My Giessen connection

- 1985-87: postdoc (nuclear fragmentation, QGP)
- 1987-93: C1 (Eff. Lagrangians, hadron & gamma production, transport, QCD Hartree)
    - 1991: habilitation
- 1993-94: C2 (Chaos in GFT, s production, hadronization MD)
- 2005: Mercator visit (non-extensive thermodynamics)

Thermodynamics with abstract composition rules

- **Non-extensive thermodynamics**
- **Zeroth Law compatibility** **(disputes)**

arXiV: 1102.0536

- **Abstract composition rules**
- **High energy particle spectra**

arXiV: 1101.3522

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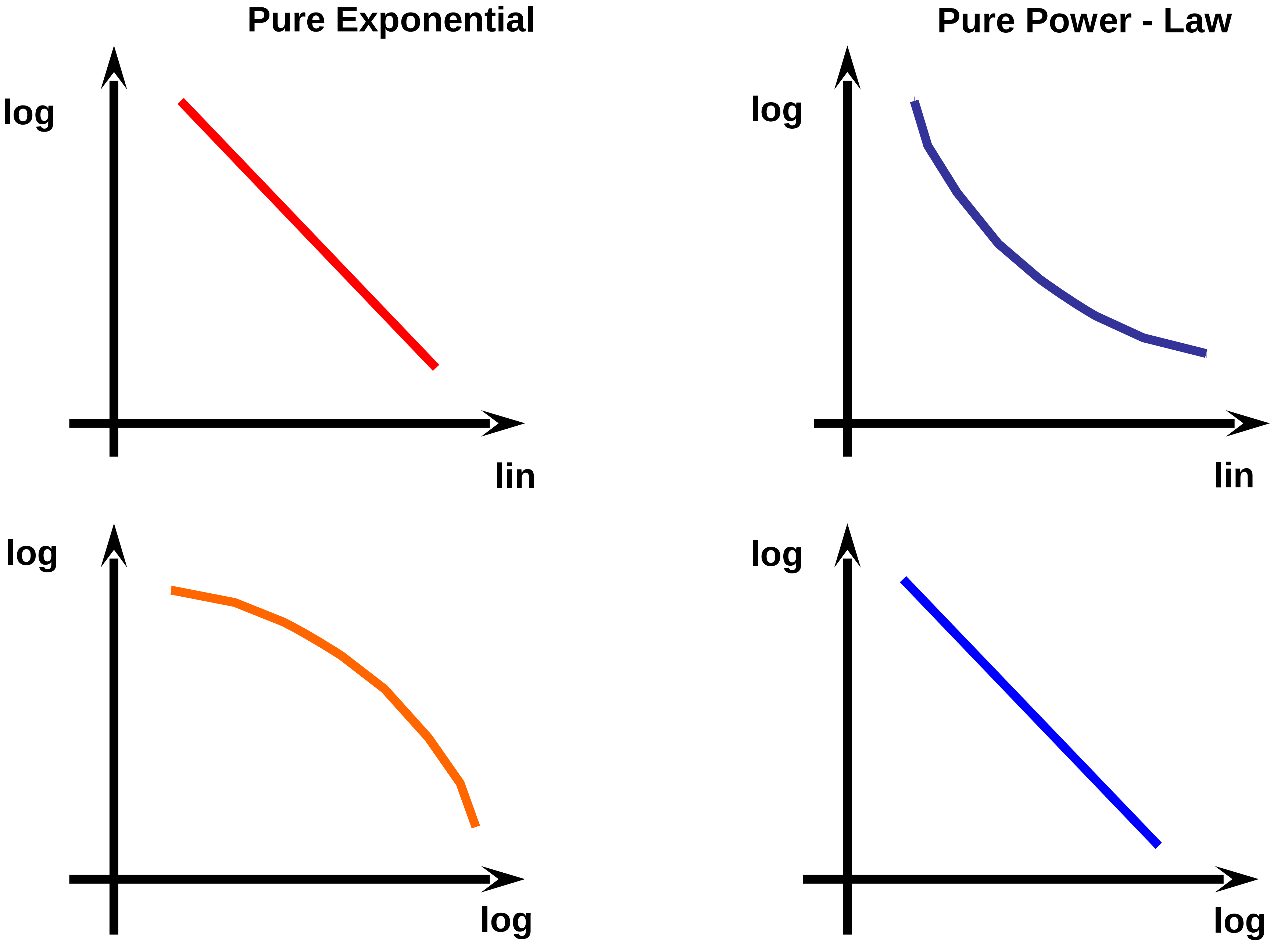
invited talk by T.S.Biro at the Winter Meeting Feb.21-25, 2011, Obergurgl, Austria - in honor of Prof. Dr. Ulrich Mosel

Not everything is exp(-E/T)

- particles and heavy ions: (SPS) RHIC, LHC
- fluctuations in financial returns
- natural catastrophes (earthquakes, etc.)
- fractal phase space filling
- network behavior
- turbulence, critical behavior
- near Bose condensates
- citation of scientific papers….



Pure Exponential
Pure Power - Law
log
lin
log
lin
log
log
log
log

”Thus a foundation of equilibrium thermodynamics from transport theory including interaction is possible without ever using exp(- H / T).”

Siegfried Grossmann: On Transport Theory in Real Gases,
Il Nuovo Cimento, Vol. XXXVII, No. 2, 16 Maggio 1965,
p. 698

But what is the (is there a) more general expression replacing exp(- H / T) ?

Energy distributions and composition rules

$$e^{-\beta E_1} \cdot e^{-\beta E_2} = e^{-\beta \left(E_1 + E_2 \right)}$$

$$E_{12} = E_1 + E_2$$

$$\left(1 + a E_1 \right)^{-v} \cdot \left(1 + a E_2 \right)^{-v} = \left(1 + a \left(E_1 + E_2 + a E_1 E_2 \right) \right)^{-v}$$

$$E_{12} = E_1 + E_2 + a E_1 E_2$$

Abstract thermodynamics

- S(E1,E2,…) = max (Jaynes-) principle
- E(E1,E2) = fix composition of traditionally additives
- 0-th law in factorizing form

  $$T1(E1) = T2(E2)$$

- ***This is equivalent to the existence and use of additive functions, the formal logarithms, L(E) etc!***
- ***Repeated compositions asymptotically lead to such a form!*** (formal logarithm)
- Entropy formulas and canonical distributions vary

Zeroth Law compatibility of non-additive thermodynamics

T.S.Biró, P.Ván, arXiv: 1101.3023

- Factorizability ⇐
- Classical concept: additive formulas
- Factorizing with non-additive energy
- Factorizing with non-additive entropy
- Temperature in the general case ⇐
- Transitivity
- Generalizing the Tsallis composition ⇐

Jaynes' entropy maximum principle

$$S\left(E_{12},V_{12},N_{12},\ldots\right)=\max$$

$$E_{12}=E_1\oplus E_2$$

$$V_{12}=V_1\oplus V_2$$

$$N_{12}=N_1\oplus N_2$$

Differentials are NOT independent!

Zeroth Law: θ(E1,…)=θ(E2,…)

empirical temperature

$$dS_{12} = \frac{\partial S_{12}}{\partial E_1} dE_1 + \frac{\partial S_{12}}{\partial E_2} dE_2 + \ldots = 0$$

$$dE_{12} = \frac{\partial E_{12}}{\partial E_1} dE_1 + \frac{\partial E_{12}}{\partial E_2} dE_2 = 0$$

$$\frac{\partial E_{12}}{\partial E_2} \frac{\partial S_{12}}{\partial S_1} S_1' = \frac{\partial E_{12}}{\partial E_1} \frac{\partial S_{12}}{\partial S_2} S_2'$$

general

For which composition laws does this factorize ?

Non-additive entropy

Non-additive energy

The temperature in general

$$\frac{\partial S_{12}}{\partial S_1}\frac{\partial E_{12}}{\partial E_2}S_1'(E_1)=\frac{\partial S_{12}}{\partial S_2}\frac{\partial E_{12}}{\partial E_1}S_2'(E_2)$$

$$F_1 G_2 H_1 \cdot A_2 B_1 C_2 \cdot S_1' = F_2 G_1 H_2 \cdot A_1 B_2 C_1 \cdot S_2'$$

$$\frac{H_1(S_1,S_2)}{H_2(S_1,S_2)}=\frac{C_1(E_1,E_2)}{C_2(E_1,E_2)}=\mathrm{const.}$$

The temperature in general

$$\frac{H_1(S_1,S_2)}{H_2(S_1,S_2)}=\frac{C_1(E_1,E_2)}{C_2(E_1,E_2)}=\text{const.}=1$$

$$\frac{F_1(S_1)}{G_1(S_1)}\cdot\frac{B_1(E_1)}{A_1(E_1)}S_1^{'}(E_1)=\frac{F_2(S_2)}{G_2(S_2)}\cdot\frac{B_2(E_2)}{A_2(E_2)}S_2^{'}(E_2)$$

$$\frac{1}{T_1}=\frac{\partial\hat{L}_1(S_1)}{\partial L_1(E_1)}=\frac{\partial\hat{L}_2(S_2)}{\partial L_2(E_2)}=\frac{1}{T_2}$$

The absolute temperature

$$\frac{1}{T}=\frac{\partial \hat{L}(S)}{\partial L(E)}$$

$$\hat{L}(S)=\int \frac{F(S)}{G(S)}dS$$

$$L(E)=\int \frac{A(E)}{B(E)}dE$$

Admissible composition laws

$$H_1 = \frac{1}{G_2 F_1} \frac{\partial}{\partial S_1} S_{12} = \frac{1}{G_1 F_2} \frac{\partial}{\partial S_2} S_{12} = H_2$$

$$\frac{G_1}{F_1} \frac{\partial}{\partial S_1} S_{12} = \frac{G_2}{F_2} \frac{\partial}{\partial S_2} S_{12}$$

$$\frac{\partial}{\partial \hat{L}_1} S_{12} = \frac{\partial}{\partial \hat{L}_2} S_{12}$$

$$S_{12} = \Psi\left(\hat{L}_1 + \hat{L}_2\right)$$

$$\hat{L}_{12}\left(S_{12}\right) = \hat{L}_1\left(S_1\right) + \hat{L}_2\left(S_2\right)$$

Admissible composition laws

$$C_1 = \frac{1}{B_2 A_1} \frac{\partial}{\partial S_1} E_{12} = \frac{1}{B_1 A_2} \frac{\partial}{\partial E_2} E_{12} = C_2$$

$$\frac{B_1}{A_1} \frac{\partial}{\partial E_1} E_{12} = \frac{B_2}{A_2} \frac{\partial}{\partial E_2} E_{12}$$

$$\frac{\partial}{\partial L_1} E_{12} = \frac{\partial}{\partial L_2} E_{12}$$

$$E_{12} = \Phi\left(L_1 + L_2\right)$$

$$L_{12}\left(E_{12}\right) = L_1\left(E_1\right) + L_2\left(E_2\right)$$

The Tsallis example for entropy

$$S_{12} = S_1 + S_2 + \hat{a} S_1 S_2$$

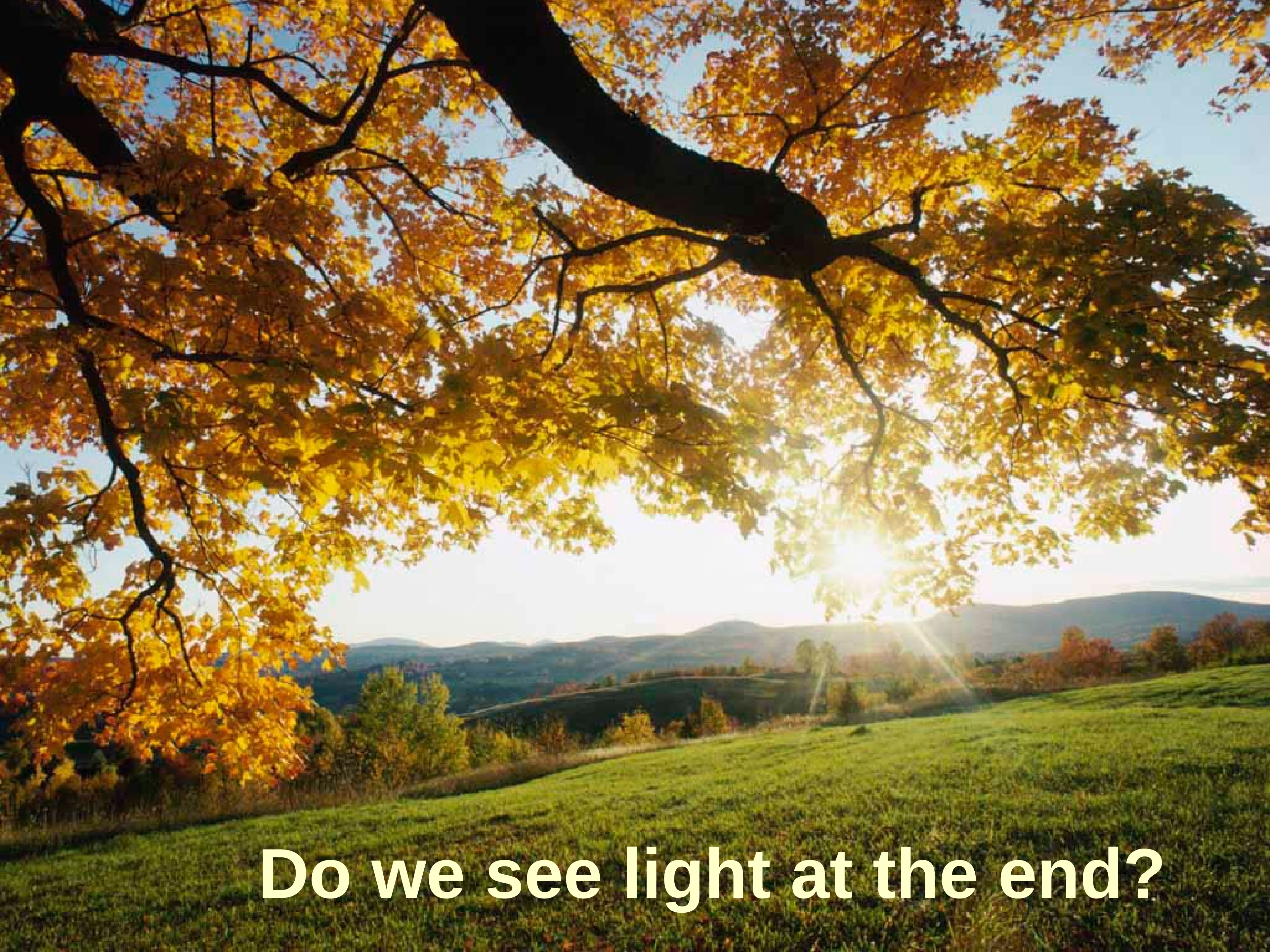
$$\left(1 + \hat{a} S_{12}\right) = \left(1 + \hat{a} S_1\right) \cdot \left(1 + \hat{a} S_2\right)$$

$$\hat{L}(S) = \frac{1}{\hat{a}} \ln\left(1 + \hat{a} S\right)$$

Heterogeneous equilibrium

$$\frac{1}{\hat{a}_{12}}\ln\left(1+\hat{a}_{12}S_{12}\right)=\frac{1}{\hat{a}_1}\ln\left(1+\hat{a}_1S_1\right)+\frac{1}{\hat{a}_2}\ln\left(1+\hat{a}_2S_2\right)$$

$$S_{12}=\frac{1}{\hat{a}_{12}}\left[\left(1+\hat{a}_1S_1\right)^{\hat{a}_{12}/\hat{a}_1}\left(1+\hat{a}_2S_2\right)^{\hat{a}_{12}/\hat{a}_2}-1\right]$$


Do we see light at the end?


Many small steps!
Many small steps!

Composition Laws: thermodynamics

In this family of entanglement all statistical phyics methods and results apply !

$$Y(S) = f\left(X(E), U(V), W(N), \ldots\right) = \max$$

- Non-extensive Boltzmann equation (Kaniadakis, Biro+Purcsel, …)
- Nonlinear Fokker-Planck equation (Borland, Quarati+Lavagno, …)
- Coupled Langevin equations (Biro+Rosenfeld, …)
- Lagrange multiplier method (Biro, ….)
- Superstatistics: shaken Monte Carlo (Cohen, Beck, Biro+Schram, …)

Canonical distribution

$$\sum_a S(X_a) - \lambda_a L_a(X_a) = \max .$$

$$p_i = \frac{1}{Z} e^{-\sum_a \frac{\mu_a}{k_B T} L_a(X_{ai})}$$

$$\sum_i p_i L_a(X_{ai}) = \langle L_a(X_a) \rangle$$

$$X_a = (E, V, N, \ldots)$$

Superstatistics

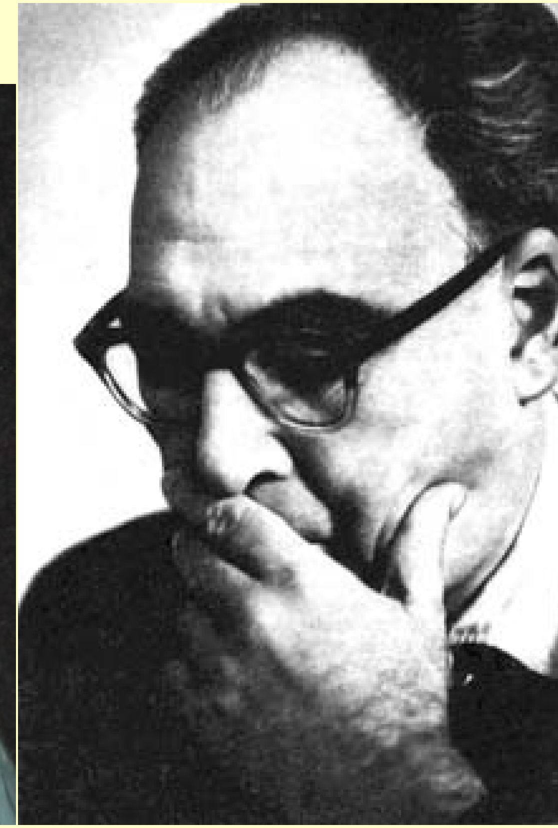
$$\int_0^\infty e^{-t\beta E} w(t)dt = e^{-\beta L(E)}$$

$$L'(E) = \frac{\langle t\, e^{-\beta tE} \rangle}{\langle e^{-\beta tE} \rangle}$$

Entropy formulas

Boltzmann

Tsallis



Rényi

… and over twenty others … (Vedral, Taneja, Aczél+Daróczy,)

Entropy formulas

$$S_B = \sum_i p_i \ln \frac{1}{p_i}$$

Boltzmann

$$S_T = \frac{1}{q-1} \sum_i \left(p_i - p_i^q \right)$$

Tsallis

$$S_R = \frac{-1}{q-1} \ln \sum_i p_i^q$$

Rényi

Formal Log of Entropy

$$S = \max. \Leftrightarrow \hat{S} = \hat{L}(S) = \max.$$

Y ( ) strict monotonic rising

$$S_T = \frac{1}{q-1} \sum_i \left(p_i - p_i^q \right)$$

Tsallis

$$S_R = \frac{-1}{q-1} \ln \sum_i p_i^q \quad a = 1 - q$$

Rényi

$$S_R = \frac{1}{a} \ln\left(1 + aS_T\right)$$

Rényi = formal logarithm (additive version) of Tsallis

Canonical Power-Law 1

$$\hat{L}(S)=\frac{1}{\hat{a}}\ln\left(1+\hat{a}\,S\right);\quad L(E);$$

$$Zp(E)=\left(1+\hat{a}\hat{\beta}\,L(E)\right)^{-\frac{1}{\hat{a}}}$$

$$\hat{\beta}=\frac{\beta}{1-\hat{a}-\hat{a}\beta\left\langle L(E)\right\rangle}$$

Footnote: integrals are incomplete Gamma functions.

Canonical Power-Law 2

$$L(E) = \frac{1}{a}\ln(1 + aE)$$

$$Zp(E) = e^{-\beta L(E)} = (1 + aE)^{-\frac{\beta}{a}}$$

Footnote: w(t) is an Euler-Gamma distribution in this case.

Canonical Power-Law 3

$$\hat{L}(S)=\frac{1}{\hat{a}}\ln\left(1+\hat{a}S\right)$$

$$Zp(E)=\left(1+\hat{a}\hat{\beta}E\right)^{-\frac{1}{\hat{a}}}$$

Footnote: The equipartition law differs for cases 2 and 3!

The cut power-law distribution is

an excellent fit to **particle spectra**

in high-energy experiments!

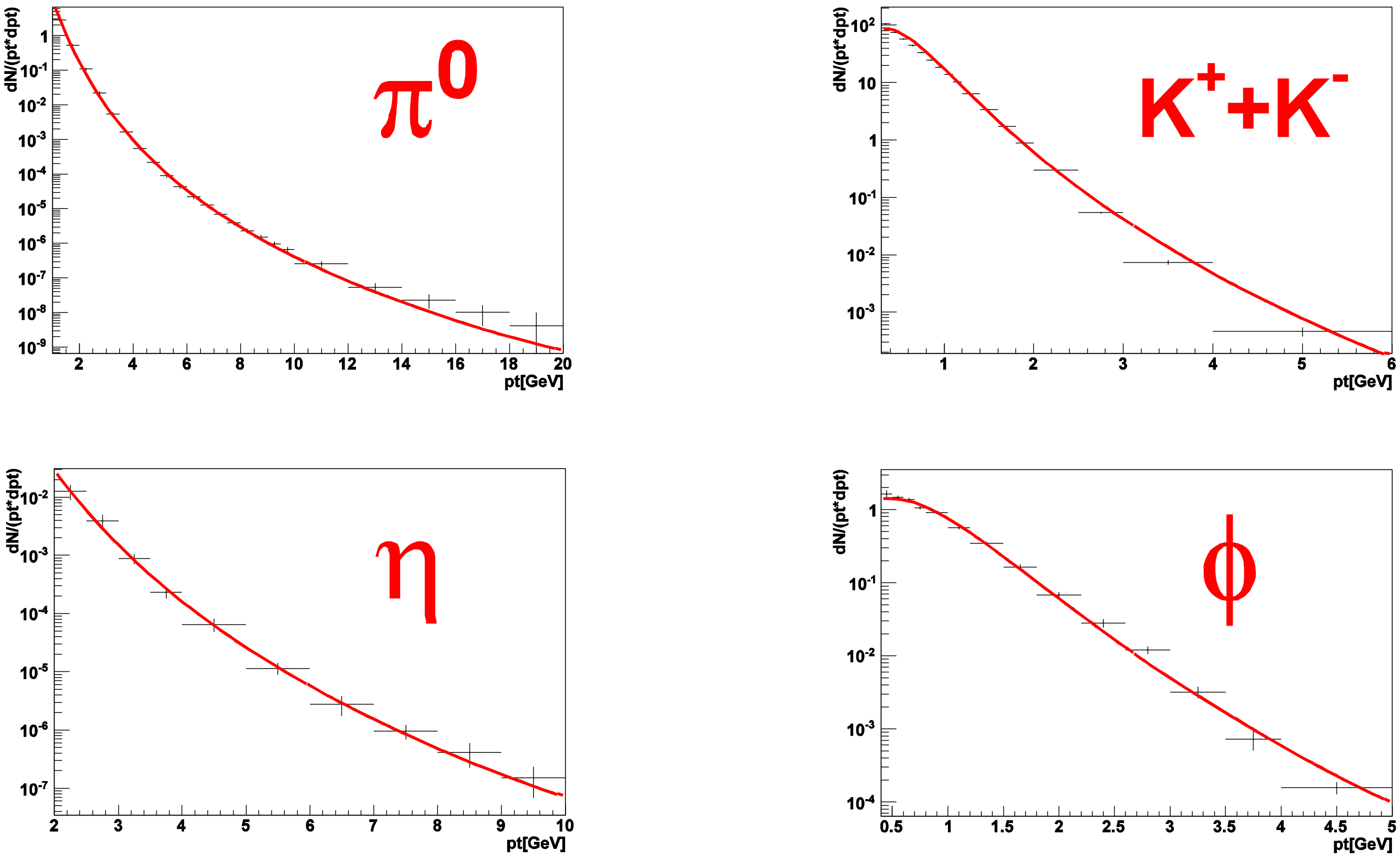
How to caluclate (predict)

T, q, etc… ?

Tsallis quark matter + transverse flow + quark coalescence fits to hadron spectra

with Károly Ürmössy

RHIC data



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with Károly Ürmössy

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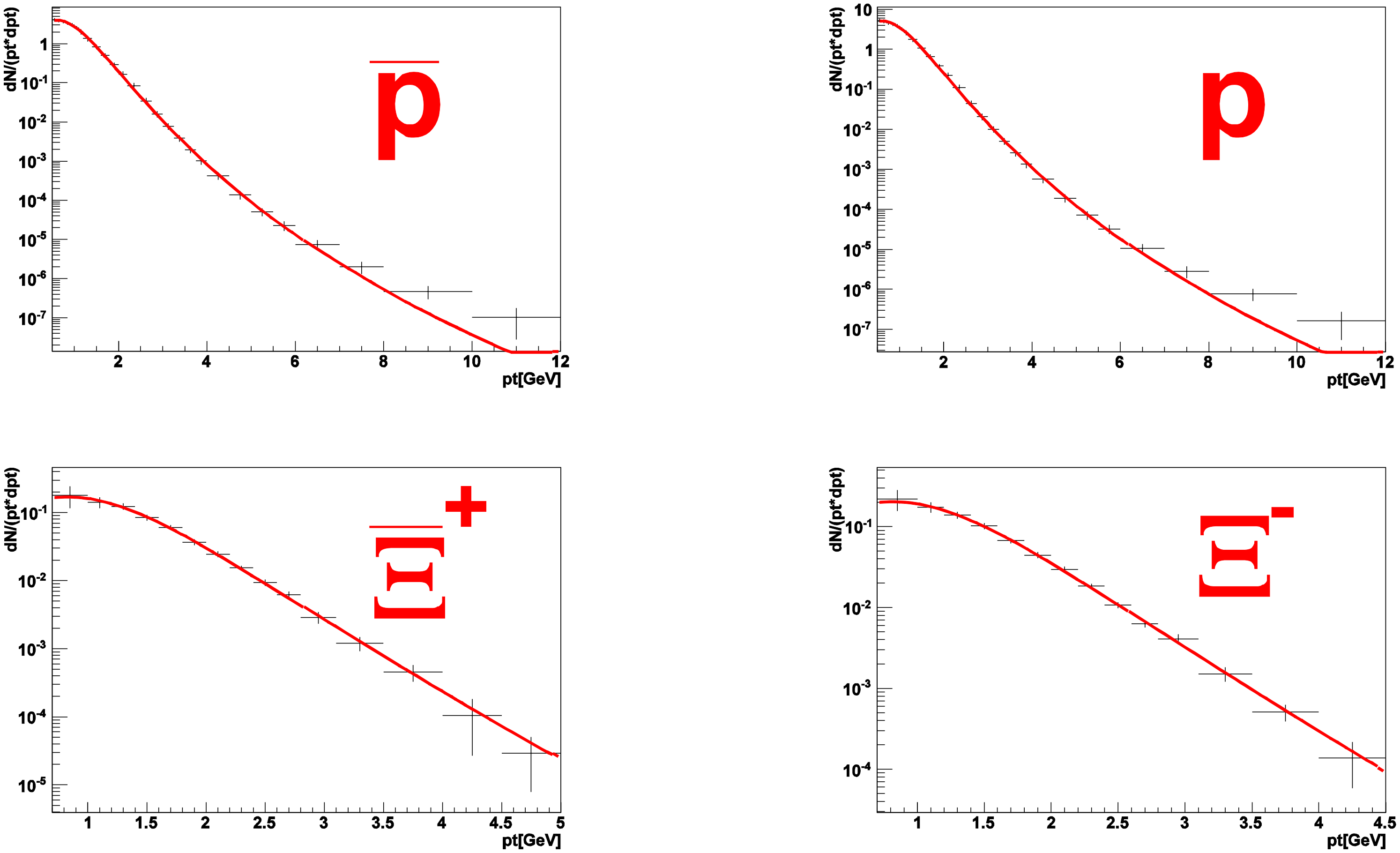

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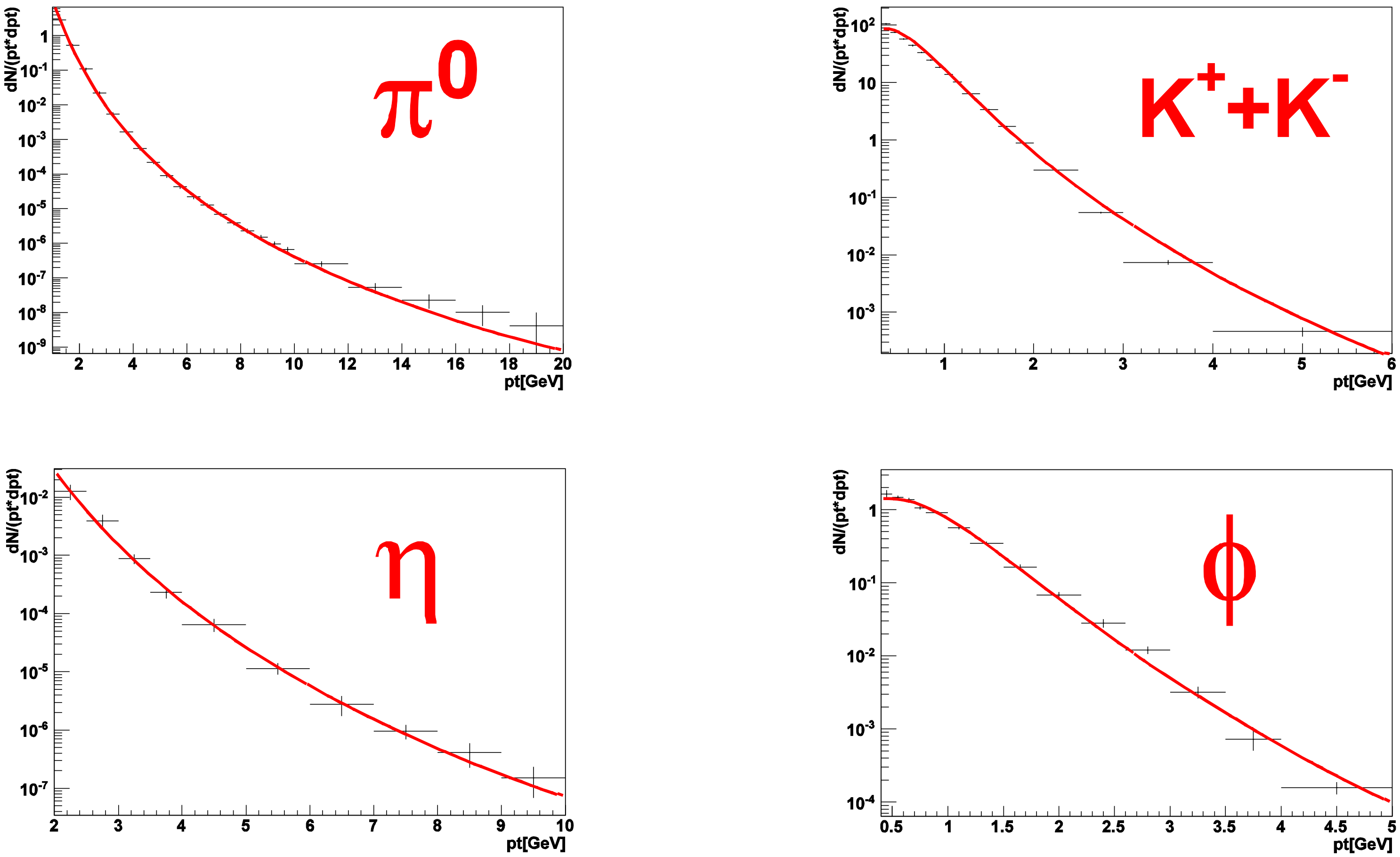

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Tsallis quark matter + transverse flow + quark coalescence fits to hadron spectra

with Károly Ürmössy

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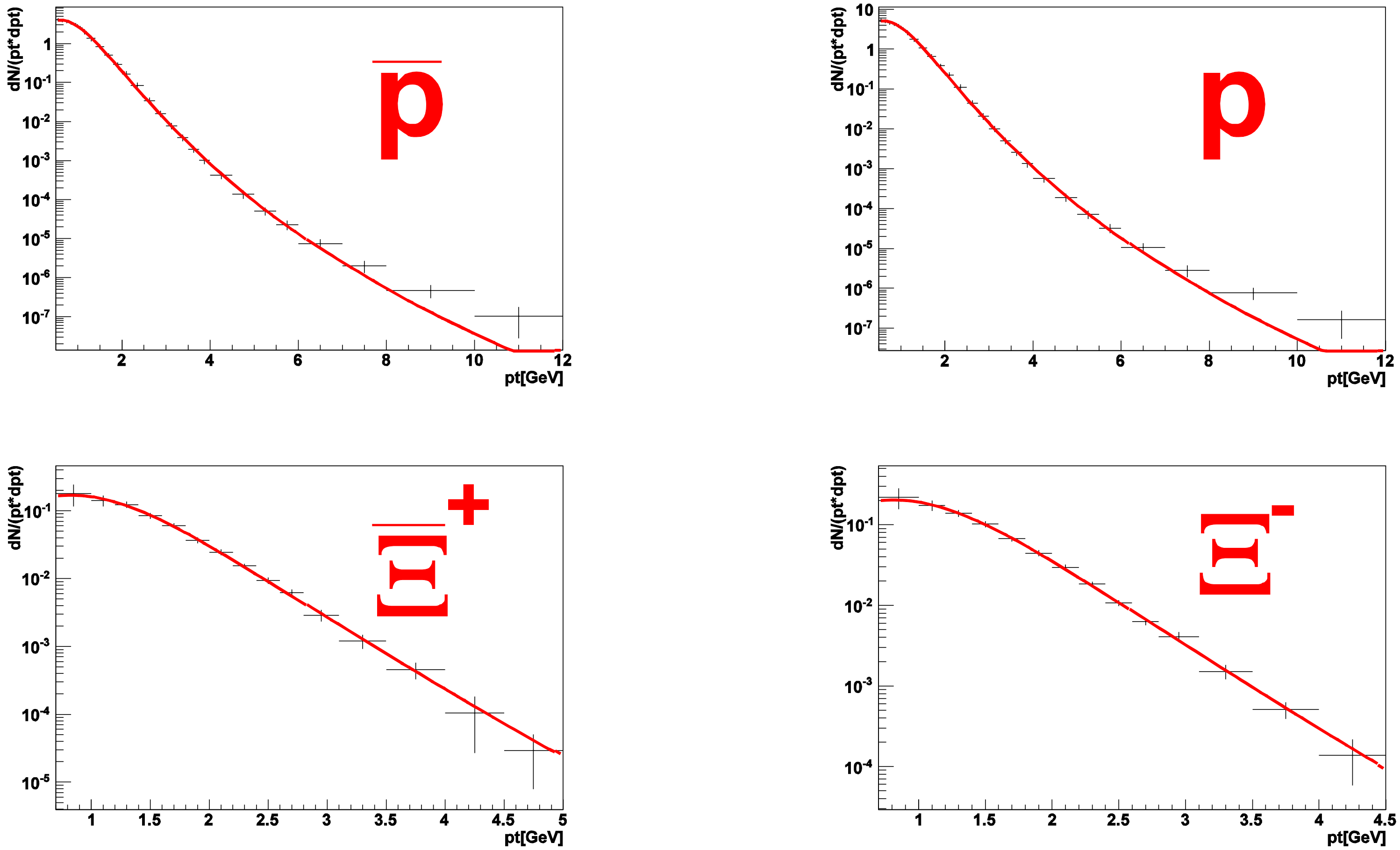

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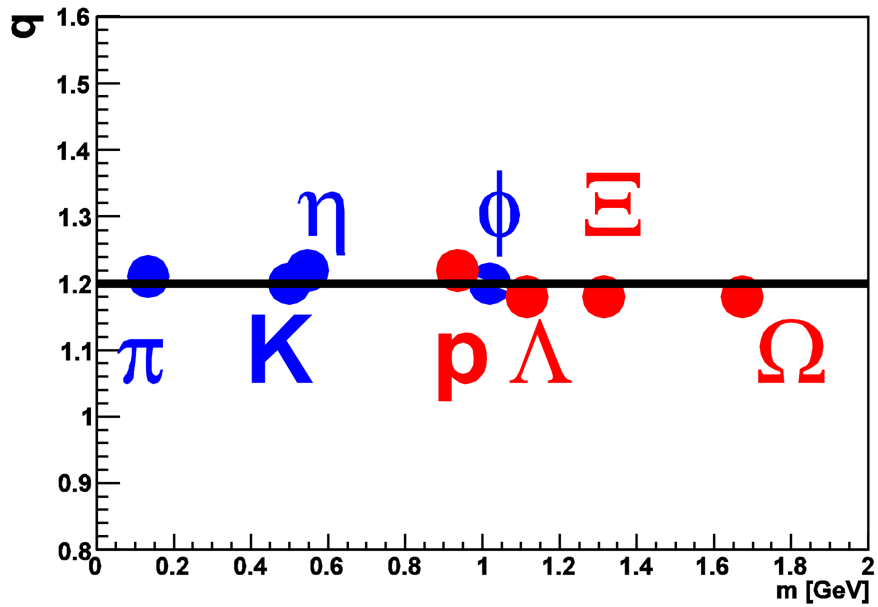
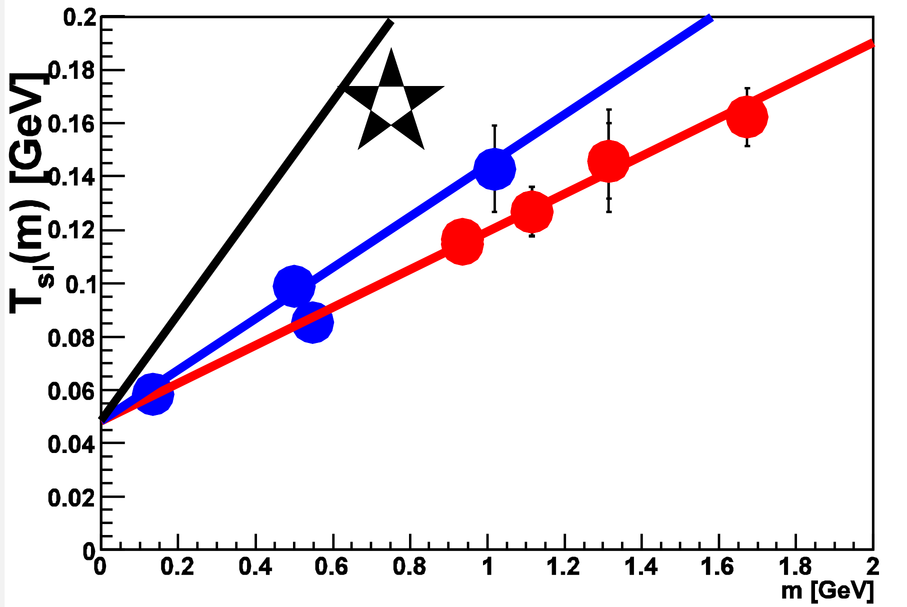
Blast wave fits and quark coalescence

with Károly Ürmössy





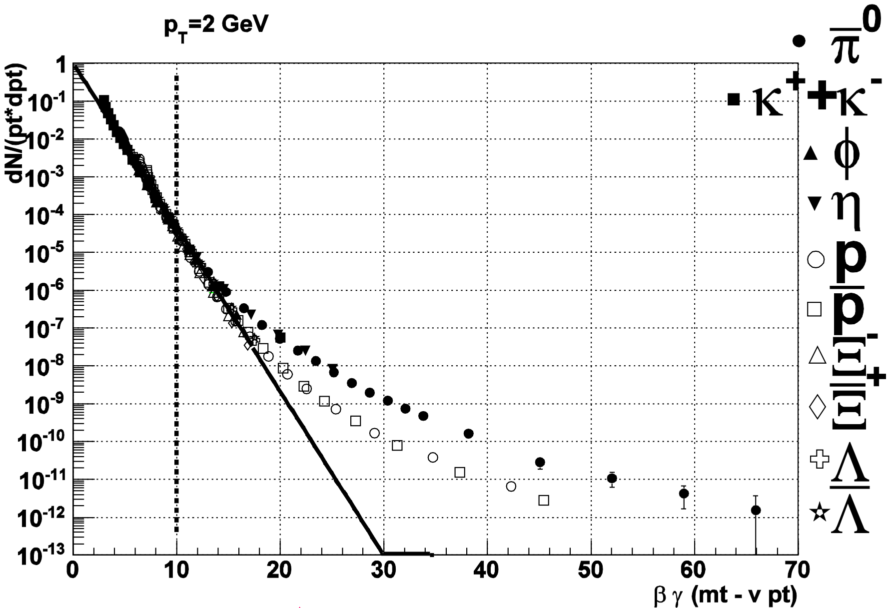
JPG 36:064044, 2009; SQM 2008, Beijing

$$q = 1 + \frac{2}{\pi^2} \approx 1.202642$$

TSB, personal speculation: arXiV: 1011.3442

All particle types follow power-law

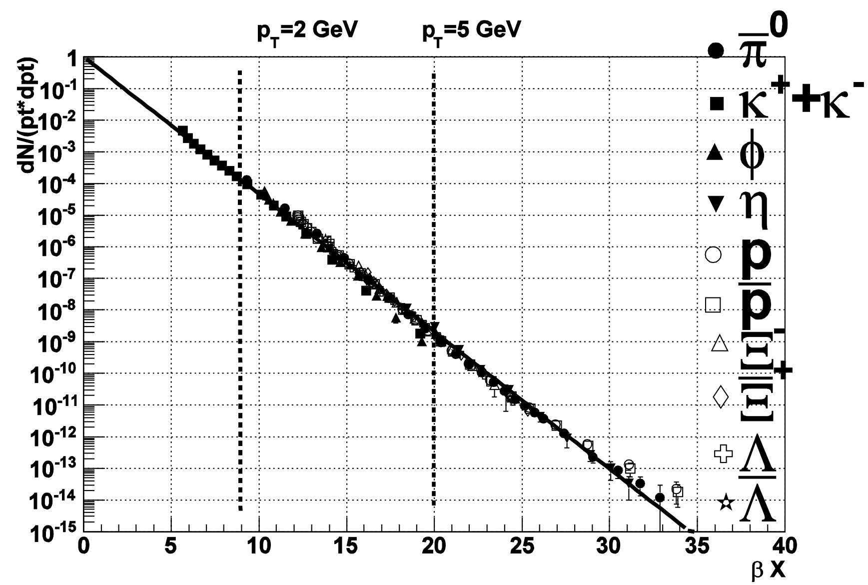
with Károly Ürmössy



E

WRONG!

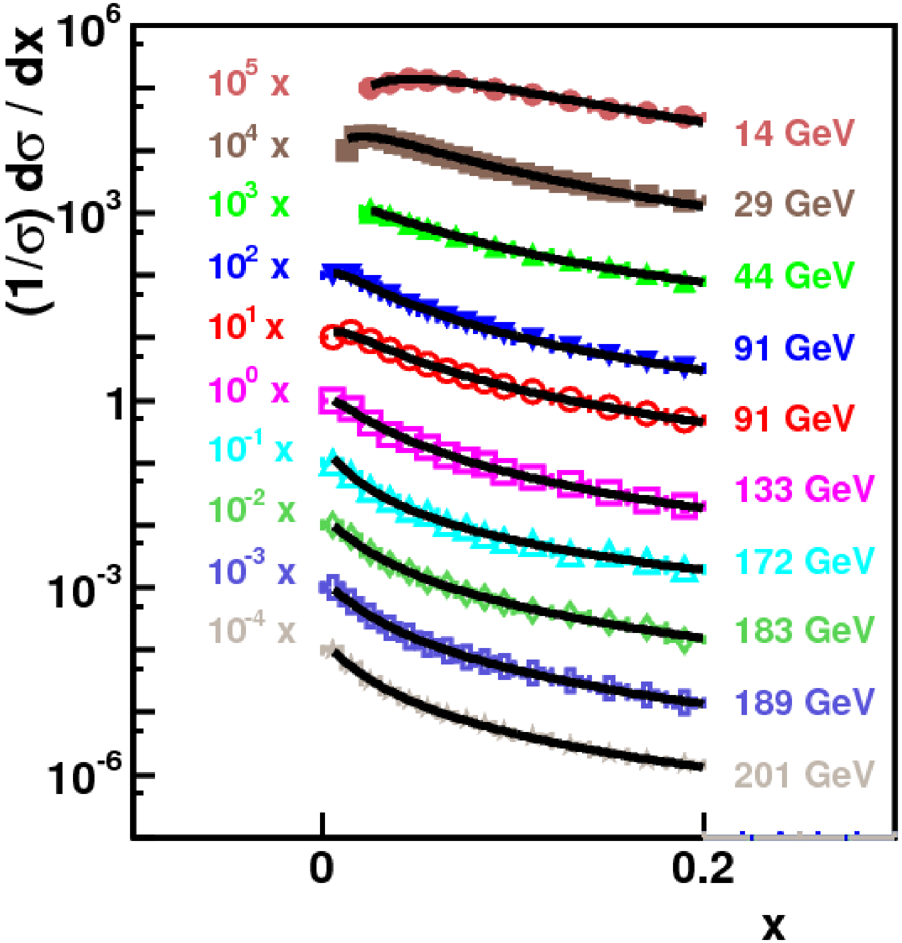
RIGHT!



RHIC

L(E)

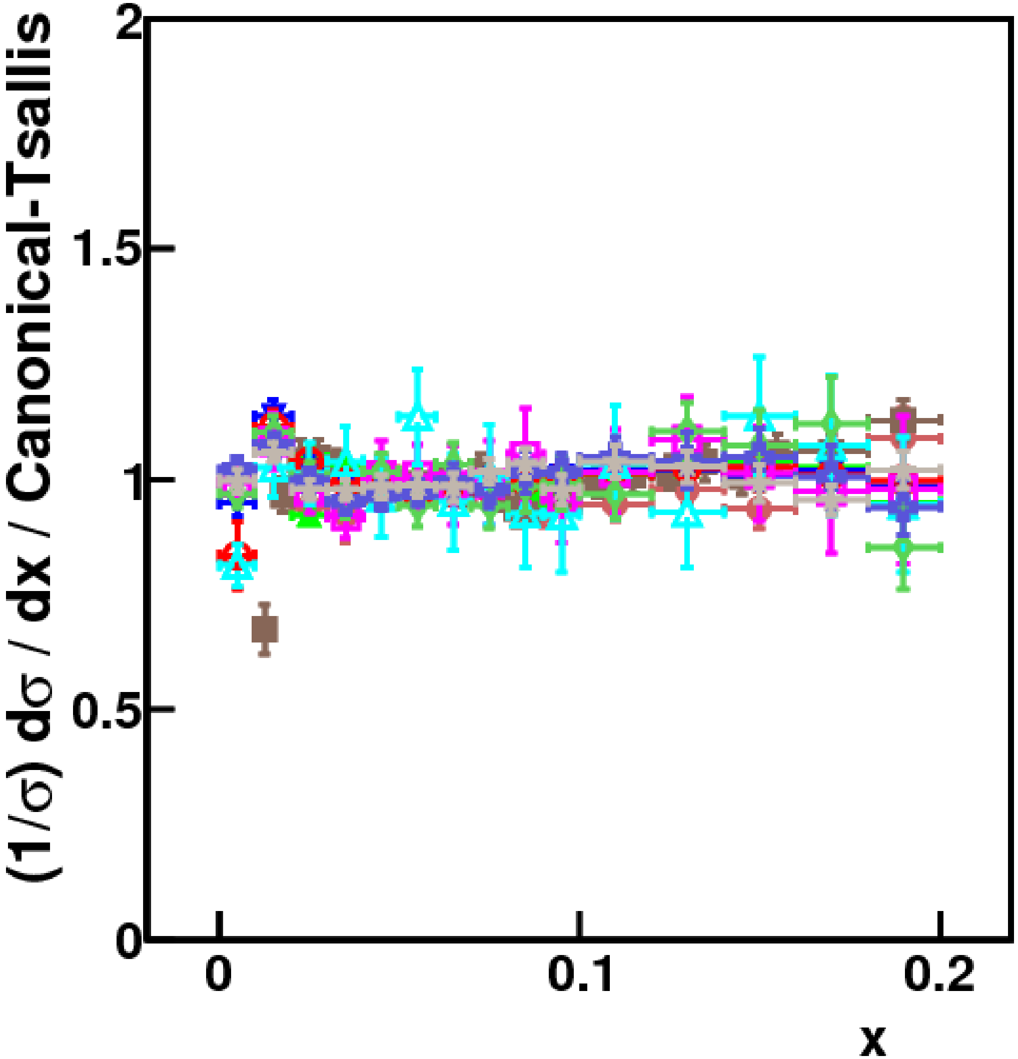
e+e- jets power of 1+log: S & E both non-additive



Ürmössy, Barnaföldi, Biró arXiv: 1101.3023

The power is sqrt(s) dependent: dimensionality reduces from 3 to 1.

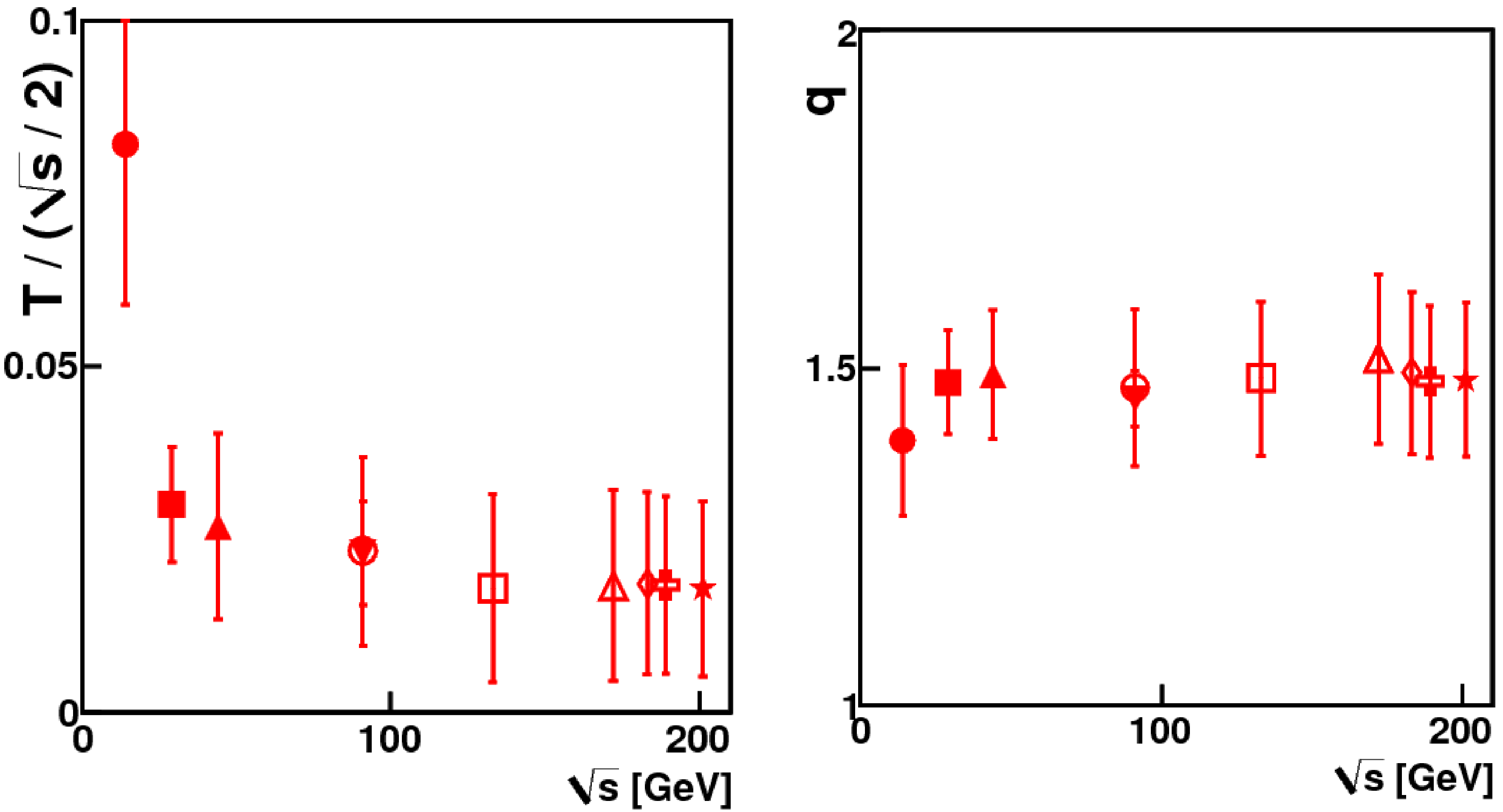
e+e- jets data / theory



Ürmössy, Barnaföldi, Biró arXiv: 1101.3023

The power is sqrt(s) dependent: dimensionality reduces from 3 to 1.

e+e- jets 1D-fit parameters



Ürmössy, Barnaföldi, Biró arXiv: 1101.3023

e+e- jets power of 1+log: S & E both non-additive

$$p(E) \propto \left(1 - \hat{\beta} \frac{\hat{a}}{a} \ln(1 - aE)\right)^{-1/\hat{a}}$$

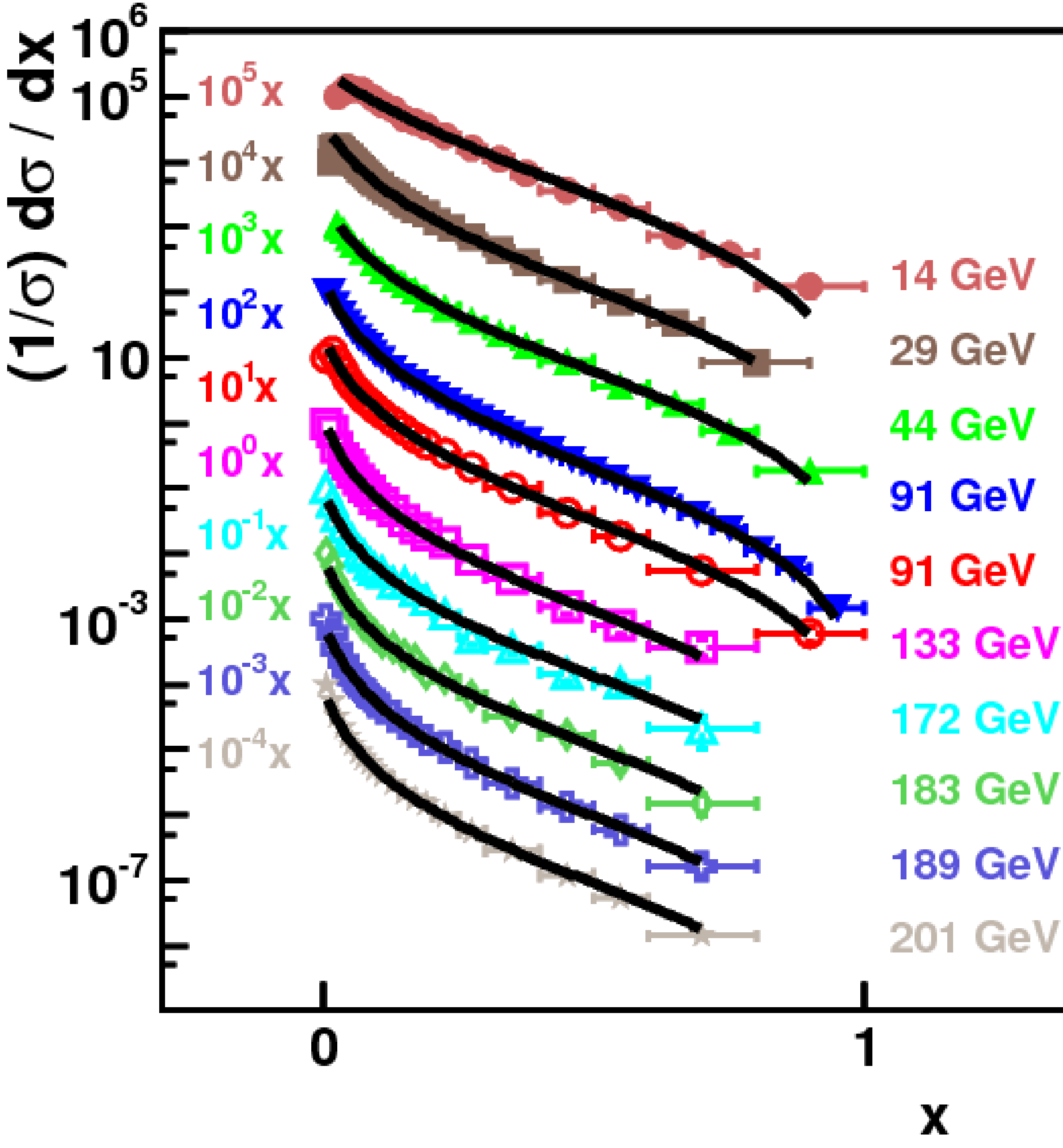
$$0 \le E \le 1/a = \sqrt{s}/2$$

$$0 \le aE \le 1 \quad 0 \le L(E) \le \infty$$

aE = x : canonical distribution is compact in E.

Superstatistics: over Euler-Gamma multiplicity distribution (KNO)

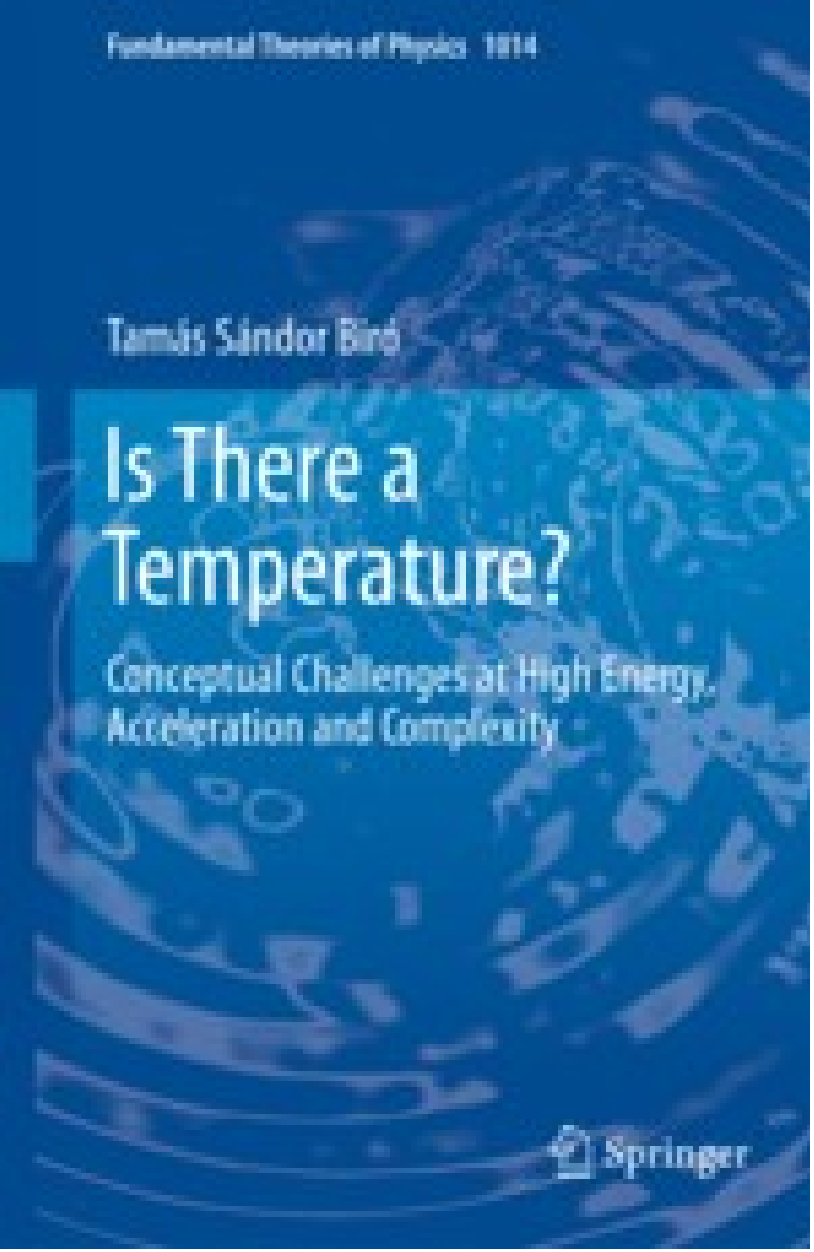
Ürmössy, Barnaföldi, Biró arXiv: 1101.3023

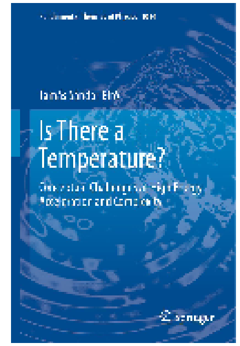


(1/σ) dσ / dx
10^6
10^5
10
10^-3
10^-7
0
1
x
10^5x
10^4x
10^3x
10^2x
10^1x
10^0x
10^-1x
10^-2x
10^-3x
10^-4x
14 GeV
29 GeV
44 GeV
91 GeV
91 GeV
133 GeV
172 GeV
183 GeV
189 GeV
201 GeV

Summary

- Thermodynamics built on composition laws is:
    - Zeroth Law compatible
    - Prefers the use of formal logarithms
    - Has canonical power-law and log power distributions
- The problem of heterogeneous (extensive – non-extensive) equilibrium is solved!
- Attempts to fit very high pT jet spectra within this generalized canonical approach seem to work → T,q





2011. XIV, 310 p. 33 illus. (Fundamental Theories of Physics, Vol. 1014) Hardcover

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T.S. Biró, Hungarian Academy of Sciences, Budapest, Hungary

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History of Entropy

1854	**Clausius**	$\Delta S = Q\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$
1856	**Clausius**	$\int \frac{\delta Q}{T} = -N$
1862	**Clausius**	$\int \frac{\delta Q}{T} \geq 0$
1865	**Clausius**	S entropy

History of Entropy

1877 **Boltzmann**

# of microstates

$$S = k \log W$$

1878 **Gibbs**

probability

$$S = -\sum_i p_i \ln p_i$$

1948 **Shannon**

information measure

$$H = -K \sum_{i=1}^{n} p(i) \log_2 p(i)$$

1957 **Jaynes**

S is an info-entropy

History of Entropy

1960	**Rényi**	generalizes Shannon
1988	**Tsallis**	generalizes Boltzmann