F1) The Cournot Model: Advanced Topic

$p = p(q_1 + q_2)$, where $p' < 0$.

The firms maximise

$\max \pi_2(q_1,q_2) = q_2[p(q_1 + q_2) - c]$,

$\Rightarrow$ FOC firm 2

$F^2(q_1,q_2) := p(q_1 + q_2) + q_2 p'(q_1 + q_2) - c = 0$

SOC: $$\frac{\partial F(q_1,q_2)}{\partial q_2} = p'(q_1+q_2) + p'(q_1+q_2) + q_2 p''(q_1+q_2) < 0$$

Slope of the reaction function via implicit function theorem:

Implicit function $F^2(q_1,q_2)$:

$$\frac{dq_2}{dq_1} = -\frac{\partial F^2(.)/\partial q_1}{\partial F^2(.)/\partial q_2} = -\frac{p'(q_1+q_2) + q_2 p''(q_1+q_2)}{2p'(q_1+q_2) + q_2 p''(q_1+q_2)}$$

Advanced topic: Derivation of second order condition gives assumption on curvature of demand function (and in general also on cost function). Must not be too concave in order to be globally satisified. If this assumption is satisified, the slope of the reaction functions derived via the implicit function theorem is negative.

Check first for linear demand: p“ = 0.

Check for which values of the elasticity epsilon in the isolelastic demand function q =A p^(-epsilon) the SOC is not globally satisfied.

In this case the reaction functions are not always downward sloping, see next page.

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- Reaction functions for the demand function: $Q = 100/P$