

## Gliederung der Vorlesung

- |  |  |
|--|--|
| <p>A) Introduction</p> <p>B) Competition and Monopoly</p> <p>C) Technology and Cost; Industry Structure</p> <p>D) Price Discrimination and Monopoly</p> <p>E) Product Variety and Quality under Monopoly</p> | <p>F) Static Games</p> <p>G) Dynamic Games, First and Second Movers</p> <div style="border: 1px solid black; background-color: #e0f2f1; padding: 5px; margin: 5px 0;"> <p>H) Horizontal Product Differentiation</p> <p>1) Price Competition &amp; Product Choice</p> <p>2) Entry &amp; Optimum Product Variety ▶</p> <p>3) Love of Variety Approach</p> </div> <p>I) Vertical Product Differentiation</p> <p>J) Advertising</p> <p>K) Research &amp; Development</p> |
|--|--|

Problem with PRN, Quantitative Analysis: This topic seems to be spread over the whole book. Shubik, Levitan: p. 301, Section 3.5.1;; Salop; p. 304, Section 11.5.2, Hotelling: p. 160, Section 7.4.2

## H1) An Example of Product Differentiation

Coke and Pepsi are nearly identical but not quite.  
As a result, the lowest priced product does not win the entire market.



$$Q_C = 63.42 - 3.98P_C + 2.25P_P$$

$$MC_C = \$4.96$$



$$Q_P = 49.52 - 5.48P_P + 1.40P_C$$

$$MC_P = \$3.96$$

There are at least two methods for solving this for  $P_C$  and  $P_P$

Easiest example of product differentiation! Degree of product differentiation exogenous. The two goods are substitutes, but not perfect ones. Changes in prices by one firm affect the other one, but does not lead to a tilting of the market

See Preis und Wettbewerb VIII for more details with linear demand for differentiated products

## H1) Bertrand and Product Differentiation

### Method 1: Calculus

Profit of Coke:  $\pi_C = (P_C - 4.96)(63.42 - 3.98P_C + 2.25P_P)$

Profit of Pepsi:  $\pi_P = (P_P - 3.96)(49.52 - 5.48P_P + 1.40P_C)$

Differentiate with respect to  $P_C$  and  $P_P$  respectively

### Method 2: MR = MC

Reorganize the demand functions

$$P_C = (15.93 + 0.57P_P) - 0.25Q_C$$

$$P_P = (9.04 + 0.26P_C) - 0.18Q_P$$

Calculate marginal revenue, equate to marginal cost, solve for  $Q_C$  and  $Q_P$  and substitute in the demand functions

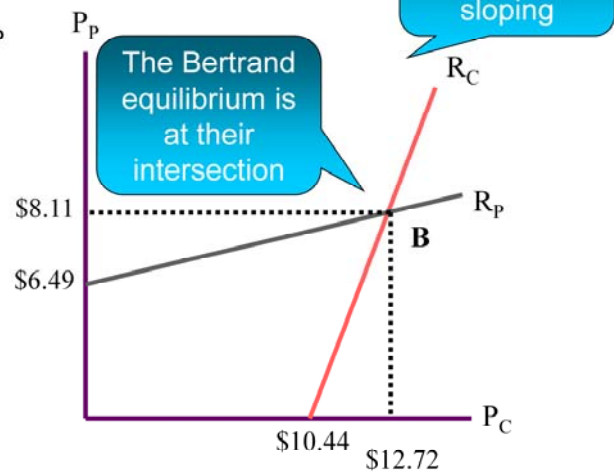
## H1) Bertrand competition and product differentiation

Both methods give the best response functions:

$$P_C = 10.44 + 0.2826P_P$$

$$P_P = 6.49 + 0.1277P_C$$

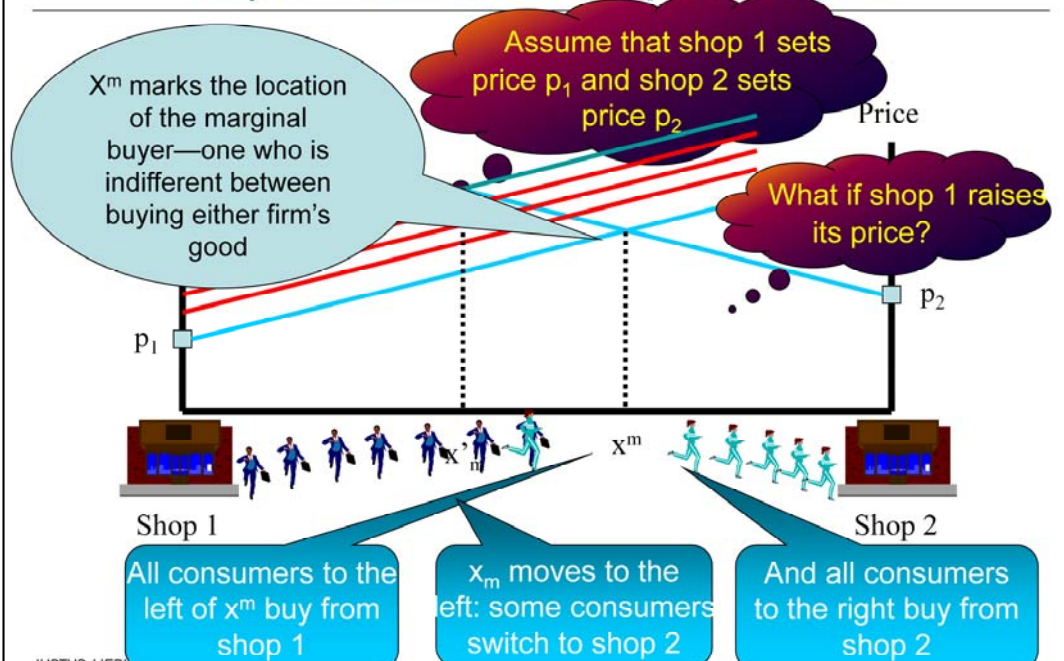
These can be solved for the equilibrium prices as indicated



## H1) Bertrand Competition and the Spatial Model

- An alternative approach is to use the spatial model from Chapter 4
  - a Main Street over which consumers are distributed
  - supplied by two shops located at opposite ends of the street
  - but now the shops are competitors
  - each consumer buys exactly one unit of the good provided that its full price is less than  $V$
  - a consumer buys from the shop offering the lower full price
  - consumers incur transport costs of  $t$  per unit distance in travelling to a shop
- ⇒ (Indirect) Utility of consumer  $i$  located at  $x^i$  and buying at a shop located at  $x_k$  charging price  $p_k$ :
$$U_k^i = V - t |x^i - x_k| - p_k$$
- What prices will the two shops charge?

## H1) Bertrand and the spatial model



## H1) Bertrand and the spatial model

$$V - t |x^m - 0| - p_1 = V - t |x^m - 1| - p_2$$

$$p_1 + tx^m = p_2 + t(1 - x^m)$$

$$\therefore 2tx^m = p_2 - p_1 + t$$

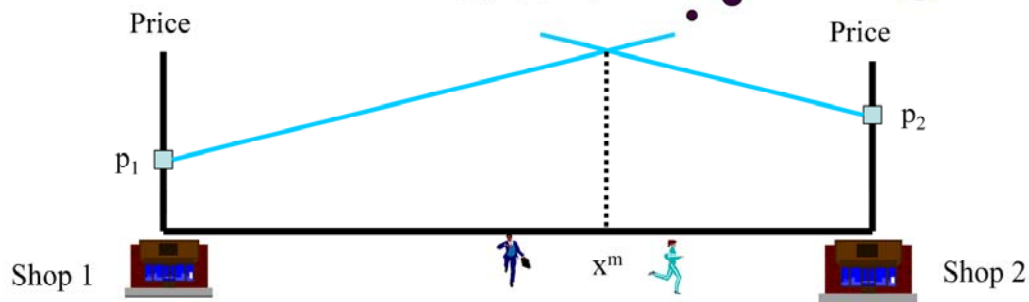
$$\therefore x^m(p_1, p_2) = (p_2 - p_1 + t)/2t$$

There are  $N$  consumers in total

So demand to firm 1 is  $D^1 = N(p_2 - p_1 + t)/2t$

This is the fraction of consumers who buy from firm 1

How is  $x^m$  determined?



Indifferent consumers  $\Leftrightarrow$  Utilities received by buying from either shop are identical  $\Leftrightarrow$  delivered prices (= price + transport costs) are identical

If prices are identical, market is shared equally!

## H1) Bertrand equilibrium

Profit to firm 1 is  $\pi_1 = (p_1 - c)D^1 = N(p_1 - c)(p_2 - p_1 + t)/2t$

$$\pi_1 = N(p_2 p_1 - p_1^2 + t p_1 + c p_1 - c p_2 - c t)/2t$$

Differentiate with respect to  $p_1$

$$\partial \pi_1 / \partial p_1 = \frac{N}{2t} (p_2 - 2p_1 + t + c) = 0$$

*Solve this  
for  $p_1$*

$$p_1^* = (p_2 + t + c)/2$$

*This is the best response  
function for firm 1*

What about firm 2? By symmetry, it has

a similar best response function.

$$p_2^* = (p_1 + t + c)/2$$

*This is the best  
response function for  
firm 2*



## H1) Bertrand and Demand

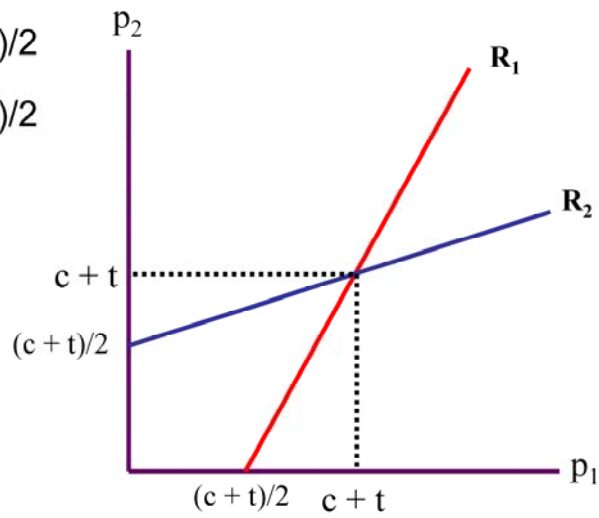
$$p_1^* = (p_2 + t + c)/2$$

$$p_2^* = (p_1 + t + c)/2$$

$$\therefore p_2^* = t + c$$

$$\therefore p_1^* = t + c$$

Symmetric equilibrium:  
Each firm has a 50%  
market share



Firms charge a constant absolute markup depending on  
transportation costs. Markup and profits increasing in  $t$ !

## H1) Product Choice

---

- In the above examples product differentiation was exogenous
- In general firms can influence the degree of product differentiation by the choice of product characteristics
- Firms typically take actions in order to differentiate their products from those of their rivals (e.g. advertising, product design)
  - ⇒ Reduces competition!
- Formal analysis of product choice in the spatial model:  
Which locations will firms choose?
  - ⇒ Maximum or minimum product differentiation

⇒ Maximum or minimum product differentiation: Firms located at endpoints or at center of market!

## H1) Location choice and pricing in the linear city

---

- Two firms, each operating one shop
- Two stage game
  1. Firms decide on the location of their shop
  2. Firms set prices (given the locations)
- Solution: Recursive:
  - Determine prices given locations
  - Substitute into profit function
  - Determine optimum locations

## H1) Location choice and pricing in the linear city

- Problem: For the spatial model with linear transport costs an equilibrium for this game does not exist!
  - Reason: If the rival is located sufficiently far away from the endpoint, firm has an incentive to undercut
  - ⇒ Undercutting wins the whole market, but cannot be an equilibrium!
  - ⇒ Given the 'equilibrium' prices (without taking account of undercutting), the firms have always an incentive to move closer to the center

Equilibrium prices if firms are located at 'sufficient' distance from each other

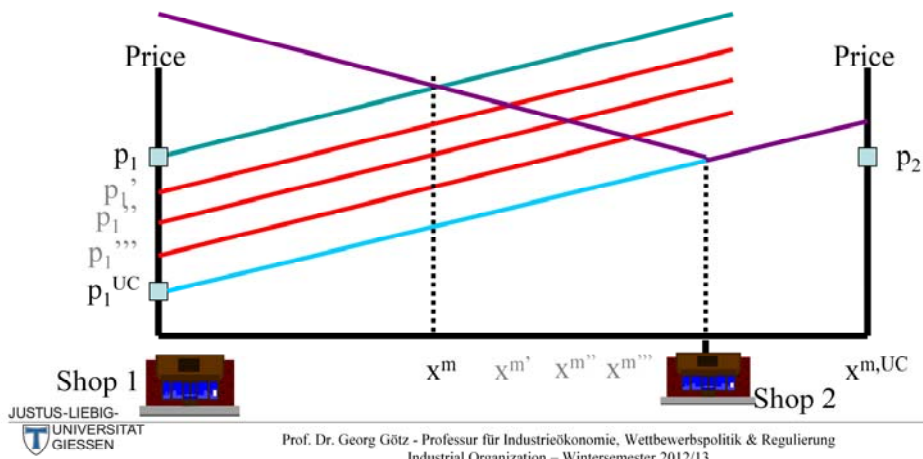
$$p_1 = c + \frac{1}{3} t (2 + x_1 + x_2), \quad p_2 = c + \frac{1}{3} t (4 - x_1 - x_2)$$

Derive reduced profit function (by inserting the equilibrium prices into the profit function) and differentiate wrt location.

⇒ It is optimal to move closer to the center!

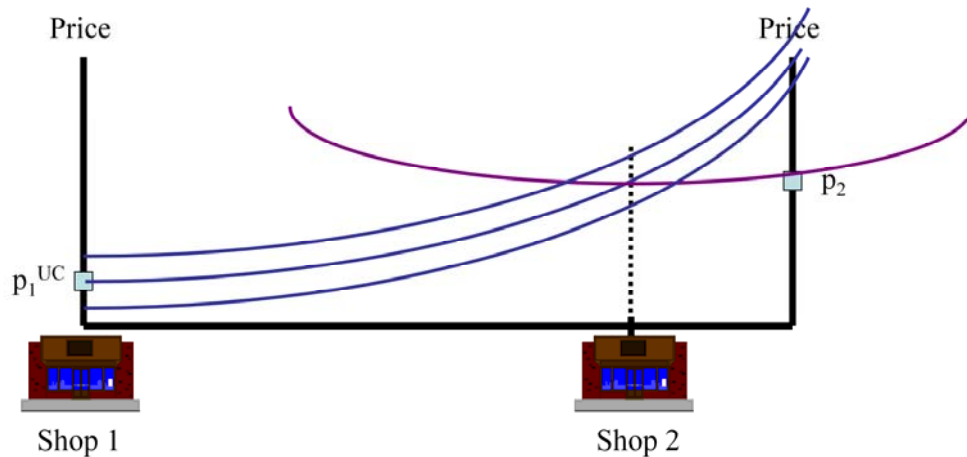
## H1) Location choice and pricing in the linear city

- Undercutting so that the consumer located at the location of the rival firm switches to our firm, leads all consumers in the backyard of the rival firm switch.
  - ⇒ Small change in price
  - ⇒ jump in demand (and profit)
- Reason for non-existence with linear transport costs:



## H1) Location choice and pricing in the linear city Quadratic Transport Costs

- Solution to non-existence problem: Transport cost increase more than proportionally with distance (convex, e.g. quadratic)
- ⇒ Small price reduction always leads to a small increase in demand



Question how transport costs look like is an empirical one.

## H1) Location choice and pricing in the linear city Quadratic Transport Costs

- Utility with quadratic transport cost:
- $U_k^i = V - t(x^i - x_k)^2 - p_k$
- Location of shops:  $x_1$  and  $x_2$  with  $x_1 < x_2$
- Solution of second stage price game:
- Indifferent consumer  $x^m$
- $V - t(x^m - x_1)^2 - p_1 = V - t(x^m - x_2)^2 - p_2$
- $\Rightarrow p_1 + t(x^m - x_1)^2 = p_2 + t(x^m - x_2)^2$

$$\Rightarrow x^m(p_1, p_2) = \frac{(x_1 + x_2)}{2} + \frac{p_2 - p_1}{2t(x_2 - x_1)}$$

Demand firm 1:  $x^m N$

Demand firm 2:  $(1 - x^m) N$

Question how transport costs look like is an empirical one.

## H1) Location choice and pricing in the linear city Quadratic Transport Costs

- Profit of firm 1 (given locations  $x_1$  and  $x_2$  as well as  $p_2$ ):

$$\Rightarrow \Pi_1(p_1, p_2, x_1, x_2) = (p_1 - c) N \left[ \frac{(x_1 + x_2)}{2} + \frac{p_2 - p_1}{2t(x_2 - x_1)} \right]$$

Analogous expression for firm 2.

Solution of second stage price game:

$\Rightarrow$  Differentiate both profits wrt (own) prices:

$$\text{FOCs } (\partial \Pi_i / \partial p_i = 0)$$

$\Rightarrow$  Solve for  $p_1$  and  $p_2$

$$\Rightarrow p_1^* = c + \frac{t(2 + x_1 + x_2)(x_2 - x_1)}{3}, \quad p_2^* = c + \frac{t(4 - x_1 - x_2)(x_2 - x_1)}{3}$$

Equilibrium prices: = marginal costs for identical locations.

Decrease (increase) if the distance between the shops decreases (increases)

$\Rightarrow$  More differentiated products are sold at higher prices!



## H1) Location choice and pricing in the linear city Quadratic Transport Costs

- Substituting the equilibrium prices into the profit functions (and manipulating) yields the reduced profit functions  $\Pi_i^*$

⇒ Profits solely as a function of locations

$$\Rightarrow \Pi_1^*(x_1, x_2) = \frac{t(2 + x_1 + x_2)^2(x_2 - x_1)}{18}$$

$$\Rightarrow \Pi_2^*(x_1, x_2) = \frac{t(4 - x_1 - x_2)^2(x_2 - x_1)}{18}$$

Choice of locations: Differentiate reduced profit function wrt own location (given location of rival)

$$\Rightarrow \partial \Pi_1^* / \partial x_1 < 0, \partial \Pi_2^* / \partial x_2 > 0$$

$$\partial \Pi_1^* / \partial x_1 = -\frac{1}{18} t (2 + 3x_1 - x_2) (2 + x_1 + x_2)$$

$$\partial \Pi_2^* / \partial x_2 = -\frac{1}{18} t (4 + x_1 - 3x_2) (-4 + x_1 + x_2)$$

⇒ Maximum product differentiation: Each firm has an incentive to move to the endpoint of the market (irresp. of what rival does)

Explanations: next slide

## H1) Location choice and pricing in the linear city Quadratic Transport Costs

- Max. product differentiation is equilibrium in dominant strategies!
- Equilibrium prices:  $p_i = c + t$ .
- If one allows locations outside the market area, firms would locate at  $1/4, 5/4$ .
- Intuition for maximum product differentiation result:
- Incentive to move closer towards rival  $\Rightarrow$  Market share gain (direct effect: given prices moving closer towards rival increases demand).
- However: Negative indirect effect of moving closer towards rival: Price competition intensifies, prices decrease!
- Indirect effect is stronger than direct one!
- If indirect effect were absent (no price competition eg due to regulation)  $\Rightarrow$  agglomeration: Both firms choose the same location!
- Socially optimal locations:  $1/4, 3/4$ .

In connection with the model with linear transport costs, the economics behind the maximum product differentiation result become obvious.

Importance of transport costs parameter  $t$ . Determines the possible degree of product differentiation

Linear transport costs: Incentive to move closer towards rival  $\Rightarrow$  Market share gain (direct effect: given prices moving closer towards rival increases demand).

However (quadratic transport costs): Negative indirect effect of moving closer towards rival: Price competition intensifies, prices decrease!

Indirect effect is stronger than direct one!

If indirect effect were absent (no price competition eg due to regulation)  $\Rightarrow$  agglomeration: Both firms choose the same location!

## H2) Entry in a spatial model: The circular city

- Question: What happens if we allow for entry (by one product/shop firms) in our spatial model?
- Is the amount of entry socially optimal, or will there be under- or over-entry?
  - ⇒ Should governments regulate entry of say shopping centers, cinemas, etc.?
- Framework to address the problem: Slightly modified spatial model: Circular city instead of linear street
  - ⇒ No endpoint problems!
  - ⇒ Circumference: 1 (⇒ length as with linear city)
- ⇒ Entry: Firms face fixed entry costs  $f$ .
- ⇒ Linear transport costs ( $t$ ), constant marginal costs  $c$

See piece of news in the Standard on shopping centers etc.

Remember: Monopolist offers too much product variety in the spacial model!

Circular city: Salop Bell J 1979

## H2) Entry in a spatial model: The circular city

- Two stage game:
- first stage: potential producers invest  $f$  or 0. Suppose  $n$  enter.
  - ⇒ 'exogenous' choice of location: equal spacing.
  - ⇒ Distance between two adjacent firms:  $1/n$ .
- Second stage: Bertrand-competition: Firms choose prices
- Demand for firm 2 given firm 1 and firm 3 charge the same price  $p$ :
  - ⇒ Indifferent consumer located at distance  $x^m$  from firm 2
  - ⇒  $p_1 + t(1/n - x^m) = p_2 + tx^m = p_3 + t(1/n - x^m)$
  - ⇒  $x^m(p_2, p) = (p - p_2 + t/n)/(2t)$
  - ⇒  $D(p_2, p) = N(p - p_2 + t/n)/t$  Demand function firm 2

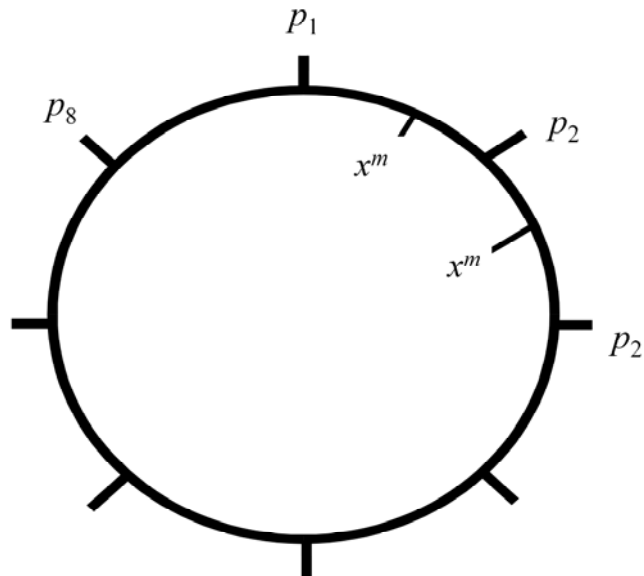
The location pattern exhibits maximum product differentiation. In the circular model there is no incentive to move closer to one (say right neighbor) rival even though there are linear transport costs. Simple reason: Moving to the right neighbor leads to a loss of market share to the left neighbor.

Assumption underlying equal spacing: Costless relocation of firms: Firms decide on locations only after the number of firms is determined.

We are interested in symmetric solutions. Therefore assume that rivals charge the same price!

Remember:  $N$  number (density) of consumers

## H2) Entry in a spatial model: The circular city



## H2) The circular city: Equilibrium prices and profits

- Profit of firm  $i$  (given all other firms charge price  $p$ ):  
 $\Rightarrow \Pi_i(p_i, p, n) = (p_i - c) N (p - p_i + t/n)/t - f$
- FOC: Differentiating wrt  $p_i$ :
- $\partial \Pi_i / \partial p_i = N (p - p_i + t/n)/t + N(p_i - c)/t = 0$   
 $\Rightarrow p_i = (p + c + t/n)/2$  (reaction function)
- Symmetric solution with  $p_i = p$ :  
 $\Rightarrow p^*(n) = c + t/n$
- Reduced profit function (depending on number of firms)  
 $\Rightarrow \Pi(n) = N t/n^2 - f$

## H2) The circular city: Entry in market system vs. socially optimal entry

- Free entry number of firms :  $\Pi(n^*) = 0$

(Ignoring integer constraint)

$$\Rightarrow \Pi(n^*) = N t / n^{*2} - f = 0 \quad \Rightarrow \quad n^* = \sqrt{\frac{N t}{f}}$$

- Socially optimal number of firms: Minimize total costs  $C$   
(Ignoring integer constraint)

$$\Rightarrow C(N, n) = N t / (4n) + n f + N c$$

$$\Rightarrow \partial C(N, n) / \partial n = - N t / (4n^2) + f = 0$$

$$\Rightarrow n^{SO} = \frac{1}{2} \sqrt{\frac{N t}{f}} = \frac{n^*}{2}$$

Market yields **excess supply of variants!**  
Business stealing effect!

Social optimum: Since all consumers buy one unit anyway, the only problem is to minimize the costs to provide the respective output. An important part of costs are of course transport costs.

Only difference to the case dealt with above (Product variety offered by monopolist (see p. 156)) is that in the above variable costs are not taken into account; they do not matter anyway! (This holds as long as the whole market is covered anyway)

Welfare result due to the fact that further entrant does not take into account that part of her profit is due to “business stealing”: Incumbents profits decrease. From a social point this is now gain, just redistribution. Incentive for too much entry.

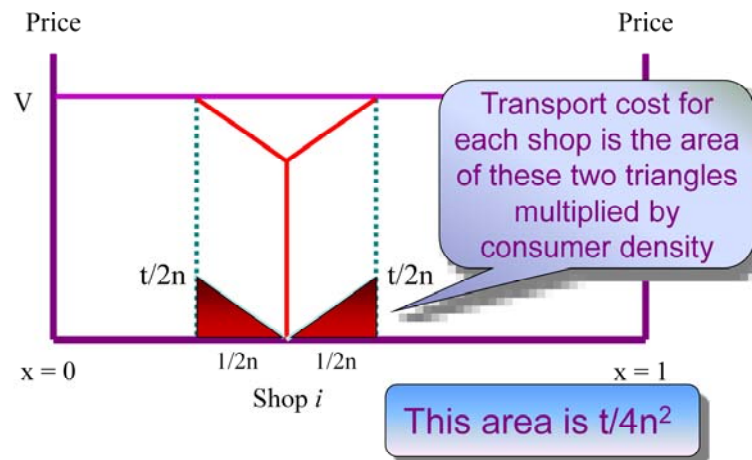
Similar effect in Cournot model when only second best solution is considered (Second best: government regulates entry, but firms are free to make production decisions as they like). Over-entry result.

However, there are also models in which the market undersupplies variety (Monopolistic competition). In particular if entry is innovative, it is hard to imagine that the so called consumer surplus effect (the gain for the consumers by having access to a new product at a price which is *below* their reservation price) could not compensate the business stealing effect (the later also called profit destruction effect). The consumer surplus effect is due to the fact that innovators/entrants typically cannot appropriate all gains they create.

Also result for Salop model may change

- (i) with different transport cost function  $t(x)$ ;
- (ii) with unequal distribution of consumers' ideal variants;
- (iii) no price competition.

## H2) Shortened reproduction of p. 156, 157 Social optimum (cont.)

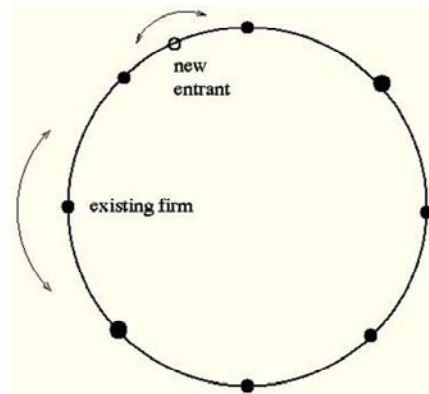


Total cost with  $n$  shops is, therefore:  $C(N, n) = n(t/4n^2)N + n.F$   
 $= tN/4n + n.F$



## Brand proliferation and entry deterrence

- Schmalensee (Bell JE 1978): „Entry Deterrence in the Ready-to-Eat Breakfast Cereal Industry”



Richard Schmalensee, 1978. "Entry Deterrence in the Ready-to-Eat Breakfast Cereal Industry," [Bell Journal of Economics](#), The RAND Corporation, vol. 9(2), pages 305-327, Autumn

Source of the notes: <http://www.econ.hku.hk/~wsuen/teaching/io/schmalensee.html>

Basic assumptions:

increasing returns

localized rivalry

relative immobility of product location

There is an asymmetry between existing and new firms in a model of localized rivalry without product relocation.

Suppose there are  $N$  existing brands in the industry. Each brand will have a market area of  $1/N$ . For the new brand located between two existing brands, the market area is only  $1/(2N)$ .

Let  $y(N)$  be the profit function of the existing brands, and let  $y^e(N)$  be the profit function of the entrant brand when there are  $N$  existing firms in the industry. Then  $y^e(N) = y(2N)$ .

We expect  $y'(N) < 0$ . Let  $N^0$  be the number of brands such that  $y(N^0) = 0$ .

The existing firms can engage in a *brand proliferation* policy in which they choose to make  $N$  brands such that  $N^0/2 < N < N^0$ .

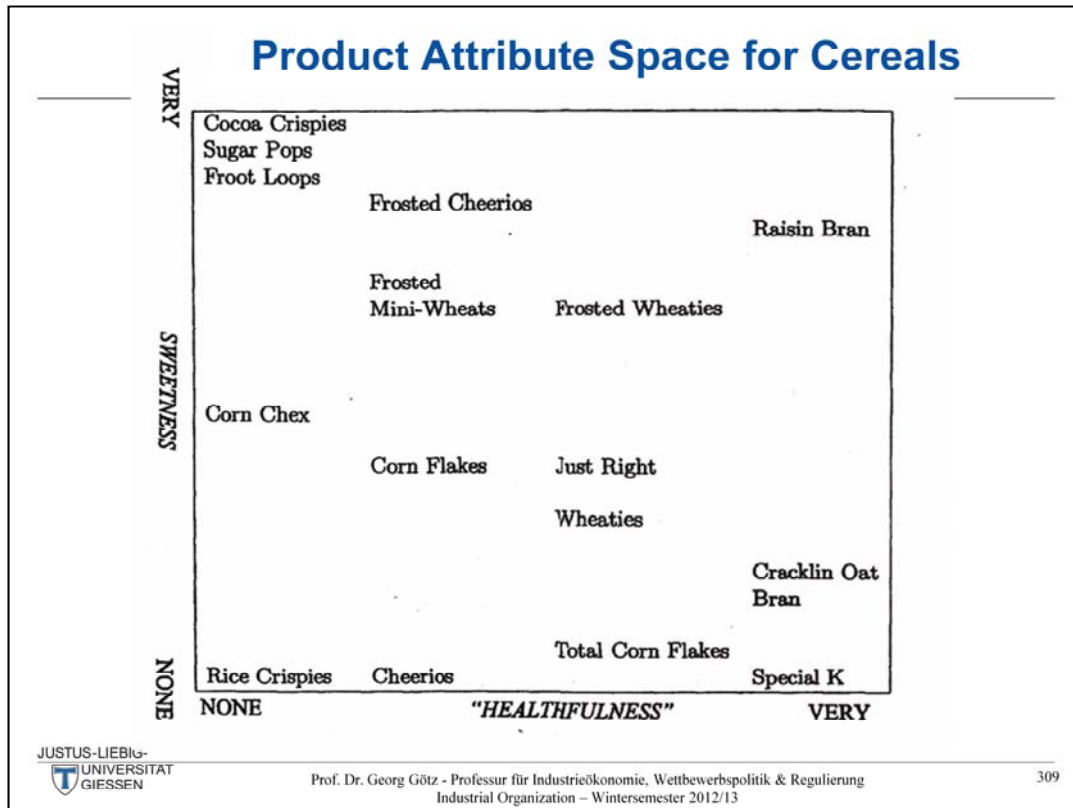
By adopting this strategy, the existing firms can deter entry and yet can make positive profits themselves because  $y(N) > y(N^0) = 0$

$y^e(N) = y(2N) < y(N^0) = 0$

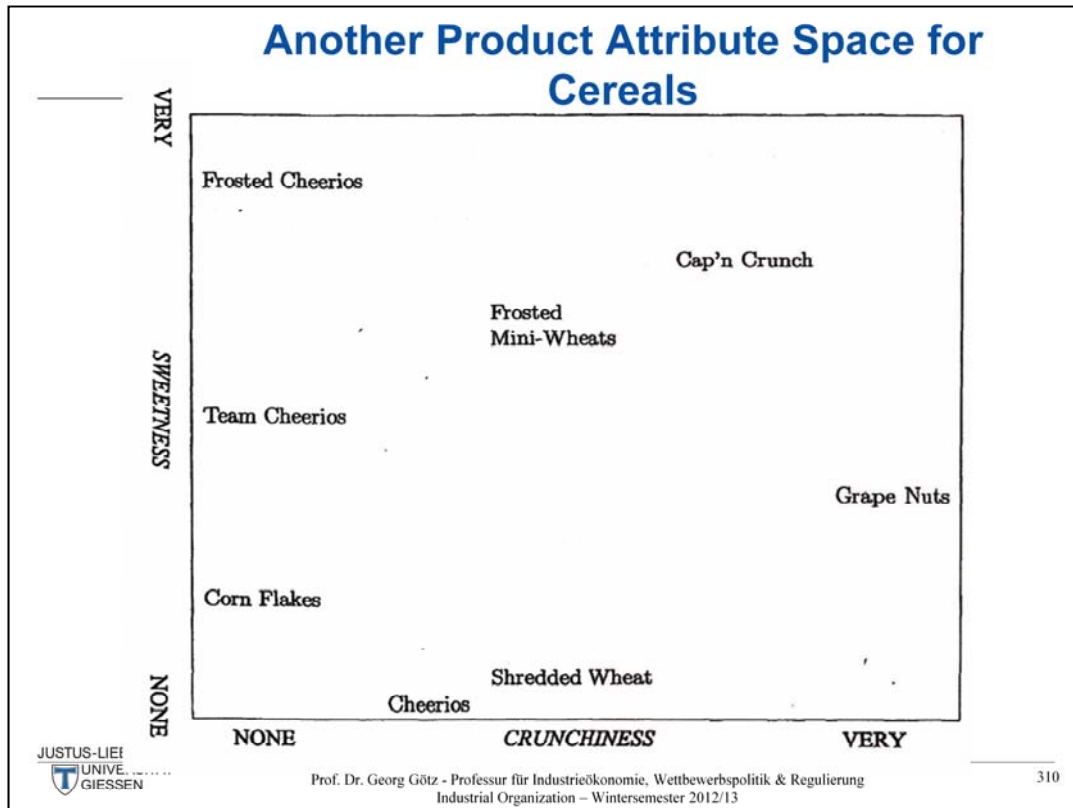
Entry deterrence through brand proliferation is more *credible* than other strategies such as limit pricing or aggressive advertising. For these other strategies, the existing firm's incentives will change once entry actually takes place. For brand proliferation, since product location is immobile and fixed costs are large, there is little that the existing firm can do to accommodate entry.

Suppose the entrant duplicate an existing brand's location instead of locating midway between two brands. Then the entrant will be in direct competition with the existing brand; and if it wipes out the existing brand, it will capture the entire  $1/N$  of the market. However, because brand proliferation implies a fairly large  $N$ , this new entrant will still suffer from a cost disadvantage compare to an existing firm that makes several brands (scale and scope economies).

The model depends on the location model in which rivalry is localized. In the cereals market, there are more than one product dimension. One brand's "neighbor" on one dimension may be different from its "neighbor" on another dimension. With many dimension, the number of these "neighbors" (i.e., direct competitors) can be very large. In the extreme, the location model with many dimensions will be just like a representative consumer model in which every brand is equally substitutable with every other brand. In this case, the asymmetry between existing and new brands will disappear.



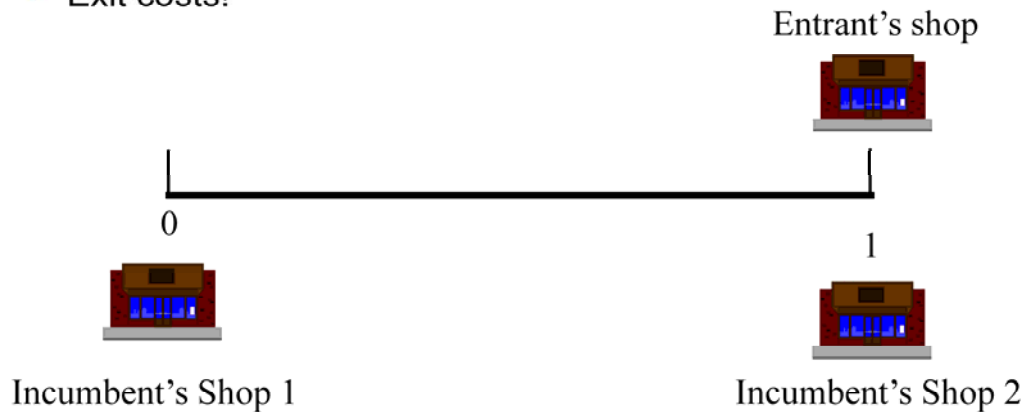
Taken from Lecture Notes on Bundling and Brand Proliferation by Professor Robert S. Pindyck ([http://web.mit.edu/rpindyck/www/Courses/BBP\\_11.pdf](http://web.mit.edu/rpindyck/www/Courses/BBP_11.pdf)).



Taken from Lecture Notes on Bundling and Brand Proliferation by Professor Robert S. Pindyck ([http://web.mit.edu/rpindyck/www/Courses/BBP\\_11.pdf](http://web.mit.edu/rpindyck/www/Courses/BBP_11.pdf)).

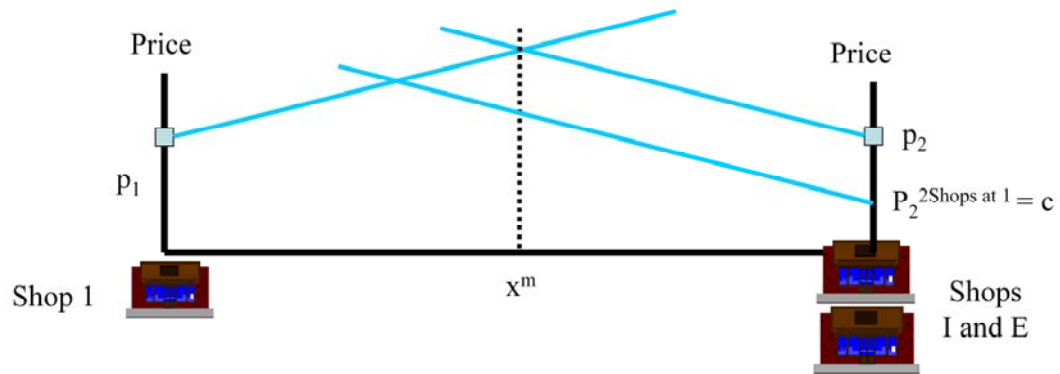
## Is spatial preemption credible?

- Can the incumbent deter entry by setting up additional shops?
- Exit costs!



Kenneth L. Judd: Credible Spatial Preemption, *The RAND Journal of Economics*, Vol. 16, No. 2 (Summer, 1985), pp. 153-166

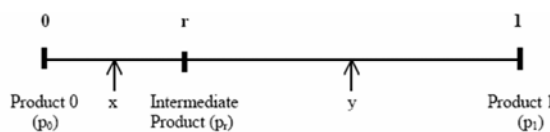
## Is spatial preemption credible?



## Is spatial preemption credible?

- Esty, Ghemawat: “Airbus vs. Boeing in Super Jumbos: A Case of Failed Preemption”
- This paper [...] concludes that Boeing attempted to preempt Airbus in introducing a new product in this space but failed to do so because of the incredibility, given the assumption of value maximization, of self-cannibalization.

Figure 2  
Products, Prices, and Customer Indifference Points with Three Locations



“Airbus vs. Boeing in Super Jumbos: A Case of Failed Preemption” Benjamin Esty (Harvard Business School) Pankaj Ghemawat (Harvard Business School)

This paper can be downloaded without charge from the Social Science Research Network electronic library at: [http://ssrn.com/abstract\\_id=302452](http://ssrn.com/abstract_id=302452)

[http://www.people.hbs.edu/besty/Esty\\_Airbus\\_Boeing.pdf](http://www.people.hbs.edu/besty/Esty_Airbus_Boeing.pdf)

pp. 20-24

Consider a model that allows for three product locations: the incumbent product at 0 (the jumbo), the entrant’s product at 1 (the superjumbo), and a possible intermediate product (the stretch jumbo) introduced by the incumbent at location  $r \in [0,1]$ . The limit point  $r = 0$  corresponds to the product market outcome if the incumbent decides not to introduce a new product at all (i.e., firm I offers a product at 0 and firm E offers a product at 1), while the limit point  $r = 1$  corresponds to the outcome, already determined to be dominated by  $r = 0$  from the incumbent’s perspective, if the incumbent offers products at both 0 and 1 and the entrant offers a product at 1. Increases in  $r$  can be thought of as decreasing substitutability within firm I’s product line while increasing it within firm E’s product line.

...

Third, while the incumbent’s market share increases with  $r$ , this increase is insufficient to offset the lower price realizations as firm E reacts by cutting prices aggressively. As a result,  $\Pi/t$  is also inversely related to  $r$ : it decreases from 0.5 at the limit point of  $r = 0$  to 0.125 at the limit point of  $r = 1$  (in which case all the operating profit is generated by the product located at 0). In other

words, the strategic effect dominates the direct effect for all values of  $r$ . The last point implies, by analogy with the argument employed above in the two product case, that the incumbent’s launch of an intermediate product (the stretch jumbo) fails exactly the same credibility test for entry-deterrence as did its option of launching the truly new product, located at 1 (the superjumbo). The incumbent’s equilibrium operating profits are higher without the intermediate product than with it. As a result, it will prefer to withdraw the product, even after it has been introduced unless, of course, there are significant exit costs. This is a striking conclusion not because of the generality of this result—which has been established only in the context of a specific demand structure—but because it demonstrates by example the unreliability of a prediction that would probably command broad assent: that large efficiency advantages for the intermediate product over the truly new product (e.g., significantly lower development costs and/or quicker speed to market) make the former an effective vehicle for an incumbent to deter entry based on the latter if the latter’s economics are sufficiently marginal to start with.

### H3) Product Differentiation: general concepts

- Horizontal product differentiation.
  - **"love of variety approach"** vs. **"ideal variety approach"**.
    - How many variants?
    - Too much or too little differentiation?
- Love of variety approach
  - Identical consumers,
  - strictly concave utility functions
  - each consumer buys *all* available varieties
  - each consumer has elastic demand

⇒ (Often) Insufficient provision of variety by the market

Love of variety approach: Consumers buy *all* available varieties. Quantities depend on prices.

### H3) Love of variety approach

- **Quadratic utility functions:**

$$u(\mathbf{x}) = \sum_{i=1}^n a_i x_i - \frac{1}{2} \sum_{i=1}^n b_i x_i^2 - \gamma \sum_{i \neq j} x_i x_j$$

- **CES-Utility functions:**

$$u(\mathbf{x}) = \left( \sum_{i=1}^n x_i^\rho \right)^{\frac{1}{\rho}}$$

- **Stone-Geary Utility functions:**

$$u(\mathbf{x}) = \prod_{i=1}^n \left( 1 + x_i \right)$$

- Outside good  $y$  (homogeneous):  $U(v(y), u(\mathbf{x}))$ ,  
often:  $U$  is quasilinear or Cobb-Douglas.

Quadratic utility function yields linear demand for differentiated products! Coke and Pepsi example (see above).

CES-utility function  $\Rightarrow$  iso-elastic demand function for each firm depending on 'average' price charged by rivals

Stone-Geary: General version of Cobb-Douglas

General remark: From these utility functions the demand functions for differentiated products industries can be derived.

Procedure: Derive demand function (standard utility maximization problem) and then proceed as in cola example (only difference:  $n$  firms).



## Gliederung der Vorlesung

- |   |   |
|---|---|
| A) Introduction                               | F) Static Games                           |
| B) Competition and Monopoly                   | G) Dynamic Games, First and Second Movers |
| C) Technology and Cost; Industry Structure    | H) Horizontal Product Differentiation     |
| D) Price Discrimination and Monopoly          | I) Vertical Product Differentiation       |
| E) Product Variety and Quality under Monopoly | J) Advertising                            |
|   | K) Research & Development                 |

Problem with PRN, Quantitative Analysis: This topic seems to be spread over the whole book. Shubik, Levitan: p. 301, Section 3.5.1;; Salop; p. 304, Section 11.5.2, Hotelling: p. 160, Section 7.4.2

## I) Vertical product differentiation: Two qualities and many consumer types

- Two firms offering two qualities  $z_k$ ,  $k = 1, 2$ .  $z_1 > z_2$
- Marginal (constant) production costs identical:  $c$
- Indirect utility of consumer  $i$  buying quality  $z_k$  at price  $p_k$   
 $V_i^k = i z_k - p_k$ ,  $i \in [0, \theta]$ .
- Consumer density:  $N / \theta \Rightarrow$  'number' of consumers:  $N$
- Consumer  $i$  who is indifferent between buying  $z_1$  and  $z_2$  if sold at  $p_1$  and  $p_2$ :  
 $V_i^1 = V_i^2 \Leftrightarrow i z_1 - p_1 = i z_2 - p_2$   
 $\Rightarrow i = (p_1 - p_2) / (z_1 - z_2)$   
 $\Rightarrow$  Demand for high quality:  $q_1 = N (\theta - i) / \theta$

Nearly identical to procedure in monopoly case, see pp. 176

Difference: identical marginal costs. Different qualities cause different fixed costs (eg R&D)

Consumer density

Same setup as in the single quality case!

If  $p_2 / z_2 = p_1 / z_1 \Rightarrow i = \bar{i}$

### I) Vertical product differentiation: Two qualities and many consumer types

- Consumer  $i$  with lowest valuation who buys a product (low quality) (Participation constraint):  
 $V_i^2 = 0 \Leftrightarrow i = p_2 / z_2$
- Demand for low quality product positive if its quality adjusted price is lower than that of high quality product, i.e.  $p_2 / z_2 < p_1 / z_1$   
 $\Rightarrow$  Demand for low quality:  $q_2 = N(i - i) / \theta$
- First problem: Derive equilibrium prices (and profits) for given qualities  
 $\Rightarrow$  Standard procedure: Solving FOCs for equilibrium prices

Interpretation of quality adjusted price eg durability, performance (razor blades, batteries):

## I) Vertical product differentiation: Two qualities and many consumer types

- Profit functions:

- Firm 1:  $\Pi_1(p_1, p_2) = (p_1 - c) N \left( \theta - \frac{p_1 - p_2}{z_1 - z_2} \right) / \theta$

- Firm 2:  $\Pi_2(p_1, p_2) = (p_2 - c) N \left( \frac{p_1 - p_2}{z_1 - z_2} - \frac{p_2}{z_2} \right) / \theta$

⇒ Differentiate both profits wrt (own) prices: FOCs ( $\partial \Pi_i / \partial p_i = 0$ )

⇒ Solve for  $p_1$  and  $p_2$

$$\Rightarrow p_1^* = \frac{z_1 (3c + 2\theta(z_1 - z_2))}{4z_1 - z_2}, \quad p_2^* = \frac{2cz_1 + z_2(c + \theta(z_1 - z_2))}{4z_1 - z_2}$$

⇒ High quality firm charges a higher price!

## I) Vertical product differentiation: Two qualities and many consumer types

- Reduced profit functions (Inserting the equilibrium prices)
  - Firm 1: 
$$\Pi_1^*(z_1, z_2) = \frac{N(z_1 - z_2)(2\theta z_1 - c)^2}{\theta(4z_1 - z_2)^2}$$
  - Firm 2: 
$$\Pi_2^*(z_1, z_2) = \frac{Nz_1(z_1 - z_2)(\theta z_2 - 2c)^2}{\theta z_2(4z_1 - z_2)^2}$$
- ⇒ The high quality firm earns higher profit
- ⇒ Profits of *both* firms increase if consumer heterogeneity ( $\theta$ ) increases (holding the number of consumers constant!)
- ⇒ Low quality firm is only viable if consumer preferences are sufficiently heterogeneous (wrt quality)  $\Leftrightarrow \theta > 2c/z_2$
- ⇒ Natural monopoly (oligopoly)! Lower bound to concentration!

With  $\theta \leq 2c/z_2$  demand for the second firm is zero even if it charges a price even marginal costs. The price of firm 1 is equal to  $c z_1/z_2$  for this range! (Limit pricing; undercutting). With a low valuation of quality (even by consumers with the high WTP for quality) the high quality firm charges a low price in order to obtain sufficient demand. This drives out the low quality firm! Again: Important assumption: Production costs do not increase much with increase in quality.

Natural oligopoly: If consumer heterogeneity increases eventually third firm is viable. Important point: Increase in  $N$ , the number of consumers, does not increase the number of firms unless heterogeneity increases also!

Sutton: If increase in quality is mainly due to fixed investments (R&D!) increases in market size do not lead to an ever increasing number of firms! Concentration does not monotonically decrease with market size! One firm could outspend the rivals in terms of R&D expenditures and earn a high profit!

## I) Vertical product differentiation: How much product differentiation?

- Again the reduced profit functions

- Firm 1:  $\Pi_1^*(z_1, z_2) = \frac{N(z_1 - z_2)(2\theta z_1 - c)^2}{\theta(4z_1 - z_2)^2}$

- Firm 2:  $\Pi_2^*(z_1, z_2) = \frac{N z_1(z_1 - z_2)(\theta z_2 - 2c)^2}{\theta z_2(4z_1 - z_2)^2}$

⇒ Profits of *both* firms increase if the level of the high quality ( $z_1$ ) increases (differentiate wrt  $z_1$ )

⇒ Firm 2 would choose a quality level which is strictly positive but smaller than  $z_1$ .

⇒ Products differentiated, but not arbitrarily low quality.

⇒ If market is 'covered', maximum product differentiation.

Market covered  $\Leftrightarrow$  all consumers buy the product (either high or low quality), in particular, consumer with lowest WTP for quality buys the low quality good at the duopoly prices (Low quality good is here the lowest quality level available).

If there is no lower bound to quality, (i.e. quality level might be zero) the low quality firm will clearly not choose the lowest available quality level!