Entire solutions and Liouville theorems

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Abstract. *A famous theorem of S. Bernstein says that every entire solution* $u = u(x)$, $x \in \mathbf{R}^2$ *of the minimal surface equation*

$$\operatorname{div}\left\{\frac{\nabla u}{\sqrt{1+\nabla u|^2}}\right\} = 0$$

is an affine function; no conditions being placed on the behavior of the solution u.

Bernstein's Theorem continues to hold up to dimension $N = 7$ *while it fails to be true in higher dimensions, in fact if* $x \in \mathbf{R}^N$, *with* $N \geq 8$, *there exist entire non-affine minimal graphs.*

Our purpose is to consider an extensive family of quasilinear elliptic-type equations which has the following strong Bernstein-Liouville property, that $u \equiv 0$ *for* **any** *entire solution* u, *no conditions whatsoever being placed on the behavior of the solution (outside of appropriate regularity assumptions). In many cases, moreover, no conditions need be placed even on the dimension* N. *We also study the behavior of solutions when the parameters of the problem do not allow the Bernstein–Liouville property, and give a number of counterexamples showing that the results are in many cases best possible.*

This is a joint work with James Serrin.