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Belief revision is the process of changing one's beliefs when a newly acquired fact contradicts the existing belief set. Psychological research on belief revision mostly used conditional reasoning problems in which an inconsistency arises between a fact, contradicting a valid conclusion, and the conditional and categorical premises. In this paper, we present a new experimental paradigm in which we explore how people change their mind about the location of objects in space. The participants received statements that described the spatial relations between a set of objects. From these premises they drew a conclusion which then, in the next step, was contradicted by a new, irrefutable fact. The participants' task was to decide which of the objects to relocate and which one to leave at its initial position. We hypothesised that this spatial revision process is based on mental models and is affected by the functional asymmetry between reference objects (RO) and the located objects (LO) of spatial relations. The results from two experiments corroborate this hypothesis. We found that individuals have a strong preference to relocate the LO of the premises, but avoid relocating the RO. This is a novel finding and opens up new avenues of research on how humans mentally revise their beliefs about spatial relations between entities in the world.

Keywords: Belief revision; Mental models; Spatial reasoning.

To understand how people change their opinion over time or in the light of new information that does not agree with their current belief is one of the most fascinating questions of psychology. In daily life, the underlying processes are highly complex and affected by several cognitive, emotional, motivational, and social factors (Gardner, 2006; Gärdenfors, 1988; Kyburg, 1983). Given this complex interplay of many factors, one might think that it is quite unsatisfying if cognitive psychologists select just one of these factors, sometimes even one of minor importance, and then squeeze it into an experimental paradigm, that often seems to be very far away from how people make decisions in daily life. However, cognitive psychologists still adopt this approach for many reasons and indeed—by using this approach—were quite successful in understanding some aspects of human belief revision. This research almost exclusively uses an experimental paradigm in which participants are confronted with conditional sentences (premises) that posit that if Proposition A is true then Proposition B is true. The work in this paradigm shows that belief revision is effected by many factors, including asymmetries between particular facts and general laws (Revlis, Lipkin, & Hayes, 1971), conditional and categorical premises (Dieussaert, Schaeken, De Neys, & d'Ydewalle, 2000; Elio & Pelletier, 1997; Girotto, Johnson-Laird, Legrenzi, & Sonino, 2000; Revlin, Cate, & Rouss, 2001), major and minor premises (Politzer & Carles, 2001), and

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reliable and unreliable information sources (Wolf, Rieger, & Knauff, 2012).

In the present paper, we seek to investigate human belief revision within an even more basic experimental paradigm. Our motivation is to pinpoint some of the most essential revision processes by using a particularly simple task that is not affected by people's prior knowledge, preexisting beliefs, or long-term convictions, etc. Another intention is that we do not want to put too much "logic" into our problems. The reason is that reasoning with conditional statements itself is a highly complex research field and that we are still far away from understanding the underlying cognitive processes (Byrne & Johnson-Laird, 2009; Oaksford & Chater, 2010; Oberauer, 2006). For this reason, we developed a spatial belief revision paradigm in which participants receive information about the location of objects in space but then have to revise their initial assumptions in the light of new information. First, the participants received a premise that described the spatial relation between two objects, e.g., "A is to the left of B". Then they received a second premise that described the spatial relation of one of these objects to a third object, e.g., "B is to the left of C". From these two premises the participants inferred that the three objects are in the arrangement A – B – C. However, they were then confronted with an additional statement, e.g., "A is to the right of C". This is the critical point in time where the participants in our experiments had to realise (and they usually did) that something must be wrong with their initial assumption about the layout of the three objects. Not all three statements can be true at the same time because the third statement contradicts the logical inference from the first two premises. One option would be to simply ignore the third statement. But that was forbidden in our experiments because the fact has been defined as indisputably true. The only option is to decide which one of the first two premises may be abandoned. If the first premise is discarded this results in the arrangement B – C – A; if the second premise is discarded this results in the arrangement C – A – B. In other words, in the arrangements B – C – A, the first statement is rejected, but the second (and third) is retained, whereas in C – A – B the second statement is rejected, but the first (and the third) statement is kept. The general structure of the problems is as follows:

Premise 1:	A is to the left of B
Premise 2:	B is to the left of C
Initial model:	A – B – C
Contradictory fact:	A is to the right of C
Revision alternative 1 [discard P1, retain P2]:	B – C – A
Revision alternative 2 [retain P1, discard P2]:	C – A – B

The two revision strategies lead to two new spatial arrangements which are illustrated in Figure 1. In the first revision strategy (left diagram in Figure 1), for instance, individuals mentally move the first object (A) in the array from the leftmost to the rightmost position in the array. In the second revision strategy (right diagram in Figure 1), individuals move the last object (C) in the array from the rightmost to the leftmost position in the array.

Do people prefer to relocate Object A or Object C? Which of the two revised layouts (in Figure 1) will be produced more often? The following studies are based on three general assumptions:

(1) Spatial reasoning relies on mental models. A mental model is a unified representation of what is true if the premises are true. Spatial relations in such models are not represented explicitly in a propositional format. Rather they are inherent in the model and thus can be (and must be) "read off" from the model by mental inspection processes.

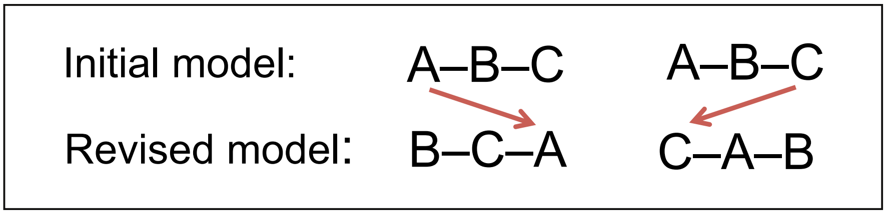

Figure 1. Two revision strategies for the contradictory fact: "A right of C." Left: LO-relocation; individuals mentally pick the first object in the array and move it to the end of the array. Right: RO-relocation; individuals choose the last object in the array and move it to the first position in the array. In both versions, the middle term B connecting the two premises remains untouched.

(2) Spatial belief revision relies on the revision of mental models. People revise a model if newly available information is inconsistent with the current model (and the new information must be taken for granted). The revision process relies on local transformations in which tokens in the model are moved to new positions. If not all available information can be true at the same time, people ''decide'' which of the information to retain and which one to discard.
(3) The model revision process is sensitive to the functional asymmetry between the reference object (RO) and the located object (LO). For instance, in the statement ''A is to the left of C'', the C is the RO and the A the object that is located in relation to the RO. The distinction has been made by several psychologists and linguists (Landau & Jackendoff, 1993; Miller & Johnson-Laird, 1972; Talmy, 1983).

The first assumption (that spatial reasoning relies on mental models) represents our general theoretical framework, the mental model theory of reasoning (Johnson-Laird, 1983, 2006; Johnson-Laird & Byrne, 1991). The theory is supported by many experimental findings in the area of relational reasoning. These studies have shown that people construct mental models from spatial and temporal relations and that reasoning with multiple spatial mental models is harder than reasoning with a single model (Boudreau & Pigeau, 2001; Carreiras & Santamaria, 1997; Schaeken & Johnson-Laird, 2000; Schaeken, Johnson-Laird, & d'Ydewalle, 1996). Reasoning also gets harder if the premises are more difficult to integrate into a unified mental model (Byrne & Johnson-Laird, 1989; Ehrlich & Johnson-Laird, 1982; Nejasmic, Krumnack, Bucher, & Knauff, 2011) or if the problem evokes irrelevant visual images (Knauff & Johnson-Laird, 2002). Other theories of relational inference cannot readily explain these findings (Carreiras & Santamaria, 1997; Goodwin & Johnson-Laird, 2005; Johnson-Laird & Byrne, 1991; Juhos, Quelhas, & Johnson-Laird, 2012; Knauff, 2009, 2013; Roberts, 2000).

The second assumption (that spatial belief revision relies on the revision of mental models) creates a new link between the model theory and spatial belief revision. Models should not be confused with beliefs, but models give rise to beliefs (see General Discussion). How people revise models is still one of the main research questions within model theory and intimately linked to the variation (and validation) of mental models. Recent empirical studies suggest that people do not create alternative models by entering a loop of model construction and model inspection (Bucher, Krumnack, Nejasmic, & Knauff, 2011; Bucher & Nejasmic, 2012; Krumnack, Bucher, Nejasmic, & Knauff, 2011). Rather, they start from the preferred model and alter this model by local transformations (Ragni, Knauff, & Nebel, 2005; Rauh et al., 2005). Annotations are used to maintain those aspects of the premise information that are necessary to vary the model (Vandierendonck, Dierckx, & De Vooght, 2004).

In the studies reported later, we focus on the third assumption (that the model revision process is sensitive to the functional asymmetry between RO and LO). In principle, it is logically possible that spatial relations could be mentally represented as propositions of the form r(A, B), where A and B are the objects to be located (Landau & Jackendoff, 1993). In human spatial cognition, however, the dominant way to represent spatial relations between objects is asymmetrical (Landau & Jackendoff, 1993; Miller & Johnson-Laird, 1976). Usually, one object is considered to be the reference object (RO), and the other is the located object (LO). Some authors refer to the located object as figure and the reference object as ground; others distinguish between locatum and relatum (Landau & Jackendoff, 1993; Talmy, 1983). The common idea of all these theories is that a spatial relation refers to the position of a particular object in focus relative to another object or area (Tenbrink, Andonova, & Coventry, 2011). The RO–LO asymmetry is also important for the present studies because we assume that the asymmetry also affects the revision of spatial mental models. In the first premise, e.g., ''A is to the left of B'', the A is the LO and the B is the RO; in the second premise, e.g., ''B is to the left of C'', the B is the LO and the C is the RO. Moving the middle term B does not solve the problem. From the remaining two objects, A is a LO, whereas C serves as a RO. This leads to the following hypothesis:

Hypothesis: Individuals prefer to relocate the object which is the LO of the premises, because the LO is considered to be more flexible and less stationary. The RO usually remains at its initial position, as it is treated as a kind of "landmark" which should not be moved (The middle term is never relocated.)

Henceforth, we refer to this as LO-relocation or LO hypothesis; the alternative hypothesis we refer to as RO-relocation or RO hypothesis. We present two experiments that tested these hypotheses. In the first experiment, the participants had to solve the revision problems just mentally by envisaging how the objects in the initial model must be rearranged. In the second experiment, the participants had to solve the problem manually by locating and relocating actual physical objects in a simple blocks world environment.

EXPERIMENT 1

Methods

Participants. We individually tested 23 students from the University of Giessen (three male; age: $M = 22.22$, $SD = 2.45$, two psychology students). They gave written informed consent and were paid for their participation.

Materials and procedure. The experiment (within-subjects design) consisted of 32 problems (and four practice trials) each of which consisted of three statements. As objects we used fruits (apple, kiwi, mango, peach) and tools (wrench, hammer, pliers, screwdriver) because for these objects no typical arrangements exist (Dutke, 1993). In half of the problems, the first premise used the "left of" relation and the "right of" relation in the second premise, whereas in the other half it was reversed. The contradictory fact also used either "left of" or the "right of" relation. The other half of problems used the same relations in all sentences. At the beginning of each problem the two premises P1 (e.g., "the apple is to the left of the peach") and P2 (e.g., "the peach is to the left of the kiwi") were presented one after the other, centred on the screen by the participants' own speed. Then two "models" were presented from which one was in agreement with the two premises (apple–peach–kiwi) and the other was not possible if the premises were taken for granted (kiwi–peach–apple). This was done to guarantee that the people have the "correct" model (e.g., apple–peach–kiwi) in mind, before they entered the revision phase, and in fact more than 95% ($M = 95.65$, $SD = 1.30$) of the models were correctly selected by the participants. The few problems in which they chose the "wrong" model (e.g., kiwi–peach–apple) were eliminated from the further analysis. In the next step, the contradictory fact (e.g., "the apple is to the right of the kiwi") was presented on the screen and the participants decided by a keypress whether this fact was in agreement with the initial statements (we explicitly told our participants that the contradictory fact is irrefutably true). In more than 90% ($M = 90.49$, $SD = 9.01$) of the problems the participants made the correct decision and the few invalid decisions were eliminated from the further analysis. Then the essential decision of the experiment followed: Two alternative revised models were presented on the screen, from which one followed the LO hypothesis (peach–kiwi–apple, i.e., left strategy in Figure 1), whereas the other followed the RO hypothesis kiwi–apple–peach, i.e., right strategy in Figure 1). The position of objects and tools were counterbalanced over the group of problems and participants.

Results and discussion

Figure 2 presents the percentages of responses corresponding to the two revision alternatives. Apparently, participants had a strong tendency to relocate the object that served as the LO in the premises ($M = 86.67\%$, $SD = 3.44$), whereas they only very seldom relocated the RO of the premises ($M = 13.33\%$, $SD = 3.44$). In Figure 1, for instance, they preferred to mentally move the first object in the array to the end of the array, but they usually did not move the last object in the array to the first position in the array. This result is statistically significant (Wilcoxon test, $z = -4.22$, $p < .001$) and corroborates the LO hypothesis, but is contrary to the RO hypothesis.

Our findings have some theoretically interesting implications. In particular, the results are difficult to explain based on purely propositional representations of spatial relations. If relations were mentally represented as propositions of the form r(A, B), where A and B are the objects to be located, we would not expect an asymmetry between LO and RO. If, however, people construct and revise mental models of the premises this might account for the asymmetrical semantic roles the objects play during the processing of spatial relational expressions. This assumption is supported by several experimental finding. Logan (1994, 1995) showed that if individuals are asked to verify spatial relations in a diagram they shift attention from the RO of the statement to the region where the LO is expected (see also Oberauer & Wilhelm 2000). Hörnig, Oberauer, and Weidenfeld (2005) studied the integration of new premise information into an already existing model and reported that reasoners integrate the LO of a relation faster if the RO of the premise was already part of the existing model. In the present experiment we were able to show that such semantic directionality effects also play an important role during model revision. The effects do not only influence how a model is constructed, as previous results suggest. Semantic directionality and the asymmetry between RO and LO obviously also have an effect when people already have constructed a model but then have to alter this model because they receive new information that does not concur with the present model. In the next experiment we test how universal such revision principles are. In particular, we wanted to explore whether the same preference for LO relation exists, if people are asked to manually move actual physical objects.

EXPERIMENT 2

What happens if people must revise an arrangement of actual physical objects? In the previous experiment, the individuals had to mentally envisage an arrangement of objects and how the objects must be relocated to account for the new fact. In the present experiment, the participants were instructed to place real physical objects on a table and then to move these objects with their hands to revise the initial arrangement. We predicted that the LO-relocation principle is not limited to the mental revision process, but also guides revisions that a person must manually execute.

Method

Participants. We tested a new sample of 22 students from the University of Giessen (five male; age: $M = 22.59$, $SD = 3.16$, five psychology students). They gave written informed consent and were paid for their participation.

Materials and procedure. The participants were sitting at a table on which they found (on a plate) a red, a blue, a green, and a yellow wooden block with 2.5 side length. The blocks were located in front of a 19“ computer monitor on which the statements (Premise 1, Premise 2, incontrovertible fact) were presented as a sequence of PowerPoint slides at the participants' own speed (the experimenter pressed a key to proceed through

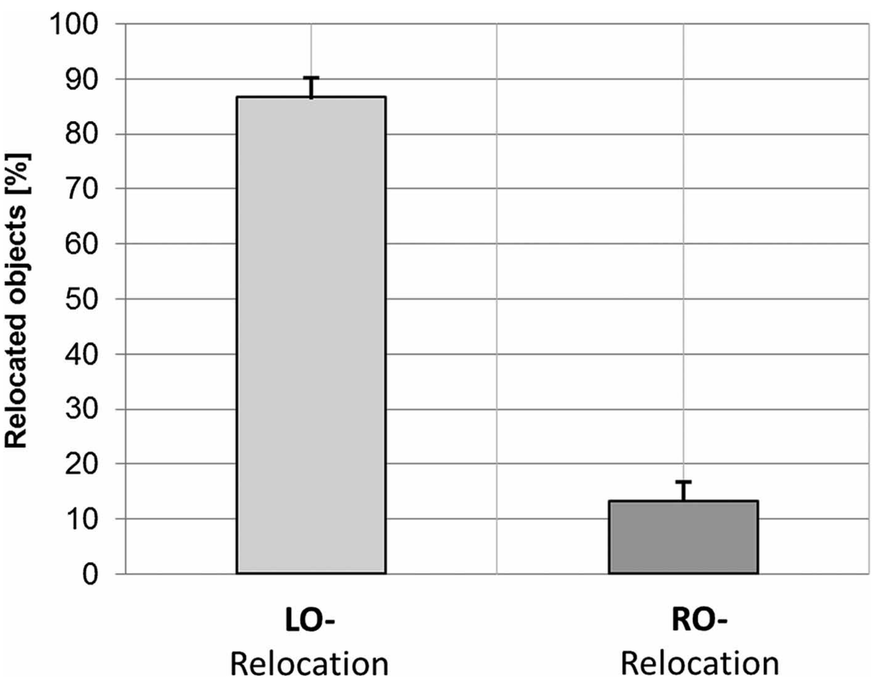

Figure 2. Relative frequency (in%) of model selections following the LO- and RO-relocation hypothesis (Experiment 1).

the experiment). Again, the experiment (within-subjects design) consisted of 32 problems (and four practice trials) each of which consisted of three sentences. The first premise described the spatial relation between two coloured blocks, e.g., ‘‘The red block is to the left of the green block’’. The participant now manually positioned the two blocks on the table. Then the second premise described the spatial relation between one of these block to a third block, e.g., ‘‘The blue block is to the right of the green block’’. The participant now positioned the third block on the table. Now the incontrovertible fact was presented. It described the relation between two blocks that contradicted the participant’s layout of objects, e.g., ‘‘The red block is to the right of the green block’’. Now the participants were instructed to pick one block of their own choice and to manually relocate this block to gain an arrangement that is consistent with the incontrovertible fact. The participants were free in their choice, as long as they just picked only one of the objects. After each problem, the blocks were put back onto the plate by the experimenter. Again the used spatial relations (‘‘left of’’, ‘‘right of’’) appeared equally often in the two premises and the contradictory fact. The participants′ actions (hand and block movements) were videotaped by a video camera that was positioned on a stand to the right of the participant.

Results and discussion

Our analysis started with the analysis of videotaped revision processes. In this analysis, for each problem the block repositioning (movement of a block to another position) was classified as either following the LO hypothesis or the RO hypothesis, respectively. In Figure 1, for instance, a block repositioning concurred with the LO hypothesis if the participant moved the first block in the array to the last position; a block repositioning counted as RO-relocation if the participants moved the last object in the array to the first position.

Figure 3 presents the percentages of block relocations following the two revision alternatives. Again, participants had a strong tendency to relocate the object that served as the LO in the contradictory fact ($M = 93.18\%$, $SD = 2.57$), whereas they only very seldom relocated the RO of the inconsistent fact ($M = 6.82\%$, $SD = 2.57$). This result is statistically significant

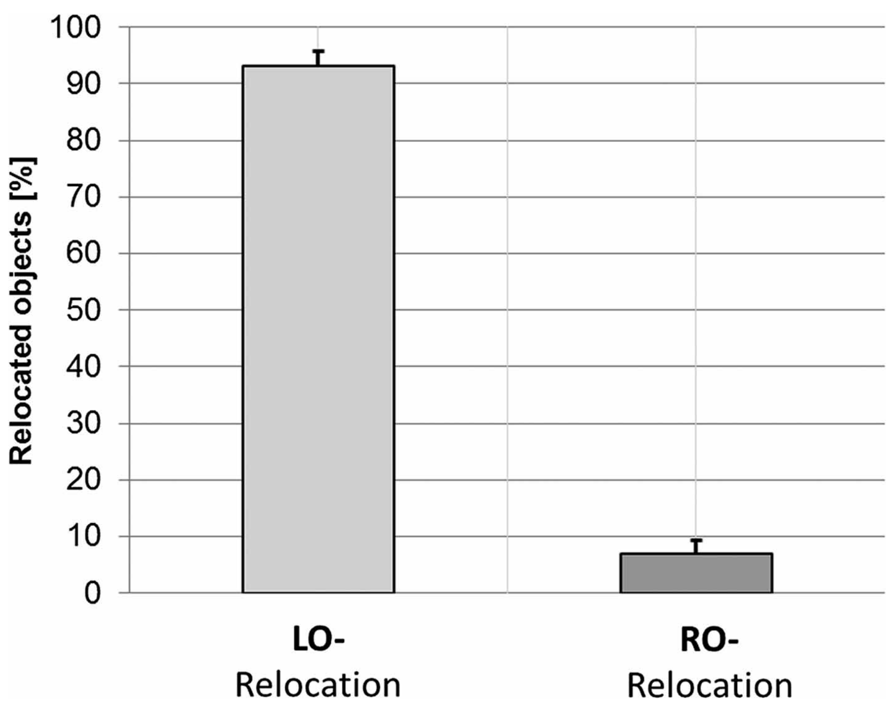

Figure 3. Relative frequency (in%) of block relocations following the LO- and RO-relocation hypothesis (Experiment 2).

(Wilcoxon test, $z = -4.27$, $p < .001$) and shows that the participants again favoured to revise the arrangement according to the LO-relocation principle. In fact, it seems as if people use the same revision principle no matter whether they solve the entire task just in their head or in a real physical environment. We are aware that this is just the first finding in this direction and that further experiments are needed to clarify how universal the LO-relocation principle is in real spaces. For example, it is possible that the physical revision of spatial arrangements differs on different spatial scales, ranging from small-scale space that we can reach by our hands to large-scale space that requires locomotion. It is also possible that the revision process is influenced by aspects of the body. For instance, in two recent experiments (which are not yet published), we found that people avoid moving large and heavy objects and this can overwrite the LO-relocation principle (Kurz, 2012; Linder, 2012). However, we did not design the experiment to answer such questions of embodiment (Barsalou, 2008) but to show that the LO principle is more universal than one might think. LO-relocation is the guiding principle if the problem is just solved mentally and it also drives the active manipulation of objects in real spatial arrangements.

GENERAL DISCUSSION

How humans revise their beliefs is one of the most challenging questions of psychology, but has so far been investigated just to a very limited extent. Most of the available studies are from the domain of conditional reasoning and have shown that a particular fact is more often abandoned than a general law (e.g., Revlis et al., 1971), people sometimes prefer to revise their belief in the conditional (Elio & Pelletier, 1997), whereas in other settings they tend to revise the categorical premise (Dieussaert et al., 2000; Girotto et al., 2000; Revlin et al., 2001). Some researchers ascribe such asymmetries to the linguistic form of the conditional (e.g., Elio & Pelletier, 1997), whereas others argue that the preference is caused by the differences between major and minor premises (e.g., Politzer & Carles, 2001). Yet others have shown that the preferences are strongly affected by the trustworthiness of the information sources (Wolf et al., 2012) and that people have difficulties to reconsider their prior beliefs (Knauff, Budeck, Wolf, & Hamburger, 2010).

The intention of the present paper was to extend the cognitive research on human belief revision to the area of relational reasoning. Our main motivation was that (1) relational inferences are probably the most frequently used form of reasoning in our daily life (Goodwin & Johnson-Laird, 2005; Knauff, 2013), (2) reasoning with relations is often easier than reasoning with conditionals (Johnson-Laird & Byrne, 1991; Knauff, 2007), (3) relations are ubiquitous in many high-order cognitive processes, such as reasoning, categorisation, planning, and language (Halford, Wilson, & Phillips, 2010), and (4) that relations and inferences on them give rise to people's choices, preferences, and attitudes (Lichtenstein & Slovic, 2006). A second motivation for the present research is that relations are intimately linked to space and that space is one of the most fundamental dimensions of our physical and psychological reality. Crucially, many spatial cognition researchers have argued that the mental space is not geometrically organised, but rather a relational space in which objects are located in relation to other objects (Gattis, 2001; Knauff, 1999). Given this prominence of space in our mind, we believe that it is sensible to develop a specific paradigm that allows us to study how people deal with conflicts in spatial representations and how they solve these inconsistencies in order to gain a consistent mental representation about the relative location of objects in space. Our main finding is that the cognitive processes underlying such revisions are guided by the construction and variation of spatial mental models and the differences between reference object and located object in the problem description. We are, however, aware that our account is by no means complete. One problem is that our results might have to do with effects of the direction of written language (from left to right versus right to left; e.g., Jahn, Knauff, & Johnson-Laird, 2007; Spalek & Hammad, 2005) and that we used the simplest sort of relational inferences with just two premises and three objects. Future research must clarify whether or not the LO-relocation principle is also so dominant in more complex reasoning problems. Another problem is that our results might have to do with one-by-one presentation of the premises and the contradictory fact. It is possible that this has triggered certain revision strategies and that individuals would use different strategies if all information would be available at the same time. Future experiments will help us to further define the boundary conditions and limitations of our present account and will also help us to identify further spatial belief revision strategies.

Another difficulty of the paper is that we are (despite better knowledge) quite careless in the use of the terms "belief" and "model". In fact, we often used the terms interchangeably, although this is theoretically problematic. A belief is usually defined as a propositional attitude, which implies the concept of intentionality. It requires a subject, who is the believer, and an object of belief, the proposition (Schwitzgebel, 2011). A proposition is a mental entity that represents meaning in a language-like code and has a truth value "true" or "false". Although propositional representations should not be identified with linguistic representations, they are language-like in the sense that they comprise abstract symbols as a language does (e.g., Anderson, 1993). As such, beliefs are not the same as models. Moreover, a specific model can be described by different propositions, and more than one model can be compatible with the same set of propositions. Strictly speaking, model revision is therefore not the same as belief revision and what we explore in the paper is primarily model revision rather than belief revision. However, this issue is very subtle, and we do not make this distinction in this short paper.

Overall, however, we believe, we could show that people are very good in detecting inconsistencies in a set of statements describing the spatial relations between objects (cf. Johnson-Laird, Girotto, & Legrenzi, 2004). They notice that a new fact does not concur with a set of forgoing statements and thus are also willing to change their putative model of the problem description. If, in a second step, they have to decide how to revise the initial model, they tend to leave the RO of the new fact where it already is and relocate the LO. The reason is the asymmetry between the RO and LO, which is well known from many studies in the area of spatial cognition research (Hayward & Tarr, 1995; Herskovits, 1986; Jackendoff & Landau, 1995; Talmy, 1983). Since the LO is considered to be the more flexible element of the model, it is easier to move this object mentally to another position than the RO, which typically is viewed as a stationary "landmark" which should be left at its initial position (e.g., Hayward & Tarr, 1995; Herskovits, 1986). This account agrees with the mental model theory, in which people reason by constructing, inspecting, and varying spatial mental models. As we have shown earlier, the model variation process in this theory follows the principle of minimal changes (Bucher et al., 2011; Harman, 1986; Krumnack et al., 2011) which says that reasoners try to keep as much as possible of a model unchanged and only vary what is absolutely necessary in order to gain a consistent mental representation of the problem scenario. The LO-relocation principle in this paper agrees with this principle of minimal changes. Further experiments are needed to explore this topic more thoroughly.

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