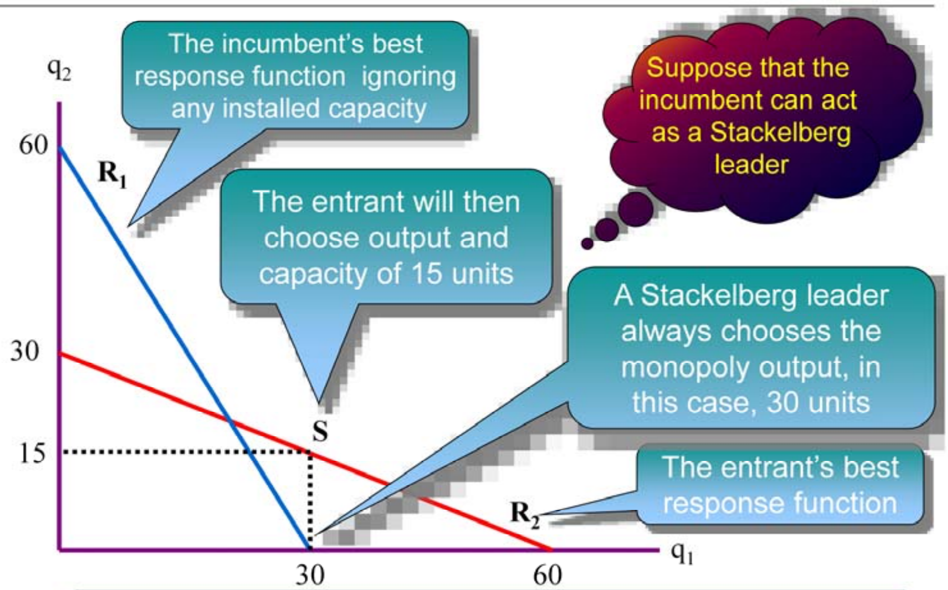
Equate marginal cost and marginal revenue

This is the entrant's best response function

- What about the incumbent?
- If we ignore any installed capacity it has a similar best response function
  $q_1 = 30 - q_2/2$
- What if the incumbent had a monopoly? ($q_2 = 0$)

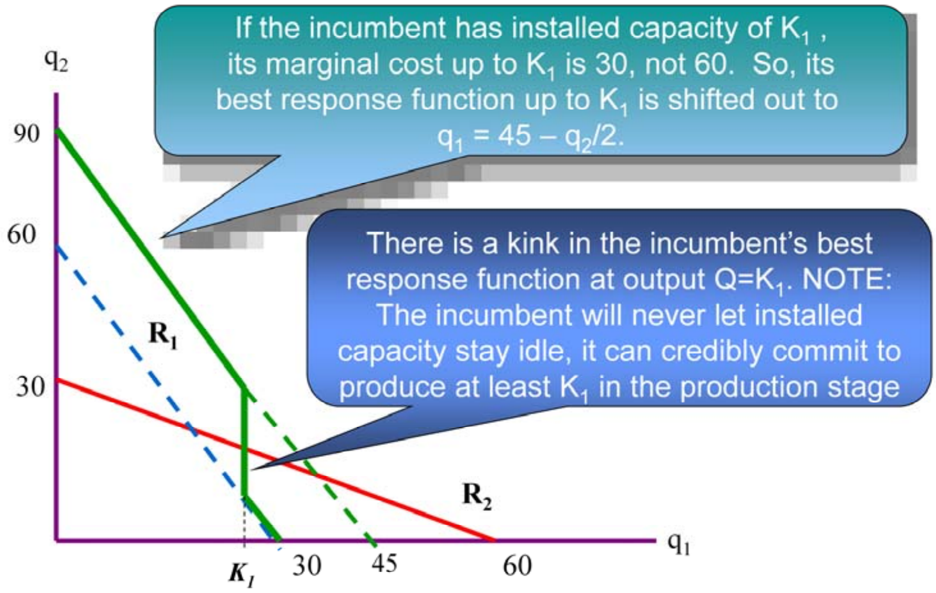
  Then with marginal costs of $60 it would produce 30 units.

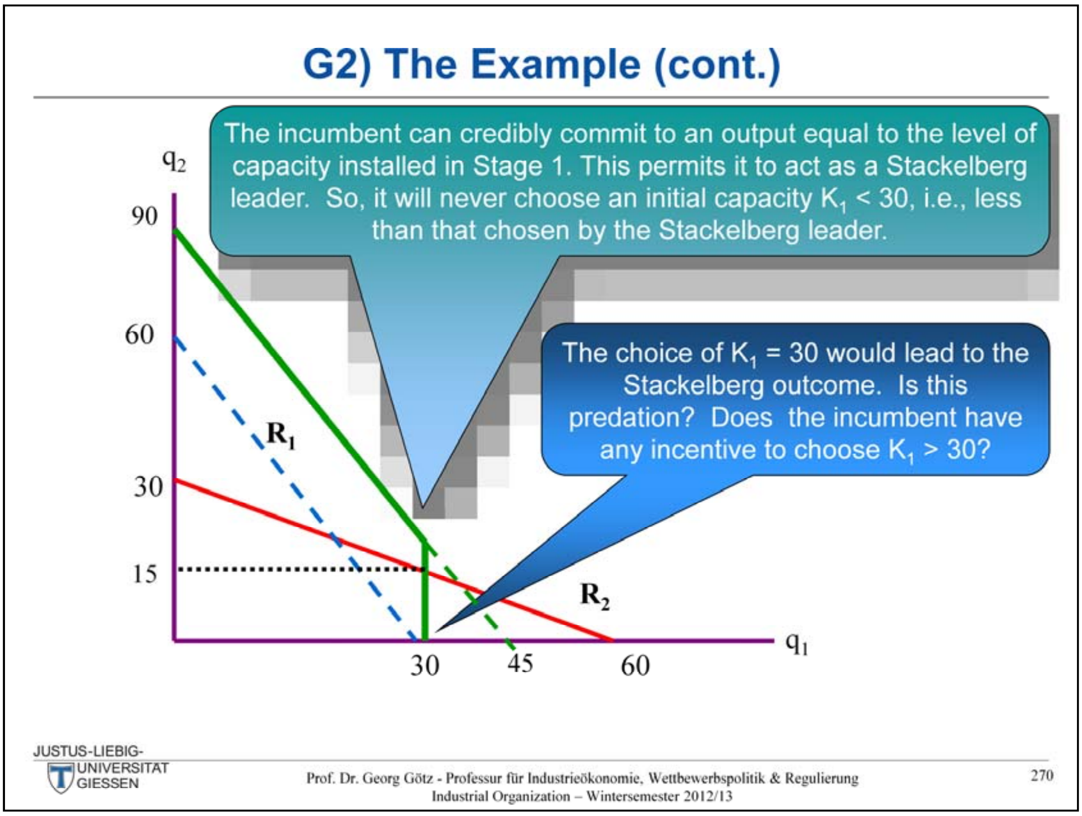
G2) The Example (cont.)



Here P=$120-$30-$15=$75. Therefore, the entrant earns a profit of ($75-$60)15= $225 less its fixed cost F in the Stackelberg equilibrium.

G2) The Example (cont.)




G2) The Example (cont.)
The incumbent can credibly commit to an output equal to the level of capacity installed in Stage 1. This permits it to act as a Stackelberg leader. So, it will never choose an initial capacity K1 < 30, i.e., less than that chosen by the Stackelberg leader.
The choice of K1 = 30 would lead to the Stackelberg outcome. Is this predation? Does the incumbent have any incentive to choose K1 > 30?
q2
90
60
30
15
R1
R2
q1
30
45
60
JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN
Prof. Dr. Georg Götz - Professur für Industrieökonomie, Wettbewerbspolitik & Regulierung
Industrial Organization – Wintersemester 2012/13
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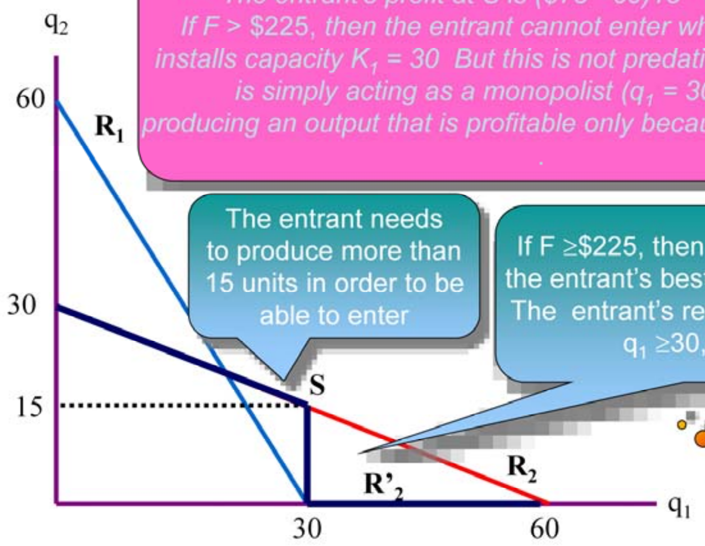
G2) The Example (cont.)

The entrant's profit at S is ($75 - 60)15 - F = $225 – F.
If F > $225, then the entrant cannot enter when the incumbent installs capacity $K_1 = 30$ But this is not predation. The incumbent is simply acting as a monopolist ($q_1 = 30$) . It is not producing an output that is profitable only because it prevents entry .

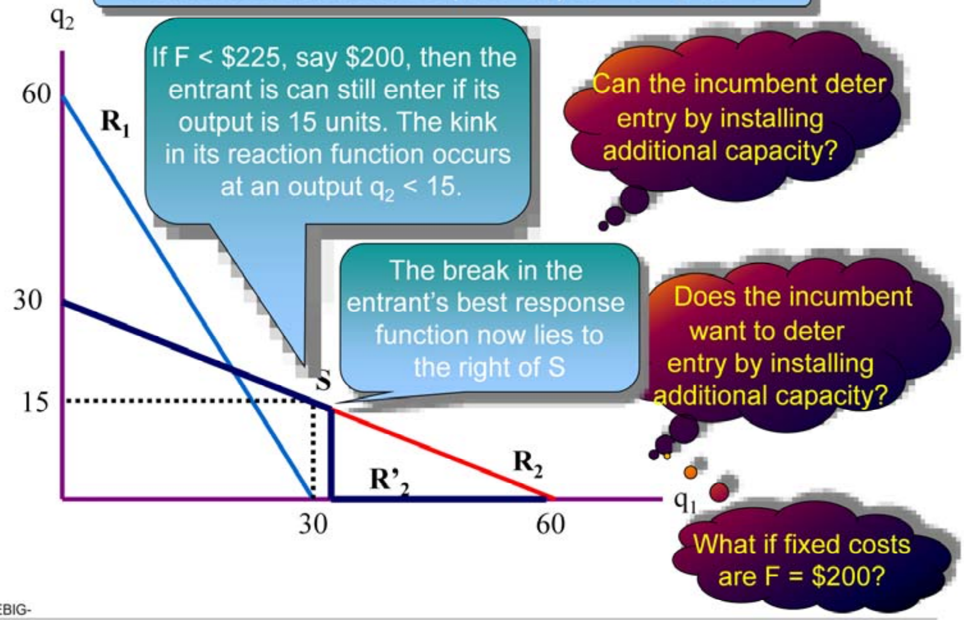


If F ≥$225, then there is a break in the entrant's bestresponse function. The entrant's response function for $q_1 \geq 30$, is $q_2 = 0$.

What if fixed costs are F > $225?

G2) The Example (cont.)

Profit of the entrant at S is ($75 - 60).15 - F = $225 - F



G2) The Example (cont.)

Up until the break-even point, the entrant's best response function is described by:

$q_2 = 30 - q_1/2$

So if the incumbent has capacity q_1 total output is

$Q = q_1 + q_2 = 30 + q_1/2$

Price is then $P = 120 - Q = 90 - q_1/2$

Profit of the entrant is $(P - 60)q_2 - F$

$= (30 - q_1/2)(30 - q_1/2) - 200$

$= 700 - 30q_1 + q_1^2/4$

Suppose $q_1 = 32$.

Then the entrant's profit would be

$700 - 960 + 256 = \$-4$

If the incumbent installs 32 units of capacity it will deter entry

What does q_1 have to be for this to be negative?

Does the incumbent want to do this?

G2) The Example (cont.)

Suppose that F = \$200 and that the incumbent accommodates entry.

Acting as a Stackelberg leader, the incumbent installs 30 units of capacity in the first period.

Profit to the incumbent in the production stage will then be:

(\$75 - \$60)x30 = \$450 *less* F = \$250. NOTE: the incumbent's cost per unit is also \$60 but \$30 of that is the per unit cost of installed capacity

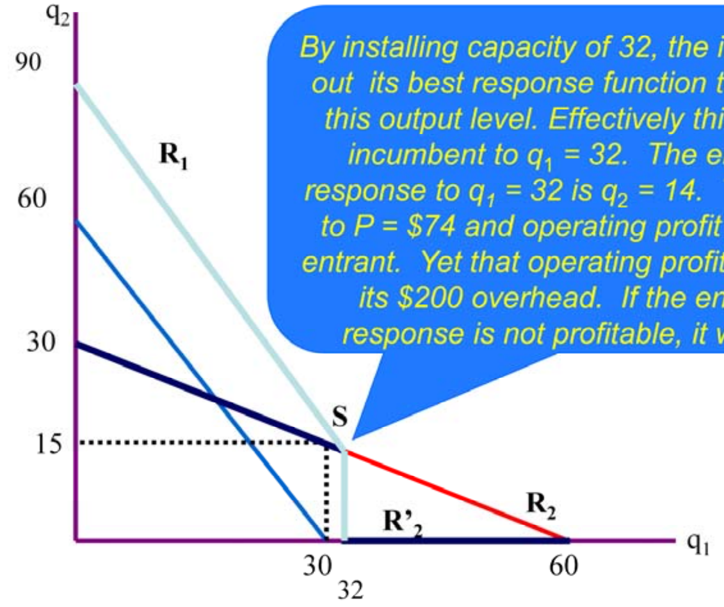
If the incumbent deters entry by installing 32 units, profit in the production stage is then:

(\$88 - \$60)x32 = \$896 *less* F = \$696

Entry deterrence by adding capacity is profitable in this case.

G2) The Example (cont.)

By installing capacity of 32, the incumbent shifts out its best response function to pass through this output level. Effectively this commits the incumbent to $q_1 = 32$. The entrant's best response to $q_1 = 32$ is $q_2 = 14$. This would lead to P = \$74 and operating profit of \$196 to the entrant. Yet that operating profit does not cover its \$200 overhead. If the entrant's best response is not profitable, it will not enter.



G2) The Example (cont.)

- Note that entry deterrence by adding capacity is not always profitable.
- Suppose that the entrant has no fixed costs.
- Then the incumbent would need to install 60 units of capacity to deter entry.
- Incumbent profit in the operating stage would be
  (\$60 - \$60)x60 – *F* = -\$200
- Now entry deterrence is not an attractive option.
- However, at a minimum, the incumbent can always act as a Stackelberg leader (q_1 = 30)

G2) Capacity Expansion and Entry Deterrence

- An example (**Stackelberg-Spence-Dixit** model):
- Three-stage game:
  1. Firm 1 (Incumbent/leader) chooses capacity K_1 at (capacity) cost c_0
  2. Firm 2 (Entrant/follower) chooses capacity K_2 at (capacity) cost c_0
  3. Firms compete (simultaneous) in quantities (q_1, q_2) with $q_i \leq K_i$. Production unit cost c. Firms can also add capacity at costs c_0
- Assumptions:
  1. Capacity investment is sunk.
  2. Firm 2 observes the leader's capacity choice
- Example:
  1. Inverse demand function: $p = 2 - q_1 - q_2$
  2. Costs: $c_0 = 1$, $c = 0$.

Capacity choice. Important: Credibility: Binding commitment.

Problem: neither costs of capacity nor costs of production

Reinterpret as Dixit model! Assume inverse demand p = 2 – k1-k2; Marginal costs c (inclusive capacity costs) = 1 (of course per unit). Marginal costs once investment in capacity is made equal to.

=> sub-game perfect: Incumbent invest in capacity in stage 1: Any capacity investment up to K1 = 1 is credible in the sense that it is optimal to fully utilize the capacity (Note that at K1 = 1; MR =0 =MC (given capacity)!

G2) Capacity Expansion and Entry Deterrence

- Solving backwards from stage 3: Different assumptions/cases
- Case 1: Firms didn't invest in capacity in stages 1 and 2

  $\Rightarrow$Standard Cournot problem (where capacity K = quantity q) with identical costs $c_0 + c$

  - $\pi^1(K_1,K_2) = (2 - K_1 - K_2)\, K_1 - 1\, K_1,$
  - $\pi^2(K_1,K_2) = K_2(1 - K_1 - K_2).$
  - Solution: $K_1 = K_2 = 1/3$, $p = 4/3$
  - Consequence: If firms choose $K_i \leq 1/3$ in stage 1 and 2, this will be the result.

Capacity choice. Important: Credibility: Binding commitment.

Problem: neither costs of capacity nor costs of production

Reinterpret as Dixit model! Assume inverse demand p = 2 – k1-k2; Marginal costs c (inclusive capacity costs) = 1 (of course per unit). Marginal costs once investment in capacity is made equal to.

=> sub-game perfect: Incumbent invest in capacity in stage 1: Any capacity investment up to K1 = 1 is credible in the sense that it is optimal to fully utilize the capacity (Note that at K1 = 1; MR =0 =MC (given capacity)!

G2) Capacity Expansion and Entry Deterrence

- Case 2: Firm 1 has "large" (unlimited) capacity from stage 1; firm 2 would not invest stage 2 (but in 3)
  - ⇒Standard Cournot problem with asymmetric costs: Firm 1's cost c (=0), firm 2's costs $c_0 + c$ (=1)
  - $\pi^1(q_1,K_2) = (2 - q_1 - K_2)\, q_1$,
  - $\pi^2(K_1,K_2) = K_2(1 - q_1 - K_2)$.
  - Solution: $q_1 = 1$, $K_2 = 0$, $p = 1$
  - Consequence: Since firm 1 wouldn't make any profit, it would not choose a capacity in stage 1 which is 1 or higher. Firm 1 will choose a capacity between 1/3 and 1!

Capacity choice. Important: Credibility: Binding commitment.

Problem: neither costs of capacity nor costs of production

Reinterpret as Dixit model! Assume inverse demand p = 2 – k1-k2; Marginal costs c (inclusive capacity costs) = 1 (of course per unit). Marginal costs once investment in capacity is made equal to.

=> sub-game perfect: Incumbent invest in capacity in stage 1: Any capacity investment up to K1 = 1 is credible in the sense that it is optimal to fully utilize the capacity (Note that at K1 = 1; MR =0 =MC (given capacity)!

G2) Capacity Expansion and Entry Deterrence

- Case 3: Firm 1 choose $K_1 \in [1/3,1]$
  - $\Rightarrow$Since firm 1 has marginal costs $c = 0$ in stage 3 and the optimal solution would be $q_1 = 1$, it is capacity constrained and chooses $q_1 = K_1$.
  - $\Rightarrow$Firm 2 knows that. Its optimal reaction (=reaction function) is: $K_2 = [1 - K_1]/2$
  - $\Rightarrow$ Stackelberg game!
- Stackelberg:
- $\max \pi^1(K_1) = K_1(1 - K_1 - [1 - K_1]/2) \Rightarrow$
- $q_1 = K_1 = 1/2$, $q_2 = K_2 = 1/4$, $p = 5/4$,
- $\pi^1 = 1/8$, $\pi^2 = 1/16$.

Capacity choice. Important: Credibility: Binding commitment.

Problem: neither costs of capacity nor costs of production

Reinterpret as Dixit model! Assume inverse demand p = 2 – k1-k2; Marginal costs c (inclusive capacity costs) = 1 (of course per unit). Marginal costs once investment in capacity is made equal to.

=> sub-game perfect: Incumbent invest in capacity in stage 1: Any capacity investment up to K1 = 1 is credible in the sense that it is optimal to fully utilize the capacity (Note that at K1 = 1; MR =0 =MC (given capacity)!

G2) Capacity Expansion and Entry Deterrence

- What happens if entrant/ follower incurs fixed costs f?
- f > 1/16 => entry blockaded => monopoly!
- f < 1/16: Incentive to deter entry by investing in capacity: K_1^d

Capacity choice. Important: Credibility: Binding commitment.

Problem: neither costs of capacity nor costs of production

Reinterpret as Dixit model! Assume inverse demand p = 2 – k1-k2; Marginal costs c (inclusive capacity costs) = 1 (of course per unit). Marginal costs once investment in capacity is made equal to.

=> sub-game perfect: Incumbent invest in capacity in stage 1: Any capacity investment up to K1 = 1 is credible in the sense that it is optimal to fully utilize the capacity (Note that at K1 = 1; MR =0 =MC (given capacity)!

G2) Capacity Expansion and Entry Deterrence

- Level of K_1^d which just prevents the follower from breaking even:
- $\pi^2(K_1^d, K_2 = [1 - K_1^d]/2\ ) = 0.$

$\Leftrightarrow \{[1 - K_1^d]/2\}[1 - K_1^d - [1 - K_1^d\ ]/2] = f$

$\Leftrightarrow\ [1 - K_1^d]^2 = 4f$ $f < 1/16$

$\Rightarrow K_1^d = 1 - 2f^{1/2} \qquad > 1/2.$

$\Rightarrow$ The incumbent‘s profit from investing in entry deterrence: (monopolist!)

- $\pi^1(K_1^d) = [1 - 2f^{1/2}][1 - 1 + 2f^{1/2}]$
- Deterrence (accomodation) $\Leftrightarrow \pi^1(K_1^d) \overset{(<)}{>} 1/8 = \pi^{1SB}$

$\Leftrightarrow f \overset{(<)}{>} (3 - 2\sqrt{2})/32 = .0054 \Leftrightarrow K_1^d \overset{(>)}{<} ½ + 1/(2\sqrt{2}) = .853$

Capacity choice.

G2) Preemption as predation

- A distinct but related issue is an incumbent investing early to prevent new entry
- Now we have an issue of *timing*
- Is it in the interests of an incumbent to preempt by
  - building new plants prior to a rival's entry
  - adding new products prior to a rival's entry
- Related to another issue
  - incumbent may race to innovate to preempt entry
- General: Incumbent's incentive to expand in 1st period/to innovate exceeds that of the entrant's. The incumbent is fighting to hang on to monopoly profit while the best that entrant can hope for is the Cournot profit of a typical industry firm