Medium dependence; are all hadrons alike?

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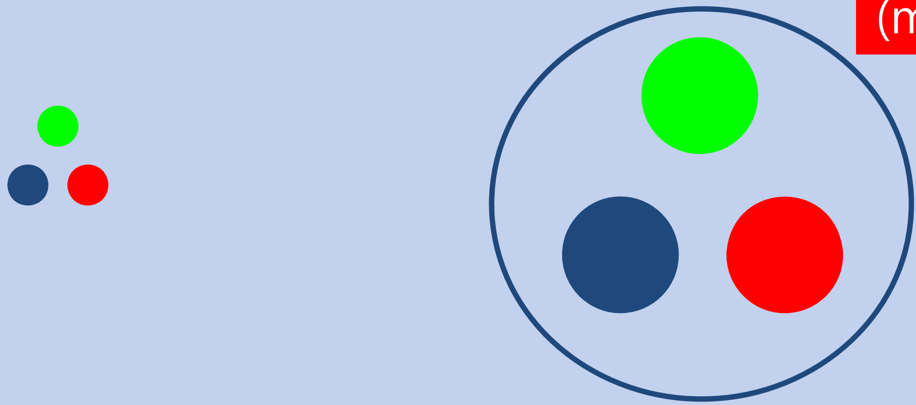
1. Looking at the world with operators
2. Light quark system – QCD sum rule constraints
   - quark operators
3. Heavy quark system - $\langle E^2 \rangle$
4. η' – $\langle G^2 \rangle_{n.p.}$

QCD is non-perturbative in vacuum

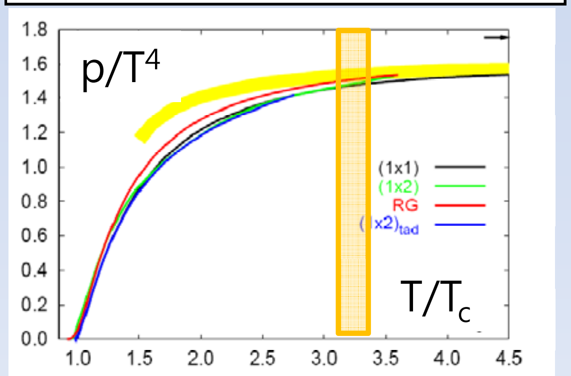


Chiral symmetry breaking (mass generation)

Confinement

QCD is non-perturbative up to 3 Tc

F. Karsch *hep-lat/0106019*

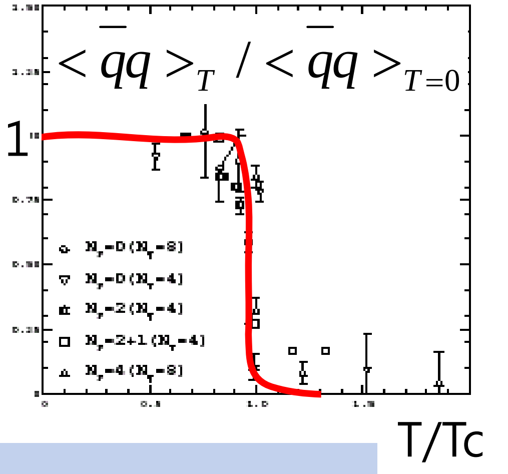


1. Looking at the world with operators

1. Model independent property of the vacuum and medium
2. Can link different physics through same operator
3. Light quark operators, $\langle E^2 \rangle$, $\langle G^2 \rangle_{\text{non-perturbative}}$

Finite temperature

Lattice gauge theory

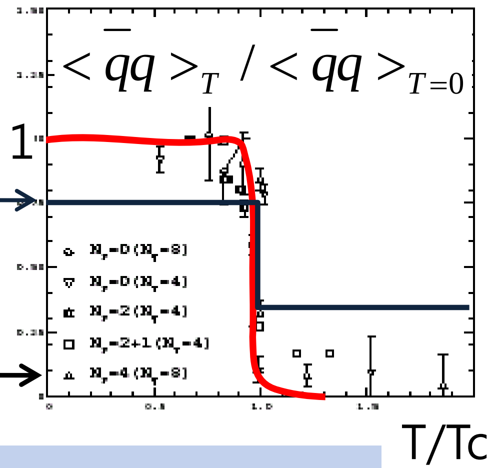


Finite temperature

$$<\bar{s}s>_T / <\bar{q}q>_{T=0} = 0.8$$

$$\langle \bar{c}c \rangle = -\frac{1}{12m_c}\left\langle \frac{\alpha}{\pi} G^2 \right\rangle$$

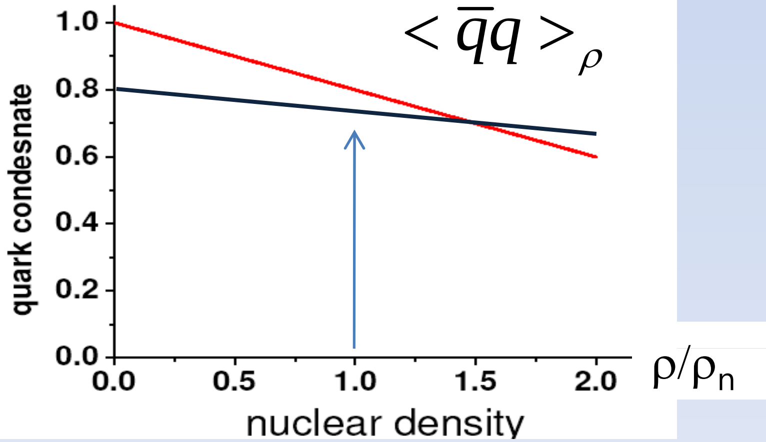
Lattice gauge theory



$$<\bar{s}s>_T \propto m_s \exp(-m_s / T)$$

Finite density

Linear density approximation



More Quark operators – chiral or non chiral

2-quark

$< \bar{q}\gamma^0 \tau^a q >$ densities

$< \bar{q}\gamma^\mu D^\nu q >$ Quark part of energy momentum tensor

4-quark

$$\Omega^g_{\alpha\beta} = ig\left(\bar{\psi}\left\{D_\alpha, F^*_{\beta\mu}\right\}\gamma_\mu\gamma_5 Q^2\psi\right)$$

$$\Omega^V_{\alpha\beta} = g^2\left(\bar{\psi}Q^2\gamma_\alpha[D_\mu, F_{\mu\beta}]\psi\right)$$

$$= g^2\left(\bar{\psi}Q^2\gamma_\alpha\lambda^a\psi\right)\left(\bar{\psi}\gamma_\beta\lambda^a\psi\right) = g^2 V^a_\alpha V^a_\beta$$

$$\Omega^A_{\alpha\beta} = g^2\left(\bar{\psi}Q\gamma_\alpha i\gamma_5\lambda^a\psi\right)\left(\bar{\psi}Q\gamma_\beta i\gamma_5\lambda^a\psi\right)$$

$$= g^2 A^a_\alpha A^a_\beta$$

J-Lab 12 GeV upgrade → higher twist effect

- Two independent operators

Gluon condensate $\left\langle \frac{\alpha}{\pi} G^2 \right\rangle = G_0$

Twist-2 Gluon $\left\langle \frac{\alpha_s}{\pi} G^{\alpha\mu} G^{\beta\mu} \right\rangle = \left(u^\alpha u^\beta - \frac{1}{4} g^{\alpha\beta} \right) G_2$

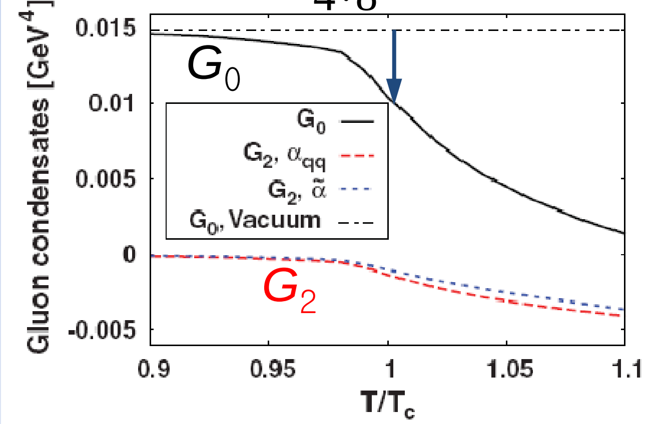
or

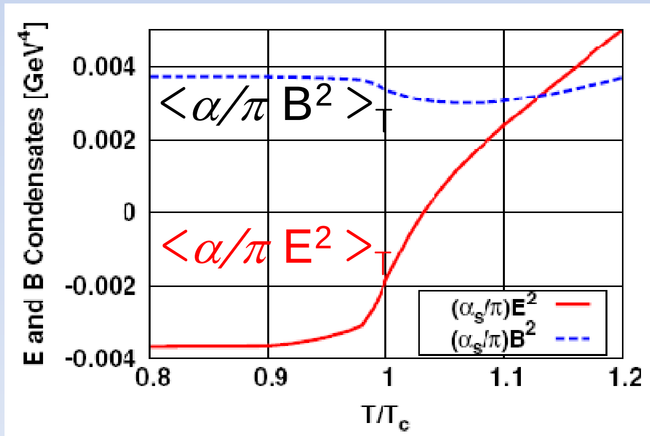
$\left\langle \frac{\alpha}{\pi} E^2 \right\rangle$

$\left\langle \frac{\alpha_s}{\pi} B^2 \right\rangle$

- At finite temperature: from $G_0 = -\frac{8}{9}(\varepsilon - 3p), \quad G_2 = \frac{\alpha}{\pi}(\varepsilon + p)$

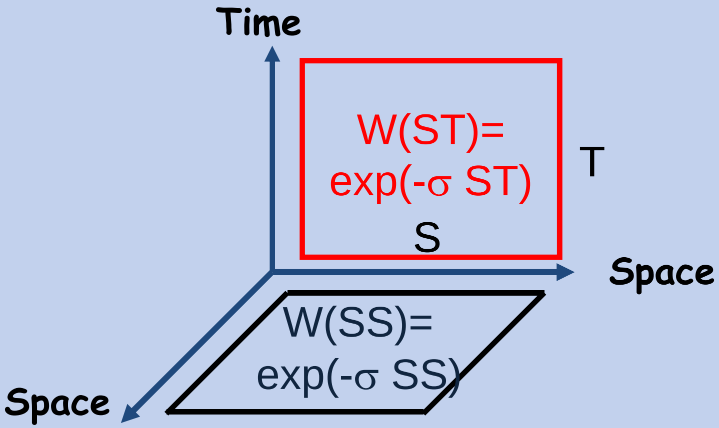
$$\mathrm{B} = -\frac{9}{4 \cdot 8}\left(G_0(T_c) - G_0\right) \approx (189\ \mathrm{MeV})^4$$





$\langle E^2\rangle$, $\langle B^2\rangle$ vs confinement potential

- Local vs non local behavior



OPE for Wilson lines: Shifman NPB73 (80)

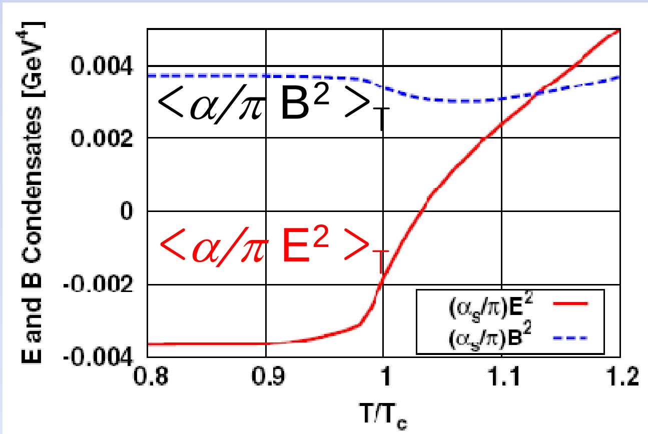
$W(S\text{-}T) = 1 - \langle \alpha/\pi\, E^2 \rangle (ST)^2 + \ldots$

$W(S\text{-}S) = 1 - \langle \alpha/\pi\, B^2 \rangle (SS)^2 + \ldots$

- Behavior at T>Tc

$W(SS) = \exp(-\sigma\, SS)$

$W(ST) = \exp(-\, g(1/S)\, S)$



Operators in at finite density and hadronic phase

- Linear density approximation

$$\langle \mathrm{Op} \rangle_\rho = \langle \mathrm{Op} \rangle_0 + \frac{\rho_N}{2m_N} \langle \mathrm{N} | \mathrm{Op} | \mathrm{N} \rangle, \qquad \Delta G_0 \propto \langle \mathrm{N} | \mathrm{T}^\mu_\mu (\mathrm{Chiral}) | \mathrm{N} \rangle = m_N^0 \rightarrow 750\,\mathrm{MeV}$$

$$G_2 \propto 2m_N \int dx x G(x,\mu^2) \qquad \rightarrow \; 0.9\, m_N$$

- Condensate at finite density

$$\begin{cases} G_0(\rho) = G_0 - \frac{8}{9} m_N^0 \rho = G_0 \left(1 - 0.061 \frac{\rho}{\rho_{\mathrm{n.m.}}} \right) \\ G_2(\rho) = -\frac{\alpha_s}{\pi} 0.9 \rho \end{cases}$$

$$\begin{cases} \Delta \left\langle \frac{\alpha}{\pi} E^2 \right\rangle_\rho = (\alpha_s \times 0.2 + 0.167)\rho \\ \Delta \left\langle \frac{\alpha}{\pi} B^2 \right\rangle_\rho = (\alpha_s \times 0.2 - 0.167)\rho \end{cases}$$

- At ρ = 5 x ρ n.m.

$$\left\langle \frac{\alpha}{\pi} G^2 \right\rangle_{5\rho_{n.m.}} = 0.7 \times \left\langle \frac{\alpha}{\pi} G^2 \right\rangle_0 \approx \left\langle \frac{\alpha}{\pi} G^2 \right\rangle_{\mathrm{Tc}}$$

Gluon condensate – Non perturbative

- Lattice calculation (Lee 89)

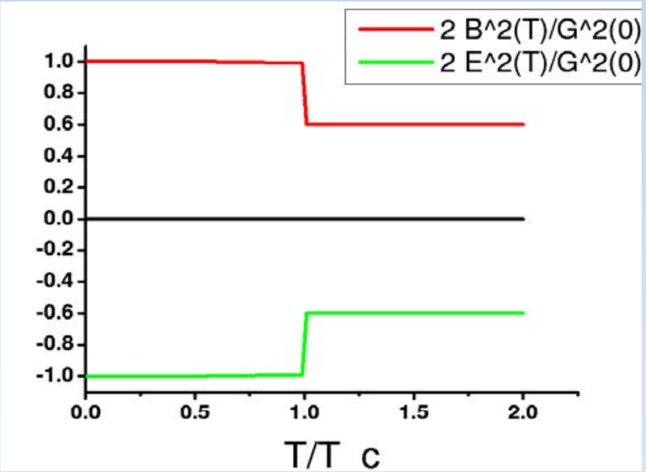
Non-perturbative Gluon condensate ;

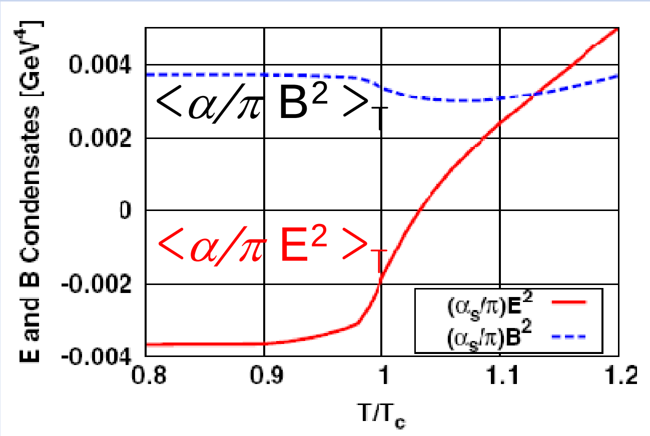
$$\left\langle \frac{\alpha}{\pi} G^2 \right\rangle = G_0(T) + a g^4 T^4 ....$$

Non zero above Tc

$$G_0(T) \approx 0.6 \times G_0(T=0)$$

- Hatsuda, Adami, Brown.. (90)
- Morita, Lee (08)





2. Light quark system

1. Nucleon
2. Vector meson
3. Phi meson

Nucleon

- Nucleon in matter : RMFT

Average potential $m^*_{n,p} = m + \Sigma_\sigma \pm \Sigma_\delta, \quad k^{*\mu}_{n,p} = k^\mu - (\Sigma^\mu_\omega \pm \Sigma^\mu_\rho)$

Symmetry Energy (Greco et al)

$$E_{sym} \approx \frac{1}{6}\frac{k_f^2}{E_F^*} - \left\langle \Sigma_\rho^0 \right\rangle + \frac{m^*}{E_F^*}\left\langle \Sigma_\delta \right\rangle$$

- QCD sum rules

$$\Pi(k) = i\int dx e^{ikx}\left\langle \eta(x)\bar\eta(0)\right\rangle \xrightarrow{OPE} \Pi_1 + \not{k}\Pi_k + \not{u}\Pi_u$$

$$\Pi(k) \xrightarrow{Phen} \frac{1}{\left(k^\mu - \Sigma_V^\mu\right)\gamma_\mu - \left(m_s + \Sigma_s\right)}$$

Cohen, Griegel, Furnstahl 92

$$\Sigma_s = -\frac{8\pi^2}{M^2}\Delta\left\langle \bar q q\right\rangle$$

$$\Sigma_V = \frac{64\pi^2}{3M^2}\left\langle q^+ q\right\rangle$$

Jeong, Lee 11

$$\Sigma_\delta = \frac{8\pi^2}{M^2}\left\langle \bar u u - \bar d d\right\rangle$$

$$\Sigma_\rho = \frac{48\pi^2}{3M^2}\left\langle u^+ u - d^+ d\right\rangle$$

Light vector meson

- Hatsuda and Lee (92)

Correlation function

$$\Pi(k) = i\int dx e^{ikx}\langle J^{\mu}(x)J_{\mu}(0)\rangle \xrightarrow{OPE} \Pi_{pert} + \langle(\bar{q}\Gamma q)(\bar{q}\Gamma q)\rangle$$

QCD sum rules

$$\mathrm{BT}(\Pi^{\mathrm{OPE}}) = \int ds e^{-s/M^2}\rho(s) \qquad \rho(s) = f\delta(s - m_V^2) + cont$$

$$m_V = m_{V0}\left(1 - 0.2\frac{n}{n_{nm}}\right) \qquad \langle(\bar{q}\Gamma q)(\bar{q}\Gamma q)\rangle \propto \langle\bar{q}q\rangle^2$$

$$m_\phi \approx m_{\phi 0}\left(1 - 0.05\frac{n}{n_{nm}}\right) \qquad m_s\langle\bar{s}s\rangle$$

- Asakawa, Ko (93)

Rho modified by pion through delta hole

$$\rho(s) = \frac{f}{(s - m_V^2)^2 + s\Gamma^2} + cont$$

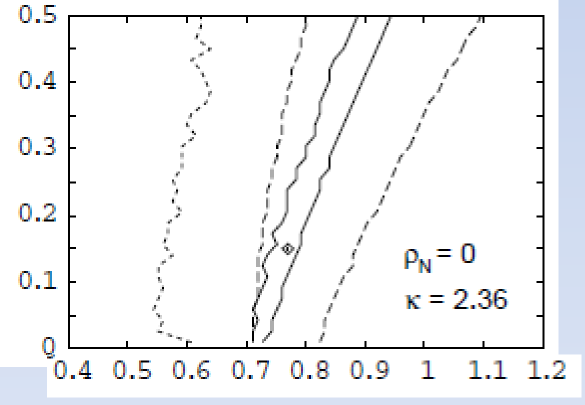
- Leupold, Peters, Mosel (98)

Mass and width $$\rho(s) = \frac{f}{\left(s - m_V^2\right)^2 + s\Gamma^2} + cont \qquad \langle(\bar{q}\Gamma q)(\bar{q}\Gamma q)\rangle \propto \langle\bar{q}q\rangle^2$$

QCD sum rule constraint $$\mathrm{BT}\left(\Pi^{\mathrm{OPE}}\right) = \int ds e^{-s/M^2} \rho(s)$$

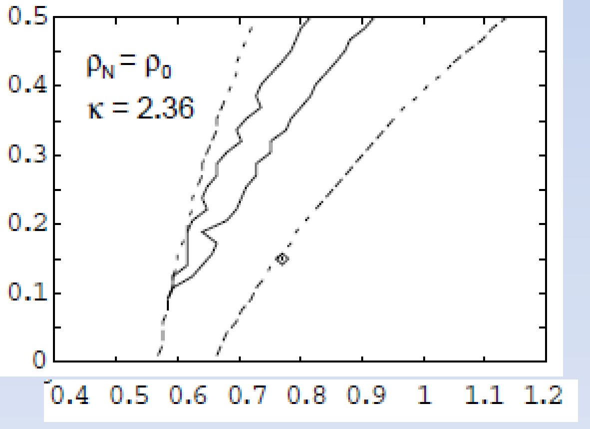
Vacuum

Width



Mass

Nuclear matter



Light vector meson – QCD sum rule constraints

- Momentum dependence (Lee 98)

In medium

$$\Pi_{\mu\nu}(\omega,k)=\Pi_T(\omega,k)P^T_{\mu\nu}+\Pi_L(\omega,k)P^L_{\mu\nu}$$

momentum

$$\Pi_{L,T}(\omega,k)=\Pi^0(\omega)+\Pi^1_{L,T}(\omega)k^2..$$

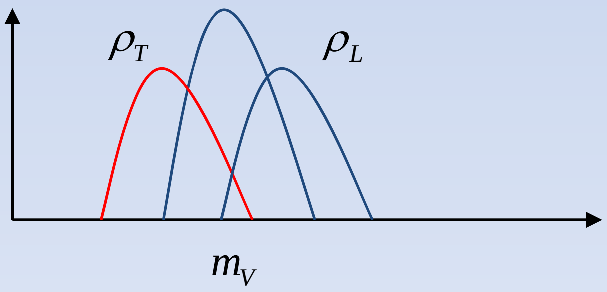
$$\mathrm{BT}\left(\Pi^1_{L,T}\right)=\int ds e^{-s/M^2}\rho^1_{L,T}(s)$$

Dominant OPE

$$\langle \bar{q}\gamma_\mu D_\nu q\rangle \propto Twist-2$$

$$\langle (\bar{q}\Gamma q)(\bar{q}\Gamma q)\rangle \propto Twist-4$$

Assuming no k dependence on width



3. Heavy quark system

1. Charmonium system
2. Panda

Heavy quark system – QCD sum rule constraints

- Morita and Lee (08→ present)

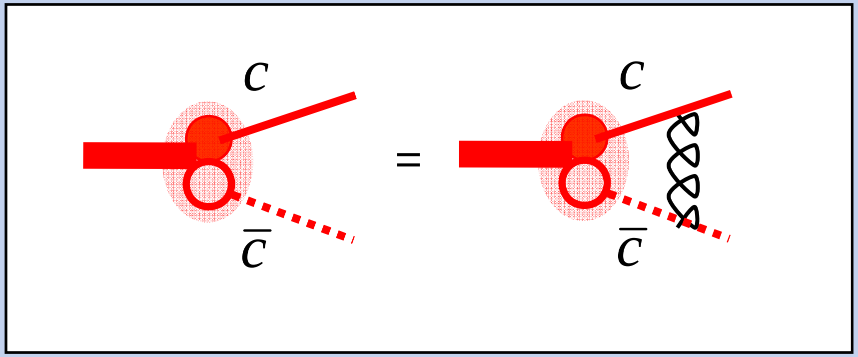
In medium $J/\psi, \eta_c, \chi_c,$

In Vacuum $X(3872), Z(4430)$

Mass shift: QCD 2^{nd} order Stark Effect : Peskin 79 $\varepsilon > \Lambda_{qcd}$

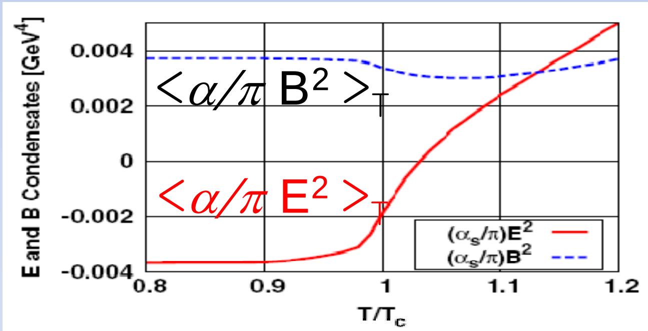
- OPE for bound state: m→ infinity

$$\varepsilon_0 = m\left(N_c g^2/16\pi\right)^2 \rightarrow O(mg^4), \qquad |\vec{k}| \rightarrow O(mg^2)$$



$$g^2 \frac{mg^4(mg^2)^3}{(mg^4)(mg^4)(mg^2)^2} \rightarrow O(1)$$

- Attractive for ground state

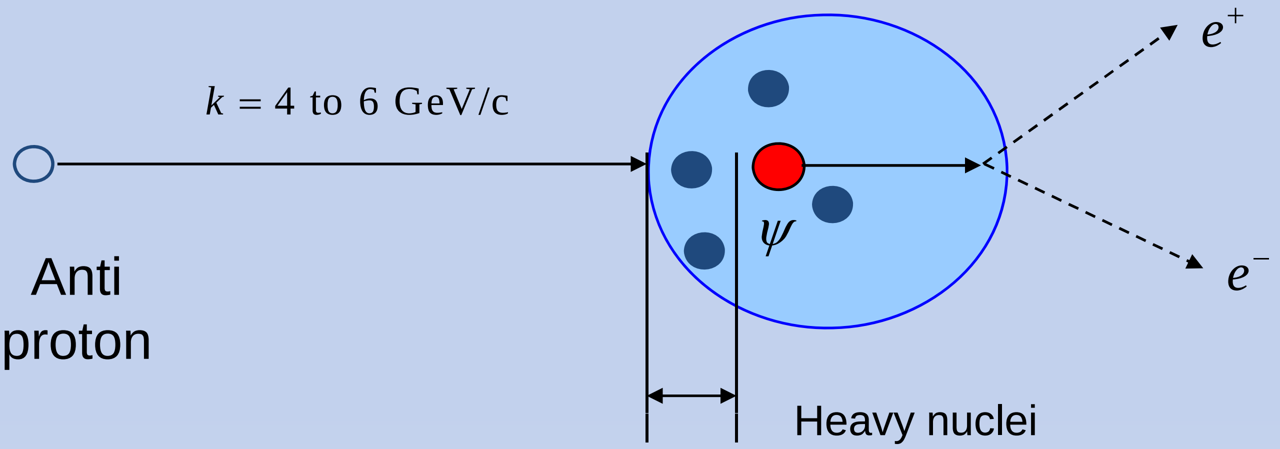


$$\Delta m_{J/\psi} = \sum_n \frac{\left|\langle i|zE|n\rangle\right|^2}{E_i - E_n} = -\frac{128}{9\pi^2}\frac{a_0^2}{\varepsilon_0}\int dx \frac{x^{3/2}}{(1+x)^6}\frac{1}{x+a_0^2\varepsilon m} \times \left\langle \frac{\alpha}{\pi}E^2 \right\rangle_{\text{Medium}}$$

Other approaches for mass shift in nuclear matter

	Quantum numbers	QCD 2nd Stark eff.	Potential model	QCD sum rules	Effects of DD loop
η_c	0^{-+}	**–8 MeV**		**–5 MeV** (Klingl, SHL ,Weise, Morita)	No effect
J/ψ	1^{--}	**–8 MeV** (Peskin, Luke)	**-10 MeV** (Brodsky et al).	**–7 MeV** (Klingl, SHL ,Weise, Morita)	<2 MeV (SHL, Ko)
χ_c	$0,1,2^{++}$	**-20 MeV**		**-15 MeV** (Morita, Lee)	No effect on χ_{c1}
$\psi(3686)$	1^{--}	**-100 MeV** (SHL, Ko)			< 30 MeV
$\psi(3770)$	1^{--}	**-140 MeV** (SHL, Ko)			< 30 MeV

Observation of Δm through p̄-A reaction



$$\frac{1}{0.17\ fm^{-3} \times 5\ fm^{2}} = 1.2\ fm$$

Can be done at J-PARC

Expected luminosity at GSI 2x $10^{32} cm^{-2} s^{-1}$ →

Table 2: Summary of parameters and resultant cross sections.

	J/ψ	η_c	χ_{c0}	χ_{c1}	χ_{c2}
m[MeV]	3097	2980	3415	3511	3556
δm[MeV]	-7	-4	-15	-15	-15
Γ_{tot} [MeV]	0.0934	25.5	10.4	0.89	2.05
Final State	e^+e^-	$\gamma\gamma$	$J/\psi\,\gamma$	$J/\psi\,\gamma$	$J/\psi\,\gamma$
$\langle\sigma_{BW}\rangle_{peak}$ [pb]	0.435	10.7	17.0	4.25	18.8
Events/day	7.5	184	294	74	326

4. η' meson

1. Witten – Veneziano formula
2. At finite temperature and density

Witten-Veneziano formula

- *Correlation function* $P(k) = -i\int dx e^{ikx} \langle G\tilde{G}(x), G\tilde{G}(0) \rangle$
- *Purge glue case* $P_0(k=0) \neq 0$ *with massless quarks* $P(k=0) = 0$
- *Large Nc argument*

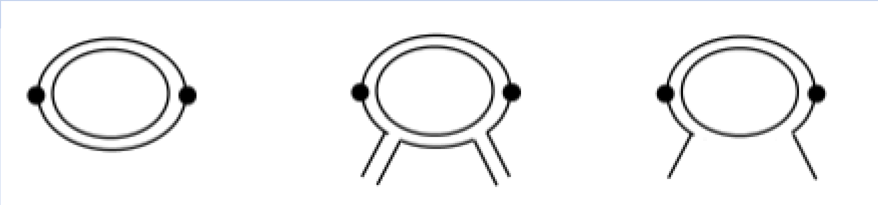
$$P(k) = \sum_{glueballs} \frac{N_c^2 a_n^2}{k^2 - m_n^2} + \sum_{mesons} \frac{N_c c_n^2}{k^2 - m_n^2} + \frac{N_c c_{\eta'}^2}{k^2 - m_{\eta'}^2} \qquad \text{with} \quad m_{\eta'}^2 \approx O\left(\frac{1}{N_c}\right)$$

- *Constraint*

$$\frac{N_c^2}{4N_F} m_{\eta'}^2 f_\pi^2 \left(\frac{16\pi^2}{g^2}\right)^2 = P_0(0)$$

Witten-Veneziano formula in medium

- *Large Nc counting* $$U_m(k) = -i\int dx e^{ikx}\left\langle G\tilde{G}(x), G\tilde{G}(0)\right\rangle_m$$



N_c^2 N_c^2 N_c

- *LET at finite temperature for S(k): Ellis, Kapusta, Tang (98)*

$$\frac{d}{d\left(-1/4g_0^2\right)}\langle Op\rangle = -i\int dx e^{ikx}\left\langle Op(x), g_0^2 GG(0)\right\rangle$$

$$\langle Op\rangle_T = const\left[M_0 \exp\left(-\frac{8\pi^2}{bg_0^2}\right)\right]^d + c'T^d = \langle Op\rangle_{T0} + c'T^d$$

$$\frac{d}{d\left(-1/4g_0^2\right)}\langle Op\rangle_T = \frac{32\pi^2}{b}\left(d - T\frac{\partial}{\partial T}\right)\langle Op\rangle_T = \frac{32\pi^2}{b}\left(d - T\frac{\partial}{\partial T}\right)\langle Op\rangle_{T0}$$

- *LET at finite temperature for P(k) : Lee, Zahed (01)*

- *W-V formula at finite temperature:*

$$\left\langle \frac{\alpha}{\pi} G^2 \right\rangle = a g^4 T^4 + \left\langle \frac{\alpha}{\pi} G^2 \right\rangle_{Non-perturbative}$$

$$\frac{N_c^2}{4N_F} m_{\eta'}^2 f_\pi^2 = \left(d - T \frac{\partial}{\partial T} \right) \left\langle \frac{\alpha}{\pi} G^2 \right\rangle \approx \left(d - T \frac{\partial}{\partial T} \right) \left\langle \frac{\alpha}{\pi} G^2 \right\rangle_{Non-perturbative}$$

Summary

1. One can look at the world with Operators

2. Light quark system – QCD sum rule constraint

3. Heavy quark system - <E2>

4. η‘ mass - <G2> non perturbative

Summary

1. Operators

2. Light quark system – QCD sum rule constraint

3. Heavy quark system - <E2>

4. η‘ mass - <G2> non perturbative

Ulrich,

Thank you for your inspiring works, please keep on inspiring us…