QCD phase transitions with functional methods

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JLU Giessen

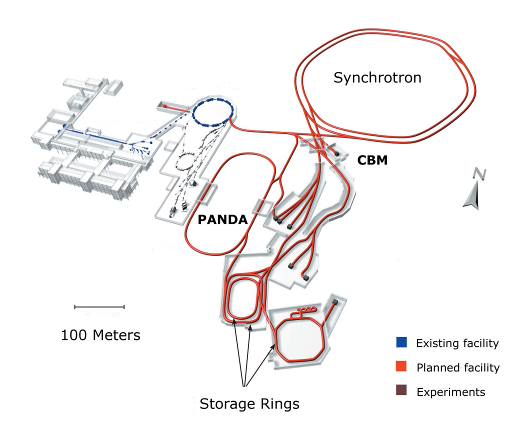
Feb. 2011

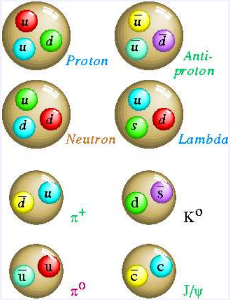
C.F., A. Maas and J. A. Mueller, EPJ **C68** (2010) 165-181.
J. A. Mueller, C.F. and D. Nickel, EPJ **C70** (2010) 1037
J. Luecker, C.F., J. A. Mueller, in preparation

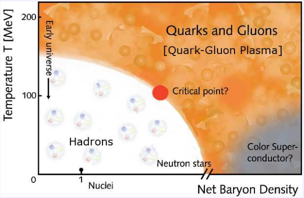
Content

Content

FAIR: CBM and PANDA

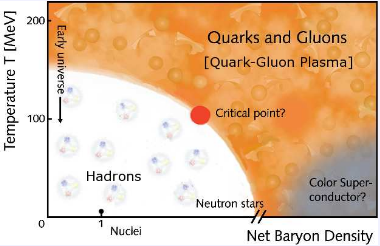






QCD phase transitions

- Existence and location of CEP
- Propagation of gluons in QGP
- Properties of quarks in QGP



- Chiral limit ($M_{weak} \to 0$): order parameter chiral condensate

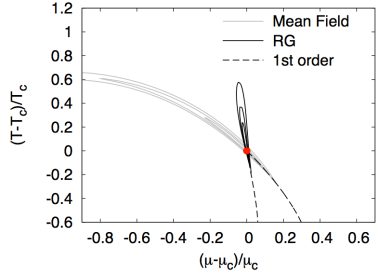
$$\langle \bar{\psi}\psi \rangle = Z_2 N_c Tr_D \int \frac{d^4p}{(2\pi)^4} S(p)$$

- Static quarks ($M_{weak} \to \infty$): order parameter Polyakov-loop

$$\Phi \sim e^{-F_q/T}$$

Why we need Functional Methods like the RG

Quark-meson model:



B.-J. Schaefer, J. Wambach PRD 75 (2007) 085015

- determine size of critical region from quark number susceptibility

$$\chi_q(T,\mu) = \partial n(T,\mu)/\partial\mu$$

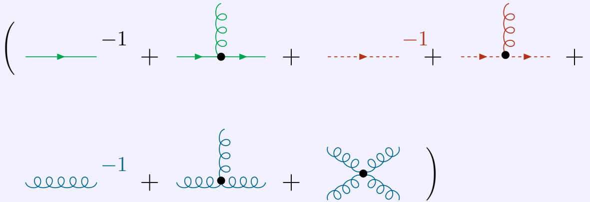
- RG: dramatic decrease in size !
  $\rightarrow$ relevant for CBM

QCD in covariant gauge

quarks, gluons and ghosts:

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A, c] \; \exp\left\{ - \int d^4x \left(\overline{\Psi}\,(i\not{D} - m)\,\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \frac{(\partial A)^2}{2\xi} + \bar{c}(-\partial D)c \right) \right\}$$

$$S_{QCD} = \int d^4x$$



$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A, c] \; \exp\left\{ -\int_0^{1/T} dt \int d^3x \left(\bar{\Psi}\,(i\not{D} - m)\,\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \frac{(\partial A)^2}{2\xi} + \bar{c}(-\partial D)c \right) \right\}$$

Landau gauge ($\xi = 0$) propagators in momentum space, $q = (\vec{q}, \omega_q)$:

$$D^{\text{Gluon}}_{\mu\nu}(q) = \frac{Z_T(q)}{q^2} P^T_{\mu\nu}(q) + \frac{Z_L(q)}{q^2} P^L_{\mu\nu}(q)$$

$$S^{\text{Quark}}(q) = \frac{1}{-i\,\vec{\gamma}\vec{q}\;A(q) - i\,\gamma_4\omega_n\;C(q) + B(q)}$$

The Goal:

Gauge invariant information from gauge fixed functional approach

Green's functions

QCD Green's functions

- are connected to confinement:
  - Gribov-Zwanziger and Kugo-Ojima scenarios
  - Running Coupling
  - Positivity
  - Polyakov Loop
- encode DχSB
- are ingredients for hadron phenomenology
  - Bound state equations:
    Bethe–Salpeter equation / Faddeev equation
  - Form factors, decays etc.

The Goal:

Gauge invariant information from gauge fixed functional approach

The Tool:

Dyson-Schwinger and Bethe-Salpeter-equations (DSE/BSE)

Lattice QCD vs. DSE/FRG: Complementary!

- Lattice simulations
  - Ab initio
  - Gauge invariant

- Functional approaches:
  Dyson-Schwinger equations (DSE)
  Functional renormalisation group (FRG)
  - Analytic solutions at small momenta
  - Space-Time-Continuum
  - Chiral symmetry: light quarks and mesons
  - Multi-scale problems feasible: e.g. (g-2)$_\mu$

    T. Goecke, C.F., R. Williams, arXiv:1012.3886 [hep-ph]
  - Chemical potential: no sign problem

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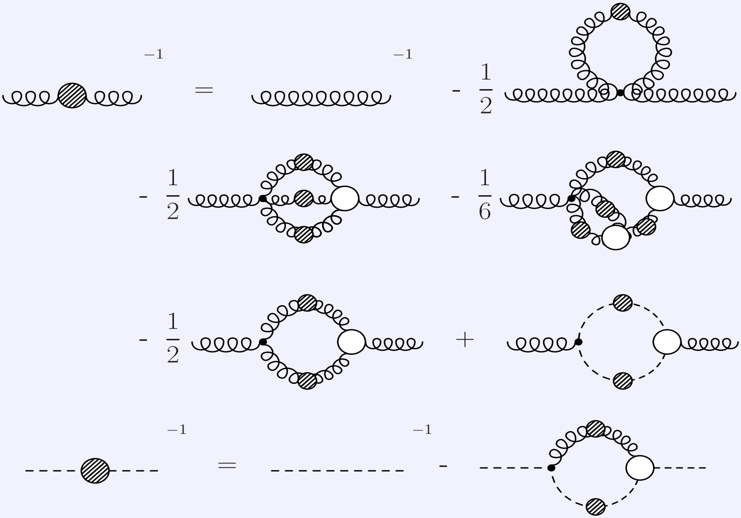
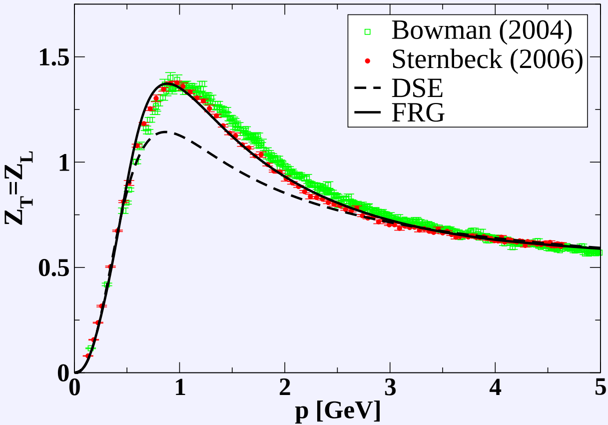
Dyson-Schwinger equations (DSEs)



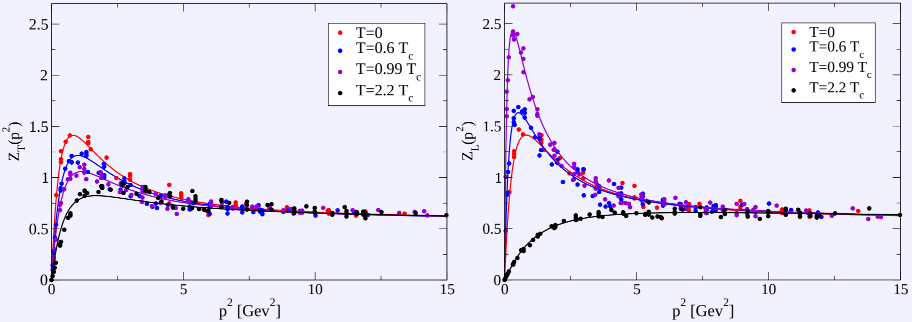
DSEs vs Lattice ($T = 0$)



- Deep infrared: Interesting and subtle questions
- Physics: positivity violation, glueball states

L. von Smekal, R. Alkofer, A. Hauck, PRL **79** (1997) 3591-3594.
C.F., A. Maas and J. M. Pawlowski, Annals Phys. **324** (2009) 2408-2437.

T-dependent gluon propagator from lattice simulations:

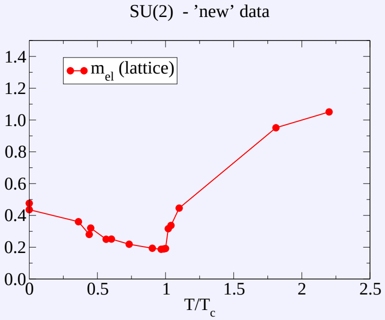


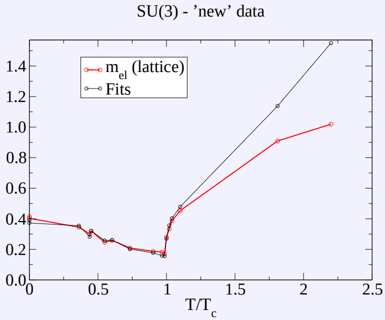
- Difference between electric and magnetic gluon
- Maximum of electric gluon at T_c

Cucchieri, Maas, Mendes, PRD 75 (2007)

C.F., Maas and Mueller, EPJC 68 (2010)

Gluon screening mass





C.F., Maas, Mueller, EPJC 68 (2010)

$$Z_{T,L}(q,T)\frac{q^2\Lambda^2}{(q^2+\Lambda^2)^2}\left\{\left(\frac{c}{q^2+\Lambda^2 a_{T,L}(T)}\right)^{b_{T,L}(T)}+\frac{q^2}{\Lambda^2}\left(\frac{\beta_0\alpha(\mu)\ln[q^2/\Lambda^2+1]}{4\pi}\right)^{\gamma}\right\}$$

- SU(2): 2nd order phase transition not yet visible
- Fits more reliable at large T than lattice: $m \propto T$

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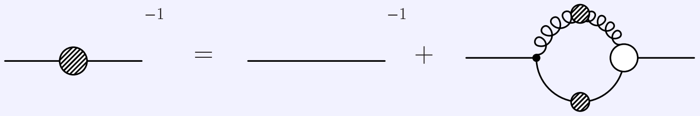
The ordinary chiral condensate



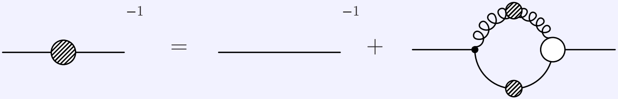
- gluon propagator from DSE/lattice
- $T = 0$: quark-gluon vertex studied via DSEs

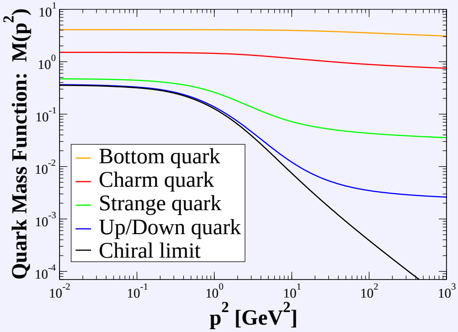
  Alkofer, C.F., Llanes-Estrada, Schwenzer, Annals Phys.324:106-172,2009.

  C.F, R. Williams, PRL **103** (2009) 122001
- $T \neq 0$: temperature and mass dependent ansatz
- Order parameter for chiral symmetry breaking:

$$\langle\bar{\psi}\psi\rangle = Z_2 N_c T \sum_{n_p} \int \frac{d^3p}{(2\pi)^3} \, Tr_D \; S(\vec{p}, \omega_p)$$

$T = 0$: Explicit vs. dynamical chiral symmetry breaking



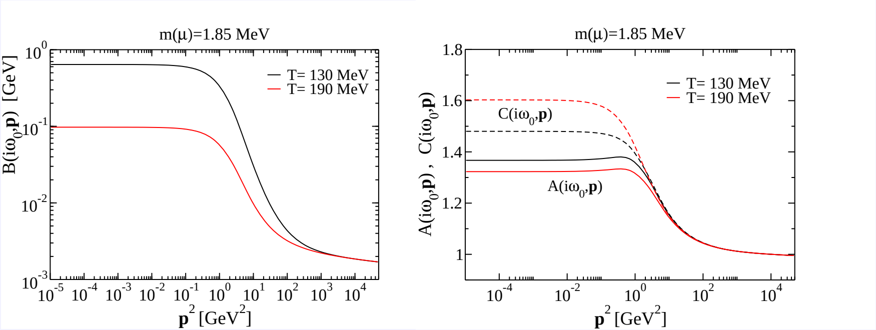


C.F. J.Phys.G G32 (2006) R253-R291

- $M(p^2) = B(p^2)/A(p^2)$: momentum dependent!
- Dynamical masses $M_{strong}(0) \approx 350$ MeV
- Flavour dependence because of M_{weak}
- $\langle\bar{\psi}\psi\rangle \approx (250\text{MeV})^3$

$T \neq 0$: Chiral symmetry restauration

$$S^{\text{Quark}}(q) = \frac{1}{-i\,\vec{\gamma}\vec{q}\,A(q) - i\,\gamma_4\omega_n\,C(q) + B(q)}$$

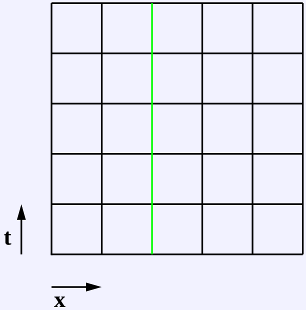


- dynamical effects below T_c $\leftrightarrow$ 'HTL-ish' above T_c

The Polyakov Loop

$$\Phi = \left\langle \frac{1}{N_c} Tr_D \mathcal{P} \exp\left\{ i \int_0^{1/T} A_4 \, dt \right\} \right\rangle \sim e^{-F_q/T}$$

Lattice:



Order parameter for center symmetry breaking:

$\Phi = 0$: confined

$\Phi \neq 0$: deconfined

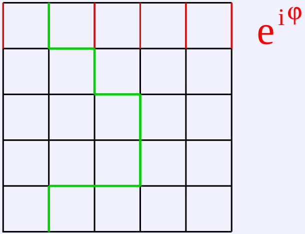
The dual condensate I

Consider general $U(1)$-valued boundary conditions in temporal direction for quark fields ψ:

$$\psi(\vec{x}, 1/T) = e^{i\varphi}\psi(\vec{x}, 0)$$

Matsubara frequencies: $\omega_p(n_t) = (2\pi T)(n_t + \varphi/2\pi)$

Lattice:



$$\langle\bar{\psi}\psi\rangle_\varphi \sim \sum \frac{exp[i\,\varphi\, n]}{(am)^l} \text{ Closed Loops}$$

E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007.
F. Synatschke, A. Wipf and C. Wozar, PRD **75**, 114003 (2007).

The dual condensate II

Then define dual condensate Σ_n:

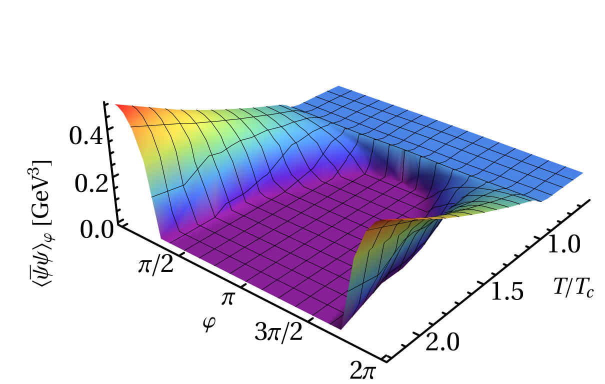
$$\Sigma_n = -\int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-i\varphi n} \langle\overline{\psi}\psi\rangle_\varphi$$

- $n = 1$ projects out loops with $n(l) = 1$: dressed Polyakov loop
- transforms under center transformation exactly like ordinary Polyakov loop: order parameter for center symmetry breaking
- Σ_1 is accessible with functional methods

  C.F., PRL **103** (2009) 052003

C. Gattringer, PRL **97**, 032003 (2006)
F. Synatschke, A. Wipf and C. Wozar, PRD **75**, 114003 (2007).
E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** 094007 (2008).
F. Synatschke, A. Wipf and K. Langfeld, PRD **77**, 114018 (2008).
J. Braun, L. Haas, F. Marhauser, J. M. Pawlowski, PRL 106 (2011)

Condensate: angular dependence in chiral limit



-

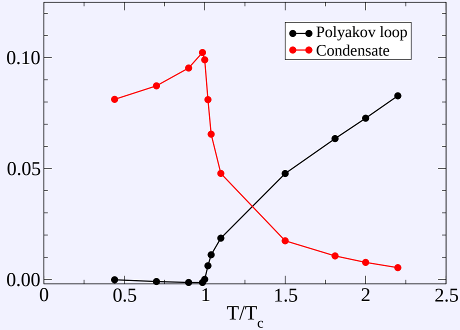
$$\Sigma_1 \quad = \quad -\int_0^{2\pi} \frac{d\varphi}{2\pi}\, e^{-i\varphi} \,\langle\bar{\psi}\psi\rangle_\varphi$$

- Width of plateau is T-dependent, $\langle\bar{\psi}\psi\rangle_\varphi(\varphi = 0) \sim T^2$

C.F. and Jens Mueller, PRD **80** (2009) 074029.

Transition temperatures, quenched

quenched, DSE

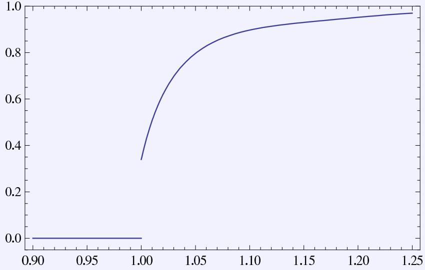


C.F., Maas, Mueller, EPJC 68 (2010).
Mueller, PhD-thesis.

- SU(2): $T_c \approx 305$ MeV
  SU(3): $T_c \approx 270$ MeV
- increasing condensate due to electric part of gluon

cf. Buividovich, Luschevskaya, Polikarpov, PRD 78 (2008) 074505.

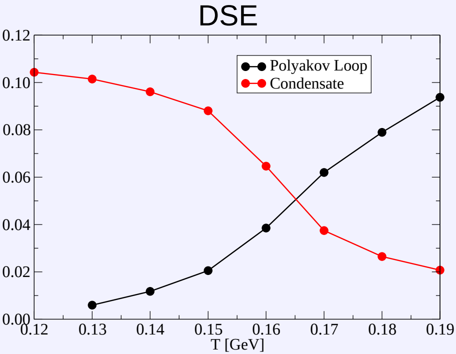
quenched, FRG



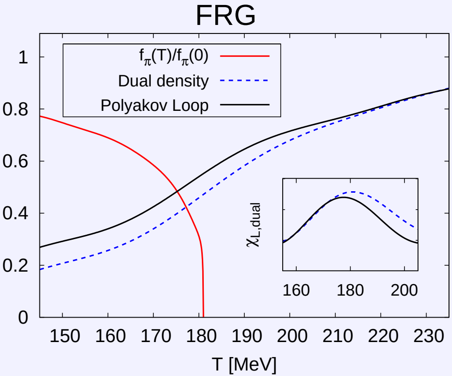
J. Braun, H. Gies, J. M. Pawlowski, PLB 684 (2010) 262-267.

- Polyakov loop potential via gluon/ghost propagators

Transition temperatures, $N_f = 2$



Luecker, C.F., Mueller, in preparation
Mueller, PhD-thesis TU Darmstadt 2010.

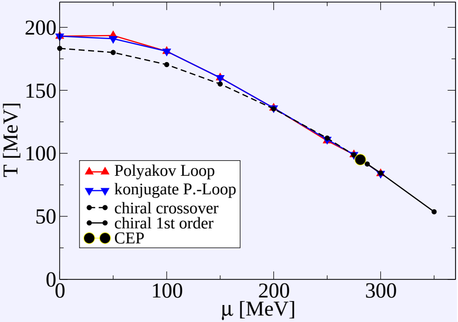


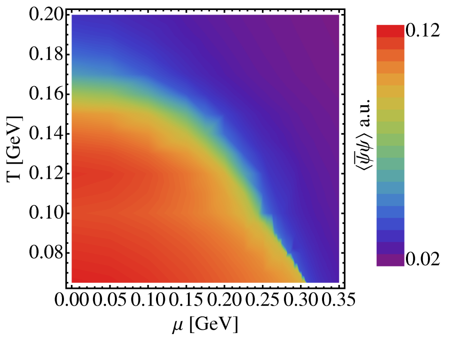
J. Braun, L. Haas, F. Marhauser, J. M. Pawlowski, PRL 106 (2011) 022002

- reduction of T_c: $T_c \approx 165$ MeV
- 'usual' behaviour of condensate

- chiral limit
- $T_\chi \simeq T_{conf} \simeq 180$ MeV

$\mu \neq 0$: Chiral and deconfinement transition





- approximation: backcoupling of quarks onto glue via HTL
- CEP at $(T, \mu) \simeq (85, 275)$ MeV
- Small μ: difference between Φ and $\bar{\Phi}$ with $\Phi < \bar{\Phi}$

C.F., Luecker, Mueller, in preparation
J. Mueller, PhD-thesis, TU Darmstadt, 2010

Content

Framework

- Vital input into transport approaches (HSD,...)
- Computation of quark-loop contribution of dilepton production

  Braaten, Pisarski, Yuan, PRL **64**, 1990

Idea: Fit spectral representation to quark propagator

F. Karsch and M. Kitazawa, PRD **80**, 056001 (2009).
F. Karsch and M. Kitazawa, PLB **658**, 45 (2007).

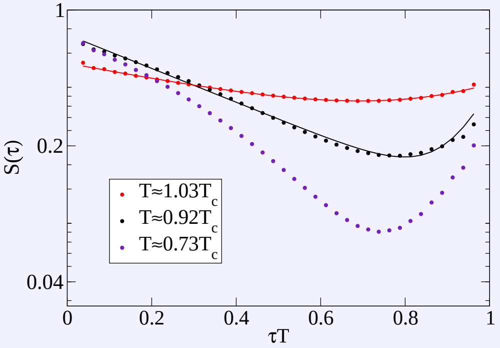
$$S(\omega_p, \vec{p}) = \int d\omega' \, \frac{\rho(\omega', \vec{p})}{\omega_p - \omega'}$$

Use ansatz for spectral function:

$$\rho(\omega) = Z_1 \; \delta(\omega - E_1) + Z_2 \; \delta(\omega + E_2)$$

Pseudoparticles: Quark and Plasmino (idealized!)

Results I: Deconfinement and quark analytic structure

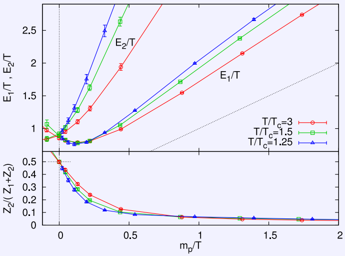


Mueller, C.F., Nickel, EPJ **C70** (2010) 1037

- $T > T_c$: Two pole ansatz works: quark and plasmino
- $T < T_c$: No fit with only real poles; positivity violations
- Alternative criterion for deconfinement

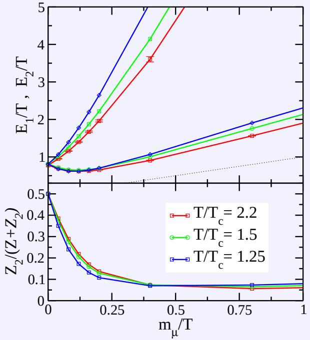
Results II: Mass dependence of quark and plasmino

Lattice



Karsch and Kitazawa, PRD **80**, 056001 (2009).

DSE

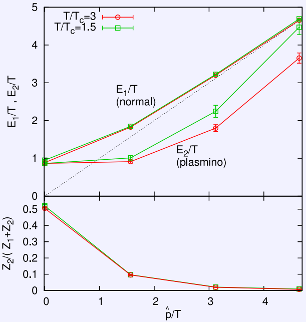


Mueller, C.F., Nickel, EPJ **C70** (2010) 1037

- Qualitative agreement with lattice results
- Large quark masses: plasmino disappears

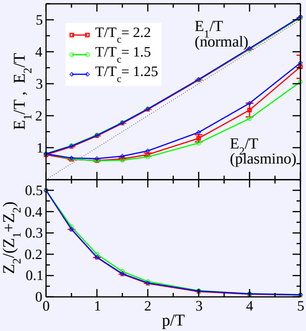
Results III: Dispersion Relation

Lattice



Karsch and Kitazawa, PRD **80**, 056001 (2009).

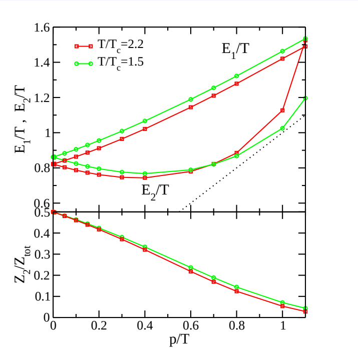
DSE

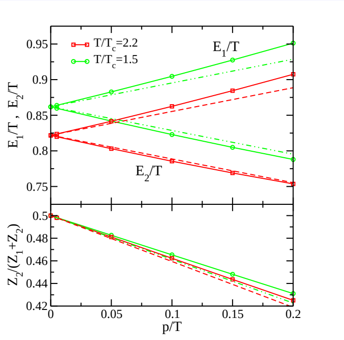


Mueller, C.F., Nickel, EPJ **C70** (2010) 1037

- Min for Plasmino at $p \neq 0$
- Plasmino enters spacelike region $\rightarrow$ fit function incomplete!

Results IV: Dispersion Relation - Details





- Include continuum part (Landau damping) into fits:

$$\rho^P_{\pm}(\omega, |\boldsymbol{p}|) = 2\pi \left[Z_1 \delta(\omega \mp E_1) + Z_2 \delta(\omega \pm E_2) \right]$$

$$+ \frac{\pi}{|\boldsymbol{p}|} m_T^2 \, (1 \mp \frac{\omega}{|\boldsymbol{p}|}) \Theta(1 - (\frac{\omega}{|\boldsymbol{p}|})^2) \left[\left(|\boldsymbol{p}| \, (1 \mp \frac{\omega}{|\boldsymbol{p}|}) \pm \frac{m_T^2}{2\,|\boldsymbol{p}|} \left[(1 \mp \frac{\omega}{|\boldsymbol{p}|}) \ln \left| \frac{\frac{\omega}{|\boldsymbol{p}|} + 1}{\frac{\omega}{|\boldsymbol{p}|} - 1} \right| \pm 2 \right] \right)^2 + \frac{\pi^2 m_T^4}{4 |\boldsymbol{p}|^2} (1 \mp \frac{\omega}{|\boldsymbol{p}|})^2 \right]^{-1}$$

- Dashed lines: HTL-result of slope at $p \to 0$.

Mueller, C.F., Nickel, EPJ **C70** (2010) 1037

Summary: QCD phase transitions

Summary:

- Temperature dependent gluon propagator:
  characteristic behavior of electric screening mass at T_c
- Similar T_c from dressed Polyakov-loop calculated from DSEs
- Similar chiral T_c from ordinary quark condensate
- Finite chemical potential beyond mean field
- Quark spectral functions: quarks and plasminos

Outlook:

- Quark spectral functions at finite chemical potential
- Thermodynamic observables

Thank you for your attention!

Happy Birthday and Happy Retirement, Ulrich!

Helmholtz Young Investigator Group ”Nonperturbative Phenomena in QCD”

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

HELMHOLTZ GEMEINSCHAFT

GSI

HIC for FAIR

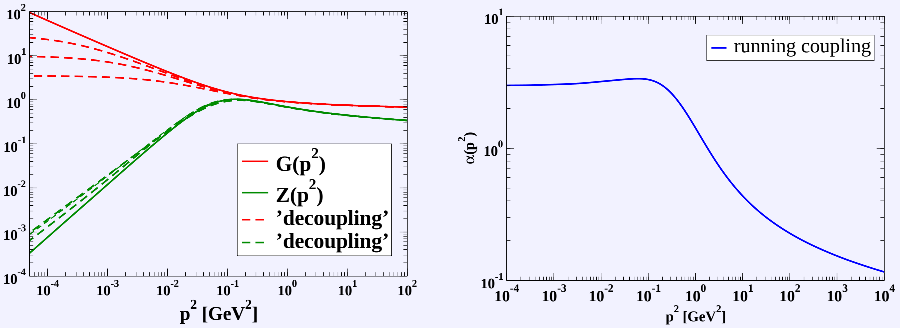
LOEWE – Landes-Offensive zur Entwicklung Wissenschaftlich-ökonomischer Exzellenz

Gluon and Quark-Gluon-Vertex

Ansatz for Quark-Gluon-Vertex:

$$\Gamma_\nu(q,k,p) = \widetilde{Z}_3 \left(\delta_{4\nu}\gamma_4 \frac{C(k)+C(p)}{2} + \delta_{j\nu}\gamma_j \frac{A(k)+A(p)}{2} \right) \times \\ \times \left(\frac{d_1}{d_2+q^2} + \frac{q^2}{\Lambda^2+q^2} \left(\frac{\beta_0 \alpha(\mu) \ln[q^2/\Lambda^2+1]}{4\pi} \right)^{2\delta} \right) .$$

Ghost, Glue and Coupling



- dynamically generated scale
- fixed point of coupling $\alpha(p^2) = g^2/(4\pi)Z(p^2)G(p^2) \approx 9/N_c$
- deep infrared ($p < 50$ MeV): scaling vs. decoupling

CF and Alkofer, PLB 536 (2002) 177.

C. Lerche and L. von Smekal, PRD **65**, 125006 (2002)

C.F., A. Maas and J. M. Pawlowski, Annals Phys. **324** (2009) 2408-2437.