

ENHANCED APPENDIX

Predicting stock return volatility: can we benefit from regression models for return intervals?

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Abstract

We study the performance of recently developed linear regression models for interval data when it comes to forecasting the uncertainty surrounding future stock returns. These interval-data models use easy-to-compute daily return intervals during the modeling, estimation, and forecasting stage. They have to stand up to comparable point-data models of the well-known capital asset pricing model (CAPM) type –which employ single daily returns based on successive closing prices and might allow for GARCH effects– in a comprehensive out-of-sample forecasting competition. The latter comprises roughly 1,000 daily observations on all 30 stocks that constitute the DAX, Germany’s main stock index, for a period covering both the calm market phase before and the more turbulent times during the recent financial crisis. The interval-data models clearly outperform simple random walk benchmarks as well as the point-data competitors in the great majority of cases. This result does not only hold when one-day-ahead forecasts of the conditional variance are considered, but is even more evident when the focus is on forecasting the width or the exact location of the next day’s return interval. Regression models based on interval arithmetic thus prove to be a promising alternative to established point-data volatility forecasting tools.

Keywords: volatility forecasting · time-varying variance · interval data · interval regression · range data · CAPM

JEL classification: C22 · C53 · G17 · G32

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Notes

This enhanced appendix contains detailed tables with individual figures supplementing the aggregate figures reported in the tables A.4, A.5, and A.6 of the original paper's (short) appendix.

As regards specific comparisons, the tables E.1 and E.2 constitute in-depth versions of the results presented in table A.4 of the original paper. The tables E.3, E.4, and E.5 contain the more detailed results corresponding to the aggregate results given in table A.5 of the original paper. Table E.6 exhibits the non-summarized versions of the results in table A.6 of the original paper.

Table E.1: Results of the variance-based forecast evaluation

		Adidas	Allianz	BASF	Bayer	Beiersdorf	BMW	Commerzbank	Daimler	Deutsche Bank	Deutsche Börse	Deutsche Lufthansa	Deutsche Post	Deutsche Telekom	Eon	Fresenius	Fresenius (pref.)	
QLIKE	rw	433.4874	204.3316	1773.916	1558.683	18931.88	685.6213	896.8532	30697.81	1162.776	234.9715	288.3323	211.2537	523.5098	388.4712	775.5399	1149502	
	m ₁	1.9625	3.0415	2.4152	2.2098	1.8083	2.4546	3.2258	2.8810	3.1863	2.7092	2.2921	2.1158	1.7602	2.2868	1.6731	2.0708	
	m ₂	2.3009	3.9363	3.3843	2.5898	2.0097	2.8573	3.9680	3.5867	4.0440	3.0448	2.5878	2.4685	1.9898	3.3621	1.7992	2.1113	
	m ₃	1.8559	2.6868	2.3548	2.1453	1.7563	2.3355	2.9930	2.7714	2.7072	2.5359	2.2381	2.0875	1.6090	2.0917	1.6017	2.0101	
	m ₄	1.9633	3.0454	2.4196	2.2162	1.8072	2.4586	3.2325	2.8798	3.1926	2.7085	2.2948	2.1206	1.7609	2.2907	1.6733	2.0712	
	m ₅	1.8615	2.7229	2.3875	2.1559	1.7710	2.3688	3.0322	2.8457	2.7607	2.5486	2.2726	2.0993	1.6245	2.1124	1.6079	2.0026	
	RW	2.2754	2.4059	2.1402	2.2114	2.3605	2.5091	3.1360	2.6579	2.4993	2.9527	2.4675	2.3997	1.7704	2.2667	2.3637	4.6835	
	M ₁	1.8730	2.2272	1.9122	1.9615	1.7581	2.2126	2.9793	2.5161	2.4320	2.6458	2.1363	2.0455	1.5926	2.0378	1.6649	2.0646	
	M ₂	1.8710	2.2250	1.9131	1.9541	1.7636	2.2138	2.9765	2.5149	2.4320	2.6408	2.1353	2.0516	1.5909	2.0369	1.6646	2.0699	
	MAR ₁	1.9062	2.2791	1.9362	1.9862	1.7587	2.1976	2.8814	2.4995	2.4155	2.6056	2.1497	2.0617	1.6264	2.0671	1.6528	2.0536	
	MAR ₂	1.9072	2.2653	1.9349	1.9835	1.7589	2.2000	2.8862	2.5019	2.4245	2.6004	2.1486	2.0645	1.6385	2.0514	1.6560	2.0565	
	MSE	rw	154.7190	395.9399	128.2059	89.9031	36.8645	192.9112	1548.131	396.6084	898.7170	326.3473	68.2395	213.0101	118.4724	173.6703	44.0396	178.1243
		m ₁	24.5603	100.1700	41.0357	28.5863	9.9260	62.2317	417.3797	89.1900	223.3569	73.6203	21.1242	24.5668	23.3768	43.7619	12.2535	31.2124
m ₂		43.1084	160.5957	59.6522	41.0270	12.6527	78.4902	494.8290	136.4333	288.1264	103.4592	30.6605	35.7942	26.9035	53.7281	12.2577	31.2721	
m ₃		22.1098	109.2331	32.1508	29.5960	9.1874	56.7648	407.7550	85.5915	166.3396	59.0320	20.2139	32.9246	22.7982	37.0626	11.4274	35.4811	
m ₄		24.5671	100.2300	41.0250	28.6195	9.9171	62.2980	417.1847	89.1743	223.3217	73.6087	21.1394	24.6419	23.3898	43.7850	12.2500	31.2228	
m ₅		21.6174	109.5649	70.6467	30.0392	9.2175	57.6477	401.5406	86.0571	168.7159	59.7821	20.5801	30.6282	22.7765	41.7750	11.5627	35.2938	
RW		26.3302	59.8583	33.4492	31.1074	14.2554	68.2122	381.4006	73.4185	193.6065	63.5525	25.6746	29.3453	30.3620	36.7839	13.5718	43.1058	
M ₁		20.6666	69.9324	32.1930	21.1455	9.2680	51.1869	374.0504	65.9641	189.4194	59.8008	16.5817	21.1399	18.9072	36.4665	11.5212	28.8538	
M ₂		19.9314	69.0286	32.1671	21.0699	9.2610	51.1861	371.3317	65.8347	189.4194	59.4880	17.0240	21.1358	18.8721	36.0746	11.4974	28.8725	
MAR ₁		22.6614	81.3489	34.2575	24.0988	9.3347	53.5939	342.9451	70.4612	188.6051	60.1724	18.6781	22.8230	21.0986	38.3356	11.5058	28.5394	
MAR ₂		22.6078	77.8836	33.4844	23.0451	9.3182	53.5611	344.8644	70.4287	183.2602	58.4682	18.5251	22.7875	21.1396	36.4591	11.4928	28.4183	

continued below

		Henkel	Infinion	K+S	Linde	MAN	Merck	Metro	Münchener Rück	RWE	Salzgitter	SAP	Siemens	ThyssenKrupp	VW	average stock	
QLIKE	rw	740.5202	774.1146	970.9957	1492.277	742.2609	376.8289	928442.5	5810.22	1993.347	1161.610	407.5450	739.9942	714.9692	221.8899	17161.95	
	m ₁	1.6564	3.1756	3.1134	1.9410	2.9394	1.8328	1.9242	2.0791	1.8483	3.1277	1.9357	2.4963	2.8707	2.7686	2.3934	
	m ₂	1.8108	3.4584	3.5645	2.3658	3.2838	1.8556	2.0742	2.5686	2.2797	3.5771	2.3419	3.1996	3.4557	2.9548	2.8277	
	m ₃	1.6684	3.0282	2.9657	1.8556	2.7267	1.7933	1.8096	1.8902	1.7378	2.9763	1.8188	2.5613	2.6856	2.5511	2.2616	
	m ₄	1.6558	3.1751	3.1133	1.9410	2.9453	1.8337	1.9254	2.0825	1.8481	3.1289	1.9405	2.4994	2.8711	2.7665	2.3954	
	m ₅	1.6711	3.0320	2.9807	1.8730	2.7559	1.7890	1.8094	1.9015	1.7484	2.9942	1.8234	2.5759	2.7171	2.5559	2.2801	
	RW	2.2682	3.2667	3.5596	2.3765	3.0038	2.5726	2.6962	1.9677	1.9617	3.4238	2.0439	2.4191	2.7551	2.9158	2.6110	
	M ₁	1.6686	3.1761	3.0950	1.8599	2.7170	1.7716	1.8571	1.7565	1.6640	3.0760	1.8242	2.1728	2.6968	2.7127	2.2036	
	M ₂	1.6742	3.1788	3.0947	1.8677	2.7134	1.7712	1.8567	1.7545	1.6635	3.0758	1.8137	2.1731	2.6949	2.6970	2.2028	
	MAR ₁	1.6704	3.1400	3.0987	1.8858	2.6984	1.7592	1.8795	1.7712	1.6956	3.0242	1.7837	2.2035	2.6463	2.6761	2.2003	
	MAR ₂	1.6736	3.1205	3.1029	1.8824	2.6947	1.7636	1.8728	1.7712	1.6914	3.0268	1.7853	2.2043	2.6528	2.6495	2.1990	
	MSE	rw	52.7582	7836.914	559.9511	104.0939	398.8082	105.8785	349.7134	131.7977	118.3228	950.5941	157.6677	394.4589	332.1239	1029.227	582.8738
		m ₁	7.7829	1298.190	277.7465	17.6773	113.7868	13.3212	60.9898	34.5164	14.7465	200.4517	42.0881	71.7345	60.4862	327.1700	125.5680
m ₂		12.1160	1186.483	266.4214	29.2994	140.3146	14.0558	64.2704	40.7716	19.7731	254.5585	49.7814	116.8387	106.1274	302.9263	140.4243	
m ₃		8.3524	3062.142	225.9144	14.1254	90.9321	15.9373	209.3851	29.1505	14.8898	165.4291	34.2602	93.9513	51.9418	332.4502	182.8843	
m ₄		7.7802	1297.376	277.4942	17.6938	114.0621	13.3411	61.1032	34.5949	14.7572	200.4922	42.1786	71.8135	60.4495	326.4689	125.5327	
m ₅		8.1155	3730.891	225.8935	14.3178	92.7723	15.0180	104.6184	28.2433	14.0969	169.0557	34.1290	93.9837	53.6880	329.4825	203.0584	
RW		11.3648	1891.813	365.9504	20.8541	115.0583	17.5032	69.9877	31.5520	11.8750	203.5576	32.6681	70.6494	44.2286	364.6199	145.8572	
M ₁		7.1782	1196.874	250.2631	15.4402	90.8462	11.7908	52.7996	29.3141	10.6298	188.1960	38.8862	52.9277	44.9851	301.5116	110.6247	
M ₂		7.0686	1196.720	250.6689	15.6295	88.0720	11.7532	52.8148	27.9025	10.5674	182.2558	38.0407	52.9354	44.6506	298.8402	110.0038	
MAR ₁		7.4820	1181.433	272.8721	16.4436	96.3016	11.7699	52.7654	29.7704	12.1071	174.1324	36.6996	59.8841	50.0905	299.9482	111.0053	
MAR ₂		7.4727	1178.556	273.0229	16.2448	96.1241	11.7754	52.1770	29.7960	11.8858	171.7740	36.7202	59.4670	50.0538	294.4678	110.1760	

The QLIKE and MSE values shown are calculated as defined in equation (23) and equation (24) in the original paper, respectively. The point-data random walk model is labeled rw, while RW denotes the interval-data random walk model. Each figure in the last column entitled “average stock” is computed as an equally-weighted average over the 30 individual figures for all stocks considered. Within each column, bold figures denote the best (i.e. the smallest) value for the QLIKE or MSE statistic, respectively, when comparing the forecasts of all eleven models for the variance of the particular stock’s excess return.

Table E.2: Results of the spread-based forecast evaluation

		Adidas	Allianz	BASF	Bayer	Beiersdorf	BMW	Commerzbank	Daimler	Deutsche Bank	Deutsche Börse	Deutsche Lufthansa	Deutsche Post	Deutsche Telekom	Eon	Fresenius	Fresenius (prof.)
QLIKE	rw	4.9719	3.9946	6.4344	6.4947	8.7298	4.2961	5.0527	9.5061	4.9876	3.8164	3.8728	3.8986	4.8586	4.1033	3.9242	36.9863
	m ₁	1.1428	1.3801	1.2129	1.2215	1.0631	1.3005	1.6453	1.4624	1.4619	1.4918	1.2853	1.2148	1.0233	1.2328	1.0010	1.2071
	m ₂	1.1851	1.4928	1.3298	1.2725	1.0818	1.3552	1.7275	1.5556	1.5586	1.5278	1.3326	1.2597	1.0446	1.3578	0.9948	1.1816
	m ₃	1.1227	1.3345	1.2073	1.2103	1.0483	1.2811	1.6030	1.4457	1.3865	1.4520	1.2754	1.2066	0.9939	1.2024	0.9803	1.1844
	m ₄	1.1429	1.3806	1.2136	1.2222	1.0627	1.3010	1.6459	1.4623	1.4628	1.4917	1.2856	1.2155	1.0235	1.2330	1.0006	1.2069
	m ₅	1.1232	1.3394	1.2103	1.2119	1.0484	1.2848	1.6077	1.4550	1.3947	1.4536	1.2797	1.2073	0.9953	1.2049	0.9809	1.1825
	RW	1.1783	1.3069	1.1895	1.2234	1.1166	1.3103	1.6121	1.4423	1.3692	1.5095	1.3125	1.2491	1.0147	1.2329	1.0468	1.3198
	M ₁	1.1180	1.2845	1.1526	1.1829	1.0425	1.2687	1.6050	1.4186	1.3637	1.4725	1.2631	1.1980	0.9869	1.1997	0.9785	1.1593
	M ₂	1.1176	1.2841	1.1528	1.1822	1.0430	1.2689	1.6047	1.4181	1.3637	1.4712	1.2628	1.1991	0.9865	1.1995	0.9784	1.1593
	MAR ₁	1.1215	1.2901	1.1565	1.1871	1.0404	1.2664	1.5855	1.4170	1.3611	1.4657	1.2647	1.1999	0.9889	1.2030	0.9759	1.1555
	MAR ₂	1.1212	1.2876	1.1561	1.1867	1.0398	1.2663	1.5863	1.4168	1.3618	1.4641	1.2646	1.2003	0.9899	1.2003	0.9764	1.1554
MSE	rw	1.5288	2.0145	1.3934	1.3410	1.0102	1.7165	3.8951	2.3421	2.8395	2.3869	1.3447	1.6249	1.1007	1.4329	0.8367	1.6256
	m ₁	0.5587	1.2368	0.6765	0.5652	0.4133	0.8237	2.4562	1.0921	1.7325	1.0408	0.5261	0.5449	0.4812	0.6779	0.4420	0.7861
	m ₂	0.7377	1.4351	0.8986	0.7063	0.4575	0.9389	2.5445	1.3901	1.9496	1.2277	0.6760	0.7149	0.4530	0.8529	0.3859	0.6408
	m ₃	0.4569	1.0138	0.5872	0.5584	0.3654	0.7029	1.9010	0.9800	1.0638	0.7549	0.4887	0.5274	0.3789	0.4889	0.3892	0.7321
	m ₄	0.5592	1.2387	0.6769	0.5665	0.4124	0.8259	2.4556	1.0918	1.7339	1.0408	0.5269	0.5472	0.4819	0.6787	0.4406	0.7848
	m ₅	0.4551	1.0277	0.6240	0.5643	0.3628	0.7200	1.9296	1.0229	1.0893	0.7667	0.5011	0.5183	0.3817	0.5121	0.3929	0.7213
	RW	0.5692	0.7186	0.5133	0.5425	0.4766	0.7199	1.6969	0.8546	1.0569	0.8954	0.6146	0.5914	0.4012	0.5567	0.4228	0.7280
	M ₁	0.4288	0.7962	0.4729	0.3904	0.3448	0.6231	1.9771	0.7793	1.2674	0.8303	0.4250	0.4475	0.3080	0.5225	0.3592	0.5331
	M ₂	0.4212	0.7881	0.4724	0.3879	0.3451	0.6233	1.9635	0.7742	1.2674	0.8214	0.4282	0.4488	0.3065	0.5180	0.3587	0.5327
	MAR ₁	0.4630	0.8878	0.5083	0.4302	0.3411	0.6458	1.6843	0.8153	1.2479	0.8110	0.4559	0.4754	0.3429	0.5478	0.3539	0.5180
	MAR ₂	0.4611	0.8449	0.4998	0.4206	0.3389	0.6439	1.6982	0.8118	1.2303	0.7906	0.4529	0.4756	0.3445	0.5223	0.3548	0.5156

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		Henkel	Infinion	K+S	Linde	MAN	Merck	Metro	Münchener Rück	RWE	Salzgitter	SAP	Siemens	ThyssenKrupp	VW	average stock
QLIKE	rw	4.4337	5.5799	5.5573	5.3666	4.7104	3.9689	33.8639	6.1859	6.7087	5.8095	3.5132	5.0114	5.5924	3.8254	7.2018
	m ₁	1.0189	1.7004	1.6753	1.1050	1.5423	1.0819	1.1027	1.0919	1.0822	1.6948	1.1007	1.3202	1.5255	1.4577	1.2949
	m ₂	1.0336	1.7256	1.7243	1.1572	1.5865	1.0441	0.9965	1.1535	1.1350	1.7436	1.1421	1.4285	1.6018	1.4594	1.3430
	m ₃	1.0199	1.6671	1.6468	1.0787	1.5061	1.0685	1.0722	1.0532	1.0543	1.6548	1.0820	1.3270	1.4959	1.4188	1.2693
	m ₄	1.0188	1.7003	1.6751	1.1051	1.5430	1.0822	1.1030	1.0923	1.0822	1.6950	1.1011	1.3204	1.5256	1.4572	1.2951
	m ₅	1.0197	1.6675	1.6473	1.0804	1.5087	1.0664	1.0700	1.0544	1.0556	1.6573	1.0802	1.3288	1.5002	1.4192	1.2712
	RW	1.0855	1.6950	1.7186	1.1475	1.5460	1.1131	1.1497	1.0654	1.0936	1.7037	1.0990	1.3183	1.5097	1.4568	1.3045
	M ₁	1.0027	1.6852	1.6682	1.0830	1.5096	1.0209	1.0569	1.0476	1.0486	1.6756	1.0682	1.2799	1.4989	1.4251	1.2588
	M ₂	1.0040	1.6856	1.6677	1.0845	1.5086	1.0210	1.0568	1.0470	1.0486	1.6748	1.0672	1.2800	1.4984	1.4233	1.2586
	MAR ₁	1.0039	1.6745	1.6658	1.0852	1.5071	1.0170	1.0590	1.0472	1.0525	1.6644	1.0615	1.2838	1.4923	1.4158	1.2570
	MAR ₂	1.0044	1.6721	1.6672	1.0848	1.5065	1.0172	1.0582	1.0476	1.0516	1.6643	1.0615	1.2834	1.4933	1.4128	1.2566
MSE	rw	1.0904	6.8167	3.2812	1.3516	2.6175	1.3503	1.8372	1.2778	1.0838	3.5213	1.2977	2.1263	2.3986	2.9418	2.0475
	m ₁	0.3478	3.6260	1.6892	0.5199	1.2533	0.6334	0.9519	0.6638	0.4308	1.6573	0.6256	0.8248	0.9432	1.9998	1.0074
	m ₂	0.3645	3.5761	1.3135	0.3875	0.9501	0.6364	1.0062	0.4864	0.3469	1.2288	0.5607	0.8430	0.7781	1.6246	0.8497
	m ₃	0.3587	3.7741	1.3064	0.3920	0.9673	0.6225	0.8630	0.4800	0.3445	1.2615	0.5544	0.8516	0.8077	1.6040	0.8593
	m ₄	0.4330	2.9653	1.8300	0.7061	1.4480	0.5085	0.8212	0.7141	0.5120	2.0594	0.6828	1.1596	1.2701	1.6536	1.0924
	m ₅	0.3478	3.6218	1.6841	0.5206	1.2579	0.6347	0.9549	0.6657	0.4314	1.6586	0.6277	0.8264	0.9430	1.9903	1.0076
	RW	0.4468	3.0344	1.8588	0.5326	1.1462	0.5700	0.7311	0.4793	0.3481	1.5104	0.4614	0.6977	0.7338	1.5110	0.8474
	M ₁	0.2968	2.7773	1.5502	0.4248	0.9688	0.4149	0.5954	0.4933	0.2910	1.4685	0.4851	0.5721	0.7257	1.4968	0.7689
	M ₂	0.2970	2.7767	1.5493	0.4310	0.9476	0.4143	0.5956	0.4795	0.2903	1.4341	0.4747	0.5724	0.7207	1.4759	0.7639
	MAR ₁	0.3069	2.5921	1.5904	0.4456	1.0067	0.4076	0.6019	0.5013	0.3144	1.3341	0.4496	0.6388	0.7514	1.4504	0.7640
	MAR ₂	0.3080	2.5586	1.6007	0.4424	1.0033	0.4082	0.5947	0.5018	0.3098	1.3265	0.4502	0.6319	0.7546	1.4117	0.7569

The QLIKE and MSE values shown are calculated as defined in equation (23) and equation (24) in the original paper, respectively. The point-data random walk model is labeled rw, while RW denotes the interval-data random walk model. Each figure in the last column entitled “average stock” is computed as an equally-weighted average over the 30 individual figures for all stocks considered. Within each column, bold figures denote the best (i.e. the smallest) value for the QLIKE or MSE statistic, respectively, when comparing the forecasts of all eleven models for the spread of the particular stock’s excess return interval.

Table E.3: Results of the interval-based forecast evaluation – midpoint forecasts

		Adidas	Allianz	BASF	Bayer	Beiersdorf	BMW	Commerzbank	Daimler	Deutsche Bank	Deutsche Börse	Deutsche Lufthansa	Deutsche Post	Deutsche Telekom	Eon	Presenius	Presenius (pref.)
MAE	rw	1.7608	2.0571	1.7427	1.7743	1.6119	1.9319	2.7928	2.1747	2.2756	2.4373	1.8167	1.8179	1.4313	1.7340	1.4085	1.9648
	m ₁	1.4063	1.8553	1.4476	1.4163	1.1721	1.5700	2.2942	1.8703	1.9851	1.9875	1.4500	1.4194	1.0998	1.4682	0.9600	1.2967
	m ₂	1.4063	1.8553	1.4476	1.4163	1.1721	1.5700	2.2942	1.8703	1.9851	1.9875	1.4500	1.4194	1.0998	1.4682	0.9600	1.2967
	m ₃	1.3867	1.8263	1.4270	1.3922	1.1639	1.5736	2.2927	1.8487	1.9349	1.9721	1.4596	1.4089	1.0956	1.3974	0.9569	1.2959
	m ₄	1.0191	1.2240	0.9898	0.9942	0.8842	1.1440	1.7369	1.2812	1.3582	1.5079	1.0525	1.0787	0.7984	1.0306	0.7660	1.0648
	m ₅	1.3988	1.8463	1.4668	1.3975	1.1755	1.5828	2.3037	1.8785	1.9481	1.9873	1.4662	1.4275	1.1064	1.4124	0.9625	1.3045
	RW	1.4292	1.7280	1.3995	1.4255	1.2901	1.6058	2.3400	1.7648	1.8714	2.0525	1.4544	1.4671	1.0964	1.3756	1.1021	1.5270
	M ₁	1.2181	1.5559	1.2075	1.1690	1.0549	1.3915	2.0462	1.5786	1.6843	1.7782	1.2559	1.2289	0.9039	1.2043	0.8341	1.2096
	M ₂	1.2290	1.5620	1.2087	1.1695	1.0608	1.3916	2.0528	1.5901	1.6834	1.7862	1.2619	1.2361	0.9148	1.1988	0.8341	1.2131
	MAR ₁	1.0144	1.2240	0.9912	0.9872	0.8927	1.1477	1.7232	1.2816	1.3623	1.5007	1.0440	1.0716	0.7989	1.0296	0.7593	1.0600
	MAR ₂	1.0193	1.2311	1.0025	0.9949	0.8915	1.1536	1.7329	1.2857	1.3660	1.5073	1.0466	1.0756	0.8052	1.0349	0.7619	1.0726
	MSE	rw	6.7494	10.3630	6.9074	6.4364	4.6238	7.5990	18.5244	9.8832	13.3557	12.0557	6.0520	7.0621	4.7761	6.9725	3.9536
m ₁		4.3991	7.5443	4.4973	3.9804	2.6134	5.2301	11.9642	7.4041	9.1367	8.2789	4.1395	4.2239	2.9070	4.5786	1.8095	3.2208
m ₂		4.3991	7.5443	4.4973	3.9804	2.6134	5.2301	11.9642	7.4041	9.1367	8.2789	4.1395	4.2239	2.9070	4.5786	1.8095	3.2208
m ₃		4.2462	7.3244	4.3027	3.8375	2.5795	5.2626	11.9547	7.1547	8.4612	8.2975	4.1836	4.1148	2.8835	3.9329	1.8019	3.2057
m ₄		2.3666	3.8817	2.3983	2.1436	1.4758	2.8939	7.3782	3.6674	4.8847	4.5698	2.1704	2.7293	1.6151	2.4849	1.2440	2.3153
m ₅		4.3357	7.5537	5.7340	3.8970	2.6124	5.3396	11.9830	7.3194	8.7563	8.4308	4.2124	4.3676	2.9569	4.1140	1.8370	3.3204
RW		4.6366	7.7809	4.9699	4.3422	3.0813	5.5241	12.9825	6.8974	9.0735	8.6649	4.0612	4.9159	3.0431	4.6908	2.5038	4.6882
M ₁		3.3671	5.7794	3.3372	2.7640	2.1472	4.2641	10.0735	5.5702	7.2309	6.7810	3.1678	3.2679	2.0581	3.2850	1.3904	2.8836
M ₂		3.4282	5.8420	3.3861	2.7899	2.1625	4.2856	10.1372	5.5898	7.2396	6.8580	3.1835	3.3506	2.0911	3.3134	1.4071	2.8930
MAR ₁		2.3697	3.9378	2.4439	2.1242	1.5057	2.9353	3.1146	3.7080	4.9358	4.5584	2.1734	2.7803	1.5916	2.5124	1.2001	2.2576
MAR ₂		2.4051	4.0230	2.5162	2.1633	1.5081	2.9915	7.4254	3.7508	5.0572	4.6095	2.1903	2.8096	1.6227	2.5779	1.2143	2.3105

continued below

		Henkel	Infinion	K+S	Linde	MAN	Merck	Metro	Münchner Rück	RWE	Salzgitter	SAP	Siemens	ThyssenKrupp	VW	average stock
MAE	rw	1.6149	3.1356	2.8234	1.7338	2.4244	1.6354	1.7826	1.5886	1.5006	2.9903	1.6330	2.0660	2.4223	2.3099	2.0131
	m ₁	1.2543	2.2513	2.1025	1.3845	1.9443	1.0890	1.3146	1.3518	1.2741	2.4293	1.3164	1.7749	1.9782	1.6673	1.5944
	m ₂	1.2543	2.2513	2.1025	1.3845	1.9443	1.0890	1.3146	1.3518	1.2741	2.4293	1.3164	1.7749	1.9782	1.6673	1.5944
	m ₃	1.2478	2.3721	2.0916	1.3703	1.9148	1.0701	1.3118	1.3253	1.2356	2.3745	1.2923	1.7612	1.9475	1.6406	1.5796
	m ₄	0.9401	1.8592	1.6597	0.9632	1.4189	0.9294	1.0180	0.9280	0.9084	1.7181	0.9357	1.1745	1.4581	1.3772	1.1740
	m ₅	1.2488	2.4230	2.0936	1.3749	1.9347	1.0786	1.3060	1.3284	1.2447	2.3759	1.2962	1.7691	1.9616	1.6967	1.5932
	RW	1.3352	2.5314	2.3060	1.3770	1.9674	1.3057	1.3919	1.2878	1.2128	2.4120	1.3359	1.6674	2.0072	1.8466	1.6305
	M ₁	1.1083	2.0487	1.8978	1.1796	1.6844	0.9981	1.1539	1.1630	1.0551	2.0945	1.1213	1.4688	1.7347	1.5325	1.3854
	M ₂	1.1120	2.0600	1.9041	1.1819	1.6858	1.0008	1.1585	1.1696	1.0574	2.0995	1.1229	1.4738	1.7401	1.5360	1.3898
	MAR ₁	0.9444	1.8501	1.6563	0.9589	1.4244	0.9309	1.0229	0.9216	0.9093	1.7153	0.9332	1.1750	1.4595	1.3724	1.1721
	MAR ₂	0.9479	1.8571	1.6541	0.9660	1.4307	0.9365	1.0260	0.9271	0.9146	1.7314	0.9375	1.1822	1.4644	1.3700	1.1776
	MSE	rw	5.0147	25.6820	15.8439	6.2007	11.8365	5.4548	7.3942	6.1873	5.3215	18.4025	5.8581	9.3129	11.4537	12.8375
m ₁		3.1042	13.1092	8.8892	3.7614	7.4920	2.5625	3.7642	4.1404	3.4612	11.6457	3.7810	6.6666	7.7322	5.5909	5.7209
m ₂		3.1042	13.1092	8.8892	3.7614	7.4920	2.5625	3.7642	4.1404	3.4612	11.6457	3.7810	6.6666	7.7322	5.5909	5.7209
m ₃		3.1028	14.6423	8.8254	3.6647	7.3157	2.4768	3.7610	3.9457	3.1793	11.1502	3.5873	6.4678	7.4933	5.5395	5.6232
m ₄		1.8140	8.9880	5.4297	1.9098	3.9582	1.9547	2.4345	2.1323	1.9763	5.8922	2.2353	3.2614	4.2608	4.1844	3.2880
m ₅		3.1230	15.9302	8.8410	3.7075	7.4728	2.4849	3.7034	3.9693	3.2508	11.1538	3.6338	6.5636	7.6411	6.6632	5.8303
RW		3.5217	16.6606	10.5901	3.9893	7.5632	3.8428	4.5625	4.3375	3.8001	11.9737	4.3151	6.4236	7.9352	7.6883	6.3020
M ₁		2.4220	11.2180	7.3391	2.7491	5.5729	2.1889	3.0877	3.2318	2.5740	8.9516	2.9289	4.8153	6.1035	4.8558	4.5135
M ₂		2.4445	11.3199	7.3905	2.7719	5.6068	2.2113	3.1412	3.2783	2.6134	9.0067	2.9551	4.8576	6.1414	4.8354	4.5511
MAR ₁		1.8427	8.9190	5.3363	1.9013	3.9543	1.9755	2.4792	2.1570	2.0269	5.8698	2.2547	3.2619	4.2614	4.1602	3.2916
MAR ₂		1.8603	8.9976	5.3611	1.9200	4.0220	1.9939	2.5343	2.2128	2.0874	5.8956	2.2691	3.3131	4.3251	4.1571	3.3375

The MAE and MSE values shown are calculated as defined in equation (26) and equation (24) in the original paper, respectively. The point-data random walk model is labeled rw, while RW denotes the interval-data random walk model. Each figure in the last column entitled “average stock” is computed as an equally-weighted average over the 30 individual figures for all stocks considered. Within each column, bold figures denote the best (i.e. the smallest) value for the MAE or MSE statistic, respectively, when comparing the forecasts of all eleven models for the midpoint of the particular stock’s excess return interval.

Table E.4: Results of the interval-based forecast evaluation – infimum forecasts

		Adidas	Allianz	BASF	Bayer	Beiersdorf	BMW	Commerzbank	Daimler	Deutsche Bank	Deutsche Börse	Deutsche Lufthansa	Deutsche Post	Deutsche Telekom	Eon	Presenius	Presenius (pref.)
MAE	rw	1.8636	2.1782	1.8566	1.8666	1.6748	2.0435	2.9925	2.3323	2.4100	2.5376	1.9403	1.9462	1.5204	1.8571	1.4418	2.0395
	m ₁	1.4758	1.9464	1.5016	1.4625	1.2639	1.6462	2.4932	1.9352	2.0696	2.0777	1.4855	1.4641	1.1858	1.5340	1.0490	1.4897
	m ₂	1.4697	1.9480	1.5341	1.4697	1.2372	1.6545	2.4355	1.9692	2.0695	2.0718	1.5073	1.4533	1.1486	1.5766	0.9956	1.3944
	m ₃	1.4547	1.9051	1.4809	1.4413	1.2257	1.6341	2.4296	1.9017	2.0093	2.0490	1.4977	1.4679	1.1354	1.4510	1.0122	1.4491
	m ₄	1.1480	1.4942	1.1030	1.1186	1.0174	1.2990	2.1164	1.4352	1.6053	1.6947	1.1543	1.2136	0.9484	1.1711	0.9162	1.3168
	m ₅	1.4580	1.9220	1.5304	1.4465	1.2300	1.6513	2.4449	1.9250	2.0241	2.0599	1.5062	1.4731	1.1434	1.4739	1.0113	1.4443
	RW	1.5082	1.8730	1.4781	1.5003	1.3378	1.6873	2.5747	1.8800	1.9722	2.1767	1.5407	1.5500	1.1605	1.4742	1.1284	1.6075
	M ₁	1.2856	1.6921	1.2758	1.2324	1.1307	1.4824	2.2685	1.6802	1.7776	1.8860	1.3262	1.3014	0.9745	1.3094	0.8981	1.2960
	M ₂	1.2979	1.7016	1.2772	1.2336	1.1361	1.4906	2.2771	1.7007	1.7907	1.8932	1.3313	1.3051	0.9884	1.2997	0.8983	1.2986
	MAR ₁	1.1172	1.4720	1.1134	1.1046	0.9782	1.2962	2.0038	1.4455	1.5629	1.6571	1.1549	1.1874	0.8808	1.1558	0.8465	1.1717
	MAR ₂	1.1235	1.4885	1.1215	1.1156	0.9694	1.2912	2.0004	1.4539	1.5671	1.6640	1.1658	1.1787	0.8931	1.1586	0.8467	1.1907
	MSE	rw	8.3547	11.8743	8.4627	7.9361	5.5185	8.8908	22.7147	12.5050	15.5009	13.8604	7.6592	9.2268	6.1847	8.7771	4.3378
m ₁		4.8953	8.3887	5.1121	4.4387	3.0221	5.8538	14.5853	8.2401	9.8632	8.9900	4.5821	4.6586	3.2688	5.2189	2.1544	4.2426
m ₂		5.2178	8.8369	5.5001	4.7505	3.2006	6.1489	14.9106	9.0546	10.3584	9.5012	4.9329	4.8231	3.3624	5.6277	2.0998	4.2278
m ₃		4.6834	8.1347	4.9314	4.3140	2.9142	5.7567	14.3867	7.9200	8.9231	8.8605	4.6357	4.6608	3.1857	4.4711	2.0303	4.2319
m ₄		3.0230	5.6972	3.2030	2.8822	1.9640	3.8797	11.1193	4.8925	6.5164	5.7397	2.8694	3.3185	2.2181	3.4024	1.6466	3.3765
m ₅		4.7301	8.3929	6.8243	4.3893	2.9294	5.8653	14.3406	8.1385	9.1912	9.0405	4.6754	4.9039	3.2494	4.7243	2.0397	4.2476
RW		5.3285	9.0110	5.6091	4.9817	3.5741	6.2278	15.8405	8.1638	9.6304	9.6523	4.8453	5.3505	3.6416	5.3637	2.6426	6.0353
M ₁		3.9165	6.6870	3.8680	3.2279	2.5692	4.8731	12.6346	6.5233	7.9308	7.5316	3.6800	3.7244	2.3970	3.9599	1.6693	3.7270
M ₂		4.0017	6.7680	3.8814	3.2251	2.5952	4.9378	12.7611	6.6119	8.0617	7.6849	3.7281	3.7518	2.4386	3.9150	1.6639	3.7320
MAR ₁		2.9935	5.3121	3.1018	2.7689	1.9059	3.6483	9.9879	4.7253	6.0768	5.5474	2.7901	3.2840	2.0569	3.3245	1.5264	3.1173
MAR ₂		3.0437	5.4007	3.1241	2.7845	1.9007	3.6194	10.1323	4.8406	6.2536	5.6436	2.8360	3.2762	2.1173	3.3263	1.5263	3.2098

continued below

		Henkel	Infinion	K+S	Linde	MAN	Merck	Metro	Münchener Rück	RWE	Salzgitter	SAP	Siemens	ThyssenKrupp	VW	average stock
MAE	rw	1.6647	3.3522	2.9949	1.8190	2.5627	1.7001	1.8377	1.6601	1.6367	3.1761	1.7720	2.1730	2.5535	2.4642	2.1289
	m ₁	1.2852	2.5892	2.2778	1.4522	2.0271	1.2853	1.4271	1.4073	1.3307	2.5625	1.4127	1.8054	2.0493	1.9119	1.6968
	m ₂	1.2713	2.4659	2.2523	1.4552	2.0267	1.1844	1.3552	1.4096	1.3376	2.5652	1.3968	1.8294	2.0737	1.8004	1.6786
	m ₃	1.2913	2.6071	2.2517	1.4256	1.9888	1.2676	1.4102	1.3525	1.2853	2.5010	1.3948	1.8164	2.0148	1.7787	1.6643
	m ₄	1.0210	2.3357	1.9662	1.0935	1.6460	1.1783	1.2395	1.0900	1.0437	2.0288	1.1277	1.3050	1.6281	1.7040	1.3720
	m ₅	1.2892	2.6104	2.2383	1.4266	2.0090	1.2438	1.3830	1.3545	1.2776	2.5077	1.3985	1.8263	2.0283	1.8298	1.6722
	RW	1.3740	2.8451	2.5634	1.4791	2.1237	1.3683	1.4690	1.3489	1.3025	2.6706	1.4519	1.7790	2.1525	1.9647	1.7447
	M ₁	1.1415	2.3297	2.1271	1.2480	1.8078	1.1021	1.2169	1.2346	1.1292	2.2732	1.2326	1.5473	1.8517	1.6755	1.4911
	M ₂	1.1405	2.3202	2.1341	1.2496	1.8071	1.1025	1.2296	1.2540	1.1346	2.2773	1.2344	1.5524	1.8567	1.6606	1.4958
	MAR ₁	1.0094	2.1862	1.9573	1.0713	1.6356	1.0505	1.1262	1.0492	1.0129	1.9846	1.0583	1.3130	1.6302	1.5657	1.3266
	MAR ₂	1.0150	2.2100	1.9534	1.0805	1.6373	1.0515	1.1392	1.0659	1.0206	2.0037	1.0579	1.3233	1.6363	1.5390	1.3321
	MSE	rw	5.9407	37.1875	19.8289	7.6137	14.7482	6.9533	8.7411	7.2076	6.8717	23.4030	7.5626	12.0358	14.1124	17.4666
m ₁		3.2398	18.9301	10.9876	4.2395	8.5167	3.2306	4.3941	4.4492	3.7563	13.6002	4.5805	7.1184	8.6377	8.1917	6.7129
m ₂		3.4139	18.0357	11.7257	4.6819	8.8988	3.1483	4.3154	4.6710	4.0176	14.5332	4.8275	7.8608	9.4101	7.8790	6.9991
m ₃		3.2629	19.6531	10.6204	4.0721	8.2017	3.1986	4.8515	4.1660	3.5072	12.9655	4.3435	7.1932	8.2459	7.1097	6.5144
m ₄		2.1597	15.1108	8.2195	2.5411	5.7449	2.7329	3.3966	2.8501	2.4970	8.4019	3.2417	4.3396	5.5740	7.0941	4.6551
m ₅		3.2580	19.3396	10.5280	4.0730	8.4051	3.0846	4.3473	4.1469	3.4871	13.0585	4.4021	7.3684	8.3423	8.5170	6.6680
RW		3.8445	24.4661	13.8319	4.6521	9.2712	4.4239	5.2331	4.6999	4.1635	14.9374	5.2613	7.5761	9.1582	9.9491	7.5789
M ₁		2.6166	16.2949	9.8272	3.2581	6.7724	2.6854	3.5087	3.5708	2.8771	11.1085	3.7684	5.6072	7.2194	6.9707	5.5002
M ₂		2.6370	16.2961	9.9062	3.3018	6.8485	2.6931	3.5655	3.6958	2.9043	11.3354	3.7639	5.6526	7.2802	6.8049	5.5481
MAR ₁		2.1799	14.0722	8.2143	2.4657	5.4095	2.5229	3.0908	2.6684	2.3820	8.1060	3.1371	4.2931	5.4592	6.3584	4.4175
MAR ₂		2.1988	14.2973	8.2427	2.4846	5.4716	2.5278	3.2152	2.7581	2.4396	8.1312	3.0557	4.3534	5.5058	5.9557	4.4558

The MAE and MSE values shown are calculated as defined in equation (26) and equation (24) in the original paper, respectively. The point-data random walk model is labeled rw, while RW denotes the interval-data random walk model. Each figure in the last column entitled “average stock” is computed as an equally-weighted average over the 30 individual figures for all stocks considered. Within each column, bold figures denote the best (i.e. the smallest) value for the MAE or MSE statistic, respectively, when comparing the forecasts of all eleven models for the infimum of the particular stock’s excess return interval.

Table E.5: Results of the interval-based forecast evaluation – supremum forecasts

		Adidas	Allianz	BASF	Bayer	Beiersdorf	BMW	Commerzbank	Daimler	Deutsche Bank	Deutsche Börse	Deutsche Lufthansa	Deutsche Post	Deutsche Telekom	Eon	Fresenius	Fresenius (pref.)	
MAE	rw	1.8481	2.2007	1.8563	1.9112	1.7215	2.0759	2.8712	2.2811	2.4279	2.5834	1.9271	1.9129	1.5269	1.8484	1.5468	2.0427	
	m ₁	1.5065	1.9366	1.5184	1.5167	1.2678	1.6705	2.4957	1.9567	2.1251	2.1428	1.5294	1.5177	1.1985	1.5604	1.1030	1.4931	
	m ₂	1.4543	1.9384	1.5348	1.4956	1.2252	1.6415	2.4347	1.9469	2.1333	2.0827	1.5142	1.5121	1.1535	1.5309	1.0465	1.3818	
	m ₃	1.4705	1.9016	1.4903	1.4981	1.2584	1.6689	2.4026	1.9376	2.0408	2.0685	1.5262	1.4991	1.1879	1.4767	1.0938	1.4603	
	m ₄	1.1320	1.2824	1.0453	1.0828	0.9960	1.2222	1.8924	1.3513	1.4718	1.6583	1.1285	1.1652	0.8963	1.0803	0.9039	1.2928	
	m ₅	1.4845	1.9218	1.5220	1.5038	1.2723	1.6722	2.4193	1.9682	2.0555	2.0775	1.5288	1.5251	1.1949	1.4789	1.1064	1.4803	
	RW	1.4907	1.7423	1.4564	1.4870	1.3716	1.6881	2.3174	1.8114	1.9641	2.1198	1.5294	1.5648	1.1375	1.4402	1.2017	1.5992	
	M ₁	1.2754	1.6169	1.2673	1.2282	1.1162	1.4617	2.1775	1.6469	1.8099	1.8893	1.3191	1.2951	0.9511	1.2486	0.9278	1.2898	
	M ₂	1.2777	1.6156	1.2702	1.2307	1.1226	1.4551	2.1804	1.6458	1.7996	1.8884	1.3224	1.3025	0.9532	1.2528	0.9280	1.2843	
	MAR ₁	1.0732	1.2703	1.0292	1.0290	0.9577	1.2130	1.7818	1.3425	1.4391	1.5863	1.1120	1.1334	0.8381	1.0549	0.8353	1.1367	
	MAR ₂	1.0730	1.2601	1.0406	1.0371	0.9617	1.2301	1.7962	1.3434	1.4438	1.5868	1.1074	1.1487	0.8359	1.0611	0.8434	1.1366	
	MSE	rw	8.2018	12.8806	8.1389	7.6187	5.7497	9.7402	22.1242	11.9458	16.8894	15.0247	7.1341	8.1472	5.5689	8.0336	5.2429	8.4234
		m ₁	5.0203	9.1735	5.2355	4.6525	3.0312	6.2539	14.2554	8.7523	11.8751	9.6493	4.7490	4.8790	3.5074	5.2942	2.3485	3.7712
m ₂		5.0557	9.1218	5.2916	4.6229	2.9412	6.1893	14.1067	8.5338	11.8142	9.5119	4.6982	5.0545	3.3574	5.2353	2.2910	3.4952	
m ₃		4.7228	8.5417	4.8485	4.4779	2.9755	6.1744	13.3247	8.3494	10.1269	9.2442	4.7090	4.6235	3.3391	4.3724	2.3519	3.6436	
m ₄		2.8085	4.5436	2.9473	2.5380	1.8124	3.5598	8.5483	4.6261	6.7209	5.4815	2.5251	3.2344	1.9760	2.9247	1.7225	2.8237	
m ₅		4.8515	8.7698	5.8917	4.5332	3.0211	6.2539	13.4846	8.5462	10.5000	9.3544	4.7516	4.8679	3.4280	4.5279	2.4203	3.8359	
RW		5.0832	7.9881	5.3574	4.7877	3.5415	6.2603	13.5183	7.3403	10.6305	9.4683	4.5063	5.6641	3.2471	5.1313	3.2107	4.7971	
M ₁		3.6755	6.4642	3.7523	3.0810	2.4149	4.9014	11.4665	6.1757	9.0659	7.6911	3.5056	3.7064	2.3352	3.6551	1.8298	3.1064	
M ₂		3.6973	6.4922	3.8356	3.1304	2.4200	4.8801	11.4404	6.1161	8.9523	7.6738	3.4954	3.8471	2.3567	3.7477	1.8678	3.1195	
MAR ₁		2.6720	4.3392	2.8026	2.3399	1.7878	3.5138	8.0099	4.3214	6.2906	5.1914	2.4685	3.2274	1.8120	2.7959	1.5815	2.4339	
MAR ₂		2.6888	4.3352	2.9078	2.3832	1.7934	3.6514	8.1150	4.2846	6.3215	5.1567	2.4504	3.2941	1.8169	2.8742	1.6119	2.4423	

continued below

		Henkel	Infinion	K+S	Linde	MAN	Merck	Metro	Münchener Rück	RWE	Salzgitter	SAP	Siemens	ThyssenKrupp	VW	average stock	
MAE	rw	1.7061	3.3341	2.9938	1.8207	2.5713	1.7255	1.8994	1.7173	1.5758	3.0940	1.6897	2.1977	2.5479	2.4305	2.1295	
	m ₁	1.3622	2.5164	2.2655	1.4595	2.0797	1.2918	1.5443	1.4812	1.3572	2.5752	1.4117	1.8742	2.0657	1.9102	1.7244	
	m ₂	1.3206	2.4091	2.2098	1.4263	2.0589	1.1799	1.4328	1.4423	1.3173	2.4917	1.3447	1.8717	2.0485	1.7755	1.6785	
	m ₃	1.3526	2.5766	2.2277	1.4242	2.0261	1.2637	1.4785	1.4179	1.2916	2.4476	1.3886	1.8681	2.0197	1.9027	1.6889	
	m ₄	1.0652	2.1106	1.7724	1.0704	1.5144	1.1398	1.2495	1.0310	0.9752	1.8435	1.0436	1.2443	1.4967	1.6281	1.2929	
	m ₅	1.3572	2.6167	2.2346	1.4358	2.0455	1.2801	1.4945	1.4254	1.3092	2.4480	1.3809	1.8667	2.0378	1.9129	1.7019	
	RW	1.4115	2.5416	2.3568	1.4151	2.0376	1.3956	1.4824	1.3631	1.2577	2.3850	1.3472	1.7058	2.0182	1.9432	1.6861	
	M ₁	1.1762	2.1944	1.9953	1.2263	1.7685	1.0832	1.2590	1.2480	1.1014	2.1836	1.1644	1.5274	1.7800	1.6415	1.4623	
	M ₂	1.1796	2.2205	1.9987	1.2268	1.7666	1.0858	1.2551	1.2360	1.1025	2.1744	1.1590	1.5333	1.7831	1.6477	1.4633	
	MAR ₁	1.0107	1.9077	1.6972	1.0203	1.4833	0.9980	1.0965	0.9983	0.9401	1.7395	0.9606	1.2198	1.4672	1.4620	1.2278	
	MAR ₂	1.0142	1.8838	1.7022	1.0260	1.4953	1.0064	1.0878	0.9941	0.9472	1.7370	0.9720	1.2200	1.4786	1.4842	1.2318	
	MSE	rw	6.2696	27.8099	18.4214	7.4908	14.1598	6.6569	9.7216	7.7226	5.9389	20.4447	6.7490	10.8427	13.5922	14.0919	11.0259
		m ₁	3.6642	14.5403	10.1693	4.3231	8.9739	3.1612	5.0381	5.1591	4.0278	13.0059	4.2327	7.8643	8.7130	6.9898	6.7437
m ₂		3.6605	14.1132	9.7128	4.2532	8.9811	2.9938	4.8554	5.0379	3.9289	12.8772	4.1001	7.7915	8.5945	6.6100	6.6277	
m ₃		3.6716	16.7836	9.6574	4.0323	8.3299	3.0280	4.6831	4.6984	3.5453	11.7925	3.9525	7.4285	8.2969	7.2185	6.4315	
m ₄		2.1638	10.1089	6.0081	2.3196	4.6873	2.4460	3.3821	2.7459	2.3184	6.6997	2.4843	3.8360	4.8337	5.2552	3.9361	
m ₅		3.7054	20.0690	9.7669	4.1260	8.4752	3.1301	4.7855	4.7518	3.7035	11.7722	3.9744	7.4620	8.5553	8.0174	6.7111	
RW		4.0926	14.9241	11.0661	4.3916	8.1476	4.4016	5.3541	4.9337	4.1329	12.0309	4.2916	6.6664	8.1799	8.4496	6.7198	
M ₁		2.8210	11.6958	7.9514	3.0896	6.3110	2.5221	3.8576	3.8795	2.8530	9.7318	3.0595	5.1678	6.4391	5.7344	5.0647	
M ₂		2.8462	11.8972	7.9735	3.1040	6.2604	2.5580	3.9080	3.8198	2.9030	9.5462	3.0958	5.2075	6.4439	5.8177	5.0818	
MAR ₁		2.1194	8.9499	5.6390	2.2280	4.5125	2.2434	3.0713	2.6481	2.3006	6.3018	2.2715	3.5083	4.5664	4.8628	3.6937	
MAR ₂		2.1377	8.8152	5.6808	2.2402	4.5791	2.2764	3.0428	2.6711	2.3547	6.3131	2.3828	3.5367	4.6534	5.1821	3.7331	

The MAE and MSE values shown are calculated as defined in equation (26) and equation (24) in the original paper, respectively. The point-data random walk model is labeled rw, while RW denotes the interval-data random walk model. Each figure in the last column entitled “average stock” is computed as an equally-weighted average over the 30 individual figures for all stocks considered. Within each column, bold figures denote the best (i.e. the smallest) value for the MAE or MSE statistic, respectively, when comparing the forecasts of all eleven models for the supremum of the particular stock’s excess return interval.

Table E.6: Results of the interval-based forecast evaluation – whole interval forecasts

		Adidas	Allianz	BASF	Bayer	Beiersdorf	BMW	Commerzbank	Daimler	Deutsche Bank	Deutsche Börse	Deutsche Lufthansa	Deutsche Post	Deutsche Telekom	Eon	Presenius	Presenius (pref.)	
MAIE	rw	3.7117	4.3789	3.7128	3.7778	3.3962	4.1195	5.8637	4.6134	4.8379	5.1210	3.8675	3.8591	3.0473	3.7055	2.9885	4.0822	
	m ₁	2.9823	3.8829	3.0200	2.9791	2.5317	3.3167	4.9888	3.8919	4.1947	4.2204	3.0149	2.9817	2.3843	3.0944	2.1520	2.9828	
	m ₂	2.9239	3.8864	3.0689	2.9653	2.4624	3.2960	4.8702	3.9161	4.2028	4.1545	3.0215	2.9655	2.3020	3.1074	2.0421	2.7762	
	m ₃	2.9252	3.8068	2.9712	2.9394	2.4841	3.3030	4.8317	3.8393	4.0501	4.1175	3.0239	2.9670	2.3233	2.9277	2.1059	2.9095	
	m ₄	2.2800	2.7766	2.1483	2.2014	2.0133	2.5212	4.0088	2.7865	3.0771	3.3530	2.2828	2.3788	1.8447	2.2514	1.8201	2.6096	
	m ₅	2.9425	3.8437	3.0524	2.9504	2.5023	3.3235	4.8642	3.8932	4.0796	4.1374	3.0350	2.9982	2.3383	2.9528	2.1177	2.9246	
	RW	2.9989	3.6153	2.9345	2.9873	2.7094	3.3755	4.8921	3.6913	3.9363	4.2965	3.0701	3.1148	2.2979	2.9145	2.3301	3.2067	
	M ₁	2.5610	3.3091	2.5432	2.4607	2.2469	2.9440	4.4460	3.3271	3.5875	3.7753	2.6453	2.5965	1.9256	2.5579	1.8259	2.5859	
	M ₂	2.5755	3.3171	2.5474	2.4644	2.2587	2.9458	4.4575	3.3465	3.5904	3.7816	2.6537	2.6076	1.9415	2.5525	1.8263	2.5830	
	MAR ₁	2.1904	2.7423	2.1426	2.1336	1.9359	2.5092	3.7857	2.7880	3.0020	3.2434	2.2669	2.3208	1.7189	2.2108	1.6817	2.3083	
	MAR ₂	2.1965	2.7485	2.1621	2.1528	1.9311	2.5214	3.7966	2.7974	3.1019	3.2508	2.2732	2.3275	1.7289	2.2196	1.6901	2.3273	
	MIAR	rw	0.2065	0.2138	0.2106	0.2082	0.1892	0.2185	0.2196	0.2179	0.2216	0.2302	0.2235	0.2175	0.2151	0.2224	0.2144	0.1835
		m ₁	0.3580	0.2858	0.3254	0.3544	0.3521	0.3386	0.3277	0.3271	0.2934	0.3336	0.3617	0.3698	0.3928	0.3436	0.3946	0.3567
		m ₂	0.3450	0.2834	0.3130	0.3380	0.3476	0.3344	0.3214	0.3179	0.2912	0.3293	0.3514	0.3556	0.3777	0.3184	0.3895	0.3529
m ₃		0.3596	0.2837	0.3228	0.3543	0.3582	0.3400	0.3403	0.3270	0.2988	0.3401	0.3609	0.3640	0.3975	0.3479	0.4010	0.3620	
m ₄		0.4361	0.4042	0.4402	0.4540	0.4245	0.4286	0.4048	0.4392	0.4000	0.4005	0.4468	0.4442	0.4747	0.4555	0.4532	0.4004	
m ₅		0.3580	0.2797	0.3204	0.3535	0.3513	0.3359	0.3370	0.3222	0.2956	0.3381	0.3560	0.3600	0.3959	0.3459	0.3991	0.3613	
RW		0.3325	0.3385	0.3586	0.3439	0.3098	0.3347	0.3419	0.3557	0.3456	0.3232	0.3569	0.3404	0.3882	0.3710	0.3394	0.2866	
M ₁		0.3911	0.3681	0.4049	0.4166	0.3853	0.3891	0.3731	0.3974	0.3756	0.3605	0.4118	0.4097	0.4466	0.4158	0.4291	0.3652	
M ₂		0.3892	0.3677	0.4044	0.4158	0.3836	0.3896	0.3729	0.3951	0.3752	0.3604	0.4101	0.4088	0.4443	0.4173	0.4293	0.3655	
MAR ₁		0.4397	0.4337	0.4710	0.4664	0.4309	0.4408	0.4272	0.4574	0.4391	0.4074	0.4606	0.4504	0.4845	0.4680	0.4606	0.4016	
MAR ₂		0.4398	0.4329	0.4689	0.4640	0.4324	0.4399	0.4266	0.4565	0.4388	0.4071	0.4597	0.4511	0.4826	0.4676	0.4590	0.3991	

continued below

		Henkel	Infinion	K+S	Linde	MAN	Merck	Metro	Münchener Rück	RWE	Salzgitter	SAP	Siemens	ThyssenKrupp	VW	average stock	
MAIE	rw	3.3708	6.6862	5.9887	3.6397	5.1340	3.4256	3.7371	3.3774	3.2125	6.2701	3.4617	4.3706	5.1014	4.8947	4.2585	
	m ₁	2.6474	5.1056	4.5433	2.9117	4.1068	2.5771	2.9715	2.8885	2.6879	5.1377	2.8244	3.6795	4.1150	3.8221	3.4212	
	m ₂	2.5918	4.8750	4.4621	2.8815	4.0856	2.3644	2.7880	2.8519	2.6549	5.0569	2.7416	3.7011	4.1222	3.5760	3.3571	
	m ₃	2.6439	5.1838	4.4795	2.8497	4.0149	2.5314	2.8887	2.7703	2.5769	4.9486	2.7833	3.6844	4.0346	3.6813	3.3532	
	m ₄	2.0861	4.4463	3.7386	2.1639	3.1604	2.3181	2.4890	2.1210	2.0189	3.8723	2.1713	2.5493	3.1248	3.3321	2.6649	
	m ₅	2.6463	5.2272	4.4729	2.8624	4.0545	2.5239	2.8775	2.7798	2.5869	4.9556	2.7794	3.6930	4.0661	3.7427	3.3741	
	RW	2.7855	5.3867	4.9202	2.8942	4.1613	2.7639	2.9514	2.7120	2.5602	5.0556	2.7992	3.4848	4.1707	3.9079	3.4308	
	M ₁	2.3178	4.5241	4.1223	2.4743	3.5764	2.1852	2.4758	2.4826	2.2305	4.4568	2.3971	3.0747	3.6317	3.3170	2.9535	
	M ₂	2.3200	4.5407	4.1328	2.4764	3.5737	2.1883	2.4847	2.4901	2.2371	4.4517	2.3934	3.0857	3.6398	3.3084	2.9591	
	MAR ₁	2.0202	4.0939	3.6544	2.0916	3.1189	2.0485	2.2228	2.0475	1.9530	3.7241	2.0188	2.5328	3.0974	3.0277	2.5544	
	MAR ₂	2.0292	4.0938	3.6556	2.1064	3.1325	2.0579	2.2270	2.0600	1.9678	3.7408	2.0298	2.5433	3.1149	3.0232	2.5639	
	MIAR	rw	0.2055	0.2263	0.2237	0.1951	0.2329	0.1953	0.2029	0.2224	0.2215	0.2066	0.2081	0.2085	0.2097	0.2250	0.2132
		m ₁	0.3448	0.3768	0.3677	0.3328	0.3418	0.3759	0.3462	0.3232	0.3563	0.3351	0.3540	0.3113	0.3276	0.3649	0.3458
		m ₂	0.3353	0.3639	0.3577	0.3165	0.3369	0.3795	0.3420	0.3161	0.3403	0.3287	0.3378	0.2937	0.3187	0.3613	0.3365
m ₃		0.3463	0.3782	0.3675	0.3261	0.3441	0.3807	0.3495	0.3282	0.3551	0.3387	0.3549	0.2971	0.3369	0.3775	0.3480	
m ₄		0.4172	0.4301	0.4330	0.4216	0.4222	0.4065	0.4094	0.4268	0.4453	0.4189	0.4415	0.4387	0.4181	0.4207	0.4286	
m ₅		0.3455	0.3763	0.3670	0.3230	0.3370	0.3807	0.3459	0.3256	0.3539	0.3363	0.3539	0.2957	0.3330	0.3755	0.3453	
RW		0.2987	0.3493	0.3361	0.3158	0.3331	0.3086	0.3089	0.3547	0.3653	0.3212	0.3393	0.3397	0.3257	0.3306	0.3365	
M ₁		0.3615	0.3999	0.3943	0.3697	0.3858	0.3945	0.3799	0.3784	0.4157	0.3758	0.3918	0.3809	0.3783	0.3873	0.3911	
M ₂		0.3614	0.3990	0.3934	0.3701	0.3870	0.3938	0.3791	0.3785	0.4151	0.3759	0.3931	0.3801	0.3768	0.3872	0.3907	
MAR ₁		0.4143	0.4347	0.4338	0.4236	0.4335	0.4141	0.4130	0.4454	0.4550	0.4299	0.4534	0.4530	0.4301	0.4223	0.4398	
MAR ₂		0.4131	0.4364	0.4343	0.4210	0.4321	0.4116	0.4142	0.4446	0.4539	0.4272	0.4524	0.4514	0.4283	0.4245	0.4390	

The MAIE values shown are the sum of the MAE values of the respective infimum and supremum forecasts (cf. tables E.4 and E.5), while the MIAR values shown are calculated as defined in equation (27) in the original paper. The point-data random walk model is labeled rw, while RW denotes the interval-data random walk model. Each figure in the last column entitled “average stock” is computed as an equally-weighted average over the 30 individual figures for all stocks considered. Within each column, bold figures denote the best value for the MAIE (which is the smallest) or the MIAR statistic (which is the largest), respectively, when comparing the forecasts of all eleven models for the particular stock’s excess return interval as a whole.