

Facets of the QCD Phase Diagram

“Herr Koch, Intelligenz ist die geringste Voraussetzung
für wissenschaftlichen Erfolg”
(U. Mosel, 1984)



(“Intelligence is the least prerequisite for scientific success”)

Big Scientific Success



Creating the perfect liquid in heavy-ion collisions

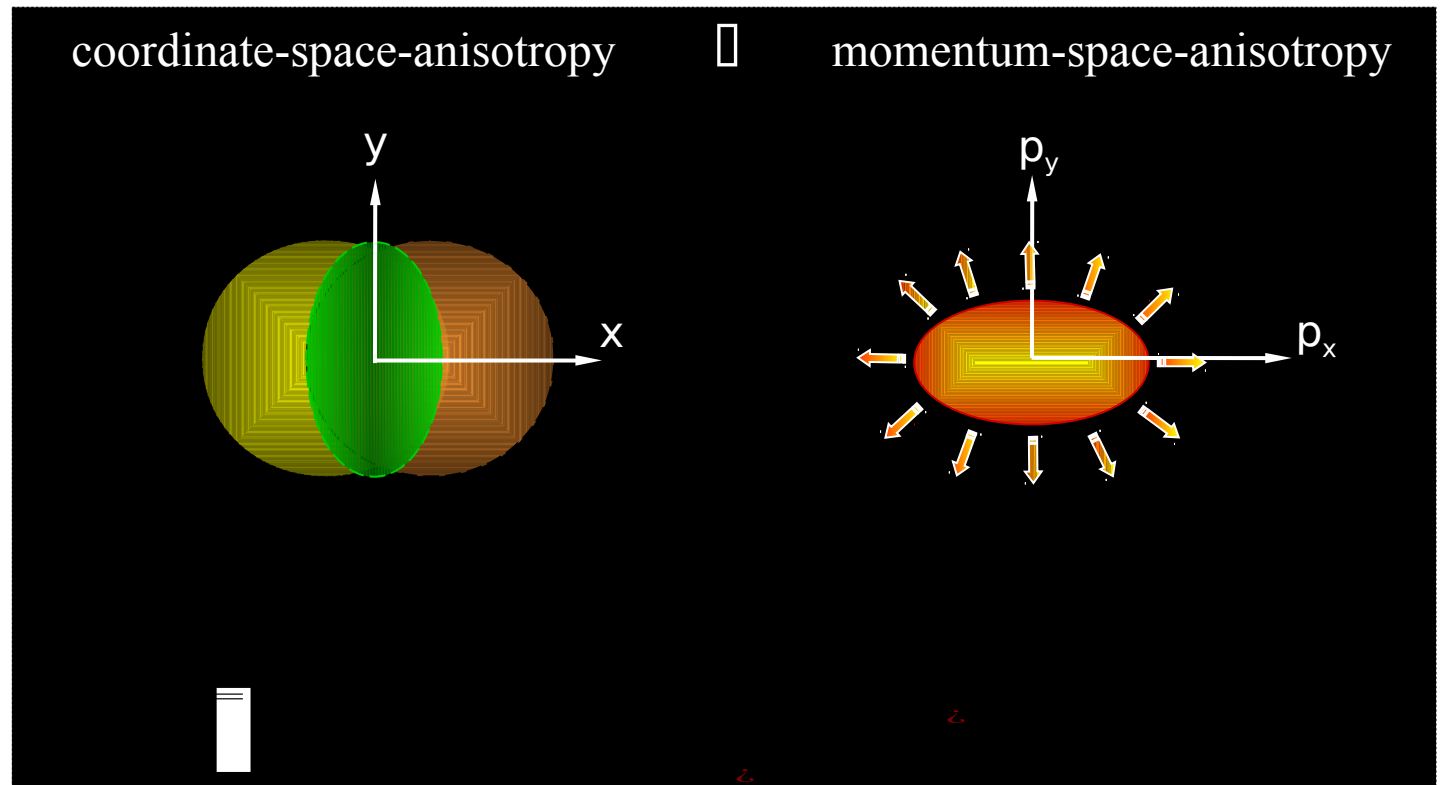
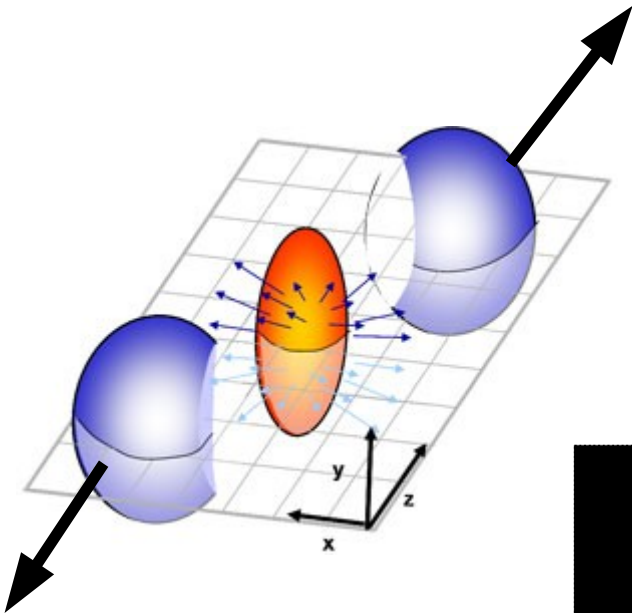
Barbara Jacak and Peter Steinberg

Expecting to find a very weakly coupled gas of quarks and gluons created in energetic collisions at Brookhaven's Relativistic Heavy Ion Collider, experimenters found instead a strongly coupled liquid with almost no viscosity.

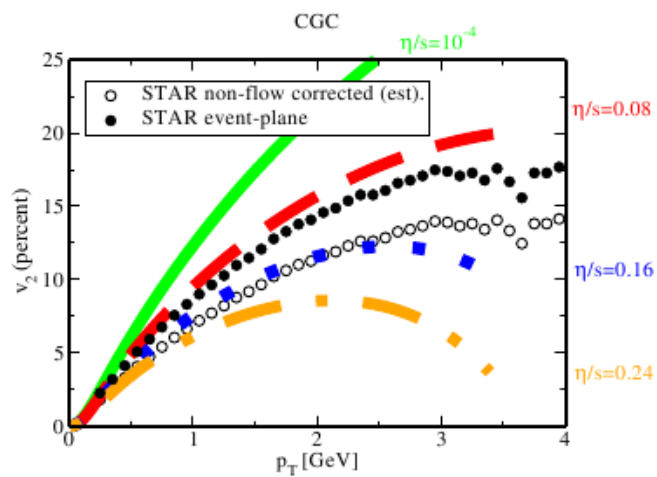
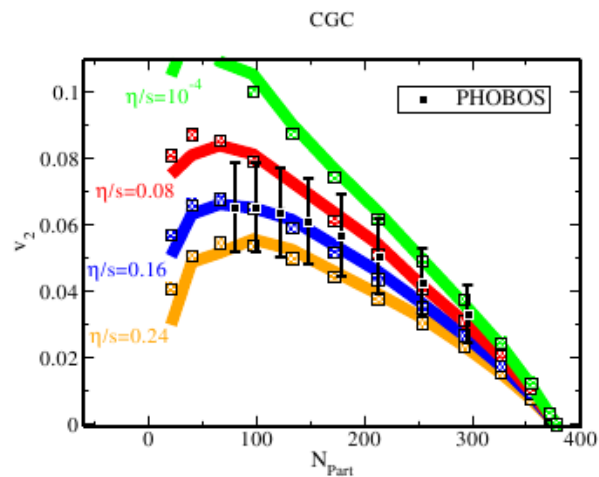
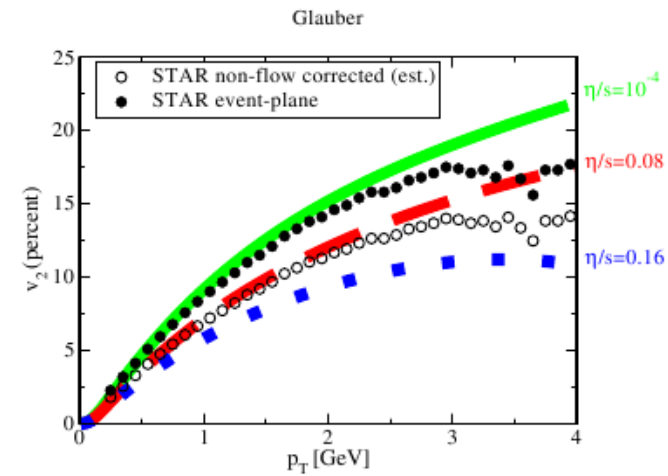
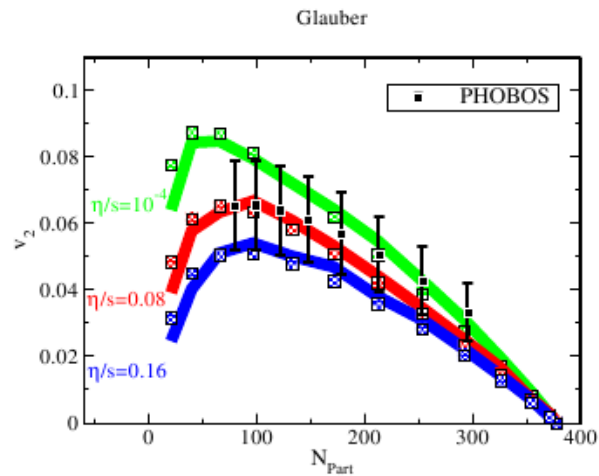
Physics Today, May 2010

Elliptic Flow

$$\frac{dN}{d\phi dp_t} = \frac{dN}{dp_t} (1 + v_1(p_t) \cos(\phi) + 2 \underline{v_2(p_t)} \cos(2\phi) + \dots)$$



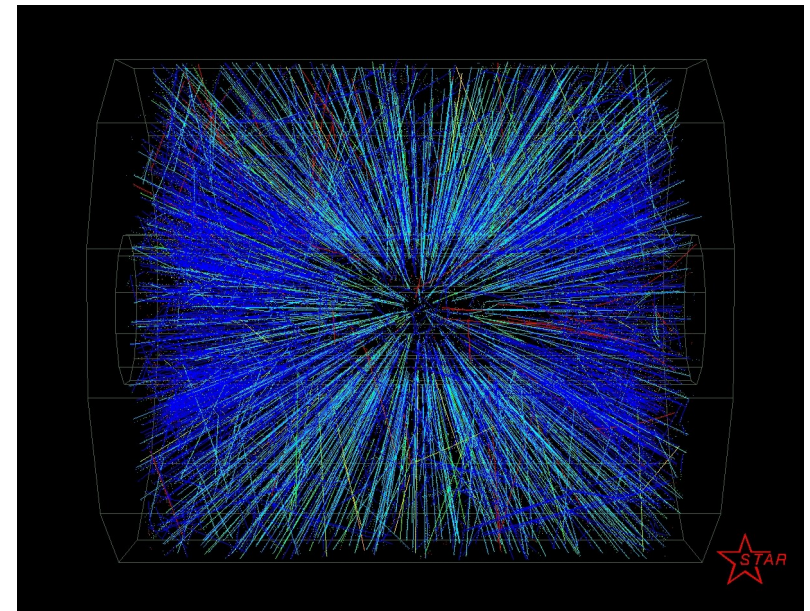
Viscous Hydrodynamics



The Perfect Liquid?



VS

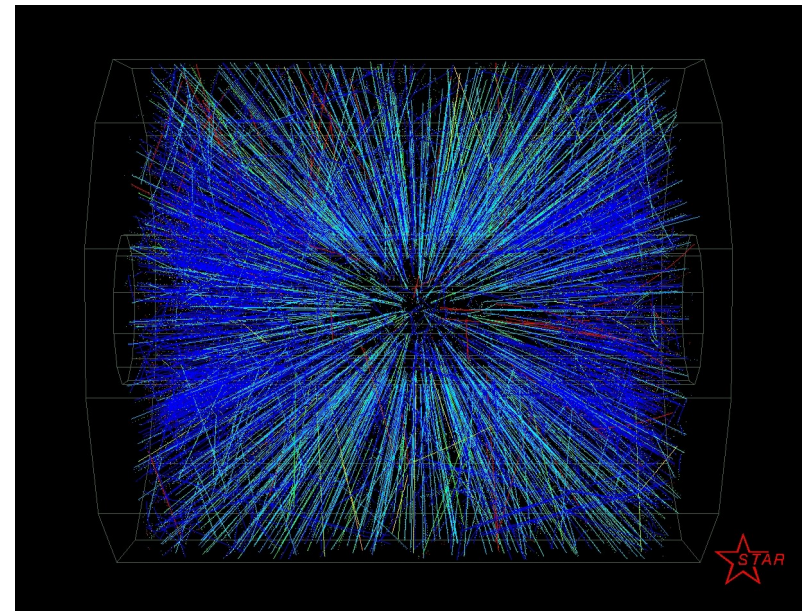


J. Liao and V.K, arXiv:0909.3105,
Phys.Rev.C80:034904,2009.

The Perfect Liquid?



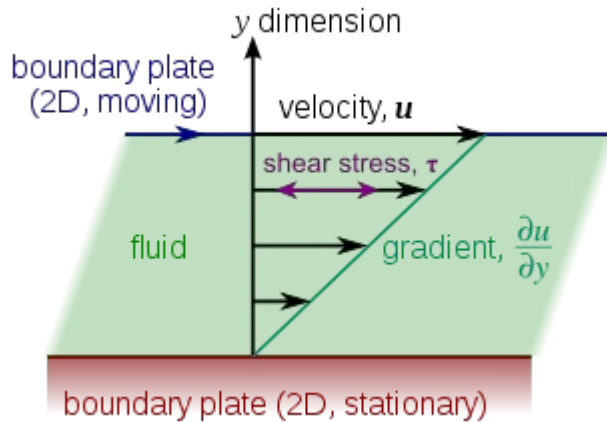
VS



J. Liao and V.K, arXiv:0909.3105,
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Viscosity

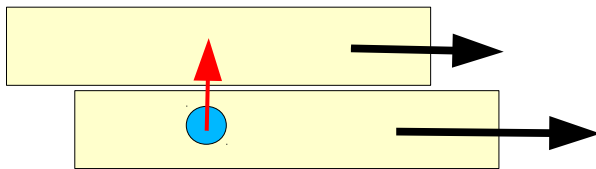
Correction to ideal fluid dynamics



Shear tensor:
$$\sigma_{i,j} = \eta \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3} \delta_{i,j} \frac{\partial v_i}{\partial x_i} \right)$$

$\eta = \textit{shear viscosity}$

Kinetic theory (dilute gases): Momentum exchange by particle transport



$$\eta \sim n \bar{v} m \lambda$$

$n = \textit{density}$

$\bar{v} = \textit{thermal velocity}$

$\lambda = \textit{mean free path}$

Viscosity ~ mean free path!

Viscosity

Navier Stokes Equation:
$$\rho \left[\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right] = -\nabla p + \eta \nabla^2 \vec{v} + \dots$$

“Inertia”
“Force”
“Friction”

Viscosity
[kg/m s]

Kinematic Viscosity $\nu = \frac{\eta}{\rho}$
[m² / s]

Water	0.001
Air	0.000018

0.10
1.5



The kinematic viscosity (friction/inertia) controls how good a fluid is

Minimum viscosity

AdS/CFT correspondence:

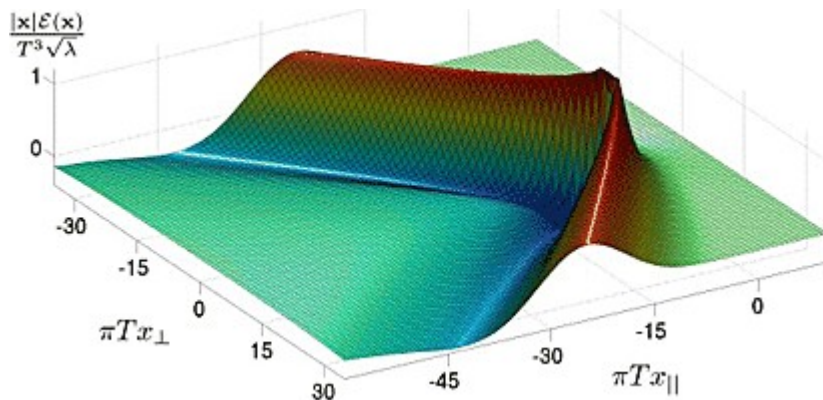
Maldacena et al, hep-th/9905111v3

Kovtun, Son, hep-th/0405231v2

$$\frac{\eta}{s} \geq \frac{1}{4\pi} \quad \eta = \text{shear viscosity}$$

$$s = \text{entropy density}$$

Holds for a large class of strongly coupled gauge theories



Kovtun, arXiv:0706.0368

Kinetic theory + waving hands:

$$\eta \sim n \bar{v} m \lambda$$

$$\bar{v} m = p, \quad p \lambda > \hbar, \quad n \sim s$$

$$\frac{\eta}{n} \sim \frac{\eta}{s} \geq 1$$

Quantum bound

More detailed derivation: Danielewicz and Gyulassy (85)

The perfect fluid?

- Is there a quantum bound on η/s ?
- Does the quantum bound provide a limit on fluidity?
- Has RHIC produced such a system? I assume so
- How about other substances
 - Water, Bio Vital, liquid Helium, cold quantum gases???
- How does one define fluidity?
- How do I compare systems on the atomic/molecular scale with those at quark/gluon scale?

Defining Fluidity

Hydrodynamics works for a big variety of systems:

- Liquids (Water)
- Gases (Air, sound)
- Interstellar Dust (Star formation)
- QGP ?

Problem: How to compare substances at vastly different length scales?

- Interstellar Dust: $n^{-1/3} \sim 10^{-4}$ m
- Water: $n^{-1/3} \sim 3 \cdot 10^{-10}$ m
- Air : $n^{-1/3} \sim 3 \cdot 10^{-9}$ m
- QGP : $n^{-1/3} < 10^{-15}$ m

Typical criterion for applicability of fluid dynamics:

Knudsen Number :
$$\frac{\text{Mean Free Path}}{\text{typical lengthscale of variation}}$$

← Kinetic theory???

← Different length-scales???

Obviously not what we need

Defining Fluidity

- 1) Extract “effective mean free path” solely from fluid-dynamics
- 2) Calibrate with “inter-particle” distance

Effective mean free path:

Analyze sound modes and determine **minimum** wavelength

$$\omega = c_s k + \frac{i}{2} k^2 \frac{\frac{4}{3} \eta}{w/c^2}$$

Enthalpy density:

$$w = \epsilon + p = Ts + \mu n \approx Ts + m n$$

$w \rightarrow m n$ Non-relativistic limit: **mass density**
controls inertia

$w \rightarrow Ts$ Relativistic limit: **entropy density**
controls inertia

Damping $\sim k^2$:

Hydro always works
in long wavelength limit

$$\frac{\eta}{S}$$

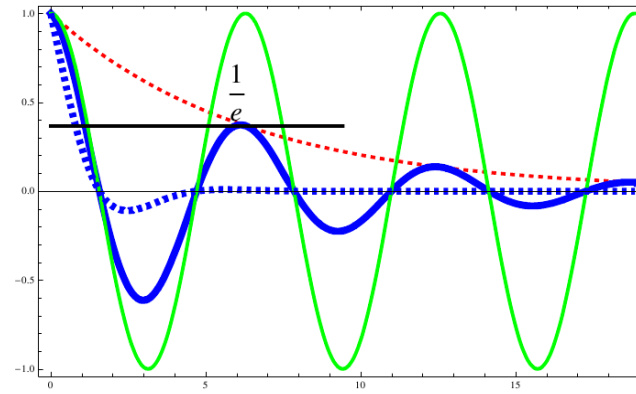
cannot be a universal quality measure

Fluidity measure

Effective mean free path: Analyze sound modes

$$\omega = c_s k + \frac{i}{2} k^2 \frac{\frac{4}{3}\eta}{w/c^2}$$

Require: $\frac{|\Im(\omega)|}{|\Re(\omega)|} \equiv \frac{L_\eta}{\Lambda} \ll 1$



Provides a minimal wavelength $\Lambda = L_\eta$

Dilute (kinetic limit): $L_\eta \rightarrow \lambda_{mfp}$

$$L_\eta = \frac{\eta}{w c_s}$$

Enthalpy density

$$w = \epsilon + p = Ts + \mu n \approx Ts + m n$$

Fluidity measure

$$L_\eta = \frac{\eta}{w c_s}$$

Calibrate with “inter-particle distance” d :

$$d \Leftrightarrow \langle \epsilon(x) \epsilon(0) \rangle \quad \text{Non-relativistic systems} \quad d = n^{(-1/3)}$$

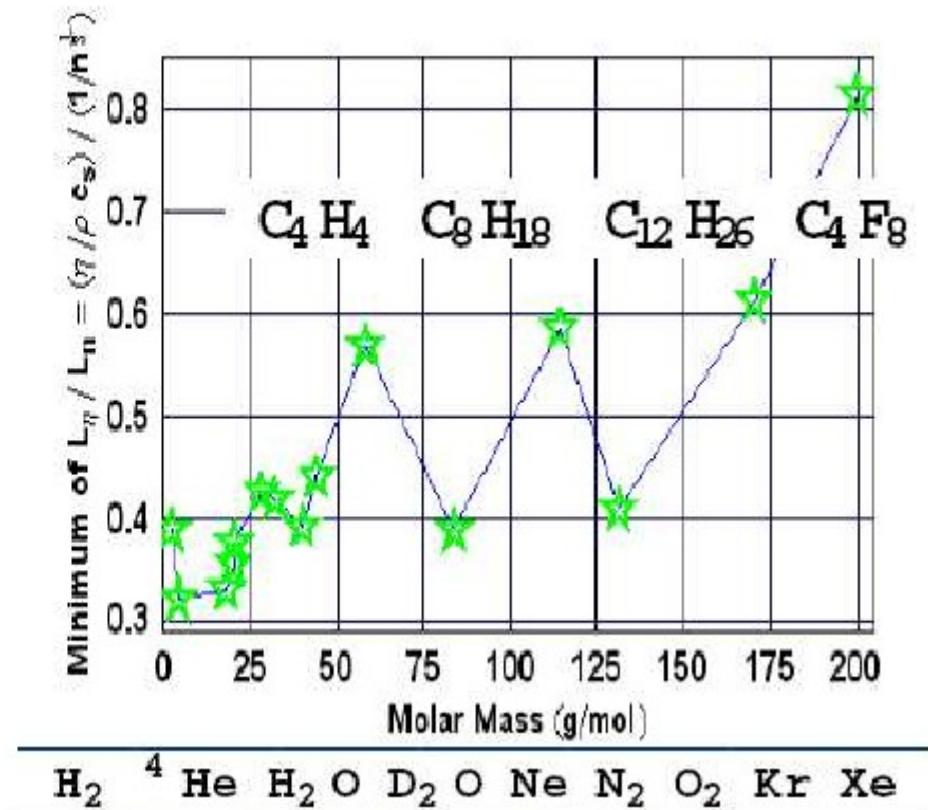
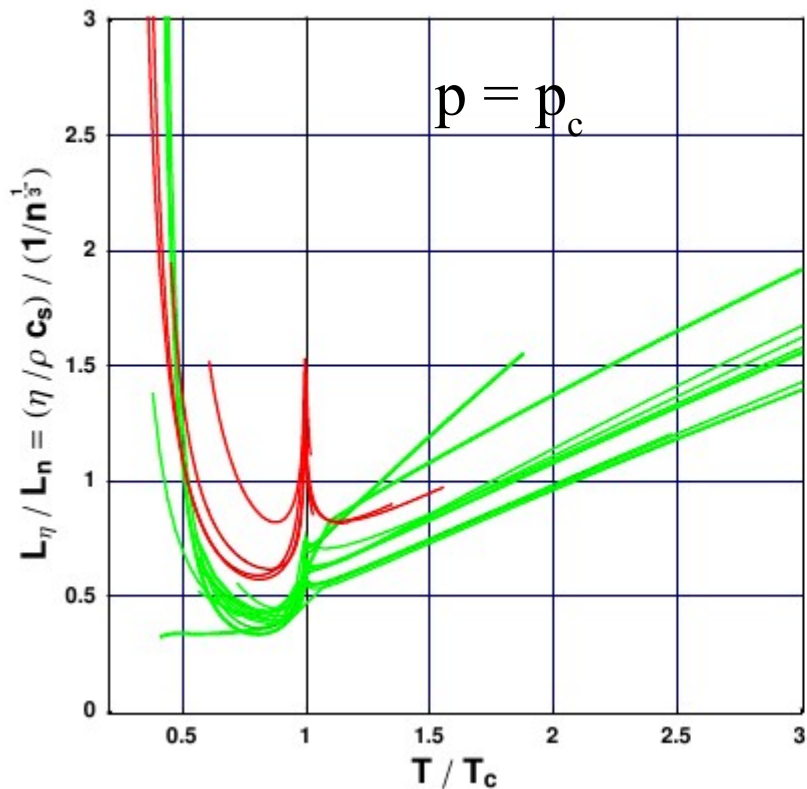
Fluidity measure:

$$F = \frac{L_\eta}{d} = \frac{\eta}{w c_s} \frac{1}{d} = \frac{\eta}{w c_s} n^{1/3}$$

Depends only on *intrinsic* properties of substance
Well defined: NO kinetic theory needed!

Fluidity measure

16 substances with M_{mol} , T_c , p_c spanning 2 Orders of Magnitude

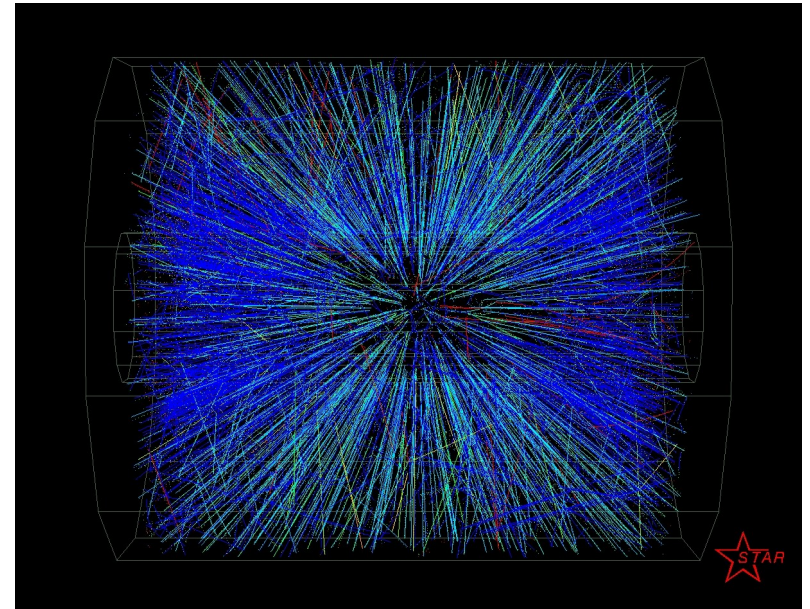


A good fluid is a good fluid!!!!

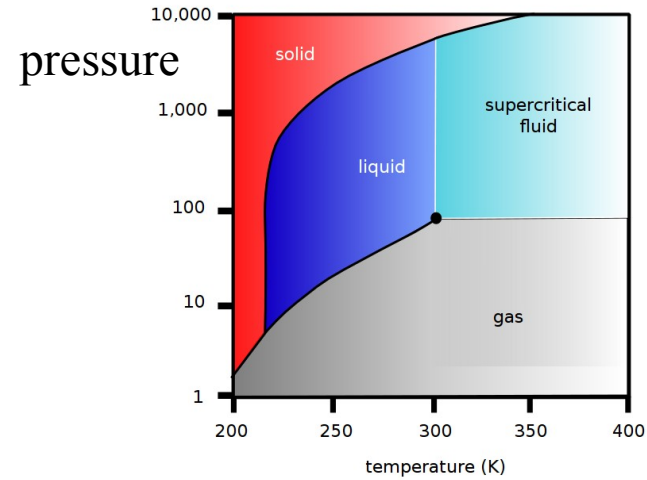
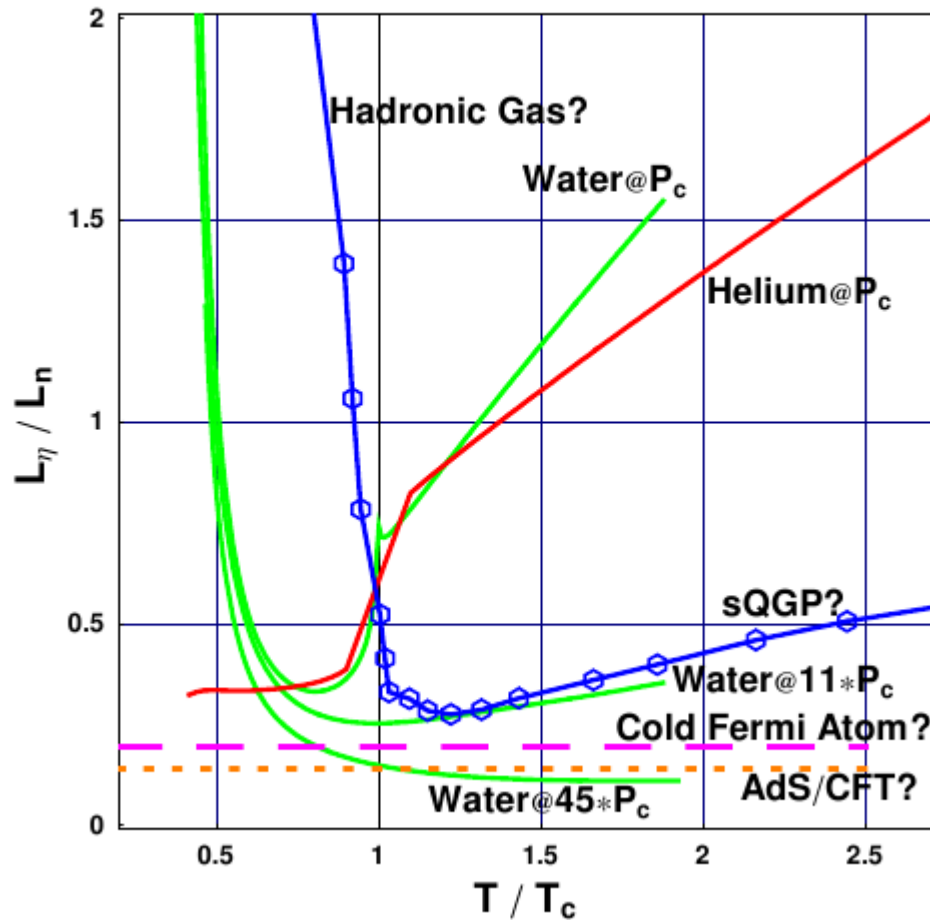
So who is the winner?



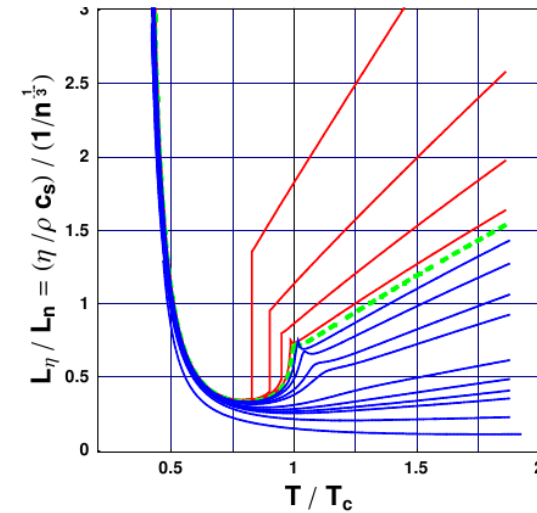
VS



None of the above Super-critical fluids!!!



CO_2



Water

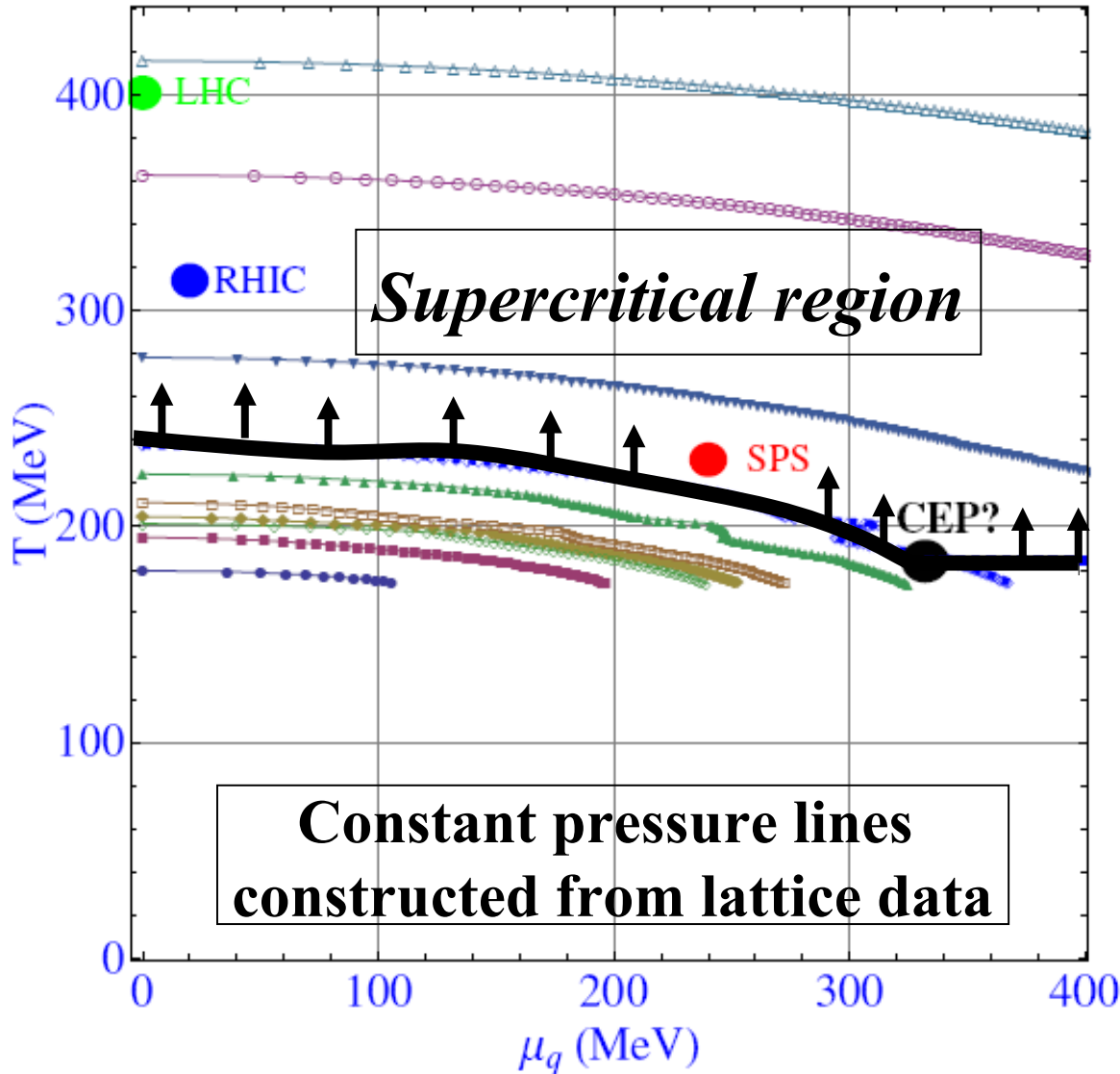
Used in dry cleaning, decaffeinating coffee,

An the winner is...



Consequences for
the QGP????

RHIC and the Dry-Cleaner



If there is a QCD critical point
RHIC-QGP would be in
Super critical region

Predict: even better hydro
behavior at LHC....

Viscosity comparison

Air:	$2 \times 10^{-5} \text{ Pa s}$
Water:	$1 \times 10^{-3} \text{ Pa s}$
Tar Pitch:	$2 \times 10^8 \text{ Pa s}$
QGP:	$2 \times 10^9 \text{ Pa s}$

$$\eta_{\text{QGP}} \simeq 10^{12} \eta_{\text{Water}}$$



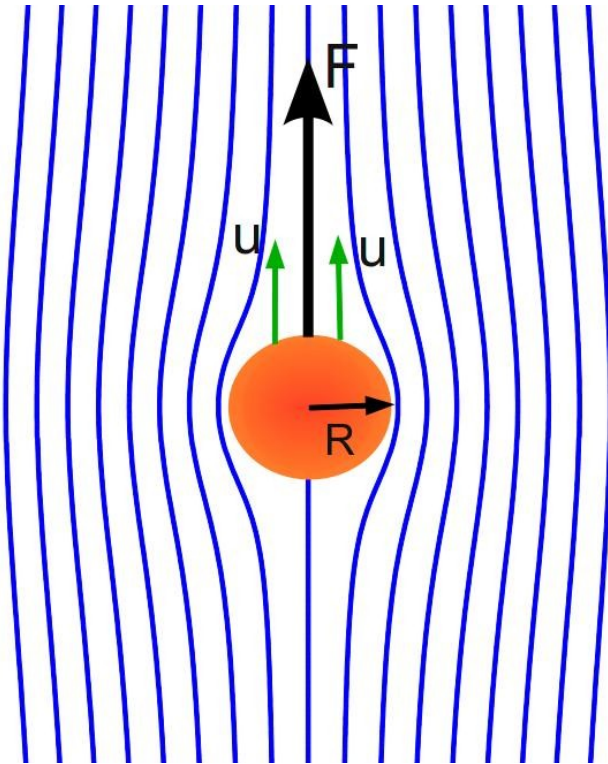
Ig-Nobel, 2005

Stirring the QGP



Stokes' Formula

$$F = 6\pi R u \eta$$



$$\eta_{QGP} \simeq 10^{12} \eta_{Water}$$

Rescale the sphere (spoon)

$$R_{QGP} \simeq 10^{-6} R_{Water}$$

$$F_{QGP} \simeq 10^6 F_{Water}$$

What have learned so far

- Universal Fluidity Measure $F = \frac{\eta}{w c_s} \frac{1}{d}$
- A good fluid is a good fluid
 - Gives $F=0$ for super-fluid component of ^4He !
 - Works also for academic cases a la Cohen et al.
- QGP nothing special
 - However, very sticky when stirred
- η/s dimensionless and meaningless
- Supercritical fluids win the race
- QGP may be a supercritical fluid
 - Predict better hydro description at LHC

And what does Lattice QCD say?

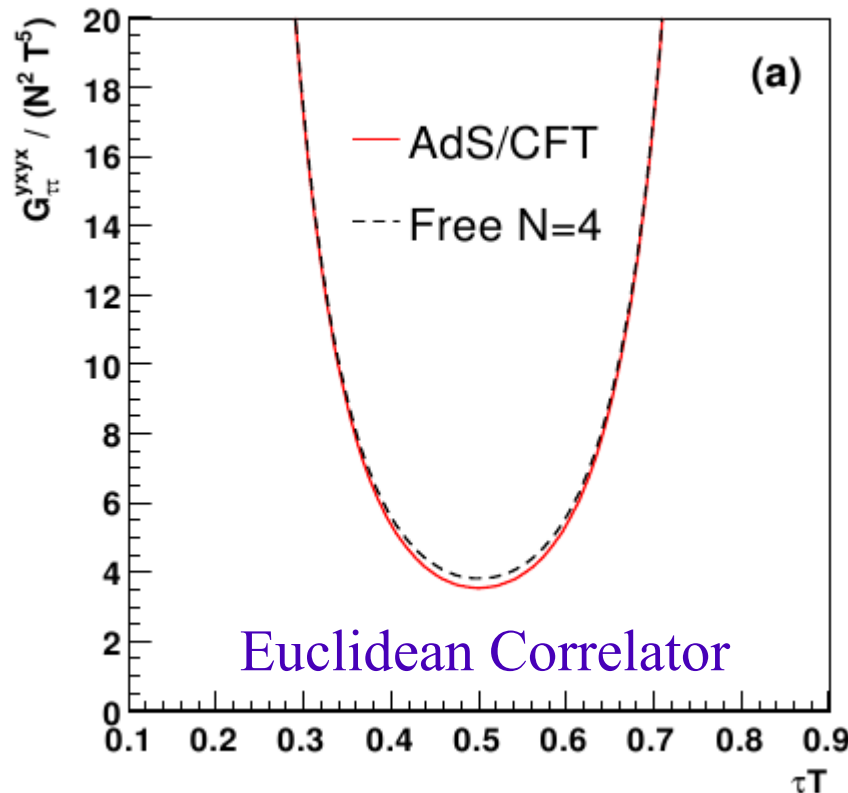
- Mixed message
 - Shear viscosity difficult to extract (time like)
 - Requires analytic continuation to real time...
 - No or very weak correlations among quarks above T_c (weak coupling ?)
 - Some indications for liquid behavior in pure glue

Euclidean Space Correlator

$$\eta = -\frac{d}{d\omega} \text{Im} \Pi^R \Big|_{\omega=0^+} \quad \Pi^R \sim \int e^{-i\omega t} \langle [T_{xy}(t), T_{xy}(0)] \rangle$$

Viscosity results from time-like correlator

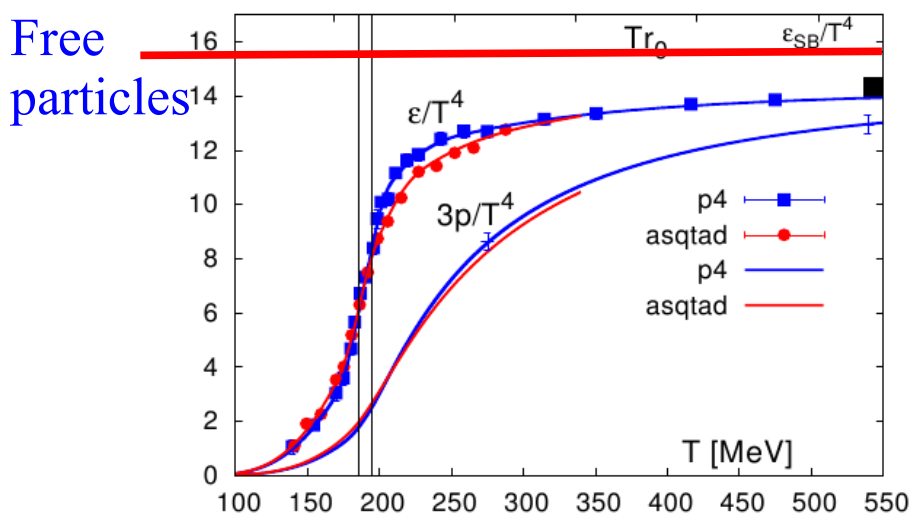
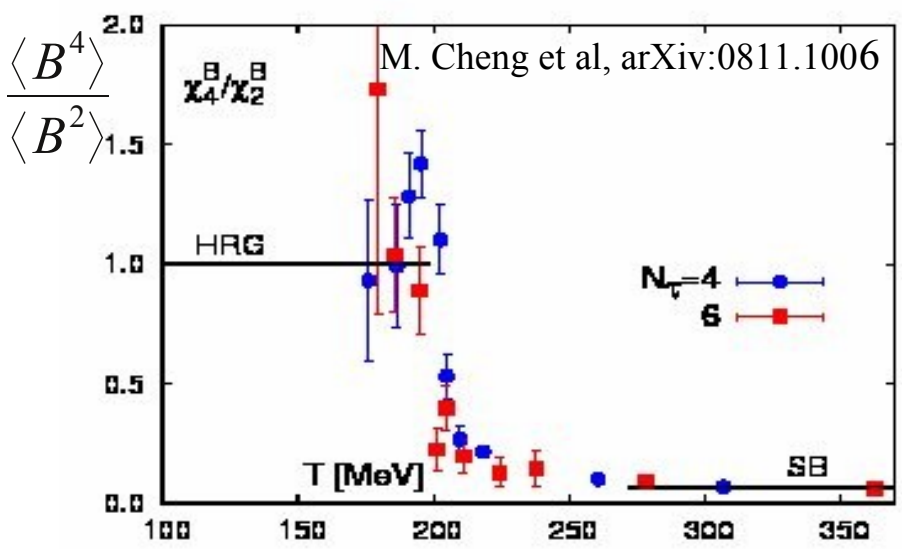
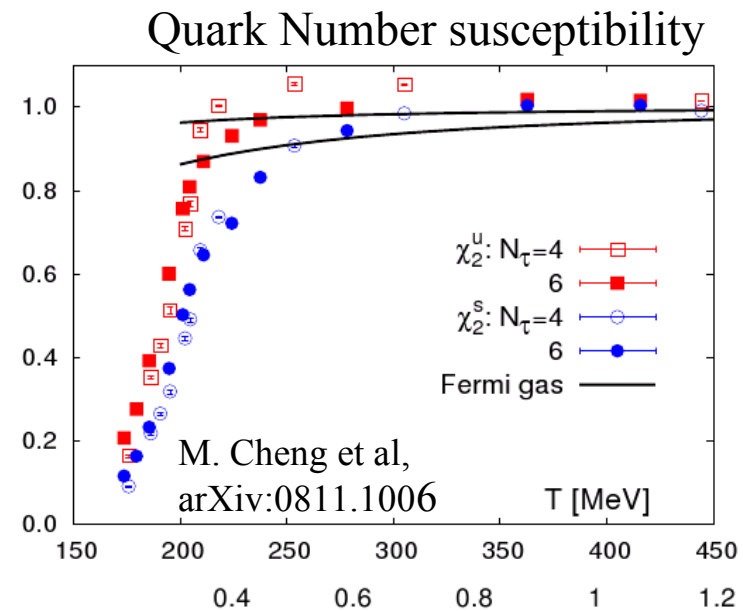
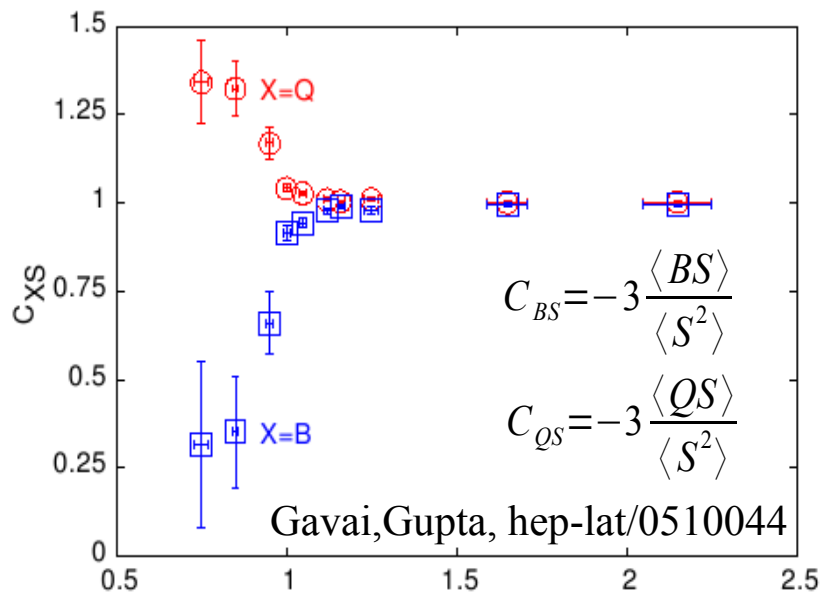
$$\langle T_{xy}(\tau) T_{xy}(0) \rangle$$



D. Teaney
Phys.Rev.D74:045025,2006.
hep-ph/0602044

Difficult to distinguish between FREE and Strongly interacting in Euclidean Space

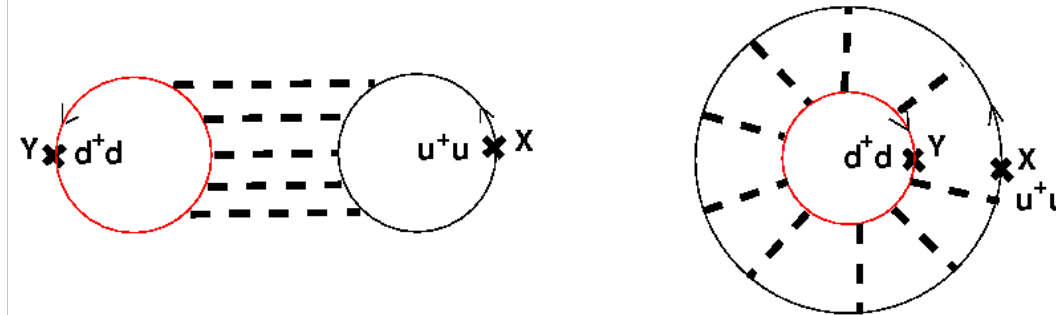
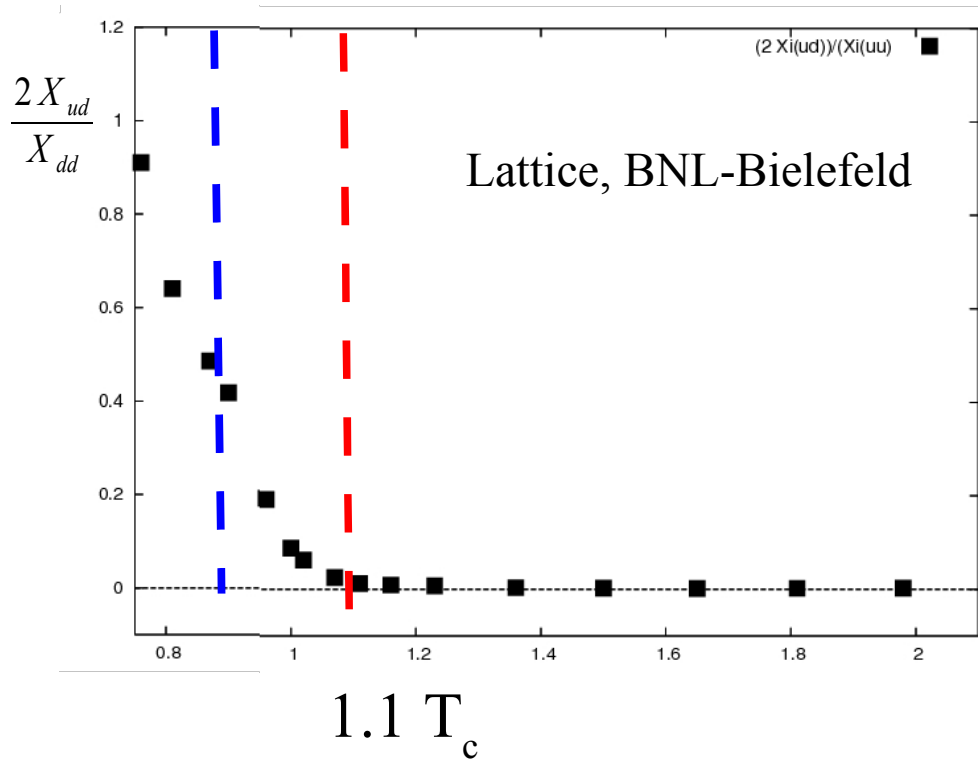
Evidence for uncorrelated flavor (quarks)



Correlations

off-diagonal
Susceptibilities

$$\chi_{ud} = \langle (\delta u)(\delta d) \rangle = T^2 \frac{\partial^2}{\partial \mu_u \partial \mu_d} \log(Z) = -T \frac{\partial^2}{\partial \mu_u \partial \mu_d} F$$



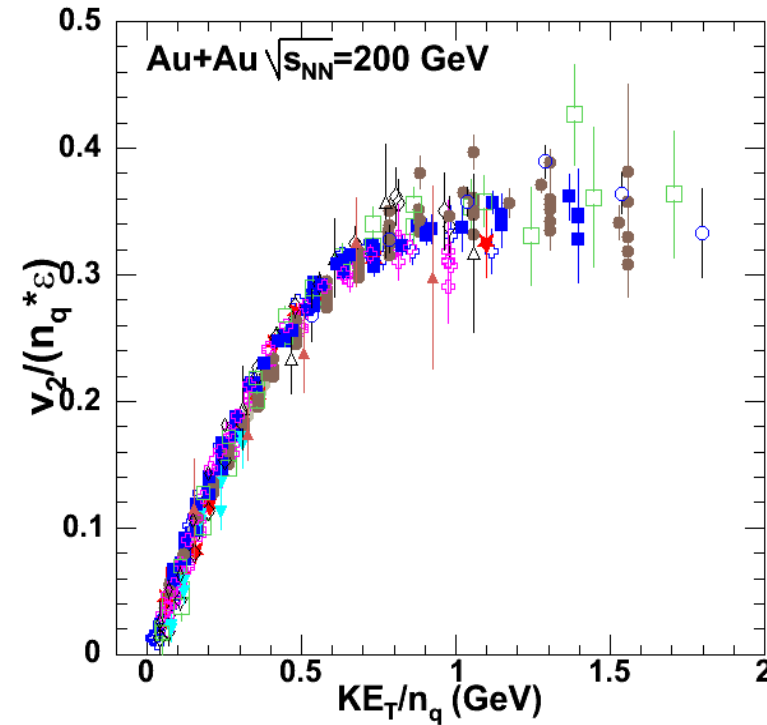
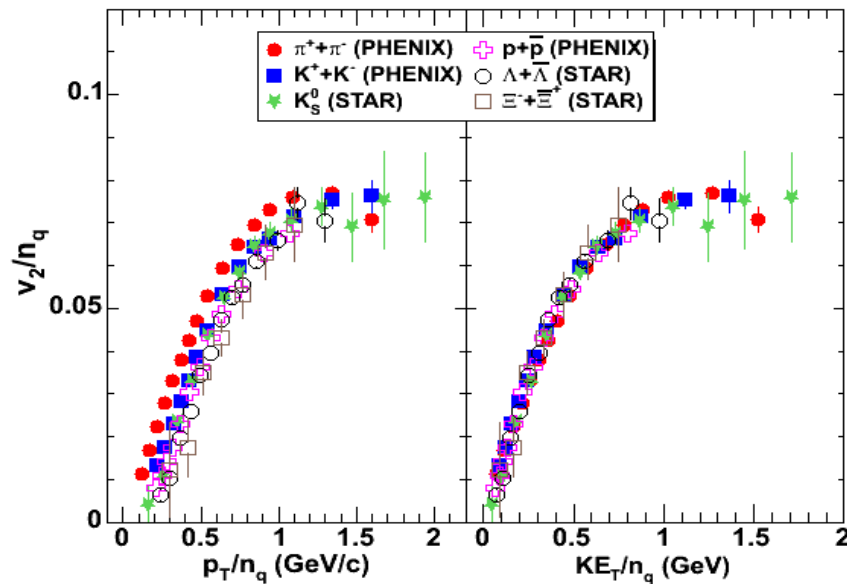
Weak Correlations above $1.1 T_c$

Consistent with hadron gas for $T < 0.9 T_c$

Consistent with quark number scaling
of elliptic flow v_2

Elliptic Flow (quark number scaling)

works even
for the pions???



Coalescence (recombination): $\frac{dN}{d\phi}(\bar{q}q) = \frac{dN}{d\phi}(\bar{q}) \frac{dN}{d\phi}(q) \sim (1+2v_2 \cos(\phi))(1+2v_2 \cos(\phi))$

$$v_2 = n_q v_2^0$$

quark number scaling

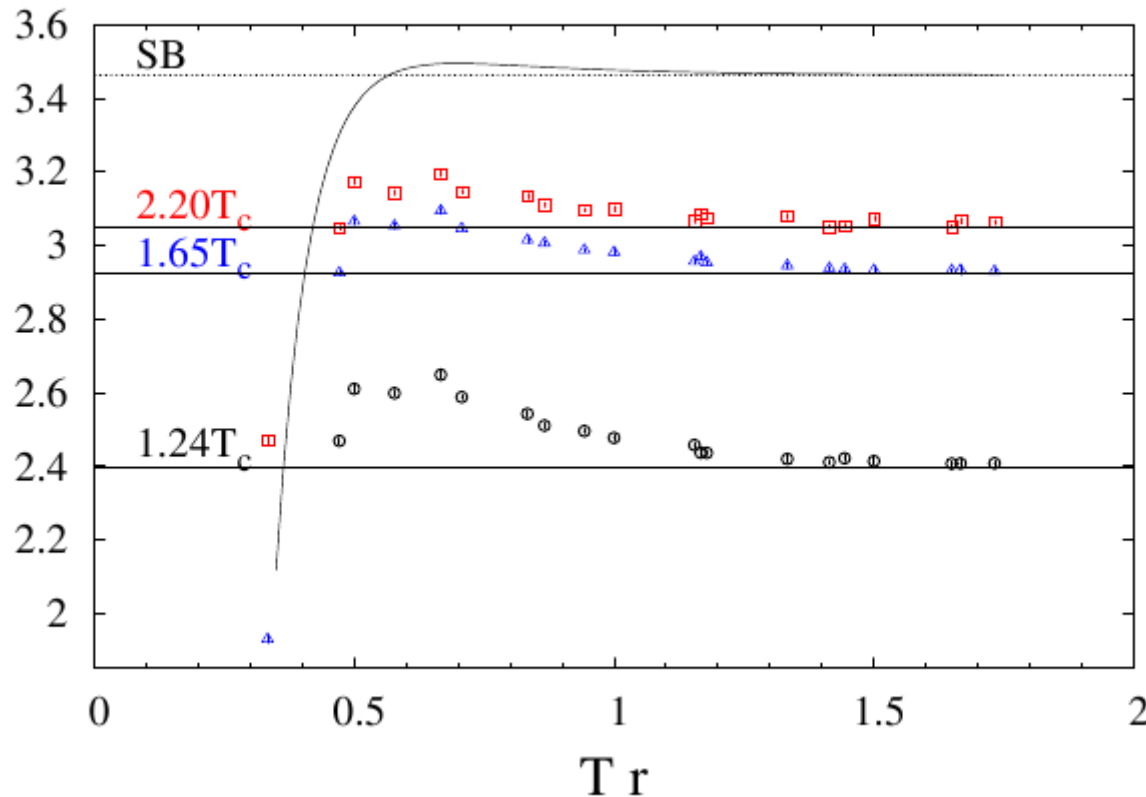
$$\sim (1+4v_2 \cos(\phi))$$

$$\rightarrow \underline{v_2(\bar{q}q) = 2v_2}$$

Density-Density correlator in pure Glue

$$G_{ee}(T, r) / (d_A T^8)$$

H.B. Meyer, arXiv:0808.1950



Not inconsistent with liquid like behaviour...

Summary

- The RHIC fluid
 - Fluidity of the RHIC “fluid” is nothing special
 - However very sticky when stirred
 - A good fluid is a good fluid
 - Is RHIC fluid in the super-critical regime?
 - If so, hydro should work even better at LHC
 - QGP has HUGE viscosity
 - Lattice input
 - Very weak flavor correlations just above T_c .
 - Strong coupling???
 - Difficult to extract viscosity
 - Density-Density correlator for pure glue show some structure



Creating the perfect liquid in heavy-ion collisions

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Expecting to find a very weakly coupled gas of quarks and gluons created in energetic collisions at Brookhaven's Relativistic Heavy Ion Collider, experimenters found instead a strongly coupled liquid with almost no viscosity.

Physics Today, May 2010



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Ulrich's advise was not so bad!



Live long and prosper!



The End