Concentration Phenomena in Nonlinear PDE's
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Solutions with mixed positive and negative spikes for some singularly perturbed elliptic problems

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Let us consider the following nonlinear elliptic equation

$$-\varepsilon^2 \Delta v + V(x)v = f(v), \quad x \in \Omega, \tag{1}$$

where Ω is a smooth domain of $\mathbb{R}^N$, ε is a small positive parameter, f is a superlinear, subcritical and odd nonlinearity, V is a uniformly positive potential. No symmetry is assumed on Ω and on V. We will discuss some recent results concerning concentration phenomena for sign changing solutions.

The Neumann and the Dirichlet problems associated to (1) in a bounded domain Ω with $V \equiv 1$ have been addressed in [1] and [3], respectively: a solution consisting of mixed positive and negative interior peaks is constructed and the peaks approach separate points of Ω as $\varepsilon \to 0^+$. The location of the concentration points depends on the geometry of the domain.

Concerning the case $\Omega = \mathbb{R}^N$, in [2] it is proved that the problem (1) admits a solution alternating positive and negative peaks with all the peaks collapsing at a local minimum of V as $\varepsilon \to 0^+$. This result has been extended in [4] to the case of a nondegenerate saddle point of V, while the case of a maximum point is treated in a work in progress.

The proof of these kind of results is based on the Lyapunov-Schmidt reduction method, which reduces the problem to finding a critical point for a finite-dimensional energy functional governing the location of the spikes. A max-min scheme is set up in order to provide a critical point for the reduced functional by defining a suitable local linking structure.

We will also consider some applications of similar min-max principles to the existence of sign-changing blow-up solutions for singularly perturbed elliptic problems involving the critical Sobolev exponent.

References

[1] T. D'Aprile. *Solutions with many mixed positive and negative interior spikes for a semilinear Neumann problem*, Calc. Var. Partial Differential Equations, to appear.

[2] T. D'Aprile, A. Pistoia. *Existence, multiplicity and profile of sign-changing clustered solutions of a semiclassical nonlinear Schrödinger equation*, Ann. Inst. H. Poincaré Anal. Non Linéaire **26** (2009), 1423–1451.

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[4] T. D'Aprile, D. Ruiz. *Positive and sign-changing clusters around saddle points of the potential for nonlinear elliptic problems*, Math. Z., to appear.

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