International Workshop on

In-Medium Effects in Hadronic and Partonic Systems

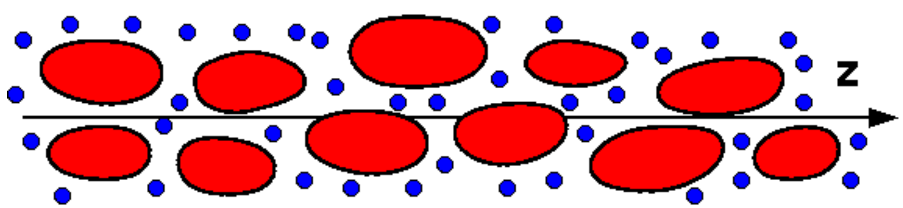
Obergurgl, Austria, 21 – 25 February 2011

HIC for FAIR

Helmholtz International Center

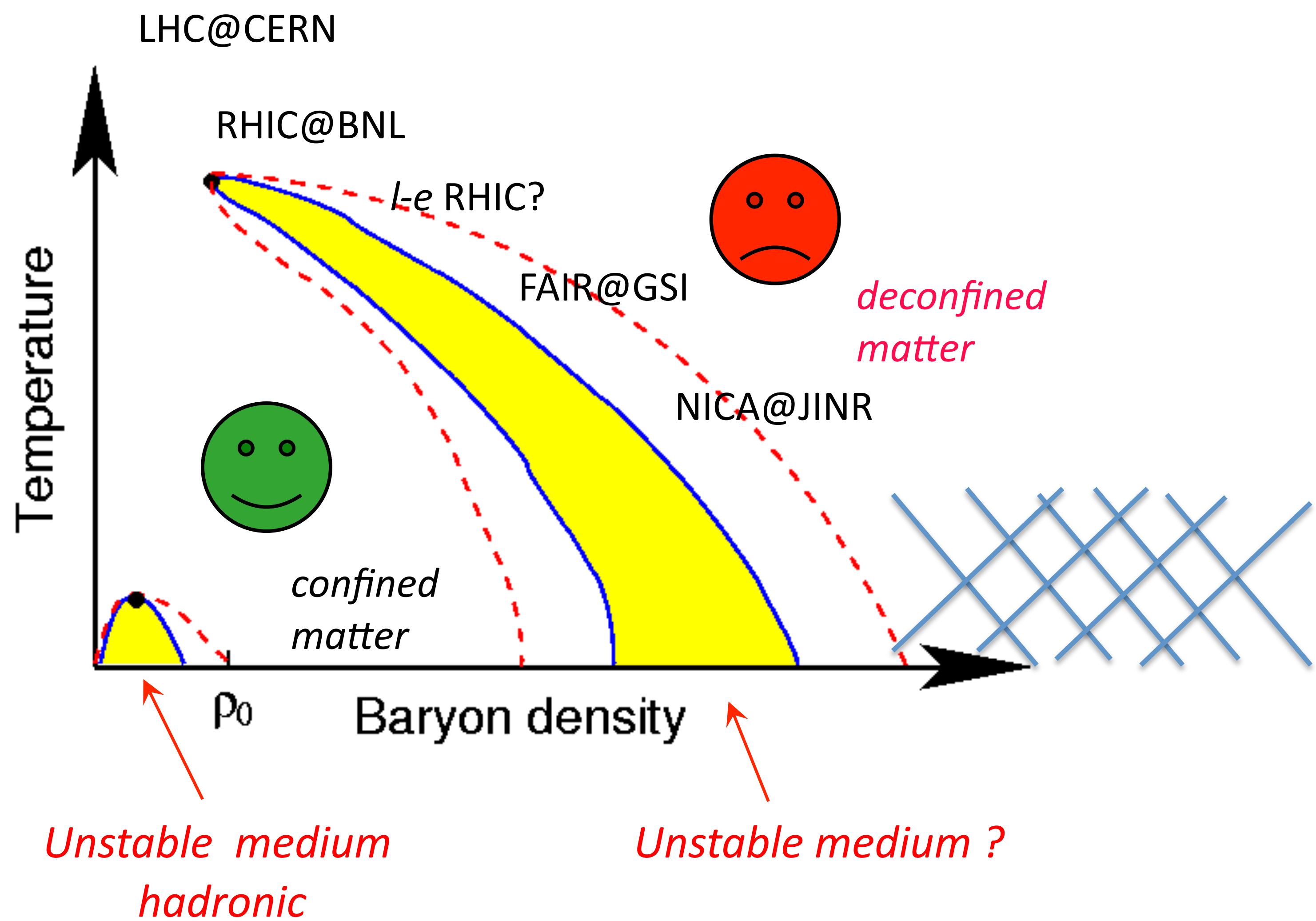
Spinodal phase decomposition with dissipative fluid dynamics

Jørgen Randrup, LBNL



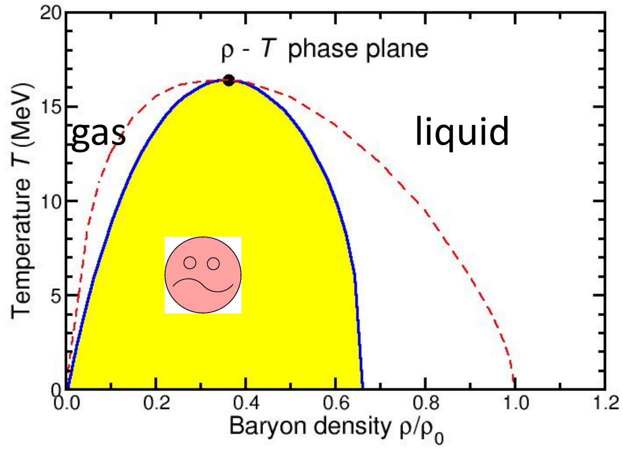
Schematic and simplified

phase diagram of strongly interacting matter

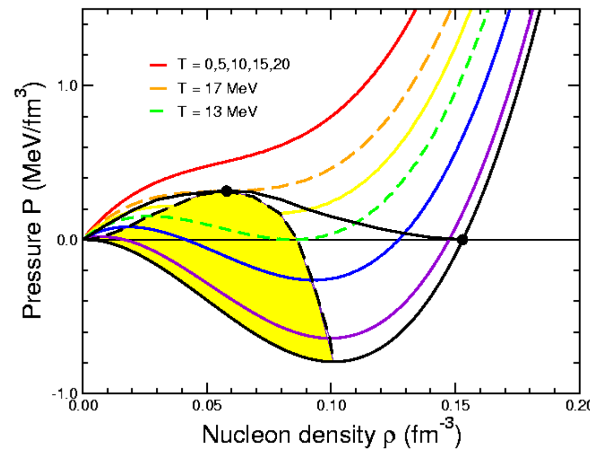


Nuclear liquid-gas phase transition

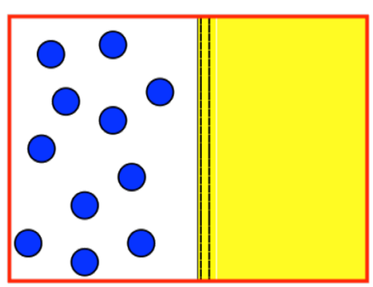
(ρ, T) phase diagram



Equation of state: $p_T(\rho)$



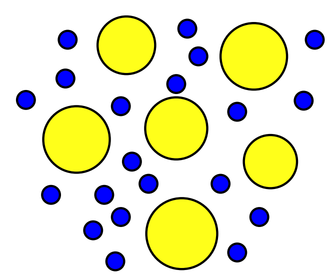
Phase coexistence



gas liquid

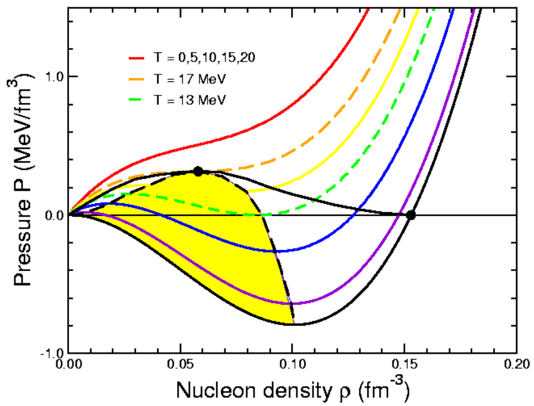
≠

Phase mixture

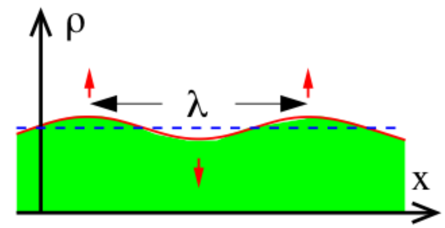


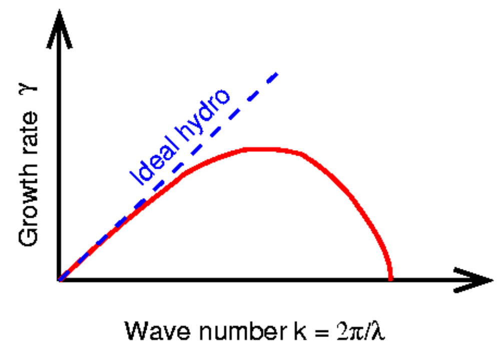
nucleons & fragments

Nuclear spinodal fragmentation



Density undulations are amplified in the spinodal region:



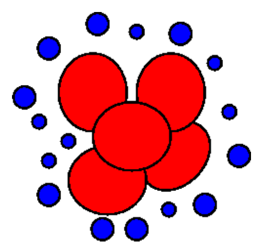


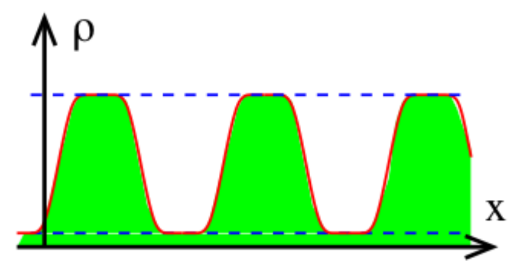
Long-wavelength distortions grow slowly (it takes time to relocate the matter)

Short-wavelength distortions grow slowly (they are hardly felt due to finite range)

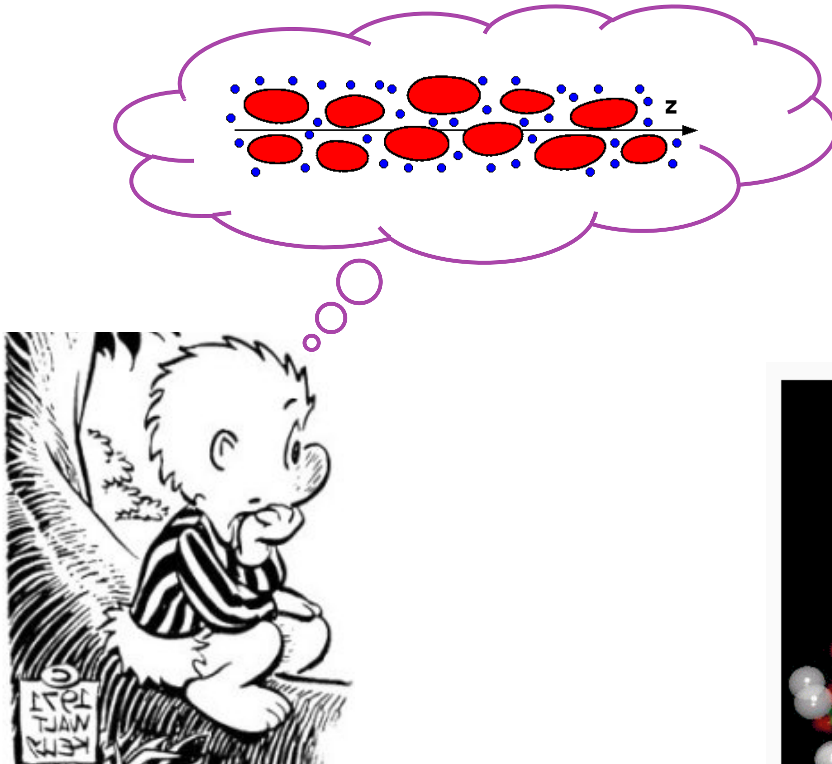
There is an *optimal length scale* that grows faster than all others:

Ph Chomaz, M Colonna, J Randrup:
Nuclear Spinodal Fragmentation,
Physics Reports 389 (2004) 263

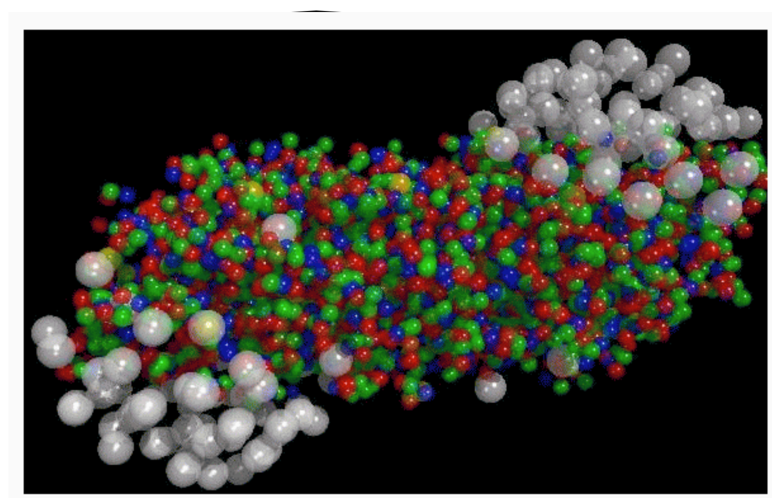




=> Equal-size fragments!



Can
spinodal phase separation
during the confinement
phase transition
happen?



Spinodal phase decomposition with dissipative fluid dynamics

Nuclear spinodal fragmentation

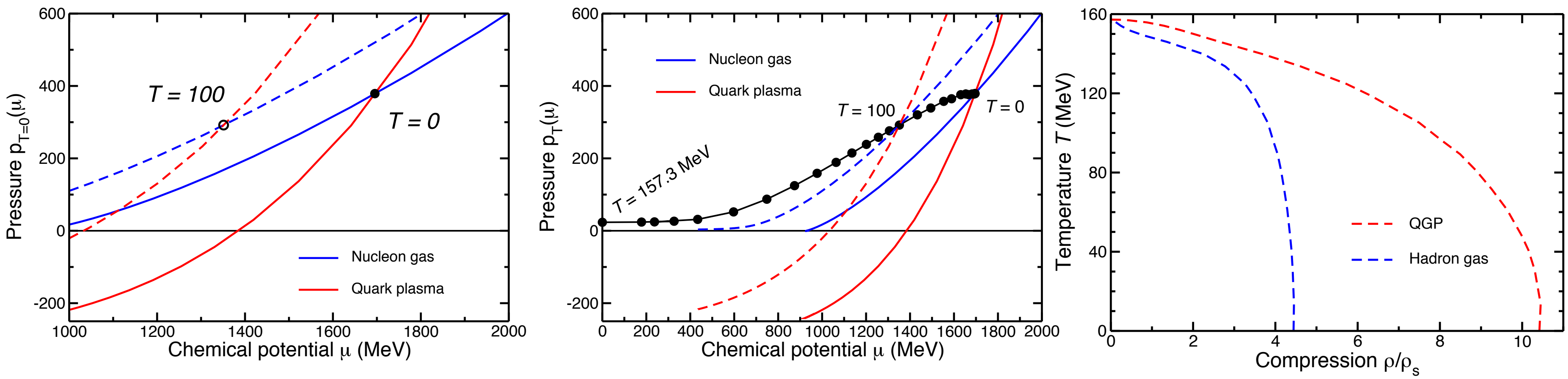
Thermodynamics

Inclusion of finite range

Dissipative fluid dynamics => dispersion relations

Dynamical phase trajectories => spinodal amplification

Idealized equations of state: HG -> QGP



Hadron Gas:

$$p^H = p_\pi + p_N + p_{\bar{N}} + p_w$$

$$p_\pi(T) = -g_\pi \int_{m_\pi}^{\infty} \frac{p\epsilon d\epsilon}{2\pi^2} \ln[1 - \mathrm{e}^{-\beta\epsilon}]$$

$$p_N(T,\mu_0) = g_N \int_{m_N}^{\infty} \frac{p\epsilon d\epsilon}{2\pi^2} \ln[1 + \mathrm{e}^{-\beta(\epsilon-\mu_0)}]$$

$$p_{\bar{N}}(T,\mu_0) = g_N \int_{m_N}^{\infty} \frac{p\epsilon d\epsilon}{2\pi^2} \ln[1 + \mathrm{e}^{-\beta(\epsilon+\mu_0)}]$$

$$p_w(\rho) = \rho\partial_\rho w(\rho) - w(\rho)$$

$$w(\rho) = \left[-A\left(\frac{\rho}{\rho_s}\right)^\alpha + B\left(\frac{\rho}{\rho_s}\right)^\beta\right]\rho$$

Quark-Gluon Plasma:

$$p^Q = p_g + p_q + p_{\bar{q}} - B$$

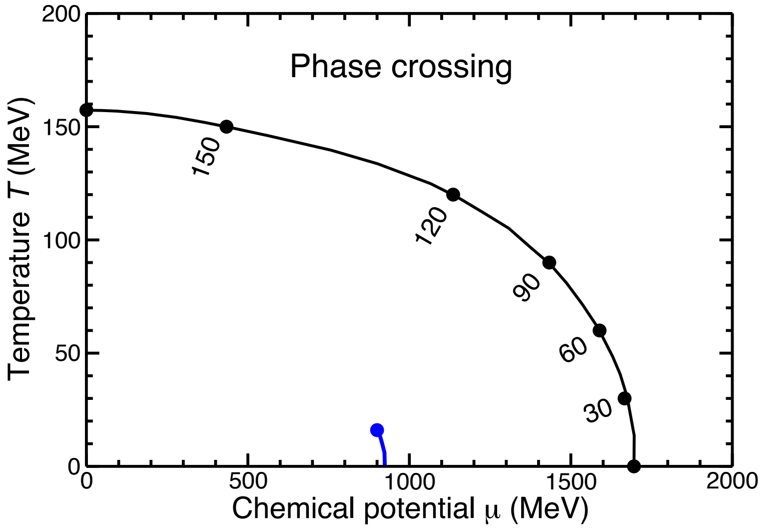
$$p_g = g_g \frac{\pi^2}{90} T^4$$

$$p_q + p_{\bar{q}} = g_q \left[\frac{7\pi^2}{360} T^4 + \frac{1}{12}\mu_q^2 T^2 + \frac{1}{24\pi^2}\mu_q^4\right]$$

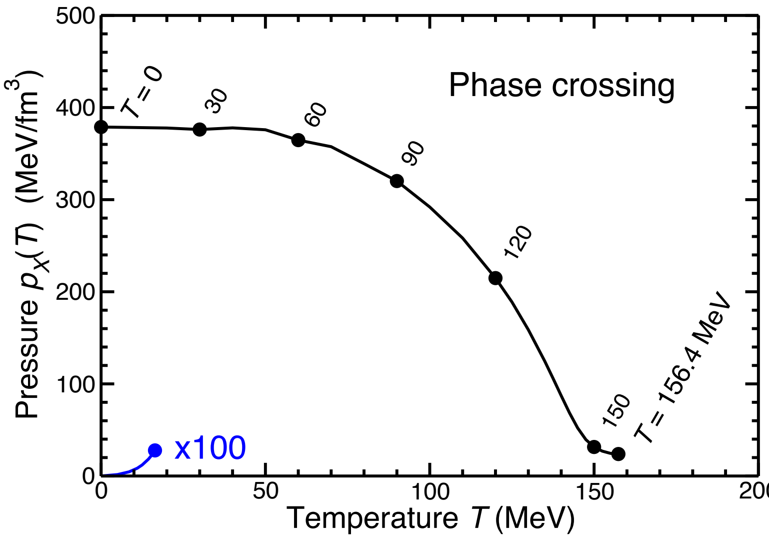
$$\mu = \mu_0 + \partial_\rho w = 3\mu_q$$

Phase coexistence: Comparison to LG

(μ,T) phase diagram



(T,p) phase diagram



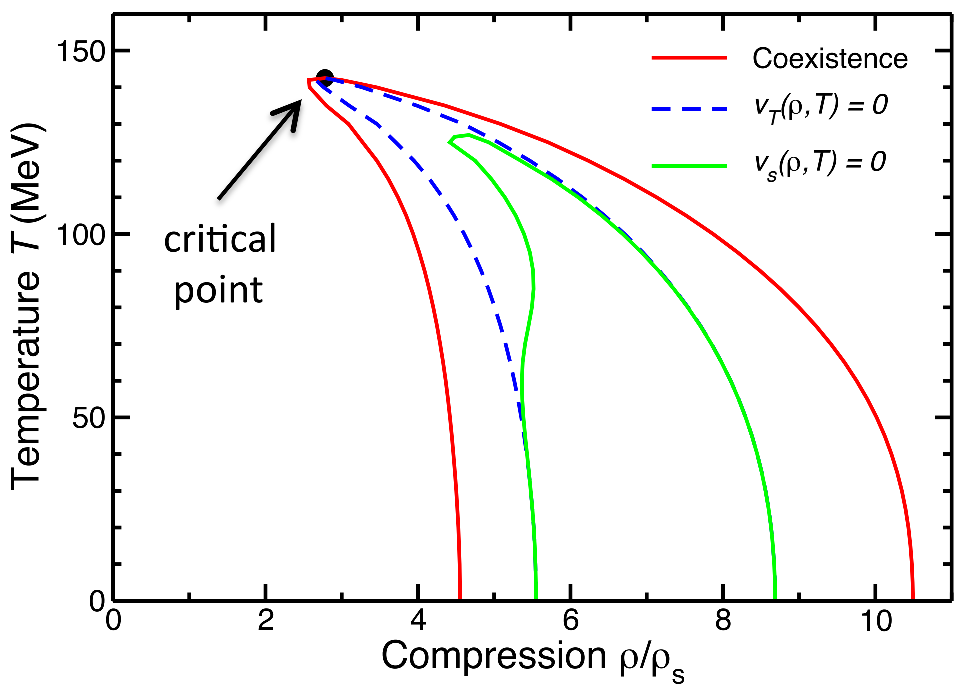
- qualitatively similar to LG

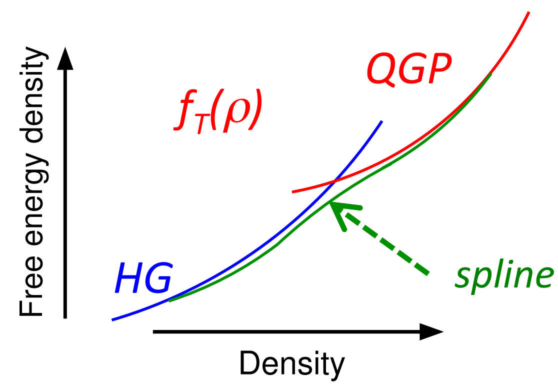
- qualitatively different from LG!

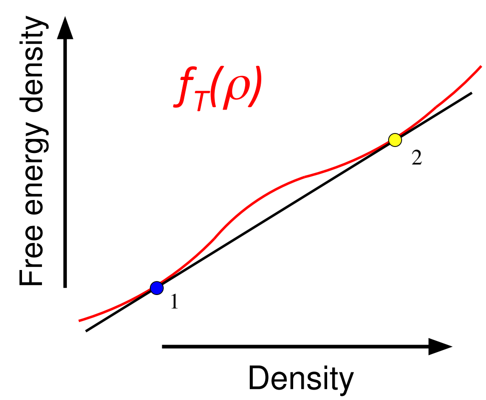
[Igor Iosilevsky, 2010]

Phase coexistence (or phase "crossing"):
Two different media have the same T, μ, p

Construct a two-phase equation of state: spline between the two idealized media







$v_T = 0$: isothermal spinodal

$v_s = 0$: isentropic spinodal

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Dissipative fluid dynamics => dispersion relations

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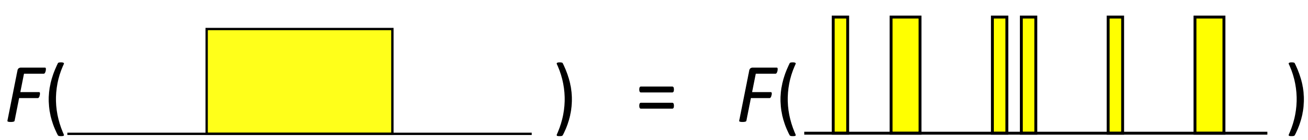
Equation of state: Finite range

Free energy density for uniform matter: $f_0(\rho, T)$

But we need to treat non-uniform systems: $\tilde{\rho}(\boldsymbol{r})$, $\tilde{T}(\boldsymbol{r})$

The local density approximation: $f[\tilde{\rho}(\cdot), \tilde{T}(\cdot)](\boldsymbol{r}) \doteq f_0(\tilde{\rho}(\boldsymbol{r}), \tilde{T}(\boldsymbol{r}))$

… implies:



No good! => Finite range *must* be taken into account

Non-uniform density $\tilde{\rho}(\boldsymbol{r})$

gradient contribution $\tilde{w}(\boldsymbol{r}) \equiv w_0(\tilde{\rho}(\boldsymbol{r})) + \frac{1}{2}C(\boldsymbol{\nabla}\tilde{\rho}(\boldsymbol{r}))^2$

local entropy density: $\tilde{\sigma}(\boldsymbol{r}) \equiv \sigma(\tilde{\varepsilon}(\boldsymbol{r}), \tilde{\rho}(\boldsymbol{r}))$

$\Rightarrow$ *total entropy:* $S[\tilde{\varepsilon}(\boldsymbol{r}), \tilde{\rho}(\boldsymbol{r})] \equiv \int \tilde{\sigma}(\boldsymbol{r}) d\boldsymbol{r}$

$$\Rightarrow \begin{cases} \tilde{\beta}(\boldsymbol{r}) \equiv \dfrac{\delta S}{\delta \tilde{\varepsilon}(\boldsymbol{r})} \Rightarrow \tilde{T}(\boldsymbol{r}) \\ \tilde{\alpha}(\boldsymbol{r}) \equiv \dfrac{\delta S}{\delta \tilde{\rho}(\boldsymbol{r})} \Rightarrow \tilde{\mu}(\boldsymbol{r}) \end{cases}$$

$\Rightarrow$ *local pressure* $\tilde{p}(\boldsymbol{r})$ *& local enthalpy density* $\tilde{h}(\boldsymbol{r})$ *& …*

Free energy density: $\tilde{f}_T(\boldsymbol{r}) = f_T(\tilde{\rho}(\boldsymbol{r})) + \frac{1}{2}C(\boldsymbol{\nabla}\tilde{\rho}(\boldsymbol{r}))^2$

J. Randrup, Phys. Rev. C 79, 054911 (2009)

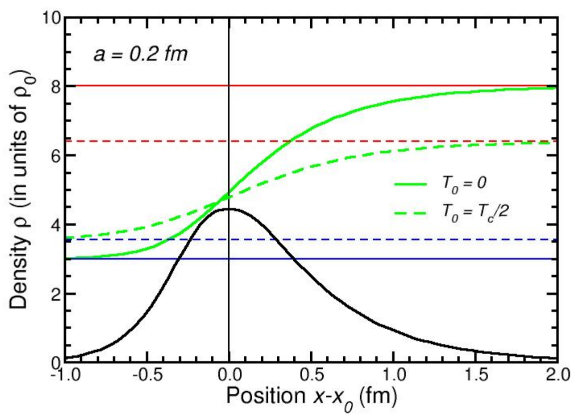
H. Heiselberg *et al.*, Phys. Rev. Lett. 70, 1355 (1993)

Interface equilibrium

Global equilibrium requires constant *T*, μ, *p*:

$$0 \doteq \delta S - \beta_0 \delta E - \alpha_0 \delta N = \int dx \left\{ [\tilde\beta(x) - \beta_0]\delta\tilde\varepsilon(x) + [\tilde\alpha(x) - \alpha_0]\delta\tilde\rho(x) \right\}$$

$$\Rightarrow \quad \tilde\beta(x) = \beta_0 \;\&\; \tilde\alpha(x) = \alpha_0 \;\Rightarrow\; \tilde p(x) = p_0$$

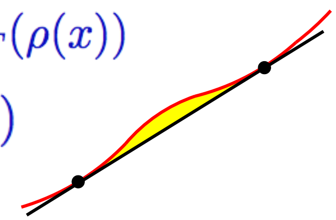


$$\tilde f_T(\boldsymbol{r}) \;=\; f_T(\tilde\rho(\boldsymbol{r})) + \tfrac{1}{2}C(\boldsymbol{\nabla}\tilde\rho(\boldsymbol{r}))^2$$ =>

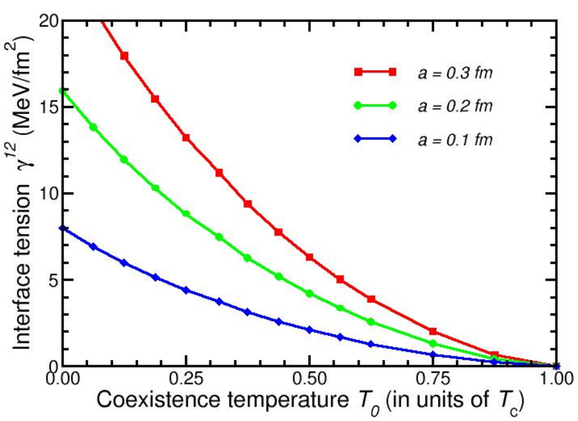
The interface density profile is determined by

$$C\partial_x^2\rho(x) \doteq \mu_T(\rho(x)) - \mu_0 = \partial_\rho \Delta f_T(\rho(x))$$

where $\Delta f_T(\rho) \equiv f_T(\rho) - f_T^M(\rho)$



$$f_T^M(\rho) \;\equiv\; f_T(\rho_i) + \mu_0(\rho - \rho_i) \;\leq\; f_T(\rho)$$



The interface tension is given by

$$\gamma_T^{12} = \int_{\rho_1}^{\rho_2} d\rho \, [2C\Delta f_T(\rho)]^{\frac{1}{2}}$$

[The density profile *ρ(x) is not* needed!]

J. Randrup, Phys. Rev. C 79, 054911 (2009)

Dynamical effect of the gradient term: The local pressure is modified

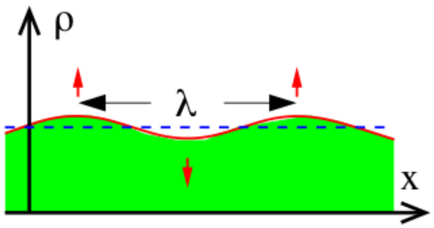
Small deviations from uniformity:

$$\tilde{p}(\boldsymbol{r}) \;\approx\; p_0(\tilde{\varepsilon}(\boldsymbol{r}), \tilde{\rho}(\boldsymbol{r})) - C\rho_0 \nabla^2 \tilde{\rho}(\boldsymbol{r})$$

Small harmonic density undulations:

$$\tilde{\rho}(\boldsymbol{r}, t) = \rho_0 + \delta\rho(x, t) \doteq \; \rho_0 + \rho_k \mathrm{e}^{ikx - i\omega t}$$

$$\boxed{p_k \;\longrightarrow\; p_k + C\rho_0 k^2 \rho_k}$$



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Transport model: Dissipative fluid dynamics

Energy-momentum tensor:

$$T^{00} \approx \varepsilon \quad \& \quad T^{0i} \approx (\varepsilon + p)v^i + q^i \quad \&$$

A Muronga, PRC 76, 014909 (2007)

$$T^{ij} \approx \delta_{ij} p - \eta[\partial_i v^j + \partial_j v^i - \tfrac{2}{3}\delta_{ij}\partial^k v^k] - \zeta \delta_{ij} \partial^k v^k$$

$$|\rho_k| \ll \rho_0 \;\Rightarrow\; |v| \ll 1$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{T} \;\approx\; \boldsymbol{\nabla} p - \eta \Delta \boldsymbol{v} - [\tfrac{1}{3}\eta + \zeta]\boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \boldsymbol{v}) \;\asymp\; \partial_x p - [\tfrac{4}{3}\eta + \zeta]\partial_x^2 v$$

Eckart frame

Equations of motion:

$$C: \quad \partial_t \rho \;\doteq -\rho_0 \boldsymbol{\nabla} \cdot \boldsymbol{v} \;\Rightarrow\; \omega \rho_k \doteq \rho_0 k v_k$$ charge

$$\boldsymbol{M}: \quad h_0 \partial_t \boldsymbol{v} \;\doteq -\boldsymbol{\nabla}[p - \zeta \boldsymbol{\nabla} \cdot \boldsymbol{v}] - \boldsymbol{\nabla} \cdot \boldsymbol{\pi} - \partial_t \boldsymbol{q}$$ momentum

$$E: \quad \partial_t \varepsilon \;\doteq -h_0 \boldsymbol{\nabla} \cdot \boldsymbol{v} - \boldsymbol{\nabla} \cdot \boldsymbol{q}$$ energy

Sound equation:

$$\partial_t E - \boldsymbol{\nabla} \cdot \boldsymbol{M}: \quad h_0 \partial_t^2 \varepsilon \;\doteq\; \Delta[p - \zeta \boldsymbol{\nabla} \cdot \boldsymbol{v}] + \boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \cdot \boldsymbol{\pi})$$

$$\omega^2 \varepsilon_k \;\doteq\; k^2 p_k - i[\tfrac{4}{3}\eta + \zeta]\frac{\omega}{\rho_0} k^2 \rho_k$$

$$\xi \equiv \tfrac{4}{3}\eta + \zeta$$

Heat flow:

$$\boldsymbol{q} \approx -\kappa[\boldsymbol{\nabla} T + T_0 \partial_t \boldsymbol{v}]: \quad q_k = -i\kappa[k T_k - \frac{T_0}{\rho_0}\frac{\omega^2}{k}\rho_k]$$

$$T_k \approx \frac{1}{1 + i\kappa k^2/\omega c_v}\frac{T_0}{\rho_0}\left(\frac{\partial p}{\partial \varepsilon}\right)_\rho \rho_k$$

Equation of state:

$$p_T(\rho) \;\Rightarrow\; p_k = \left(\frac{\partial p}{\partial \varepsilon}\right)_\rho c_v T_k + \frac{h_0}{\rho_0} v_T^2 \rho_k$$

Dispersion equation:

$$\omega^2 \;\doteq\; v_T^2 k^2 + C\frac{\rho_0^2}{h_0}k^4 - i\xi\frac{\omega}{h_0}k^2 + \frac{v_s^2 - v_T^2}{1 + i\kappa k^2/\omega c_v}k^2$$

Heiselberg, Pethick, Ravenhall, Ann. Phys. 233, 37 (1993)

Dispersion relations

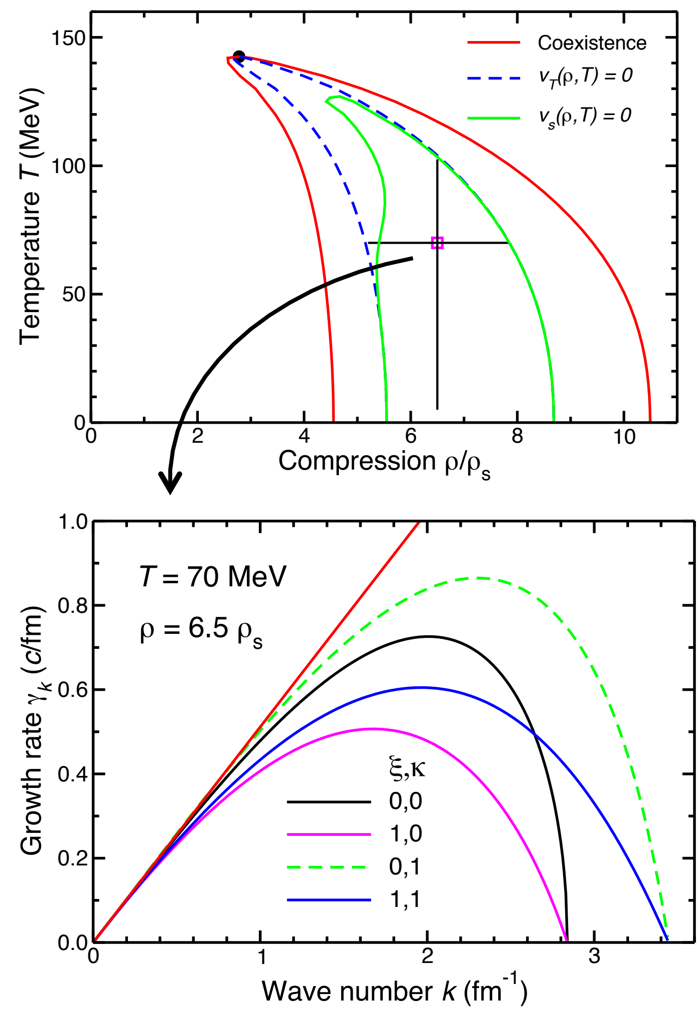
$$\omega_k = \epsilon_k + i\gamma_k$$

Ideal fluid dynamics: $\omega^2 \doteq v_s^2 k^2$

+ gradient term: $\omega^2 \doteq v_s^2 k^2 + C\dfrac{\rho_0^2}{h_0} k^4$

+ shear & bulk viscosity: $\omega^2 \doteq v_s^2 k^2 + C\dfrac{\rho_0^2}{h_0} k^4 - i\xi\dfrac{\omega}{h_0} k^2$

+ heat conduction: $\omega^2 \doteq v_T^2 k^2 + C\dfrac{\rho_0^2}{h_0} k^4 - i\xi\dfrac{\omega}{h_0} k^2 + \dfrac{v_s^2 - v_T^2}{1 + i\kappa k^2/\omega c_v} k^2$



$$\xi \equiv \tfrac{4}{3}\eta + \zeta$$

Transport coefficients

$$\eta_0 \geq 1$$
$$\kappa_0 \geq 1$$

1) Bulk viscosity ζ: Ignore

$$\zeta \ll \eta \;\Rightarrow\; \xi \equiv \tfrac{4}{3}\eta + \zeta \approx \tfrac{4}{3}\eta$$

2) Shear viscosity η:

$$\rho = 0: \quad h \equiv p + \varepsilon = T\sigma$$
$$\rho = 0: \quad \eta \geq \frac{\hbar}{4\pi}\sigma = \frac{\hbar}{4\pi}\frac{h}{T}$$

$$\eta(\rho,T) = \eta_0 \frac{c_0}{c} d(\rho,T)\, h(\rho,T)$$

$$\rho > 0,\; T \ll mc^2: \; h \asymp mc^2 n \gg T\sigma$$
$$\rho = 0: \quad n \sim T^3 \;\Rightarrow\; \frac{\hbar c}{T} = 4\pi c_0\, d$$

$$d \equiv n^{1/3}$$

$$\lambda_{\text{visc}} \equiv \frac{1}{c}\frac{\xi(\rho,T)}{h(\rho,T)/c^2} \approx \tfrac{4}{3}\eta_0\, c_0\, d(\rho,T)$$

3) Heat conductivity κ:

$$\eta \approx \tfrac{1}{3} n\bar{p}\ell$$
$$\kappa \approx \tfrac{1}{3}\bar{v}\ell c_v$$

$$\frac{\kappa}{\eta} \approx \frac{c_v}{h/c^2}$$

$$\bar{p} = m\bar{v}$$
$$c_v \equiv \partial_T \varepsilon_T(\rho)$$

$$h \asymp mc^2 n$$
$$c_v \asymp \tfrac{3}{2}n$$

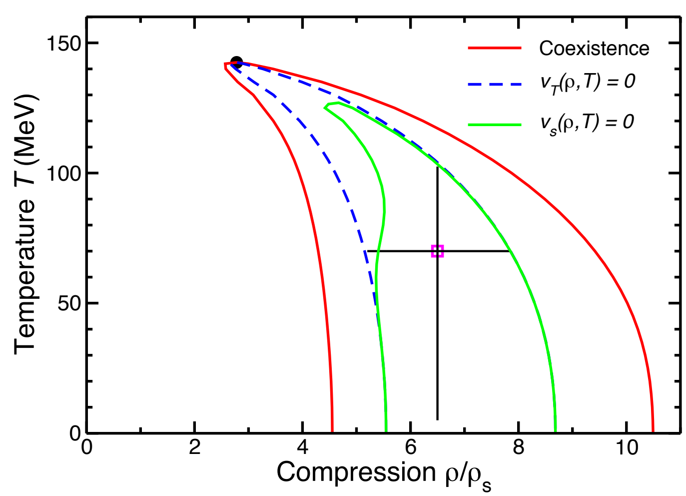
$$\kappa(\rho,T) = \kappa_0\, c_0\, c\, d(\rho,T)\, c_v(\rho,T)$$

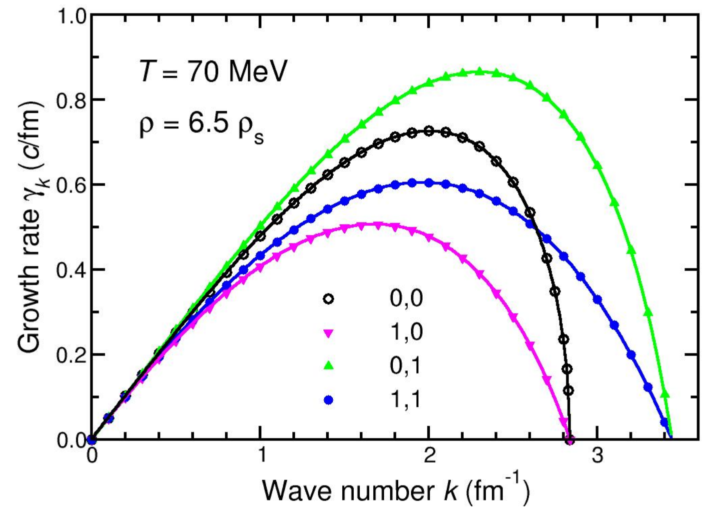
$$\lambda_{\text{heat}} \equiv \frac{1}{c}\frac{\kappa(\rho,T)}{c_v(\rho,T)} = \kappa_0\, c_0\, d(\rho,T)$$

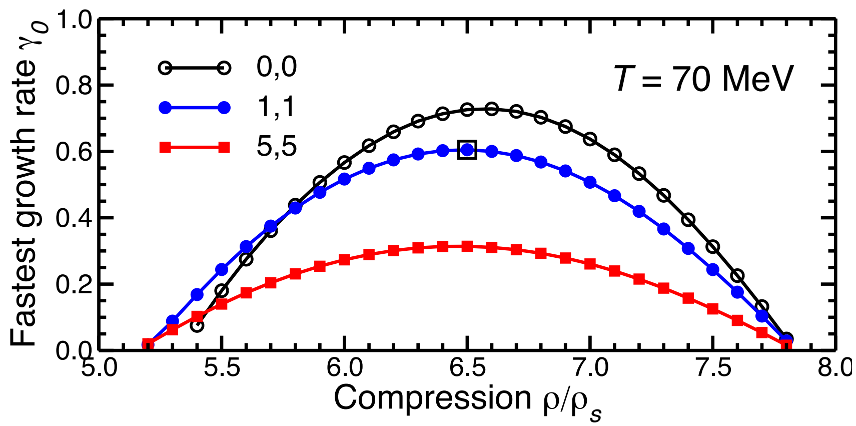
$$c_0 = \frac{1}{4\pi}\left[(g_g + \tfrac{3}{3}g_q)\frac{\zeta(3)}{\pi^2}\right]^{\frac{1}{3}} \approx 0.12779$$

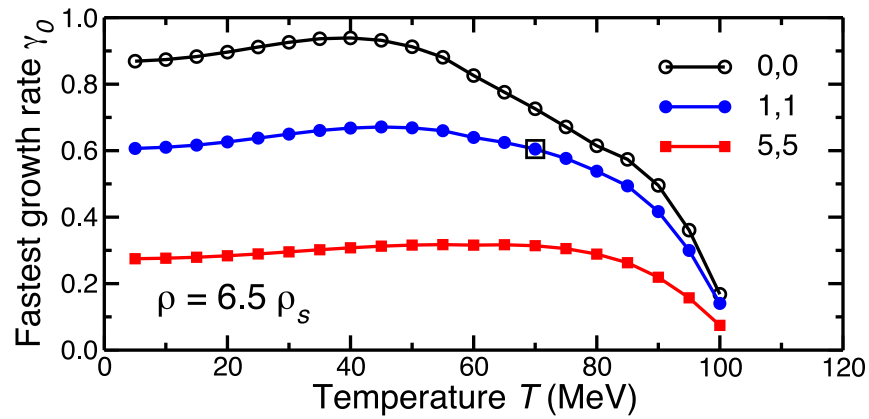
V Koch & J Liao, PRC81, 014902 (2010)

Spinodal growth rates









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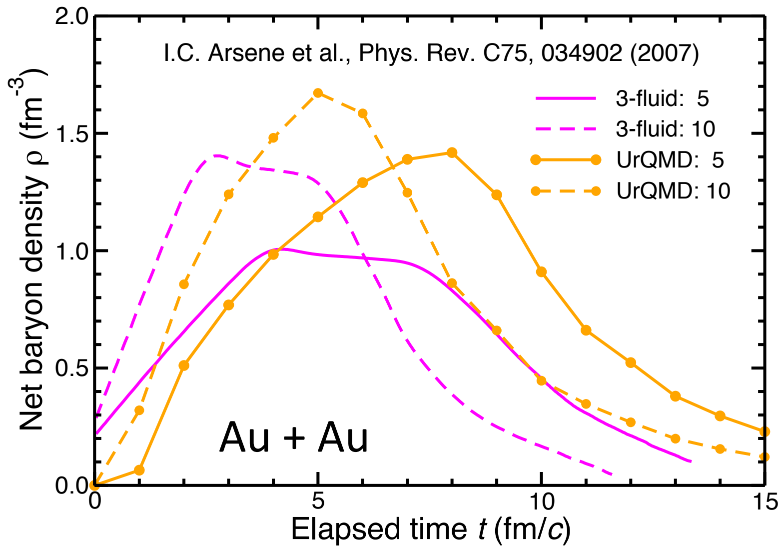
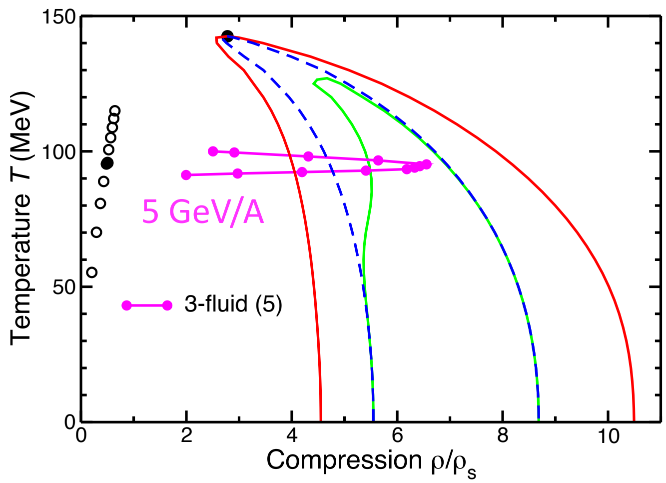
Dynamical phase trajectories

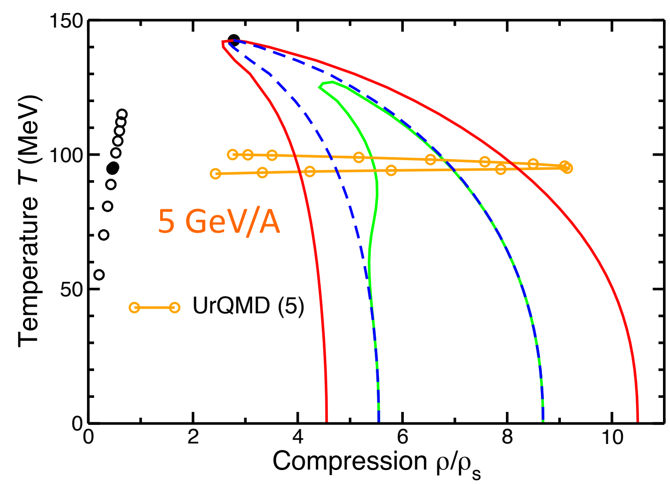


UrQMD
3-fluid
PHSD
QGSM
GiBUU

3-fluid



UrQMD



Dynamics of collective modes in many-body systems

Amplitude evolution: $\omega_\nu = \epsilon_\nu + i\gamma_\nu$

$$\frac{d}{dt}A_\nu(t) \;=\; -i\omega_\nu A_\nu(t) \;+\; B_\nu(t)$$

Correlation function:

$$\sigma_{\nu\mu}(t_1,t_2) \;\equiv\; \prec A_\nu(t_1)\,A_\mu(t_2)^* \succ$$

Markovian noise:

$$\prec B_\nu(t)\,B_\mu(t')^* \succ \;=\; 2\mathcal{D}_{\nu\mu}\,\delta(t-t')$$

Evolution:

seed *feedback*

$$\frac{d}{dt}\sigma_{\nu\mu}(t) \;=\; 2\mathcal{D}_{\nu\mu} \;-\; i(\omega_\nu - \omega_\mu^*)\sigma_{\nu\mu}$$

Variance of a single mode:

$$\frac{d}{dt}\sigma_\nu^2 \;=\; 2\mathcal{D}_\nu \;+\; 2\gamma_\nu\sigma_\nu^2$$

Lalime Equation

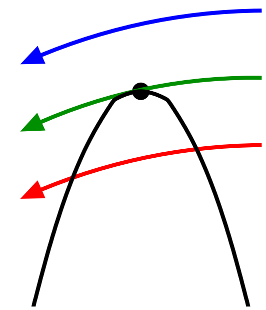
Amplification coefficient:

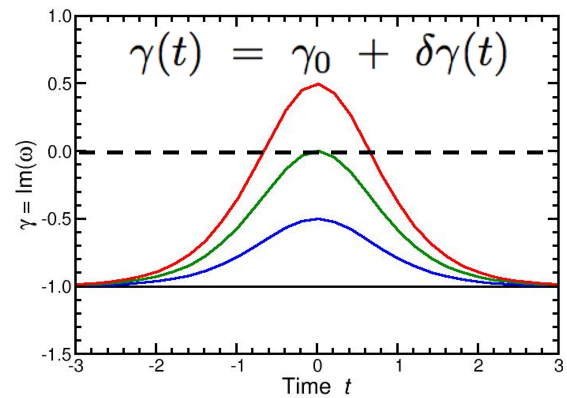
$$\Gamma_\nu(t) \equiv \int_0^t \gamma_\nu(t')dt'$$

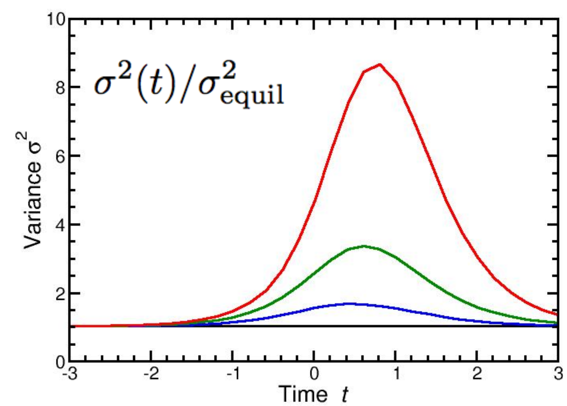
=> $$\sigma_\nu^2(t) \;=\; \left[2\mathcal{D}_\nu \int_0^t \mathrm{e}^{-2\Gamma_\nu(t')}dt' + \sigma_0^2\right]\mathrm{e}^{2\Gamma_\nu(t)}$$

$$\gamma_\nu < 0: \quad \sigma_\nu^2(t) \;\to\; -\mathcal{D}_\nu/\gamma_\nu$$

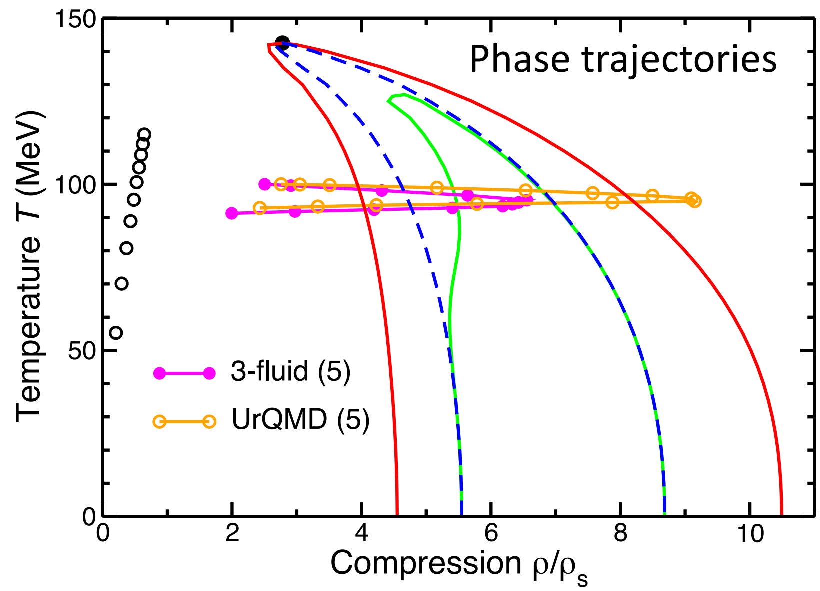
Colonna, Chomaz, Randrup, NPA 567 (1994) 637







Spinodal amplification

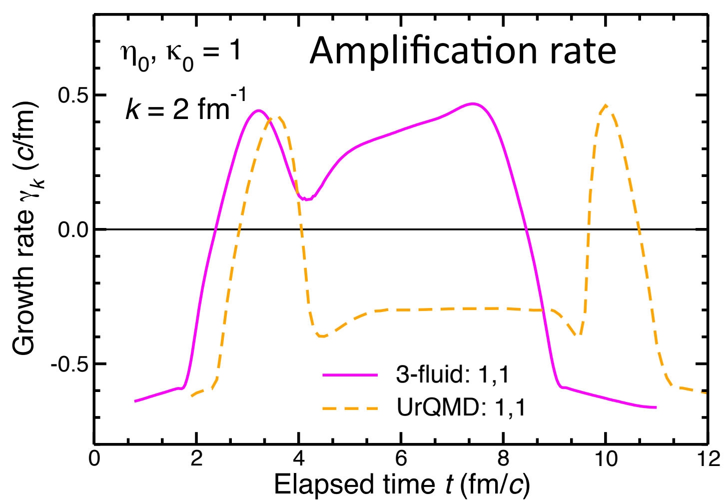


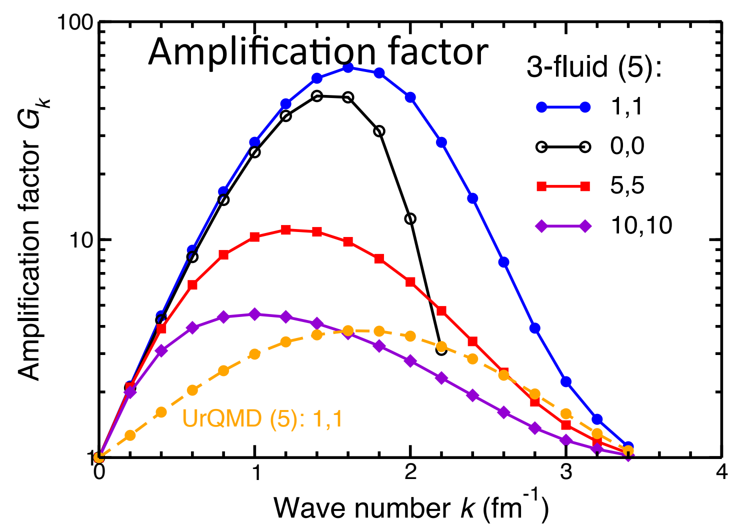
Amplification coefficient:

$$\Gamma_\nu(t) \equiv \int_0^t \gamma_\nu(t')dt'$$

Amplification factor:

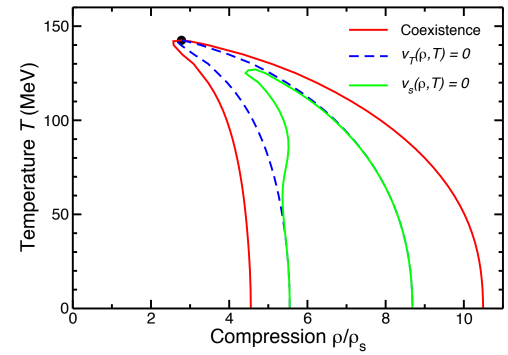
$$G_k = \mathrm{e}^{2\Gamma_k}$$





Can spinodal phase separation occur during confinement?

Conclusions



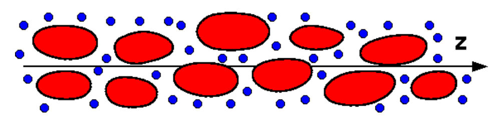
Spinodal decomposition *may* occur at $E_{lab} \approx$ 5-10 GeV/A (FAIR & NICA):

1) Exists an optimal E_{lab} range

2) Significant amplification

- but:

Full dynamical calculations are needed!



MODEL REQUIREMENTS:

EoS with phase transition

Confined & deconfined phases

Phase coexistence, incl interface*

Spinodal modes*

Dynamics with instabilities*

Suggestion: *Use fluid dynamics with finite range*

Important!

Need: *Transport coefficients $\eta(\rho,T)$, $\zeta(\rho,T)$, $\kappa(\rho,T)$ in addition to the Equation of State $p(\rho,T)$*

If it occurs: Useful signal of phase transition

** Requires finite range*