LIOUVILLE-TYPE THEOREMS FOR SEMILINEAR ELLIPTIC SYSTEMS: THE LANE-EMDEN CONJECTURE

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The so-called Lane-Emden conjecture asserts that the elliptic system

$$\begin{cases} -\Delta u = v^p, & x \in \mathbb{R}^n, \\ -\Delta v = u^q, & x \in \mathbb{R}^n, \end{cases}$$

$(p, q > 0)$ has no positive classical solution if and only if the pair (p, q) lies below the Sobolev critical hyperbola, i.e.

$$\frac{1}{p+1} + \frac{1}{q+1} > 1 - \frac{2}{n}.$$

This statement is the analogue of the celebrated Gidas-Spruck [3] Liouville-type theorem for the scalar case. Up to now, the conjecture had been proved for radial solutions [5, 8], in $n \leq 3$ space dimensions [9, 6], and in certain subregions below the critical hyperbola for $n \geq 4$ [2, 11, 5, 9, 4, 7, 1].

I will in particular report on the recent work [10], where I establish the conjecture in 4 space dimensions and obtain a new region of nonexistence for $n \geq 5$.

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