Thinking & Reasoning

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/ptar20

Authors’ response: how are beliefs represented in the mind?

Markus Knauff & Lupita Estefania Gazzo Castañeda

To cite this article: Markus Knauff & Lupita Estefania Gazzo Castañeda (2023): Authors’ response: how are beliefs represented in the mind?, Thinking & Reasoning, DOI: 10.1080/13546783.2023.2223110

To link to this article: https://doi.org/10.1080/13546783.2023.2223110

Published online: 22 Jun 2023.

THINKING & REASONING
https://doi.org/10.1080/13546783.2023.2223110

Authors' response: how are beliefs represented in the mind?

Markus Knauff and Lupita Estefania Gazzo Castañeda

Experimental Psychology and Cognitive Science, University of Giessen, Giessen, Germany

ABSTRACT

The commentators of our target article present several detailed arguments to refute the opposing theory. The real issue, however, seems to be the fundamental question of how the mind represents the content of beliefs. We distinguish between *qualitative*, *quantitative* and *comparative* approaches to modeling uncertain beliefs. We describe which theory falls into which of these classes. We also argue that the *comparative* level is the most fundamental, and challenge commentators to justify why they think that beliefs have *more* or *less* structure in the human mind than can be captured at the comparative level.

KEYWORDS Reasoning; beliefs; normativity; new paradigm; Bayes; mental models

Understanding how people arrive at (rational or irrational) beliefs, how they make inferences, and how they evaluate arguments is important for basic cognitive research, but it is also important in times when humankind is facing fundamental challenges (Knauff & Spohn, 2021a). But, why, then, are there so few researchers studying human reasoning today? At least compared to other fields of psychology such as learning, memory, emotions, or perception? Phil Johnson-Laird told us that Douglas Medin (when he was editor of *Psychological Review*) once remarked that he knew of no field as full of controversy as the psychology of reasoning. Perhaps this has hurt the field. Maybe many have been scared away by the continuing controversies and the multitude of contradictory theories and have therefore turned to other fields. We ourselves know people who have for this reason withdrawn from our field of research.

With our target article, we wanted to settle at least some disputes in the psychology of reasoning. We thank all commentators for their astute thoughts on our target article. The encouraging thing about the commentaries is that **Douven** and **Oaksford** agreed that the term "new paradigm"

CONTACT Markus Knauff Markus.Knauff@psychol.uni-giessen.de Experimental Psychology and Cognitive Science, University of Giessen, Otto-Behaghel-Str. 10F, 35394 Giessen, Germany.

This article has been corrected with minor changes. These changes do not impact the academic content of the article.

might be questionable, albeit a brilliant marketing trick. However, our critique of this notion was not the main intention of our target article. Rather, it was to show where the struggle between probabilistic theories of reasoning and mental model theory could be transformed into a fruitful collaboration. The discouraging lesson, however, is that we seem to have underestimated how large the gap is between advocates of the Bayesian approach and mental model theory. In fact, the commentators from the Bayesian camp mostly repeated their criticism at mental model theory already expressed elsewhere. Most of the criticisms are theoretical, not empirical. **Johnson-Laird and Khemlani** present a remarkable list of results the probabilistic approach cannot explain. Most of the criticisms are empirical, not theoretical.

In this reply, we still plea for more collaboration between the different camps, both on a theoretical and an empirical level. But rather than examining each of the commentators' arguments point-by-point, we will address their commentaries by focusing on a more fundamental question that applies to both approaches and allows us to understand their commonalities and differences. Identifying these can lead to progress in the field, as we still argue.

The real issue between the different approaches seems to be how the mind represents the content of beliefs. The first question is: What is it what we believe, disbelieve, consider plausible or probable, etc.? It might be uncontroversial that these are not the actual utterances, sentences, premises, or conclusions presented to people, but the *propositions* that mentally represent these linguistic entities. The next question relates to the many forms of beliefs or, more generally, epistemic states. This seems to be the main reason of the controversies. The crucial point is that epistemic states can be characterized in a *qualitative* or *quantitative* way. Computer scientists have shown that it is difficult to define what exactly "quantitative" means, since there is a spectrum of possible representations (e.g., Forbus et al., 1991). However, a representation that includes all real numbers R from the interval [0,1] can clearly be called *quantitative*, since it can be used as a continuous measure of something that is given in degrees. Using subjective probabilities is thus a *quantitative* way of characterizing beliefs, while using mental models representing possibilities is not. Model theory rather characterizes beliefs *qualitatively* by sets of distinct entities.

Certainly, the *qualitative* level is indispensable. It is too deeply rooted in everyday discourse. Not all our beliefs have the form of *quantitative* degrees. In many situations in daily life, people simply believe or disbelieve something, for example, we know for sure that we are now sitting in our office at University of Giessen and definitely not at a beach in California. Thus, a person's epistemic state can be characterized *qualitatively* simply as a set of discrete beliefs that are "accepted," "endorsed," or "maintained" by the person at that point in time. In reasoning experiments, therefore, for decades participants had only to decide whether an inference is valid or invalid (according to standard logic). Classical mental model theory has made the most important contributions to our understanding of the underlying cognitive process. It has shown that people represent the premises of reasoning problems in distinct mental models, which are then inspected for new information not explicitly given in the premises. Although model theory has recently undergone substantial revisions, it maintains the key idea that models are distinct *qualitative* representations and that inferences follow necessarily iff they have no counterexamples. Other assumptions also remain unchanged and all refer to qualitative representations: The more models that have to be taken into account, the harder an inference is and a frequent cause of errors is to overlook an alternative model of a possibility. Even **Oaksford** concedes the importance of such qualitative small-scale models of the world and their importance in human reasoning.

Certainly, the *quantitative* level is also indispensable. Many of our beliefs come in different degrees. Our beliefs are often more or less certain. For Bayesians the qualitative account is too simplistic and psychologically implausible. For example, we believe that Mike Oaksford is now in London, but we are not certain. In daily life, we have many words to express our degree of beliefs in this *quantitative* way. We speak of probabilities, plausibility, likeliness, uncertainty, and the like. The so-called "new paradigm" deserves credit for bringing this fact back to the attention of reasoning researchers. One of its key ideas is cogent. We need to account for the fact that human beliefs are inherently uncertain and reasoning is concerned with updating of uncertain beliefs. Thus, we can rarely be certain of the truth of the premises over which we reason (Chater & Oaksford, 2021; Oaksford & Chater, 2020). The other idea is that degrees of belief can be captured by Bayesian probability theory, a mathematical theory of how to reason with degrees of belief. The general form for the probability of a proposition, q, is P(q|B), that is, any probability assignment is conditional on background knowledge B (Oaksford & Chater, 2020). The key question, however, is whether the *quantitative* approach of Bayesian probability theory is the *only* way to characterize beliefs that come in degrees. Here some clarifications are necessary. Of course, **Oaksford** knows that people flounder with the easiest probability problems (e.g., Dasgupta et al., 2020; Sanborn & Chater, 2016). Therefore, **Oaksford** and other supporters of the probabilistic account say that their approach is *qualitative*, not *quantitative* (Oaksford & Chater, 2009). They justify this by assuming that the human cognitive system does not perform actual Bayesian computations, but uses, for example, learning and sampling methods to calculate probabilities (e.g., Dasgupta et al., 2020; Sanborn & Chater, 2016; see also Politzer & Baratgin, 2016). However, this claim is nevertheless mathematically incorrect. If the degree of a belief is mapped to all real numbers between 0 and 1 or percentages between 0 and 100%, this is a *quantitative* representation, even if no exact Bayesian calculations are performed. **Oaksford** might know that but nevertheless frequently uses the term "probabilistic brain" without spelling out where the brain in fact does probabilistic computations and where it does not.

Another argument from Bayesians is that the qualitative concept can be translated into a quantitative one. For instance, in his commentary, **Douven** argues that, ideally, we assign probability 1 to all truths and probability 0 to all falsehoods. Similarly, Oaksford (2015) already argued that human deductive reasoning simply is reasoning with the probabilities 0 and 1. The problem with this account, however, is that it fails to distinguish between those truths that are necessary and those that are certain. Moreover, a neuroimaging study from our group has shown that deductive and probabilistic reasoning are qualitatively distinct and also rely on distinct neuronal processes (Gazzo Castañeda et al., 2023). A further problem is that **Douven**'s commentary raises the difficult question of how weak a belief should be so that it still counts as a belief. Do you believe A when your subjective probability for it is greater than 0.5, or 0.75, or only when it is equal to 1? Several accounts in artificial intelligence and mathematical philosophy try to combine the *qualitative* and the *quantitative* beliefs in that way (e.g., Leitgeb, 2017). In psychology, Oberauer and Wilhelm (2003) conducted experiments showing that if people have a continuous degree of belief in a conditional "if p, than q" but are forced to choose between "true" or "false," they set a threshold such that a degree of belief larger than the threshold is considered sufficiently high to warrant a "true" response, and a degree lower than the threshold result in a "false" response. Unfortunately, this threshold idea found only little resonance in cognitive research. The main reason might be that the threshold between belief and disbelief is difficult to determine empirically and may vary in different areas of discourse and among different individuals.

There is, in fact, a *third way* to characterize the structure of beliefs, which we call the *comparative* level (Knauff & Spohn, 2021b; Spohn, 2012). On this level, a proposition is considered *more or less* plausible, credible, or certain than another. This level lies somehow between the quantitative and qualitative levels. For example, we can believe that it is more likely that Igor Douven is now in Paris than that he is in Tokyo. It is, however, not necessary to say how much we believe one more than the other. Most generally, a comparative conception of epistemic states is associated with a set a *plausibilities*, which are elements in a partially ordered space. The only real requirement is that if A is a subset of B then the plausibility of A is less than equal to the plausibility of B (Halpern, 2017). Computer scientists have shown that in fact every other representation of uncertainty can also be viewed as a plausibility measure on the comparative level. The comparative level corresponds to the *ordinal scale* in psychological measurement theory (Luce et al., 1990; Stevens, 1946), and it is widely recognized that most measures collected by participants in the psychological sciences achieve ordinal but not the quantitative, interval status. Thus, mathematical methods using more than the relative rank-order of data ought not to be used (Luce et al., 1990). There is ample evidence that comparisons and rank-orders are deeply rooted in human cognition (e.g., Hansson & Grüne-Yanoff, 2022; Hausman, 2012). They lead to different actions and people use comparisons to develop attitudes toward a set of objects or events that are usually reflected in an implicit or explicit decision or choice (Lichtenstein & Slovic, 2006). The comparisons typically correspond to relations, which are also omnipresent in everyday life. Reasoning with the relations has been studied extensively and there are detailed cognitive theories about how people reason with relations on the comparative level (Goodwin & Johnson-Laird, 2005; Holyoak & Lu, 2021; Knauff, 1999, 2013; Knauff et al., 2004, 2013; Krumnack et al., 2011). Philosophers have developed sophisticated formal models for representing beliefs in rank orders and their dynamics that make less strict mathematical assumptions than probability theory, but still allow us to express differences in the degree of beliefs (e.g., Spohn, 2012).

A key advantage of the comparative level is that it forces researchers to justify how much structure a belief needs. Does, for instance, the belief at the comparative level "It is more plausible that Igor Douven is in Paris than that he is in Tokyo" have enough structure? Or do we need more structure such as in "The probability that Igor Douven is in Paris is 67% and that he is in Tokyo is 33%"? Or even less structure such as in "Igor Douven is in Paris and not in Tokyo"? The key questions are thus:

- Do beliefs have *more* structure in the human mind than can be captured at the comparative level, as implied in Bayesian theories of reasoning?
- Do beliefs have *less* structure in the human mind than can be captured at the comparative level, as suggested by mental model theory?

Advocating Bayesian probability theory as a general framework for human reasoning means to answer the first question with a clear "yes." This "yes," however, has strong mathematical implications: the difference between each two beliefs should be equal, there should be an absolute zero point, and the probabilities assigned to an exhaustive set of beliefs should add up to 100%. This raises three main cognitive questions: First, what is the meaning of all these assumptions when we talk about beliefs that people hold about the objects in the world and their relations? For instance, what is the cognitive meaning of this zero-point in terms of degrees of beliefs? Which mathematics do we want to allow for a cognitive scale of beliefs? Do we want to allow researchers to compare the intervals or differences between beliefs on the probability scale? Or can we go forward with less demanding requirements? It might be enough to simply depict the order or rank of beliefs without actually establishing the degree of variation between them. Second, are the mathematical assumptions supported by empirical evidence? Do people follow all or at least some axioms of Bayesian probability theory when dealing with uncertain beliefs? Several experiments speak against this. For instance, studies have shown that the additivity axiom is often violated (Hinterecker et al., 2016; Khemlani et al., 2012, 2015). We admit that the situation is somewhat more complicated, since these issues might apply differently to the normative and descriptive levels of theory or, as Oaksford and others prefer to say, to the computational and algorithmic levels in Marr's terminology. Moreover, we already mentioned that Bayesians argue that the cognitive system does not perform actual Bayesian calculations. But, still, there are many alternatives to representing degrees of belief quantitatively as probabilities (Dubois & Prade, 2021; Halpern, 2017). So why should we use Bayes theory as a descriptive and normative theory of human rationality? Why not use another theory? And third: How general is the Bayesian approach to reasoning? In his commentary, **Oaksford** claims that there is a broad consensus that intelligent decision-making and reasoning are implemented probabilistically in the brain. He also argues that probabilistic theories of reasoning are part of a new, progressive development that spans the brain and cognitive sciences, called Bayesian cognitive science. He cites several research papers in support of this thesis. However, a growing number of scientists are already questioning the generality of the Bayesian framework. In the field of artificial intelligence (AI), Judea Pearl was a key figure in the effort to enable machines to think probabilistically. Today, he is one of the most vocal critics of this development. In his latest book, he argues that AI suffers from a complete misinterpretation of what intelligence actually is. He argues convincingly that the field of AI is stuck in probabilistic associations but unable to compute cause and effect (Pearl & Mackenzie, 2018). Other leading experts in AI concede that probabilistic machine learning methods work well on large datasets but criticize the lack of language and representation and that many stable states of the network perform the same tasks (Van Benthem et al., 2021). In psychology, a growing number of researchers are also criticizing the incompleteness of probability theory's much-vaunted claim to generality. Recently, for example, Szollosi et al. (2023) provided a theoretical analysis of what people need to do to deal with uncertainty in cognitive tasks. Their analysis found that the use of probabilistic concepts often hides essential, non-probabilistic steps that people must take to solve cognitive problems. So, certainly, if Bayesianism were as general as its proponents claim, this would lead to a laudable unification in the field. **Johnson-Laird and Khemlani** are quite right that the multiplicity of explanations for even small empirical findings is a scandal that calls into question the scientific merits of reasoning research. We are here not saying that Bayesianism is useless for the psychology of reasoning. But we are saying that its much-vaunted claim of generality is unwarranted for several reasons.

Advocating mental models as a theory of human reasoning means to answer the second question with "yes." But, again, this "yes," has strong implications: Classical model theory did not try to deal with uncertainty until it explained how naïve individuals make extensional judgments of probabilities (Johnson-Laird et al., 1999). Thus, it has been substantially revised in the last decade. The key of this revision is that models represent possibilities to which the meanings of natural expressions refer. **Oaksford** as well **Douven** and **Cruz** see here a lack of a clear normative theory.

This is astonishing, as some advocates of the new paradigm have emphatically argued against the comparison of human reasoning with any normative standards (Elqayam & Evans, 2011). In our target article, we already argued that probabilistic theorists need to achieve more agreement on the role of normativity, especially if they want to be considered a "new paradigm."

In their commentaries, **Over** and **Cruz** argue that the revised mental model is difficult to falsify. We agree with this in part, as the principle of pragmatic modulation does indeed lead to an indefinite number of meanings of conditionals (Johnson-Laird & Byrne, 2002). However, it is also a fact that **Johnson-Laird and Khemlani** present a long list of experimental results that either falsify probabilistic theory or at least cannot be explained by it. None of the new paradigms provides such a list of empirical evidence. They are concerned with completeness, consistency, etc. It is obvious that they have yet to show robust results predicted by probability theory but disproving model theory. This seems to be the reason why they argue mainly theoretically, not empirically (Hinterecker et al., 2019). On the long run, however, the key question is: does a psychological theory make corroborated predictions?

To return to the comparative level. The good news is that both frameworks are in principle open to this characterization of uncertain beliefs. Within the so-called "new paradigm," some researchers have suggested using the *Ramsey test* to determine the degree of belief in a conditional assertion by comparing the likelihood of p&q to p&not-q (Evans et al., 2003). This approach is closer to the comparative level than the Bayesian approach, and, interestingly, closer to mental model theory as it can be seen as a "simulation heuristics" (Evans & Over, 2004, p. 119) based on mental models. Within mental model theory, Khemlani et al. (2015) have suggested that people construct an iconic model based on the proportions of naïve probabilities (see also Johnson-Laird et al., 1999). This iconic model can be imagined as a line whose length represents the probability of an event. The longer the line, the greater the probability (López-Astorga et al., 2022). The interesting feature of this iconic representation is that additional evidence can make people shift this line into a higher or lower probability, allowing people to compare and average the probability of different events. In principle, this approach represent beliefs *quantitatively* in terms of probabilities, both extensionally and intensionally, but it can also be used for belief representation on the *comparative* level (Khemlani et al., 2015; see also Johnson-Laird et al., 1999).

Knauff (2013) and Ragni and Knauff (2013) developed an extension of model theory in which models are rank-ordered according to their plausibility. In *preferred model theory*, people first construct a *preferred mental model* that is most plausible based on their prior knowledge. The preferred models are even similar in different cultures (Knauff & Ragni, 2011). Alternative models are constructed only if this preferred model is inconsistent with further information. Then the second most plausible model is constructed, then the third most plausible, and so on. The ranking of the models is represented in a neighborhood graph, where the number of edges represents the similarity to the preferred model. Although the theory is currently just spelled out for reasoning with spatial and temporal relations, it shows that model theory is compatible with a non-quantitative characterization of beliefs on the comparative level (see also Jahn et al., 2007; Knauff, 1999; Knauff et al., 2004).

So, is it possible to find a *shared* view of how the mind represents the content of beliefs? Should we model beliefs qualitatively, quantitatively, or comparatively? The comparative level seems to be the most fundamental (Knauff, 2013; Spohn, 2012). Researchers should explain why they postulate more or less structure in their theories about beliefs in the mind. **Johnson-Laird and Khemlani** say that collaborations among theoretical adversaries are seldom fruitful because neither side likes to give up ideas. This is not our experience. In fact, we two (MK and LEGC) have quite different opinions on many questions of reasoning research. Yet, we do perceive this as a benefit, not a burden. In the reasoning community, we can thus imagine more fruitful collaboration within Kahneman's concept of *adversarial collaboration*, in which experiments are jointly designed and conducted by people with competing hypotheses (e.g., Mellers et al., 2001). The harder task might be on the normative side anyway. For Spohn (2011) *normativity* defines the borderline between science and humanities. We would even argue that normativity is particularity important for psychology, intersecting natural science, social science, and humanities. The history of psychology has shown us how speculative theories can be without such normative standards. But where does the psychology of reasoning stand today? Several normative features of Bayesian probability theory are psychologically implausible. Revised model theory might be too flexible in some of its normative assumptions. And Bayesians seem to assume that the Bayesian conception of probability is the *only* normative theory for human cognition—despite the fact that there are many normative alternatives for dealing with uncertainty, for example, possibility theory, imprecise probabilities theory, plausibility theory, Dempster-Shafer theory of belief functions, ranking theory, etc. (see Dubois & Prade, 2021; Halpern, 2017). Ranking theory, for example, is a normative theory of the dynamics of beliefs at the comparative level (Kern-Isberner et al., 2021; Spohn, 2012), which can also be useful for modeling human reasoning (Skovgaard-Olsen, 2016). The problem, we think, may be that psychologists are not sufficiently aware of the many normative theories for dealing with uncertain beliefs, although they could also help in developing descriptive theories of how beliefs are represented in the mind.

Acknowledgements

We thank Phil Johnson-Laird for his advice on an earlier version of this paper.

Disclosure Statement

No potential conflict of interest was reported by the author(s).

Funding

This research was supported by DFG grant KN 465/9-2 within the Priority Program "New Frameworks of Rationality" (SPP 1516) to MK.

References

Chater, N., & Oaksford, M. (2021). Bayesian rationality in the psychology of reasoning. In M. Knauff & W. Spohn (Eds.), *The handbook of rationality* (pp. 285–294). MIT Press.

Dasgupta, I., Schulz, E., Tenenbaum, J. B., & Gershman, S. J. (2020). A theory of learning to infer. *Psychological Review*, *127*(3), 412–441. https://doi.org/10.1037/rev0000181

Dubois, D., & Prade, H. (2021). Representing belief: Beyond probability and logic. In M. Knauff & W. Spohn (Eds.), *The handbook of rationality* (pp. 305–316). MIT Press.

Elqayam, S., & Evans, J. S. B. T. (2011). Subtracting "ought" from "is": Descriptivism versus normativism in the study of human thinking. *The Behavioral and Brain Sciences*, *34*(5), 233–248. https://doi.org/10.1017/S0140525X11000045

Evans, J. S. B. T., Handley, S. J., & Over, D. E. (2003). Conditionals and conditional probability. *Journal of Experimental Psychology. Learning, Memory, and Cognition*, *29*(2), 321–335. https://doi.org/10.1037/0278-7393.29.2.321

Evans, J. S. B. T., & Over, D. E. (2004). *If*. Oxford University Press.

Forbus, K., Nielsen, P., & Faltings, B. (1991). Qualitative spatial reasoning: The CLOCK project. *Artificial Intelligence*, *51*(1-3), 417–471. https://doi.org/10.1016/0004-3702(91)90116-2

Gazzo Castañeda, L. E., Sklarek, B., Dal Mas, D., & Knauff, M. (2023). Probabilistic and deductive reasoning in the human brain. *NeuroImage*, *275*, 120180. https://doi.org/10.1016/j.neuroimage.2023.12018

Goodwin, G. P., & Johnson-Laird, P. N. (2005). Reasoning about relations. *Psychological Review*, *112*(2), 468–493. https://doi.org/10.1037/0033-295X.112.2.468

Halpern, J. Y. (2017). *Reasoning about uncertainty* (2nd ed.). MIT Press.

Hansson, S. O., & Grüne-Yanoff, T. (2022). Preferences. In E. N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy (Spring 2022 Edition)*. Stanford University. https://plato.stanford.edu/archives/spr2022/entries/preferences/

Hausman, D. M. (2012). *Preference, value, choice, and welfare*. Cambridge University Press.

Hinterecker, T., Knauff, M., & Johnson-Laird, P. N. (2016). Modality, probability, and mental models. *Journal of Experimental Psychology. Learning, Memory, and Cognition*, *42*(10), 1606–1620. https://doi.org/10.1037/xlm0000255

Hinterecker, T., Knauff, M., & Johnson-Laird, P. N. (2019). How to infer possibilities: A reply to Oaksford et al. (2018). *Journal of Experimental Psychology. Learning, Memory, and Cognition*, *45*(2), 298–301. https://doi.org/10.1037/xlm0000627

Holyoak, K. J., & Lu, H. (2021). Emergence of relational reasoning. *Current Opinion in Behavioral Sciences*, *37*, 118–124. https://doi.org/10.1016/j.cobeha.2020.11.012

Jahn, G., Knauff, M., & Johnson-Laird, P. N. (2007). Preferred mental models in reasoning about spatial relations. *Memory & Cognition, 35*, 2075–2087. https://doi.org/10.3758/BF03192939

Johnson-Laird, P. N., & Byrne, R. M. J. (2002). Conditionals: A theory of meaning, pragmatics, and inference. *Psychological Review*, *109*(4), 646–678. https://doi.org/10.1037/0033-295x.109.4.646

Johnson-Laird, P. N., Legrenzi, P., Girotto, V., Legrenzi, M. S., & Caverni, J.-P. (1999). Naive probability: A mental model theory of extensional reasoning. *Psychological Review*, *106*(1), 62–88. https://doi.org/10.1037/0033-295X.106.1.62

Kern-Isberner, G., Skovgaard-Olsen, N., & Spohn, W. (2021). Ranking theory. In M. Knauff & W. Spohn (Eds.), *The handbook of rationality* (pp. 337–345). MIT Press.

Khemlani, S. S., Lotstein, M., & Johnson-Laird, P. (2012). The probabilities of unique events. *PloS One*, *7*(10), e45975. https://doi.org/10.1371/journal.pone.0045975

Khemlani, S. S., Lotstein, M., & Johnson-Laird, P. N. (2015). Naïve probability: Model-based estimates of unique events. *Cognitive Science*, *39*(6), 1216–1258. https://doi.org/10.1111/cogs.12193

Knauff, M. (1999). The cognitive adequacy of Allen's interval calculus for qualitative spatial representation and reasoning. *Spatial Cognition and Computation, 1*, 261–290. https://doi.org/10.1023/A:1010097601575

Knauff, M. (2013). *Space to reason: A spatial theory of human thought*. MIT Press.

Knauff, M., Bucher, L., Krumnack, A., & Nejasmic, J. (2013). Spatial belief revision. *Journal of Cognitive Psychology, 25*, 147–156. https://doi.org/10.1080/20445911.2012.751910

Knauff, M., Strube, G., Jola, C., Rauh, R., & Schlieder, C. (2004). The psychological validity of qualitative spatial reasoning in one dimension. *Spatial Cognition and Computation, 4*, 167–188. https://doi.org/10.1207/s15427633scc0402_3

Knauff, M., & Ragni, M. (2011). Cross-cultural preferences in spatial reasoning. *Journal of Cognition and Culture*, *11*(1–2), 1–21. https://doi.org/10.1163/156853711X568662

Knauff, M., & Spohn, W. (2021a). *The handbook of rationality*. MIT Press.

Knauff, M., & Spohn, W. (2021b). Psychological and philosophical frameworks of rationality—A systematic introduction. In M. Knauff & W. Spohn (Eds.), *The handbook of rationality* (pp. 1–55). MIT Press.

Krumnack, A., Bucher, L., Nejasmic, J., Nebel, B., & Knauff, M. (2011). A model for relational reasoning as verbal reasoning. *Cognitive Systems Research, 11*, 377–392. https://doi.org/10.1016/j.cogsys.2010.11.001

Leitgeb, H. (2017). *The stability of belief. How rational belief coheres with probability*. Oxford University Press.

Lichtenstein, S., & Slovic, P. (2006). *The construction of preference*. Cambridge University Press.

López-Astorga, M., Ragni, M., & Johnson-Laird, P. N. (2022). The probability of conditionals: A review. *Psychonomic Bulletin & Review*, *29*(1), 1–20. https://doi.org/10.3758/s13423-021-01938-5

Luce, R. D., Krantz, D. H., Suppes, P., & Tversky, A. (1990). *Foundations of measurement, Vol. 3: Representation, axiomatization, and invariance*. Academic Press.

Mellers, B., Hertwig, R., & Kahneman, D. (2001). Frequency representations of eliminate conjunction effects? An exercise in adversarial collaboration. *Psychological Science*, *12*(4), 269–275. https://doi.org/10.1111/1467-9280.00350

Oaksford, M. (2015). Imaging deductive reasoning and the new paradigm. *Human Neuroscience*, *9*, 101. https://doi.org/10.3389/fnhum.2015.00101

Oaksford, M., & Chater, N. (2009). Précis of Bayesian rationality: The probabilistic approach to human reasoning. *The Behavioral and Brain Sciences*, *32*(1), 69–84. https://doi.org/10.1017/S0140525X09000284

Oaksford, M., & Chater, N. (2020). New paradigms in the psychology of reasoning. *Annual Review of Psychology*, *71*, 305–330. https://doi.org/10.1146/annurev-psych-010419-051132

Oberauer, K., & Wilhelm, O. (2003). The meaning(s) of conditionals: Conditional probabilities, mental models, and personal utilities. *Journal of Experimental Psychology. Learning, Memory, and Cognition*, *29*(4), 680–693. https://doi.org/10.1037/0278-7393.29.4.680

Pearl, J., & Mackenzie, D. (2018). *The book of why: The new science of cause and effect*. Basic Books.

Politzer, G., & Baratgin, J. (2016). Deductive schemas with uncertain premises using qualitative probability expressions. *Thinking & Reasoning*, *22*(1), 78–98. https://doi.org/10.1080/13546783.2015.105256x

Ragni, M., & Knauff, M. (2013). A theory and a computational model of spatial reasoning with preferred mental models. *Psychological Review*, *120*(3), 561–588. https://doi.org/10.1037/a0032460

Sanborn, A. N., & Chater, N. (2016). Bayesian brains without probabilities. *Trends in Cognitive Sciences*, *20*(12), 883–893. https://doi.org/10.1016/j.tics.2016.10.003

Skovgaard-Olsen, N. (2016). Ranking theory and conditional reasoning. *Cognitive Science*, *40*(4), 848–880. https://doi.org/10.1111/cogs.12267

Spohn, W. (2011). Normativity is the key to the difference between the human- and the natural sciences. In D. Dieks, W. J. Gonzales, S. Hartmann, T. Uebel, & M. Weber (Eds.), *Explanation, prediction, and confirmation* (pp. 241–251). Springer.

Spohn, W. (2012). *The laws of belief: Ranking theory and its philosophical applications*. Oxford University Press.

Stevens, S. S. (1946). On the theory of scales of measurement. *Science (New York, N.Y.)*, *103*(2684), 677–680. https://doi.org/10.1126/science.103.2684.677

Szollosi, A., Donkin, C., & Newell, B. R. (2023). Toward nonprobabilistic explanations of learning and decision-making. *Psychological Review*, *130*(2), 546–568. https://doi.org/10.1037/rev0000355

Van Benthem, J., Liu, F., & Smets, S. (2021). Logico-computational aspects of rationality. In M. Knauff & W. Spohn (Eds.), *The handbook of rationality* (pp. 185–196). MIT Press.