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The Psychological Validity of Qualitative Spatial Reasoning in One Dimension

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One of the central questions of spatial reasoning research is whether the underlying processes are inherently visual, spatial, or logical. We applied the dual task interference paradigm to spatial reasoning problems in one dimension, using Allen's interval calculus, in order to make progress towards resolving this argument. Our results indicate that spatial reasoning with interval relations is largely based on the construction and inspection of qualitative spatial representations, or mental models, while no evidence for logical proofs of derivations or the involvement of visual representations and processes was found.

Keywords: Psychological validity, qualitative spatial reasoning, mental images, mental models, interval relations

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Research in spatial cognition offers a wide spectrum of possibilities to combine psychological and computational approaches. Sometimes, the design of computational systems may merely be based on what researchers believe to happen in their mind when solving spatial reasoning problems. In other cases, the computational model is simply inspired by existing psychological evidence. It may also happen that an already existing computational approach is enabled post hoc as cognitively adequate, or psychologically valid. The latter seems to hold for the interval calculus introduced by Allen (1983), which originated in the temporal domain and was imported into spatial reasoning (Freksa, 1991; Güsgen, 1989; Hernández, 1994; Mukerjee & Joe, 1990),[1] and has established itself among the most important formalisms to reason about space (e.g., Egenhofer, 1991; Randell, Cui & Cohn, 1992). Much of its attraction for qualitative spatial reasoning (QSR) rests on the claim that it is akin to human inference. Yet there is only little justification for this claim (Cohn, 1997). Instead, proponents of the diagrammatic reasoning (DR) approach assert that spatial inference relies on diagrams, or visual images—an equally doubtful conjecture (Glasgow, & Papadias, 1992; Glasgow, Narayanan, & Chandrasekaran, 1995). All logical approaches are, of course, by default committed to the "orthodox" view of AI, that logically framed propositional representations together with logic inference are sufficient to realize spatial reasoning.

But what happens in the human mind when people reason about spatial relations? Are the underlying cognitive processes inherently spatial, visual, or even pure "mental logic?" The aim of the present paper is to make progress towards resolving this controversy, at least as it applies to reasoning with the interval relations introduced by Allen (1983).

One might object that Allen's calculus, due to its origin in the temporal domain, is principally confined to a single dimension (with an orientation from the past to the future), whereas objective as well as experienced space comprises the three dimensions denoted by the words up-down, forward-backward, and left-right. Extensions of the calculus to more than one dimension are, in fact, restricted—although not insignificant. Rectangular-bound boxes for layout and configuration problems, for example, can be described by Allen relations in two or three orthogonal dimensions (for an application, see Schlieder & Hagen, 2000). Complex real-world objects, like routes (roads, railroads, rivers, ducts, etc.) may be described as one-dimensional objects embedded in space, yielding intervals defined by landmarks, bends, crossings, and the like.

Accordingly, Balbiani, Condotta, and Del Cerro (2003) defined the notion of a block algebra that is based upon a spatial application of Allen's interval

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[1] Recent applications of QSR can be found for instance in computer vision (Fernyhough, Cohn & Hogg, 1997), qualitative physics (Weld & De Kleer, 1990), image information systems (Chang & Jungert, 1996), robot navigation (Levitt & Lawton, 1990), document analysis (Walischewski, 1997) and geographic information systems (GIS; Egenhofer & Mark, 1995).

algebra. It consists of a set of block relations together with the elementary operations of composition, converse and intersection. The 13 basic relations of this algebra constitute the exhaustive list of the relations possibly holding between two blocks. These relations are similar to the spatial version of Allen's interval relations we investigated in the following experiments. So, while there may be many spatial relations beyond the scope of Allen's calculus, it can nevertheless be used to describe an important subset of spatial reasoning, and therefore yields a valid formal approach to spatial reasoning. In addition, Allen's interval relations have already been shown to fulfill two basic requirements for being cognitively adequate: People can easily learn and use them (Knauff, 1999). Therefore, Allen's calculus is used here to address the question whether spatial reasoning in the human mind is supported by visual imagery, by (non-visual) spatial representations, or by mental proofs alone.

The present paper starts with a brief description of Allen's interval relations and the inference based on them. We then describe the experimental methods from cognitive psychology that were used in the studies. Next, we briefly sketch the psychological theories that aim to explain human spatial reasoning. We then report two experiments that were conducted to clarify the role of proofs of derivation, visual mental imagery and (qualitative) spatial models for human reasoning.

A SHORT OVERVIEW OF ALLEN'S CALCULUS

Allen's calculus consists of intervals originally representing events (i.e., intervals in time), qualitative relations between these intervals, and an algebra for reasoning about these relations. The 13 base relations are illustrated in Table 1. They are jointly exhaustive and pairwise disjoint (i.e., exactly one relation holds between any two intervals). Furthermore, 2^{13} relations can be obtained as unions of the base relations. The original names from Allen (1983; first column in Table 1) reflect that they were introduced for temporal reasoning. In the spatial domain the intervals represent segments on a line or curve and the second column shows how the relations can be translated into natural-language expressions for the spatial domain. The symbolic notation (column 3) for the original temporal relations is maintained for the spatial relation. Note that the pictorial examples (column 4) with the two rectangles X (gray) and Y (black) are two-dimensional only for graphical reasons but are in fact one-dimensional. In the last column the relations are defined on the basis of the startpoint s and endpoint e of the two intervals (column 5).

Reasoning based on the interval relations is based on their compositions. Given the qualitative relation between X and Y and also between Y and Z, such a composition is defined as the relation between X and Z. Since each relation can be combined with any other, there are 169 compositions. In his paper, Allen (1983) proposed a constraint-satisfaction algorithm to compute the compositions and even if reasoning with the full calculus of 2^{13} relations is NP-hard, reasoning

Table 1

The 13 Qualitative Interval Relations Used in the Primary Tasks

Expressions from Allen (1983) (temporal)	Natural-language description for the spatial domain	Symbol	Graphical example	Point ordering (s = start point, e = endpoint)
before	X lies to the left of Y	X < Y		$s_X < e_X < s_Y < e_Y$
meets	X touches Y at the left	X m Y		$s_X < e_X = s_Y < e_Y$
overlaps	X overlaps Y from the left	X o Y		$s_X < s_Y < e_X < e_Y$
starts	X lies left-justified in Y	X s Y		$s_Y = s_X < e_X < e_Y$
during	X is completely in Y	X d Y		$s_Y < s_X < e_X < e_Y$
finishes	X lies right-justified in Y	X f Y		$s_Y < s_X < e_X = e_Y$
equals	X equals Y	X = Y		$s_X = s_Y < e_Y = e_X$
finishes-inverse	X contains Y right-justified	X fi Y		$s_X < s_Y < e_Y = e_X$
during-inverse	X surrounds Y	X di Y		$s_X < s_Y < e_Y < e_X$
starts-inverse	X contains Y left-justified	X si Y		$s_X = s_Y < e_Y < e_X$
overlaps-inverse	X overlaps Y from the right	X oi Y		$s_Y < s_X < e_Y < e_X$
meets – inverse	X touches Y at the right	X miY		$s_Y < e_Y = s_X < e_X$
after	X lies to the right of Y	X > Y		$s_Y < e_Y < s_X < e_X$

Note: col. 1: the original names introduced for temporal reasoning; col. 2: the natural language descriptions for the spatial domain; col. 5: the order of starting points and endpoints (adapted and augmented according to Allen, 1983).

with the 13 base relations can be performed in polynomial time (Vilain & Kautz, 1986; Vilain, Kautz & van Beek, 1990).

Reasoning with the interval relations is very similar to the most frequently used psychological paradigm for investigating human spatial reasoning, which is usually referred to as three-term series problem or linear syllogism (e.g., Johnson-Laird, 1972). Such a problem that uses the exact technical meaning from Allen's calculus is:

> *The red rectangle lies to the left of the green rectangle.*
> *The green rectangle overlaps the blue rectangle from the left.*
> *Does it follow: The red rectangle lies to the left of the blue rectangle?*

For the sample problem, the conclusion does follow and participants in a psychological study should answer the question with YES.[2] Instead, human subjects should answer NO, when asked to verify, for instance, the following statement:

> *The red rectangle touches the blue rectangle from the left. Yes/No?*

In cognitive psychology, the first two statements are called premises, and the participants' task is to decide whether the third statement, referred to as conclusion, follows from the premises. Note that the term *conclusion* is not used in the usual logical sense, but just as the third sentence of a three-term-series problem that has to be verified. The red and the blue rectangle are referred to as *end terms*, and the green rectangle is the middle term that connects the two end terms.

RELATIONAL REASONING IN PSYCHOLOGY

In the psychology of reasoning, spatial three-term-series problems have been studied extensively (Byrne & Johnson-Laird, 1989; Careirras & Santamarìa, 1997; Hagert, 1985; Johnson-Laird & Byrne, 1991; Knauff, 1999; Knauff & Johnson-Laird, 2002; Knauff, Rauh & Schlieder, 1995; Knauff, Rauh, Schlieder & Strube 1998a, 1998b; Maki, 1981; Mani & Johnson-Laird, 1982; Rauh, 2000). Yet, there is still controversy about how the findings can be integrated into a general theory of human spatial reasoning. Three accounts have received the most attention: The main idea of the *sentential account* (Braine & O'Brien, 1998; Rips, 1994) is committed to the orthodox view of AI that logic representations together with forms of logic inference be sufficient to exhibit intelligent behavior. Accordingly, human reasoning is described as application of language-like formal rules of inference. The language-based rules for *modus ponens* and transitive inference are used to solve inference problems by introducing and eliminating sentential connectives. This process is carried out by transferring the inference rules into working memory and applying them to the given premises, which are also represented in a language-like format.

Two other approaches developed very quickly, influenced by the mental imagery debate in psychology in the early 1980's, which revolved around the question whether cognitive processes rely on a single representational format or

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[2] Our experiments were done in German. In English translation, our verbalizations of the interval relations may not be fully adequate.

on different sorts of representation (see Block, 1981; Tye, 1991 for an overview). The key idea of the *visual account* is that reasoners imagine the information from the premises in a visual mental image and inspect this vivid visual image to draw a conclusion. Technically speaking, this account is reflected in the *diagrammatic reasoning approaches*. In present psychological theories, mental images are defined as structurally similar to perceptions and likewise represent colours, shapes, and spatial extent, they can be rotated and scanned, and have a limited resolution (Finke, 1989; Kosslyn, 1980). Mental operations on visual images are seen as isomorphic to corresponding operations on real perceptions. Reasoning, from this point of view, is to "look" mentally at a visual mental image to find new information not explicitly given in the premises (De Soto, London & Handel, 1965; Glenberg, Kruley & Langston, 1994; Glenberg & Langston, 1992; Huttenlocher, 1968; Kosslyn, 1994; Rinck, Hähnel, Bower & Glowalla, 1997).

The third account is *spatial*, postulating that reasoning relies on the construction and manipulation of spatially organized mental models (Johnson-Laird, 1983; Johnson-Laird & Byrne, 1991; Knauff, et al., 1998a; Knauff & Johnson-Laird, 2002). Mental models represent a possible state of affairs described in the premises. Reasoning is conceptualized as a cognitive process, in which spatially organized models of the given premises are constructed and alternative models are sequentially generated and inspected. A conclusion is true if it holds in all models that can be built from the premises. Johnson-Laird (1998) emphasized that spatially organized mental models are not to be identified with visual images. In contrast to visual images, mental models are representations in form of spatial arrays that are not restricted to the visual modality (Johnson-Laird, 1996; Johnson-Laird & Byrne, 1991; Knauff & Johnson-Laird, 2002). Such a model, according to the theory of mental models, suffices for reasoning: It captures the relevant spatial properties qualitatively. Hence, reasoning with a relation of the form: *A is to the left of B*, derives merely from the meaning of the relation and its contribution to models of assertions. Obviously, the mental model account corresponds to the central assumption of *QSR*.

So, what happens in the mind if humans reason with Allen's interval relations? Are the underlying cognitive processes inherently visual or spatial, or even pure logic? To investigate these questions in the following experiments we have used the concept of resource limitation. The underlying idea of this well-established paradigm is that cognitive subsystems have limited capacities. Hence, if tasks interfere, then they share the same cognitive subsystem. If not, they appear to be carried out in different systems (Bourke, 1997; Gopher & Donchin, 1986, Navon & Gopher, 1979). In our experiments, participants had to solve inference problems based on the spatial interpretation of the interval calculus introduced by Allen (1983). Henceforth we refer to these problems as *primary tasks*. They were sometimes solved concurrently with one of five other tasks, which we henceforth refer to as *secondary tasks*. These were maximally concordant in all relevant aspects except for the variation in the visual and spatial components. As a *verbal secondary task*, we used the well-known technique of articulatory suppression to preoccupy the subsystem usually involved in sentential proofs of derivation (e.g., Baddeley, 1986, p. 79). The other four secondary tasks were presented visually or acoustically, and were either spatial or non-spatial. *Visual secondary tasks* were visual but not spatial, *visuo-spatial secondary tasks* were visual and spatial, *spatial secondary tasks* were spatial but not visual, and a *control secondary task* was neither visual nor spatial (see the Method section of Experiment 1 below for an exact description of the tasks).

The competing hypotheses make the following predictions:

> *Sentential account*: Since reasoning with the interval relations is based on language-based proofs of derivation, articulatory suppression (1) should impede reasoning performance.
>
> *Visual account*: Since reasoning relies on visual imagery, modality-specific interference between reasoning problems and visual as well as visuo-spatial secondary tasks (2 and 3) should appear.
>
> *Spatial account*: Since the spatial subsystem is responsible for reasoning, cross-modal interference between reasoning and visuo-spatial and spatial secondary tasks (2 and 4) should appear.

None of the accounts predicts an effect of the control secondary task (5). Before the main experiments started, all participants accomplished a *learning phase* in which they learned the exact meaning of the interval relations. This was done to avoid a confusion of inferential and conceptual cognitive adequacy (Knauff, Rauh, and Renz, 1997). In addition this procedure guaranteed that participants realized the one-dimensional nature of the relations and the inferences. They learned that the inferences only hold for rectangular aligned regions and that only these inferences are in the scope of the experiments. The learning phase started with descriptions of the spatial relationship of a red and a blue interval using the 13 qualitative relations (in German). Each verbal description was presented with a short commentary about the location of the startpoint and endpoint of the two intervals (block) together with a picture with a red and blue interval that matched the description. In a subsequent phase the participants were tested for the understanding of the relations: this phase consisted of trials, during which participants are presented with the one-sentence description of the red and blue interval. They then had to determine the start points and endpoints of the intervals by mouse clicks. After confirmation of the final choices, the participants were told whether the choices were correct or false. If they were false, verbal information about the correct point ordering was given. Trials were presented in blocks of all 13 relations in randomized order. The learning criterion for one relation was accomplished if the participant gives correct answers in 3 consecutive blocks of the corresponding relation. The learning phase stops as soon as the participant reaches the learning criterion for all thirteen relations. This procedure has already been used in previously published experiments (Knauff, 1999; Knauff, et al., 1998a).

EXPERIMENT 1

Method

Participants

Forty-eight undergraduate students of the University of Freiburg participated in the experiment. They were paid for their participation. All participants successfully accomplished the learning phase and thus were familiar with the exact meaning of the relations—and their limitations.

Materials

Primary tasks. All main tasks were presented as sentences on the computer screen. The primary tasks were structured exactly as illustrated in the example given above. There were two premises and one conclusion and the participants were asked to evaluate whether or not the conclusion follows from the premises. From the 169 possible compositions of the base relations we used only those with a definite solution (i.e., a single relation, not a disjunction of several relations), and also avoided the trivial compositions with '='. From this set of problems, we selected 48 problems, so that we had eight tasks for each of the five secondary tasks conditions and control. Half of the problems were valid, the other half invalid. To ensure that the participants could not anticipate the relation about which they would be asked next, we asked for one of the two premises in 50 percent of the problems, while in the other 50 percent the participants had to verify a conclusion. The colors *red*, *green*, and so forth, were used to avoid any enumeration (such as *first, second, third* or *A, B, C*) that could have an effect on the inference.

Secondary tasks. In the *articulatory suppression* condition, participants had to continuously repeat a number of digits (1-2-3), which is an established method to produce language-based interference (Baddeley, 1990). In the visual secondary tasks, a gray square was displayed in the middle of the screen and its brightness varied in five discrete steps. Participants had to decide whether it became darker or lighter. In the visuo-spatial secondary tasks, the square displayed on the computer screen was shifted to the left or to the right, and the participants had to determine the direction. In the spatial secondary tasks, a sinus tone was presented binaurally via stereo headphones (due to relative loudness in the left and right channel, there were five apparent positions: left, mid-left, center, mid-right and right), and participants had to determine whether the tone shifted to the left or to the right. In the control secondary tasks, the tones were centered and participants had to decide whether the tone became louder or softer. All secondary tasks consisted of five different states of the stimuli and began with the middle one. After the first stimulus, the next stimulus was selected randomly according to one of the four characteristics. Participants had to compare each stimulus n to its direct predecessor $n - 1$, making a total of $n - 1$ decisions. The control secondary tasks were thus used to control for all processes that might work on top of the visual and spatial components.

Procedure

The participants were tested individually in a quiet room under constant illumination of the computer screen. They wore headphones for the entire experiment. They sat in front of a computer that administered the instructions and the tasks for all phases of the experiment. An assistant to the experimenter was in the room for the duration of the experiment, separated from the subject by a small screen on the table. The participants were not allowed to use any utilities, to draw sketches, or to use their fingers. The difficulty of the secondary tasks was controlled by separate norming trials at the beginning of the experiment.

Each premise and conclusion was presented as sentence on a separate screen and participants proceeded from one to the next by pressing a key on a response box (Potts & Scholz, 1975). Participants had at most 10 seconds to read a premise. If they did not progress to the next screen during this time, the program automatically went on. The 10-second limit was used to prevent participants from learning the premises by heart and was determined by the average premise reading time plus two standard deviations in previous experiments (Knauff, et al., 1998b). There was no time limit for the conclusion. It was presented centred on the screen and participants responded by pressing one of two keys on the response box.

The secondary tasks were presented on a separate 14-inch CRT, or via stereo headphones. The secondary tasks started after the second premise and before the conclusion appeared. In half of the problems the secondary tasks continued for 7.5 seconds, in the other half for 15 seconds. The order of these two conditions was randomized across participants. To avoid a possible confounding of the secondary tasks with other spatial aspects evoked by pressing further buttons on different positions (moving the hand from one to the other), participants responded to the secondary task verbally by saying *right* or *left*, *lighter* or *darker*, *louder* or *softer*. To code participants' responses, the job of pressing associated buttons was taken over by the assistant to the experimenter. In the articulatory suppression condition participants repeated the three digits 1, 2, 3 continuously. The number of generated digits was recorded. In the undisrupted reasoning condition, participants were not requested to solve any secondary tasks, but only to wait until the conclusion appeared. To examine the influence of individual differences in skill and strategy use, after the experiment the participants were interviewed about the strategies they applied to solve the problems.

Design

The participants acted as their own controls and solved the main tasks alone (henceforth referred to as baseline) and under all secondary task conditions in a within-subjects design. Eight trials (main tasks) were performed under each of the five secondary task conditions and baseline, making a total of 48 problems. The combinations and the order of main and secondary tasks were counterbalanced across the participants.

Table 2

Percentages of Correct Responses to the Secondary Tasks, as Performed Separately at the Beginning of the Experiments

	visual	visuo-spatial	spatial	control
% correct	96.1	96.9	97.0	95.1

Table 3

Percentages of Correct Responses and Mean Response Times (in seconds) in the Primary Tasks as a Function of the Secondary Tasks and in the Baseline Condition

	baseline	visual	visuo-spatial	spatial	control	art. sup.
% correct	85.2	84.0	79.4*	77.1*	80.0	82.6
response time	5.13	5.05	5.13	5.58*	5.08	5.31

Note: An asterisk indicates a statistically significant difference with respect to the baseline condition ($p < .05$)

Results and Discussion

The measurements taken during the norming trials at the beginning showed that the average number of correct responses for all secondary tasks was about 95 percent and there were no significant differences between the tasks themselves (see Table 2).

The first row in Table 3 shows the mean number of correct responses in the baseline condition (reasoning with interval relations alone) and under the five suppression conditions. As there was no reliable difference between the patterns of results for the two retention intervals, we pooled the results. The overall effect of the suppression conditions is statistically reliable, Friedman analysis of variance,[3] $F(3) = 12.073$; $p < .05$. Although these tests are not orthogonal, the separate comparisons show that visuo-spatial, Wilcoxon test $z = 2.117$; $p < .05$, and spatial, Wilcoxon test $z = 2.886$; $p < .05$, secondary tasks impeded reasoning with interval relations compared to the baseline condition. In contrast, the visual and the control secondary tasks, as well as articulatory suppression, did not impair participants' performance in the reasoning problems. The differences between recognition and inference problems are only marginal and therefore not presented in the table.

As can be seen in the second row in Table 3, the differences are also reflected in the time participants needed to verify the conclusions. Again, we obtained a similar pattern of results for recognition and inference problems, and pooled them. The overall effect of the secondary task conditions is statistically reliable, Friedman analysis of variance, $F(3) = 11.927$; $p < .05$. The separate comparison

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[3] Because of non-normal distributions and inhomogeneity of variances, all tests used here are nonparametric and appropriate for assessing the significance of differences in within-subjects experiments, Siegel & Castellan, 1988.

of the conditions shows that participants responded slower to the inference problems with spatial suppression, Wilcoxon test $z = 2.45$; $p < .015$, whereas visual and control secondary tasks, as well as articulatory suppression, had no significant effect on the response times in the main tasks. Finally, we also pooled the spatial and the visuo-spatial secondary task conditions and the visual and visuo-spatial secondary tasks conditions, respectively. The analyses showed a significant effect of the spatial component, Wilcoxon test $z = 1.97$; $p < .05$, but no effect of modality.

To ensure that the results were not affected by a shift of attention from the primary to the secondary tasks (Bourke, 1997; Gopher & Donchin, 1986; Navon & Gopher, 1979), we compared the relative number of correct responses in the norming trials (when no primary task was applied; see Table 2) with the dual-task conditions. These analyses did not show any reliable differences between the data in Table 2 and the performance in the main experiment (visual 95.7%, visuo-spatial 93.6%, spatial 94.9%, and control 96.6% correct).[4]

Although the interview method has strong limitations, it can provide some clues on how the participants solved the problems (or at least think that they did). In fact, all subjects reported having used a visuo-spatial strategy for solving the reasoning problems. They reported having integrated the three objects into a single model or image, and differences between subjects were just present in the level of detail of these images (e.g., whether they explicitly contained all three colours or not).

The experiment was conducted to investigate the sentential, visual, and spatial components of reasoning with the interval relations. The results of the experiment establish the importance of distinguishing between these types of processes in human spatial reasoning. First, no significant contribution of a language-based subsystem to such tasks was found—articulatory suppression that usually disrupts logic-based reasoning did not affect reasoning performance. This result is in agreement with other findings (e.g., Klauer, Stegmaier, & Meiser, 1997). Second, concurrent visuo-spatial and spatial secondary tasks significantly reduced the number of correct answers and the responses took longer compared to reasoning tasks alone. Indeed, the impairment of reasoning performance was obtained under visuo-spatial suppression as well as under audio-spatial suppression, and there was no significant difference between these two conditions. In contrast, the purely visual secondary tasks, which referred to non-spatial but visual object properties, had no significant effect on reasoning performance in the main tasks. Accordingly, when we pooled the response latencies in the two spatial and the two non-spatial conditions, a significant effect of the spatial component was found, whereas the comparison of the two visual with the acoustical suppression conditions did not yield a significant effect of modality.

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[4] Due to a technical problem, this secondary task analysis is based on only 24 of the 48 participants.

The results appear to corroborate the *spatial hypothesis* that the spatial representations and processes are utilized in reasoning and, conversely, do not support the *visual hypothesis*. An alternative account of our results, however, makes no appeal to the visual or spatial nature of reasoning processes. It is conceivable that the critical difference between spatial and non-spatial secondary tasks is that participants in the former tasks had to verbalize the prepositions *left* and *right* which were also used in the reasoning problems, whereas the premises did not use the expressions from the non-spatial secondary tasks (louder–softer, lighter–darker). Under these circumstances, the observed results can be treated also as a language-based interference effect between the spatial prepositions in the premises and the verbal response to the spatial secondary task.

EXPERIMENT 2

In this experiment, the difficulty of the problems was systematically manipulated. Our aims were (1) to avoid using problems that were invariably easy, and (2) to examine the possibility of an interaction between difficulty and the secondary tasks. The experiment also avoids the problems we encountered in Experiment 1, by making some minor changes in the experimental procedure.

Method

Participants

Forty-eight students from the same population as in Experiment 1, ranging in age from 22 to 30 years, participated in the experiment. All participants successfully accomplished the learning phase and none of the participants had attended in the first experiment.

Materials

Primary task. The same problems as in Experiment 1 were used, but we also manipulated the difficulty of the inferences by using converse relations. The reasoning problems were of the form exemplified in the following examples:

easy:	a R b	hard:	b R' a
	b R c		c R' b
	a R c?		a R c?

where R denotes the relation that holds between the two objects a and b. The recognition problems were obtained by repeating one of the two premises literally or by using the converse relation:

easy:	a R b	hard:	a R b
	b R c		b R c
	a R b or b R c?		b R' a or c R' b?

The participants again had to evaluate whether or not the conclusion followed from the premises. Half of the problems were valid, the other half invalid.

Secondary tasks. Compared with Experiment 1, only the control secondary task was changed. In this condition, now centered tones were presented and participants had to decide whether the pitch of the present tone was higher or lower than that of the previous tone. Similar to the other secondary tasks, we used five different pitches and the secondary tasks always started with the tone in the middle pitch. The difficulty of this task did not differ reliably from the other secondary tasks. Since, in agreement with the literature, articulatory suppression did not have a significant effect in Experiment 1, we dropped this condition.

Procedure

The procedure was almost identical to the first experiment except that participants also responded to the secondary task by pressing a button on the response box. Participants responded by pressing a single button once for *yes* or twice for *no*, respectively, in the primary and secondary tasks, in order to avoid a further spatial aspect (movement of the hand to different keys). The assignment of the keys and the number of key presses was counterbalanced over the set of participants. To increase the efficiency of the experimental procedure, the training phase was shortened and the block with the single primary task was presented at the beginning of the experiment. Again, after the experiment the participants were interviewed about the strategies they applied to solve the problems.

Results and Discussion

As expected, participants gave more correct answers to the easy reasoning problems (79.5%) than to the difficult reasoning problems (71.0%), but there was no interaction with the secondary tasks. We then analyzed the effect of the secondary tasks on the easy problems. The overall effect of the suppression conditions is statistically reliable, Friedman analysis of variance, $F(3) = 37.255$; $p < .001$. Moreover, as shown in the first column of Table 4, the pattern of interference matches with the first experiment. In both modalities, the interference was greater if the secondary tasks had a spatial content rather than a non-spatial content. The main effect of the spatial vs. non-spatial secondary tasks is statistically significant, Wilcoxon test $z = 2.19$; $p < .05$, whereas the modality had no significant effect on the performance in the main tasks. The (non-orthogonal) separate comparison of the four suppression conditions with baseline performance shows that reasoning errors increased under visuo-spatial suppression, Wilcoxon test $z = 2.885$; $p < .05$, and under spatial suppression, though the latter was not significant. Neither the visual nor the control secondary tasks impaired participants' reasoning accuracy.

The separate analysis of hard problems yielded the surprising result that there was neither a main effect of the spatial component nor of the modality with

Table 4

Mean Percentages of Correct Responses and Response Times (in seconds) in the Primary Task with Concurrent Execution of the Secondary Tasks, and in the Baseline Condition

	Baseline	visual	visuo-spatial	spatial	control
% correct $^+$	82.3	83.3	67.4*	78.5	85.8
response time	7.84	8.78	9.52*	8.81*	7.91

Note: An asterisk indicates a statistically significant difference with respect to the baseline condition ($p < .05$)

$^+$ This analysis is based on easy problems only.

respect to errors. This might be a floor effect (performance was around 60% correct, given a chance rate of 50%), or an effect that is specific to the interval relations and thus not yet understood.

The results for the response times corroborate the findings of the easy problems and the results of Experiment 1. Overall, the main effect of the suppression conditions is statistically reliable, Friedman analysis of variance, $F(3) = 15.415$; $p < .005$. Separate comparisons of the suppression conditions with baseline show that participants responded slower to the main tasks under visuo-spatial suppression, Wilcoxon test $z = 3.15$; $p < .002$, and under spatial suppression, Wilcoxon test $z = 2.18$; $p < .05$, whereas the other two suppression conditions had no reliable effect. The mean response times in the primary tasks as a function of the secondary tasks are presented in the second column of Table 4.

In the secondary task analysis, we compared the mean number of correct responses in the baseline condition and the dual-task conditions as in Experiment 1. Overall, participants did not carry out the secondary tasks exactly as in the first experiment, but the data again showed that the obtained results for the primary tasks were not affected by an attention shift from the main tasks to the secondary tasks. In fact, none of the four comparisons between baseline condition and the dual-task condition yielded a statistically significant difference.

In the interview, participants again reported having used a visuo-spatial strategy in which they integrated the three objects into a single model or image. Differences between subjects again were just present in the level of detail of these models.

The results again support the spatial account of reasoning with the interval relations. Although the accuracy data are indisputable only for the easy problems, it is an open question whether the null-effect in difficult problems is due to methodological problems or whether they reflect a particular kind of processes, which are not yet understood. However, the response times for the whole set of problems are unequivocal. Participants responded slower to the main tasks under visuo-spatial and spatial suppression, whereas the visual secondary task (and the control secondary task) had no reliable effect on the response time in the reasoning problems.

GENERAL DISCUSSION

We conducted two dual-task experiments to determine the sentential, visual, and spatial components underlying human spatial reasoning. The findings of Experiment 1 do not agree with the sentential account of reasoning. If reasoning is mainly a language-based process, then articulatory suppression should impair reasoning performance, whereas spatial and/or visual secondary tasks should not have a negative effect. The results of both experiments also present difficulties for a visual account of reasoning. If reasoning is inherently visual, it should be carried out in a visual cognitive subsystem and we should have found significant interference of the reasoning problems with any kind of visually presented secondary task.

The reported results agree best with the spatial account of reasoning. They also converge with other psychological findings from studies that investigated the effects of visual and spatial imageability and reasoning. Egan and Grimes-Farraw (1982) analyzed retrospective reports of participants who solved three-term series problems and found that those who used visual mental imagery were even worse in solving the problems than participants with more abstract strategies. In our studies, the interviews provided additional evidence in this direction, but due to the vagueness of the interview method should not be overestimated. However, Knauff and Johnson-Laird (2000, 2002) report results that point in a similar direction. In these studies, we investigated relational reasoning and manipulated the ease of envisaging the materials as visual images and spatial layouts. The outcome demonstrates that materials that are easy to visualize impair reasoning unless they are also easy to envisage spatially. We refer to this effect as *visual-impedance-effect* (Knauff & Johnson-Laird, 2002). The fact that visual imageability does not improve reasoning is also reported in studies that did not find differences in reasoning accuracy for imaginable and less imaginable reasoning problems (Sternberg, 1980; Richardson, 1987; Johnson-Laird, Byrne & Tabossi, 1989; Newstead, Pollard & Griggs, 1986). In Knauff and May (in press), we found that congenitally totally blind persons (who experience no visual mental images) are even better in reasoning problems that evoke visual images in the sighted.

Support for the spatial account also comes from functional brain imaging studies. In a study by Knauff, Mulack, Kassubek, Salih, and Greenlee (2002), three-term-series inferences activated a bilateral parietal-frontal network distributed over parts of the prefrontal cortex, the inferior and superior parietal cortex, and the precuneus, whereas no significant activation was measured in early visual areas, usually activated by visual imagery (Kosslyn et al., 1999; Kosslyn, Thompson, Kim, & Alpert, 1995; Sabbah et al., 1995; a contrasting result is reported in Knauff, Kassubek, Mulack, & Greenlee, 2000). In fact, reasoning activated regions of the brain that make up the *where-pathway* of spatial perception and working memory (e.g., Ungerleider & Mishkin, 1982; Smith et al., 1995). In contrast, the *what-pathway* that processes visual features such as shape, texture, and color (Landau & Jackendoff, 1993; Rueckl, Cave, & Kosslyn, 1989; Ungerleider, 1996) seems not to be involved. Other studies have corroborated these findings (Goel & Dolan, 2001; Osherson et al., 1998; Prabhakaran, Smith, Desmond, Glover, & Gabrieli, 1997).

Another conclusion of our results is that visual imagery is dispensable for reasoning. This finding, however, is probably limited to the kind of inferences studied in this article. Knauff, Fangmeier, Ruff, and Johnson-Laird (2003) have recently shown that the cognitive processes during reasoning depend on the nature of relations. In general, reasoning elicits spatial representations and processes, but highly visual problems in addition elicit visual images. Since participants in reasoning experiments usually report at least some imagery, this leaves us with the conclusion that visual imagery is not functional (i.e., helpful) in spatial reasoning with Allen's relations.

Another limitation of our studies is due to the characteristics of Allen's interval relations. As we already mentioned in the introduction we used the compositions from a one dimensional space and applied them to rectangles. This approach certainly underestimates the ability to generalize from one-dimensional to two-dimensional space. In particular, a setting with non-rectangular objects offers more degrees of freedom so that some constraints inherent in the one-dimensional setting do not hold anymore. The extension of Allen's interval calculus for other shapes in fact may lead to incorrect inferences. This could also have confused the participants, because they did not envisage only rectangles as objects. Although this is a critical point in our studies, we at least partially dealt with it by instructing and training the participants very carefully. In fact, the participants underwent a long learning session, to minimize such misunderstandings and given the relatively high rate of correct responses it is reasonable to assume that they indeed have learned the limitations of the used materials. Further evidence will be needed before a detailed and less restricted modeling of the inference process is possible. A series of ensuing experiments will be concerned with the *inferential cognitive adequacy* of more general approaches such as the RCC-8 calculus (Randell, et al., 1992) and the 9-intersection calculus by Egenhofer and colleagues (Egenhofer, 1991; Egenhofer, & Franzosa, 1991). The *conceptual cognitive adequacy* of these approaches has already been proven in our previous studies (e.g., Knauff, et al., 1997; Renz, Rauh, & Knauff, 2000). The present studies can nevertheless be used to describe an important subset of (human) spatial reasoning.

CONCLUSIONS

Reasoning with the interval relations introduced by Allen (1983) relies on spatial representations and processes. Neither formal rules of inference nor visual mental imagery appear to play an essential role in such inferences. This view is consistent both with the finding that reasoning with the interval relations interferes with spatial secondary tasks (but not with visual) and that verbal secondary tasks did not hinder reasoning. The phenomena are consistent with the theory of mental models. Reasoning is based on the construction, inspection, and variation of spatial representations. These spatial representations are models of the spatial relations among entities, that is, they represent *which* things are *where*. In inferential tasks, spatial models are likely to exclude visual detail, to represent only the information relevant to inference (Johnson-Laird, 1998; Knauff, 1999; Knauff, et al., 1998a). This account agrees with the central assumptions of QSR. But is QSR thus akin to human reasoning? On the one hand, *cognitive validity* can only be claimed in comparison to other approaches that arise with different predictions (Knauff, et al., 1997). In the light of the reported results, QSR indeed appears to be cognitively more plausible than traditional propositional approaches, which are strongly related to the psychological theories of mental proofs (or vice versa, Rips, 1994). The data also present difficulties for DR accounts of reasoning, since the main assumption that visual representations are essential to inference is not supported by the presented findings. A word of caution, however, is that the visual and spatial nature of representations in reasoning depends on the nature of the problem. Reasoning with Allen's interval relations might elicit spatial representations, but reasoning with other problems might elicit visual images in addition.

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