

Nearly Perfect Fluidity: From Atoms to Quarks

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See T. Schaefer, D. Teaney, "Perfect Fluidity" [arXiv:0904.3107]

The first piece of advice I received in my physics career

Herr Schäfer, lesen Sie nicht so viel – die Weltliteratur ist unerschöpflich – rechnen Sie lieber mal was*.

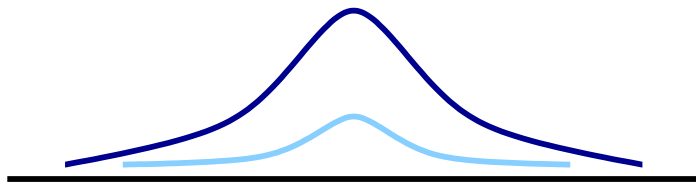
This piece of wisdom has an important corollary.

Don't just compute anything, compute things that can be confronted with high quality data.

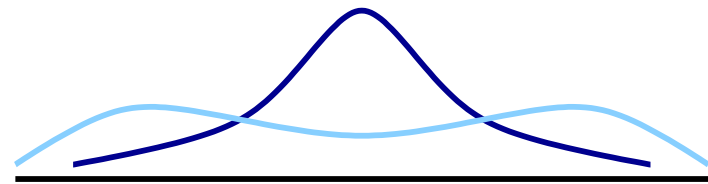
* Apparently, I was not the only person to receive this piece of advice.

Fluids: Gases, Liquids, Plasmas, ...

Hydrodynamics: Long-wavelength, low-frequency dynamics of conserved or spontaneously broken symmetry variables.



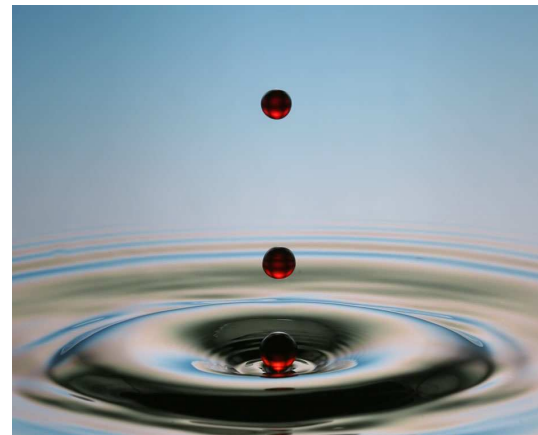
$$\tau \sim \tau_{micro}$$



$$\tau \sim \lambda$$

Historically: Water

$$(\rho, \epsilon, \vec{\pi})$$



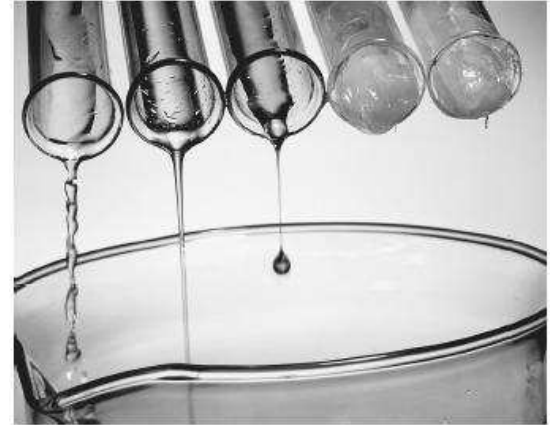
Simple non-relativistic fluid

Simple fluid: Conservation laws for mass, energy, momentum

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

$$\frac{\partial \epsilon}{\partial t} + \vec{\nabla} \cdot \vec{j}^\epsilon = 0$$

$$\frac{\partial}{\partial t} (\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0$$



Constitutive relations: Energy momentum tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta \left(\partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k \right) + O(\partial^2)$$

reactive

dissipative

2nd order

$$\text{Expansion } \Pi_{ij}^0 \gg \delta \Pi_{ij}^1 \gg \delta \Pi_{ij}^2$$

Regime of applicability

Expansion parameter $Re^{-1} = \frac{\eta(\partial v)}{\rho v^2} = \frac{\eta}{\rho L v} \ll 1$

$$Re = \frac{\hbar n}{\eta} \times \frac{m v L}{\hbar}$$

fluid property flow property

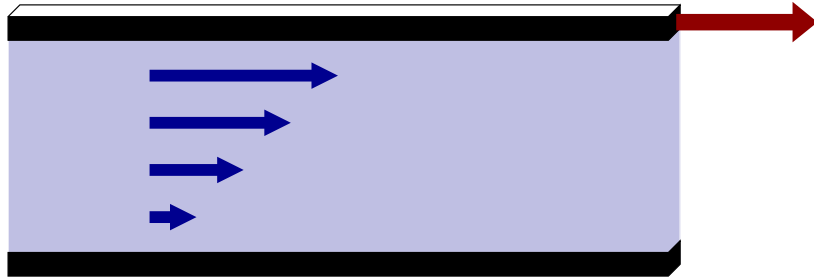
Kinetic theory estimate: $\eta \sim n p l_{mfp}$

$$Re^{-1} = \frac{v}{c_s} Kn \qquad Kn = \frac{l_{mfp}}{L}$$

expansion parameter $Kn \ll 1$

Shear viscosity

Viscosity determines shear stress (“friction”) in fluid flow

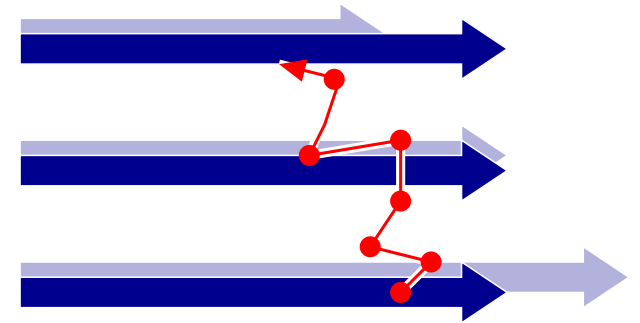


$$F = A \eta \frac{\partial v_x}{\partial y}$$

Kinetic theory: conserved quantities carried by quasi-particles

$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p + \vec{F} \cdot \vec{\nabla}_p f_p = C[f_p]$$

$$\eta \sim \frac{1}{3} n \bar{p} l_{mfp}$$



Dilute, weakly interacting gas: $l_{mfp} \sim 1/(n\sigma)$

$$\eta \sim \frac{1}{3} \bar{p} \sigma$$

independent of density!

Shear viscosity

non-interacting gas ($\sigma \rightarrow 0$):

$$\eta \rightarrow \infty$$

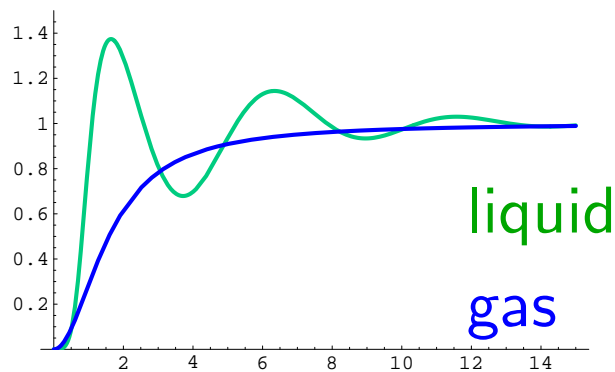
non-interacting and hydro limit ($T \rightarrow \infty$) limit do not commute

strongly interacting gas:

$$\frac{\eta}{n} \sim \bar{p} l_{mfp} \geq \hbar$$

but: kinetic theory not reliable!

what happens if the gas condenses into a liquid?



Eyring, Frenkel:

$$\eta \simeq hn \exp(E/T) \geq hn$$

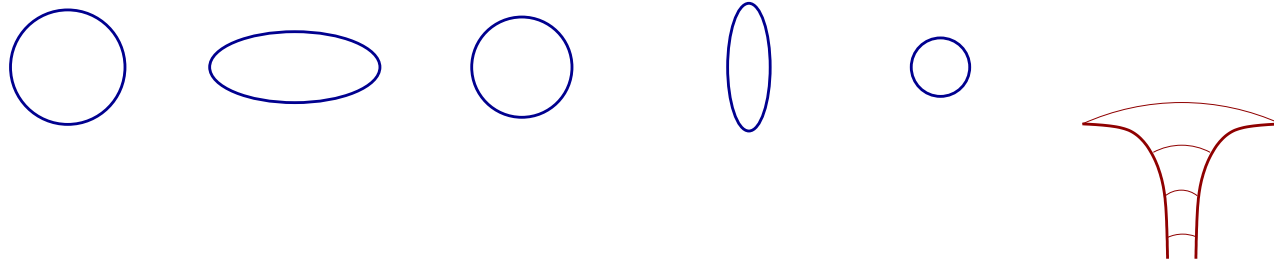
Holographic Duals: Transport Properties

Thermal (conformal) field theory $\equiv AdS_5$ black hole

| | | |
|-----------------|-------------------|---|
| CFT temperature | \Leftrightarrow | Hawking temperature |
| CFT entropy | \Leftrightarrow | Hawking-Bekenstein entropy \sim area of event horizon |
| shear viscosity | \Leftrightarrow | Graviton absorption cross section \sim area of event horizon |

$$T_{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}}$$

$$g_{\mu\nu} = g_{\mu\nu}^0 + \gamma_{\mu\nu}$$



Holographic Duals: Transport Properties

Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT entropy \Leftrightarrow

Hawking-Bekenstein entropy

\sim area of event horizon

shear viscosity \Leftrightarrow

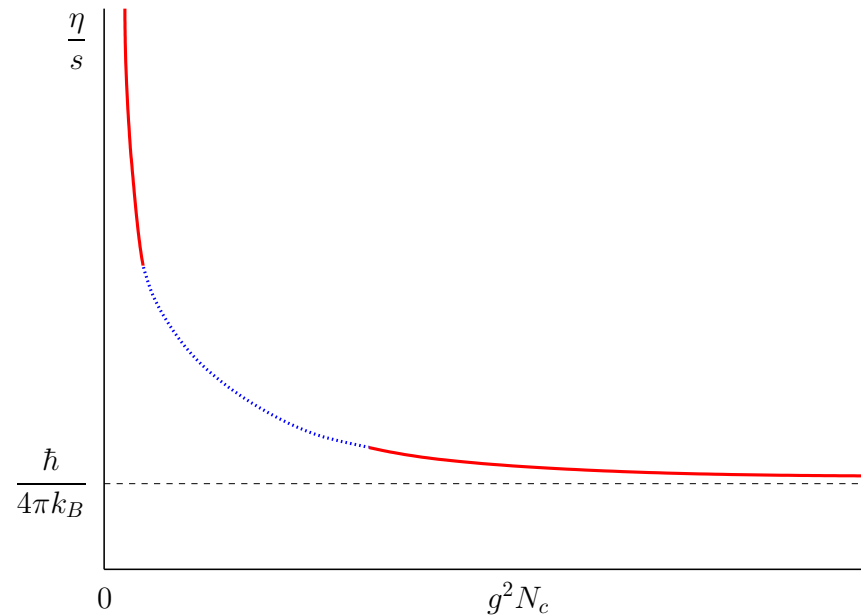
Graviton absorption cross section

\sim area of event horizon

Strong coupling limit

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

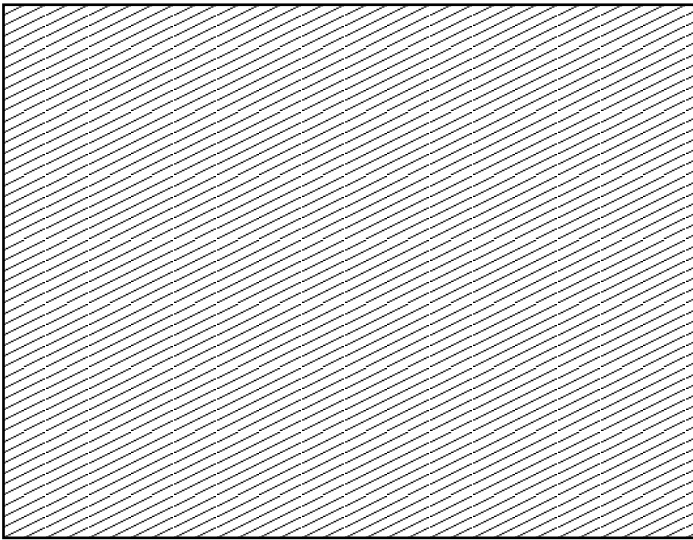
Son and Starinets (2001)



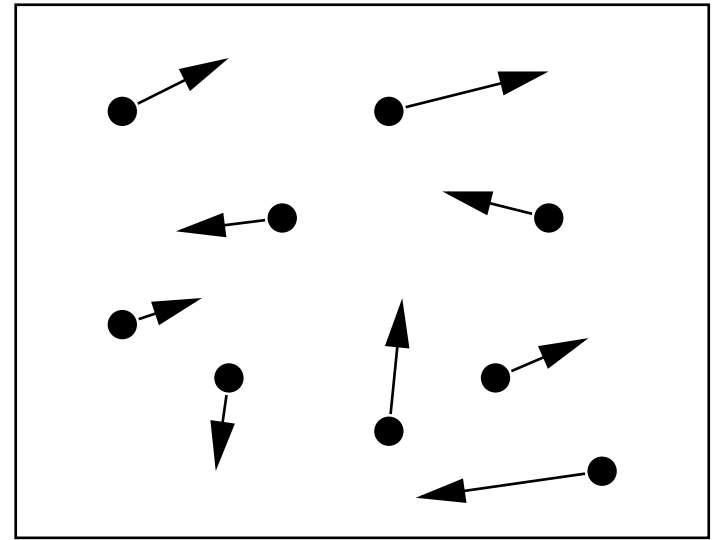
Strong coupling limit universal? Provides lower bound for all theories?

Answer appears to be no; e.g. theories with higher derivative gravity duals.

Kinetics vs No-Kinetics



AdS/CFT low viscosity goo



pQCD kinetic plasma

Effective theories for fluids (Here: Weak coupling QCD)



$$\mathcal{L} = \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a$$



$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p = C[f_p] \quad (\omega < T)$$



$$\frac{\partial}{\partial t} (\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0 \quad (\omega < g^4 T)$$

Effective theories (Strong coupling)



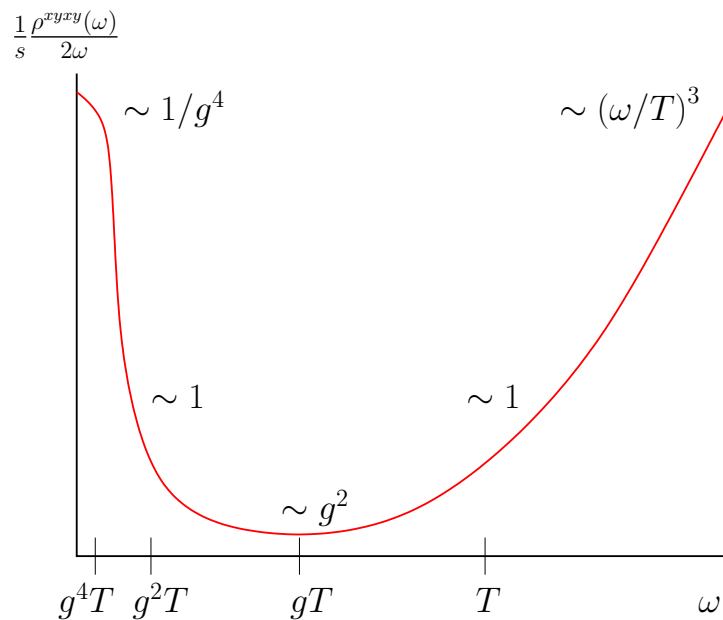
$$\mathcal{L} = \bar{\lambda}(i\sigma \cdot D)\lambda - \frac{1}{4}G_{\mu\nu}^a G_{\mu\nu}^a + \dots \Leftrightarrow S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} \mathcal{R} + \dots$$



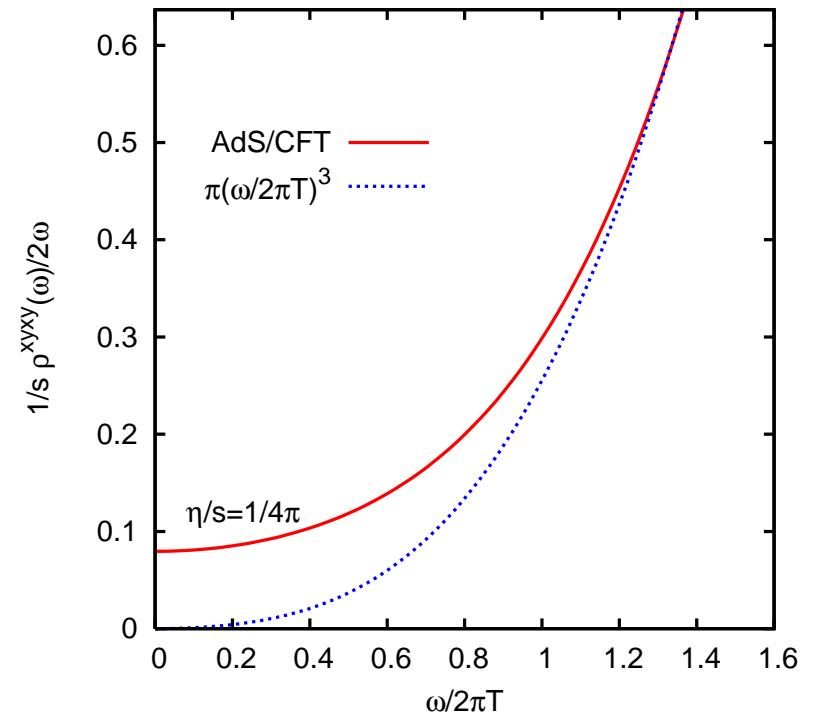
$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0 \quad (\omega < T)$$

Kinetics vs No-Kinetics

Spectral function $\rho(\omega) = \text{Im}G_R(\omega, 0)$ associated with T_{xy}



weak coupling QCD



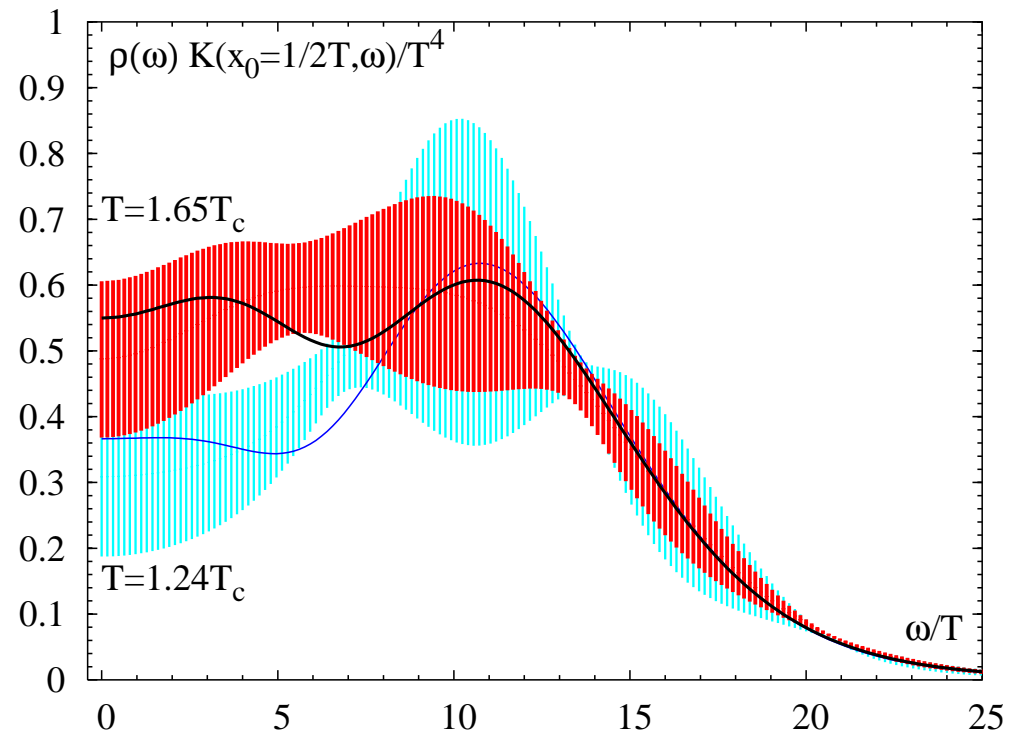
strong coupling AdS/CFT

transport peak vs no transport peak

Spectral function (lattice QCD)

$$\frac{\eta}{s} = 0.13(3)$$

$$\frac{\eta}{s} = 0.10(6)$$



H. Meyer (2007)

Perfect Fluids: How to be a contender?

Bound is quantum mechanical

need quantum fluids

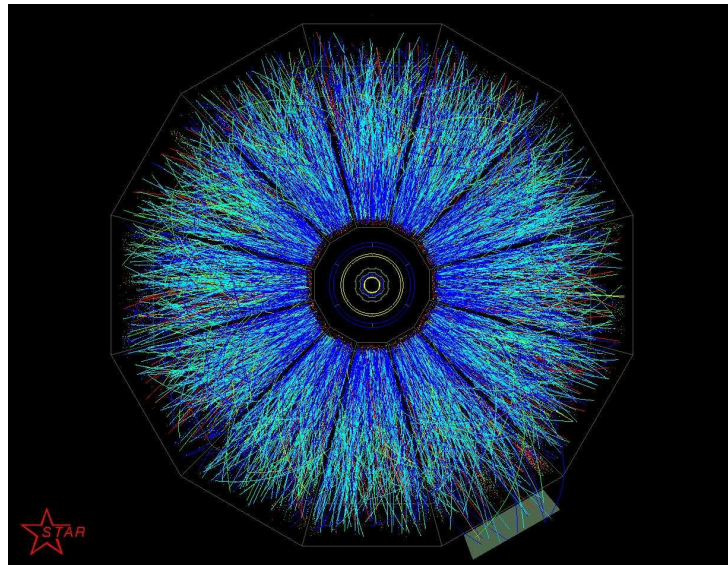
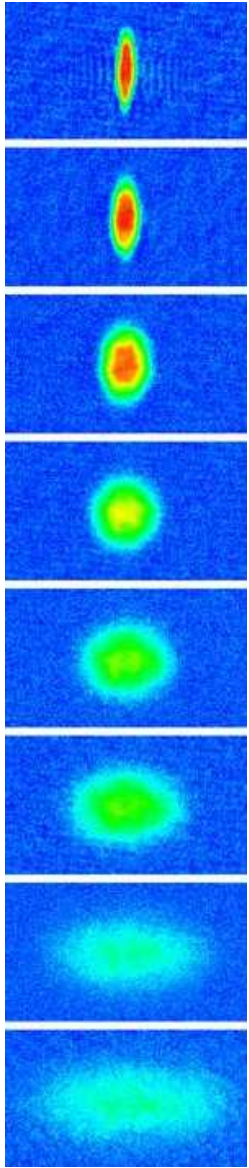
Bound is incompatible with weak coupling and kinetic theory

strong interactions, no quasi-particles

Model system has conformal invariance (essential?)

(Almost) scale invariant systems

Perfect Fluids: The contenders



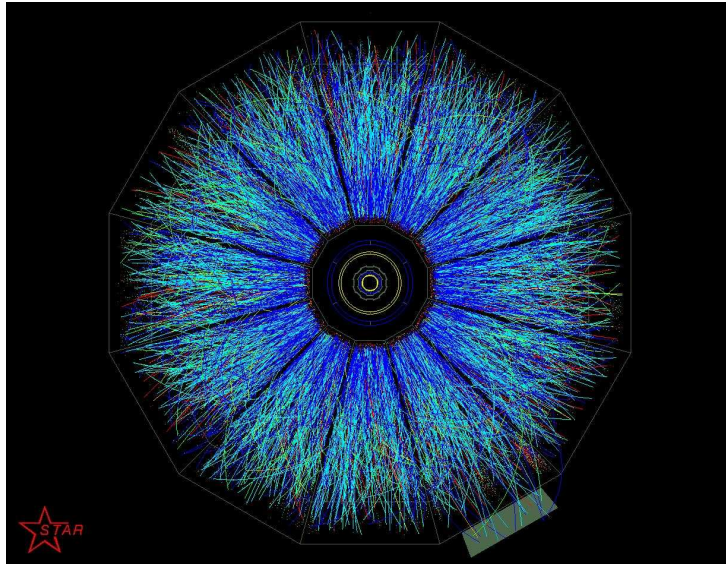
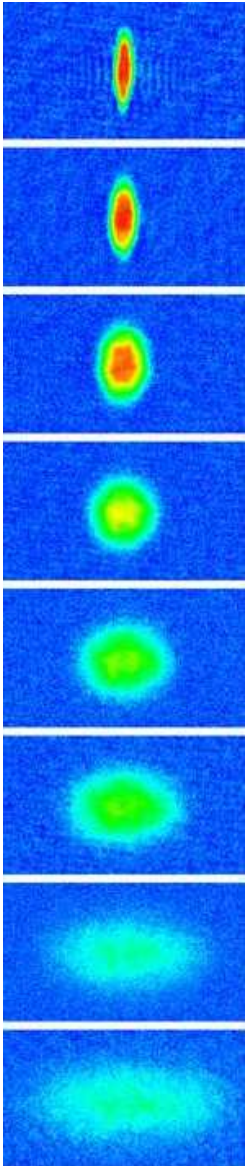
QGP ($T=180$ MeV)

Trapped Atoms
($T=0.1$ neV)



Liquid Helium
($T=0.1$ meV)

Perfect Fluids: The contenders



QGP $\eta = 5 \cdot 10^{11} Pa \cdot s$

Trapped Atoms

$\eta = 1.7 \cdot 10^{-15} Pa \cdot s$



Liquid Helium

$\eta = 1.7 \cdot 10^{-6} Pa \cdot s$

Consider ratios

η/s

Perfect Fluids: Not a contender

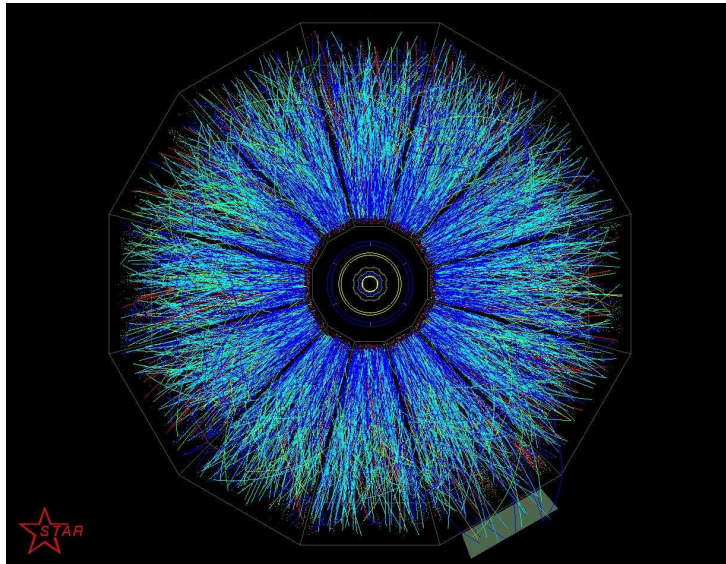
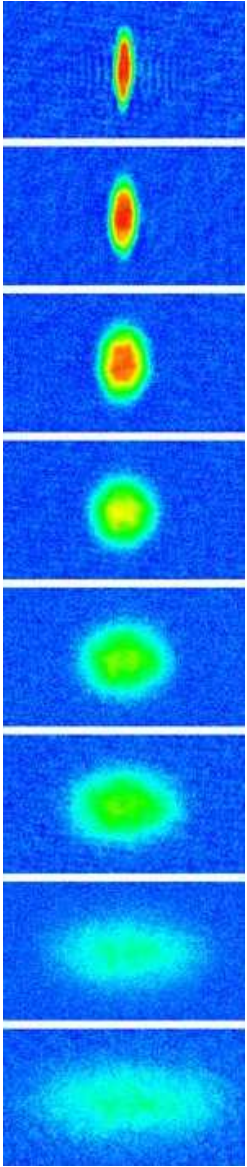


Queensland pitch-drop
experiment

1927-2011 (8 drops)

$$\eta = (2.3 \pm 0.5) \cdot 10^8 \text{ Pa s}$$

Perfect Fluids: The contenders



$$V = V_{sr}(x) + V_{vdw}(x)$$

$$\mathcal{L} = \bar{q}i\mathcal{D}q - \frac{1}{4g^2}G^2$$

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2 \quad (C_0 \rightarrow \infty)$$

Kinetic Theory: Quasiparticles

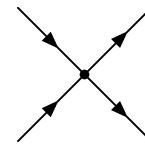
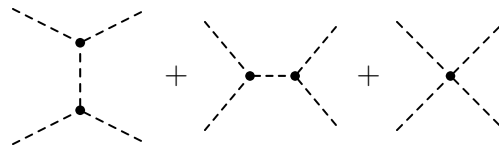
low temperature

high temperature

unitary gas

phonons

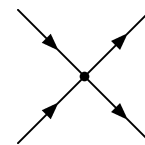
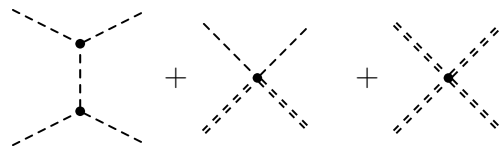
atoms



helium

phonons, rotons

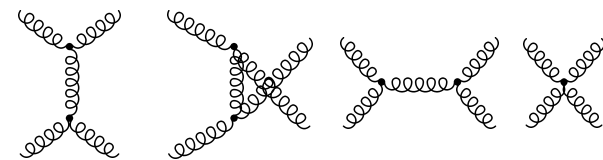
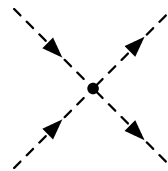
atoms



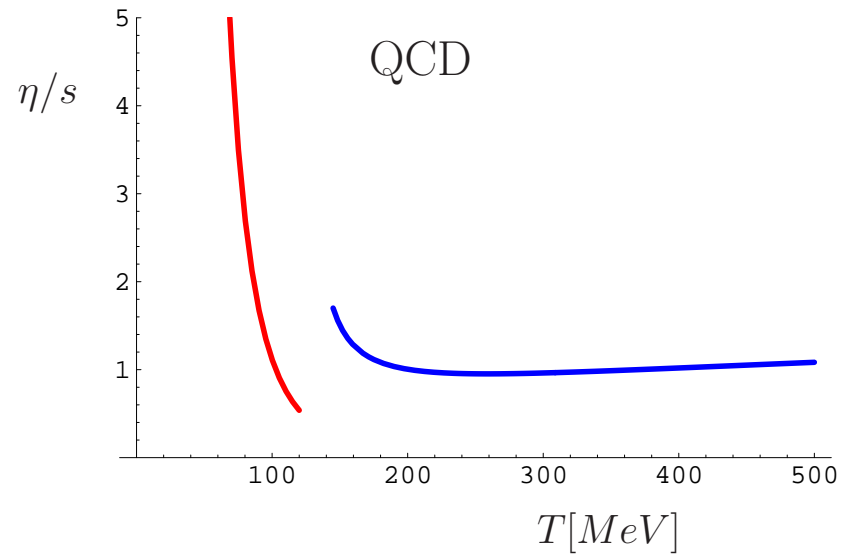
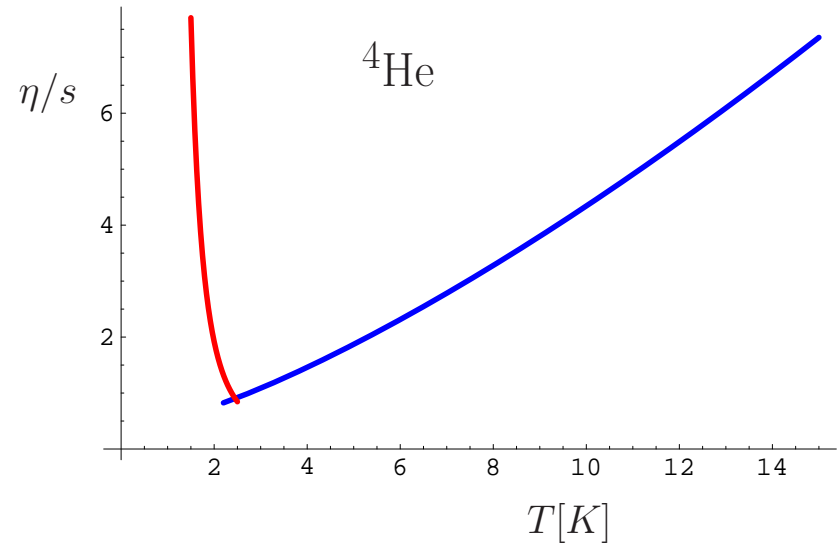
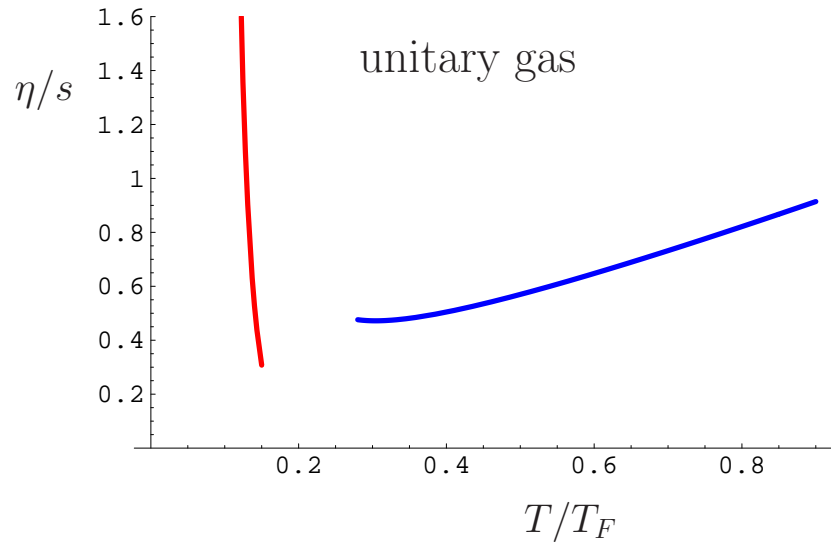
QCD

pions

quarks, gluons



Theory Summary



I. Experiment (Liquid Helium)

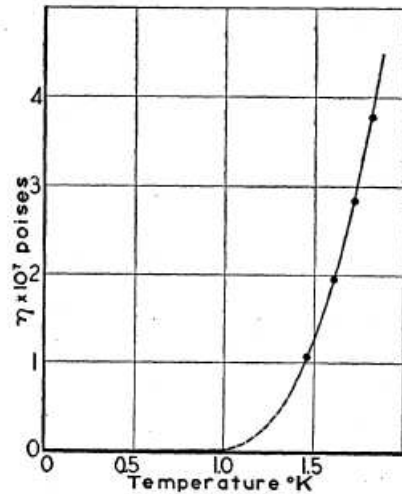
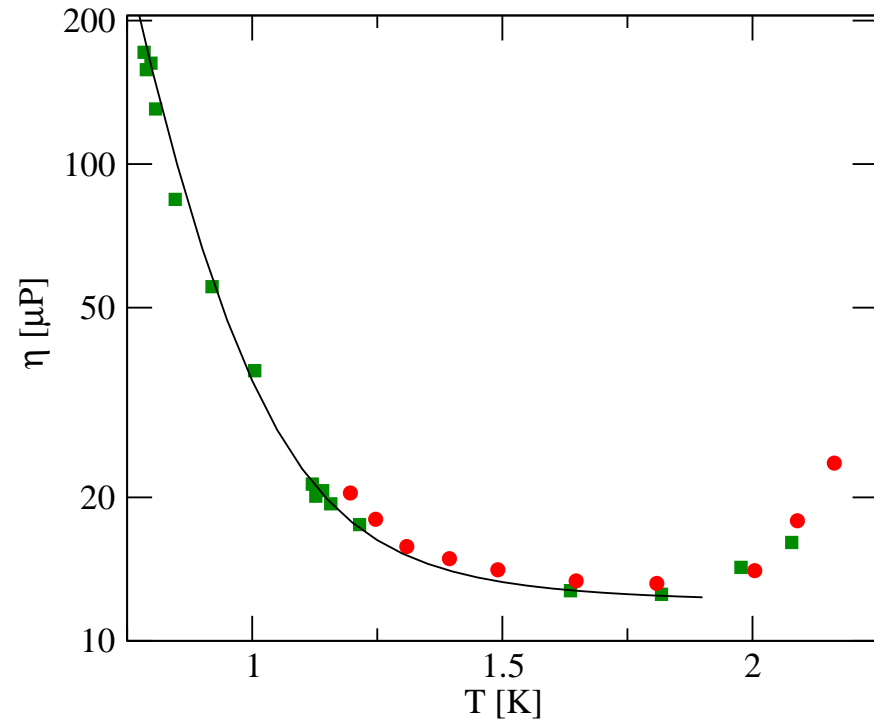


FIG. 1. The viscosity of liquid helium II measured by flow through a 10^{-4} cm channel.



Kapitza (1938)

viscosity vanishes below T_c

capillary flow viscometer

Hollis-Hallett (1955)

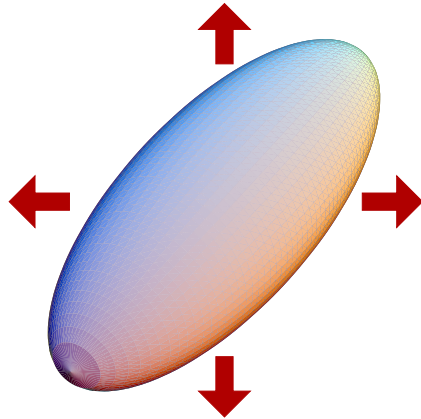
roton minimum, phonon rise

rotation viscometer

$$\eta/s \simeq 0.8 \hbar/k_B$$

II. Hydrodynamics (Cold atoms)

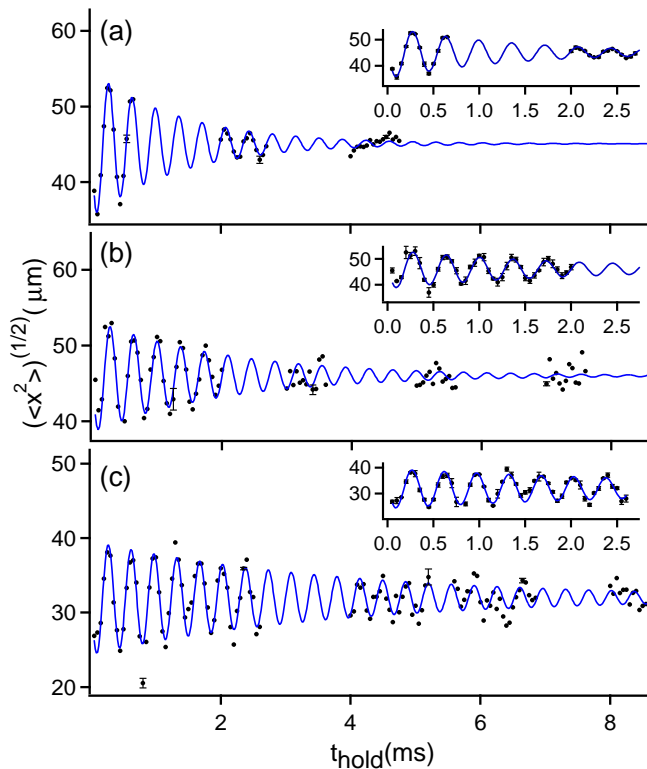
Radial breathing mode



Ideal fluid hydrodynamics ($P \sim n^{5/3}$)

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{v}) = 0$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{v} = -\frac{\vec{\nabla} P}{mn} - \frac{\vec{\nabla} V}{m}$$



Hydro frequency at unitarity

$$\omega = \sqrt{\frac{10}{3}} \omega_{\perp}$$

Damping small, depends on T/T_F .

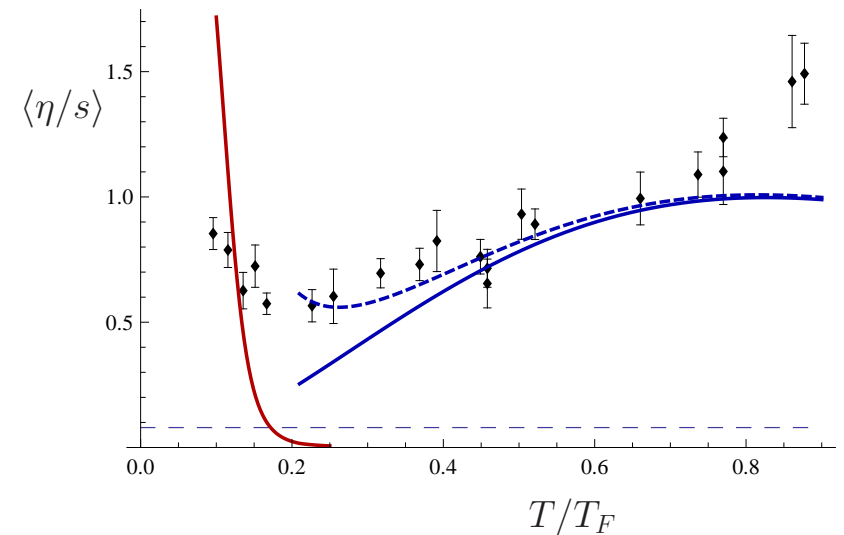
Viscous Hydrodynamics

Energy dissipation (η, ζ, κ : shear, bulk viscosity, heat conductivity)

$$\dot{E} = -\frac{1}{2} \int d^3x \eta(x) \left(\partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k \right)^2 - \int d^3x \zeta(x) (\partial_i v_i)^2 - \frac{1}{T} \int d^3x \kappa(x) (\partial_i T)^2$$

Shear viscosity to entropy ratio
(assuming $\zeta = \kappa = 0$)

$$\frac{\eta}{s} = (3\lambda N)^{\frac{1}{3}} \frac{\Gamma}{\omega_{\perp}} \frac{E_0}{E_F} \frac{N}{S}$$



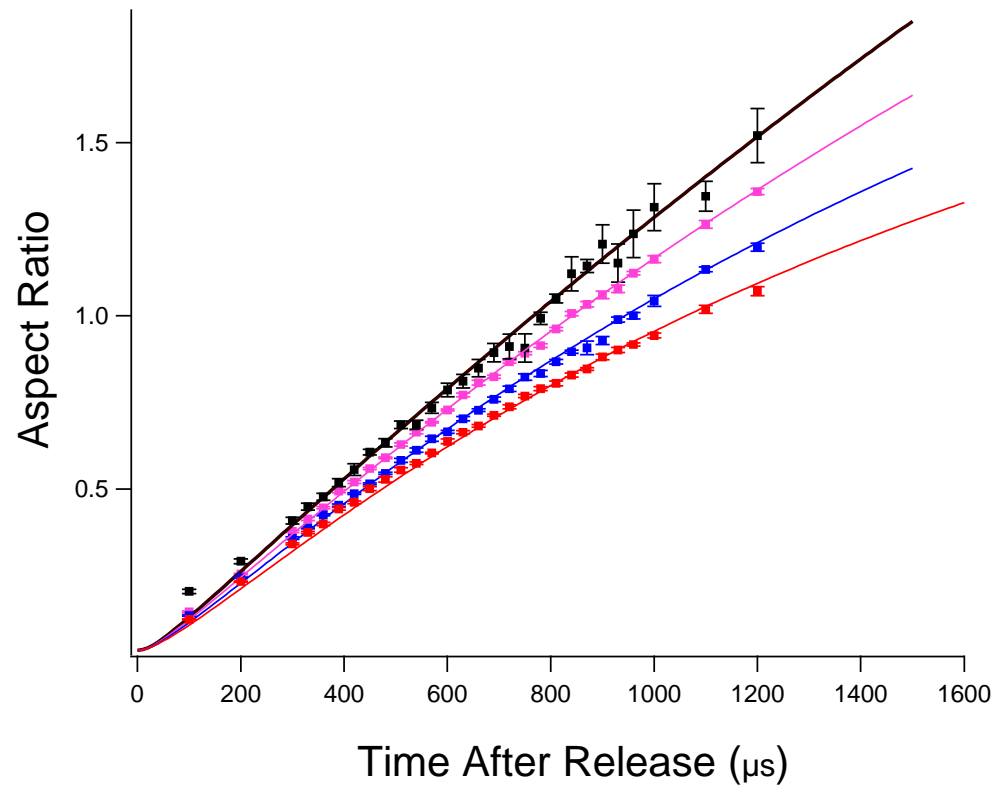
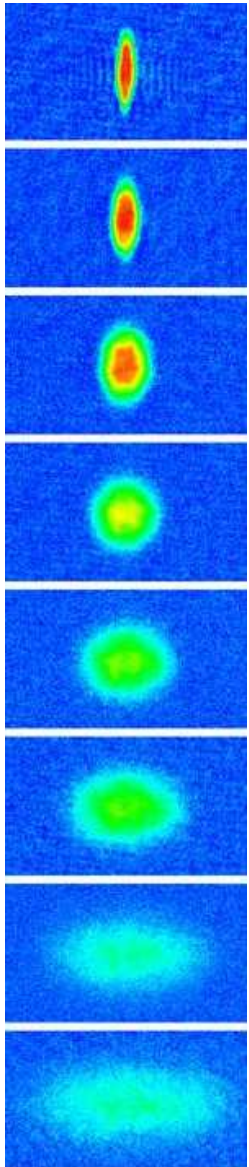
Schaefer (2007), see also Bruun, Smith

$T \ll T_F$

$T \gg T_F, \tau_R \simeq \eta/P$

Elliptic flow: High T limit

$$\text{Quantum viscosity } \eta = \eta_0 \frac{(mT)^{3/2}}{\hbar^2}$$



$$\eta = \eta_0 (mT)^{3/2}$$

$$\tau_R = \eta / P$$

Cao et al., Science (2010)

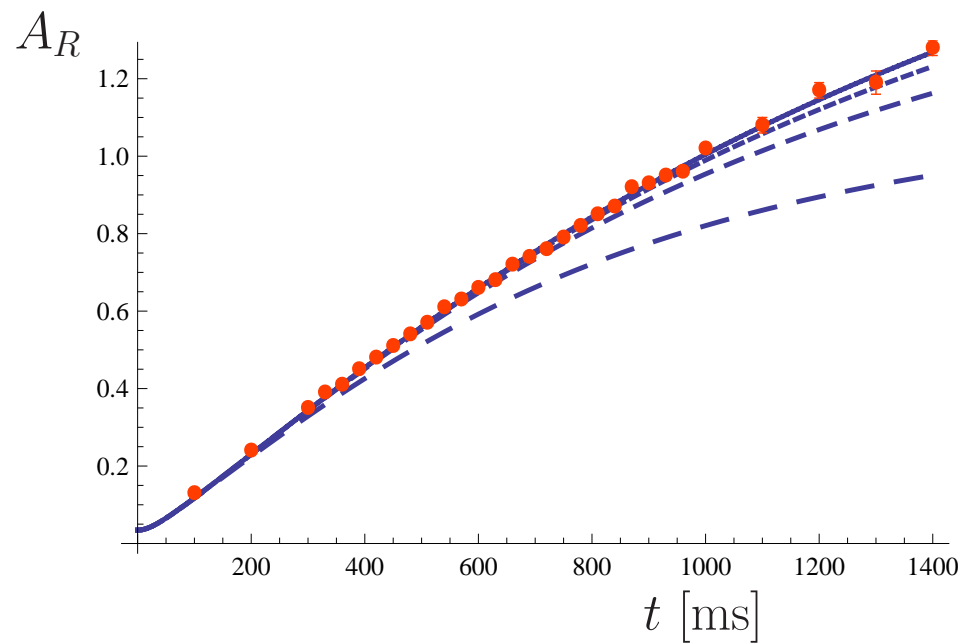
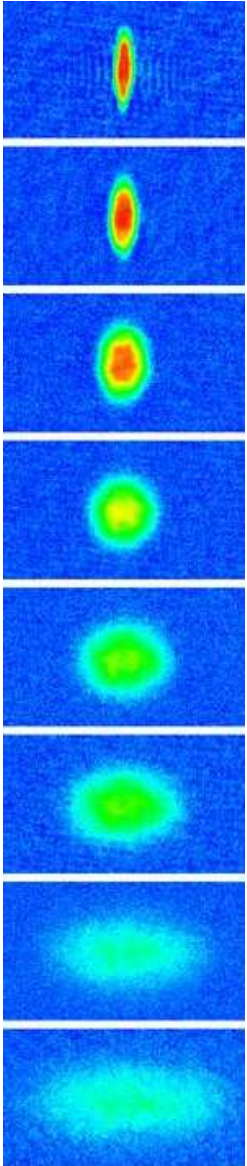
$$\text{fit: } \eta_0 = 0.33 \pm 0.04$$

$$\text{theory: } \eta_0 = \frac{15}{32\sqrt{\pi}} = 0.26$$

Elliptic flow: Freezeout?

switch from hydro to (weakly collisional) kinetics

at scale factor $b_{\perp}^{fr} = 1, 5, 10, 20$

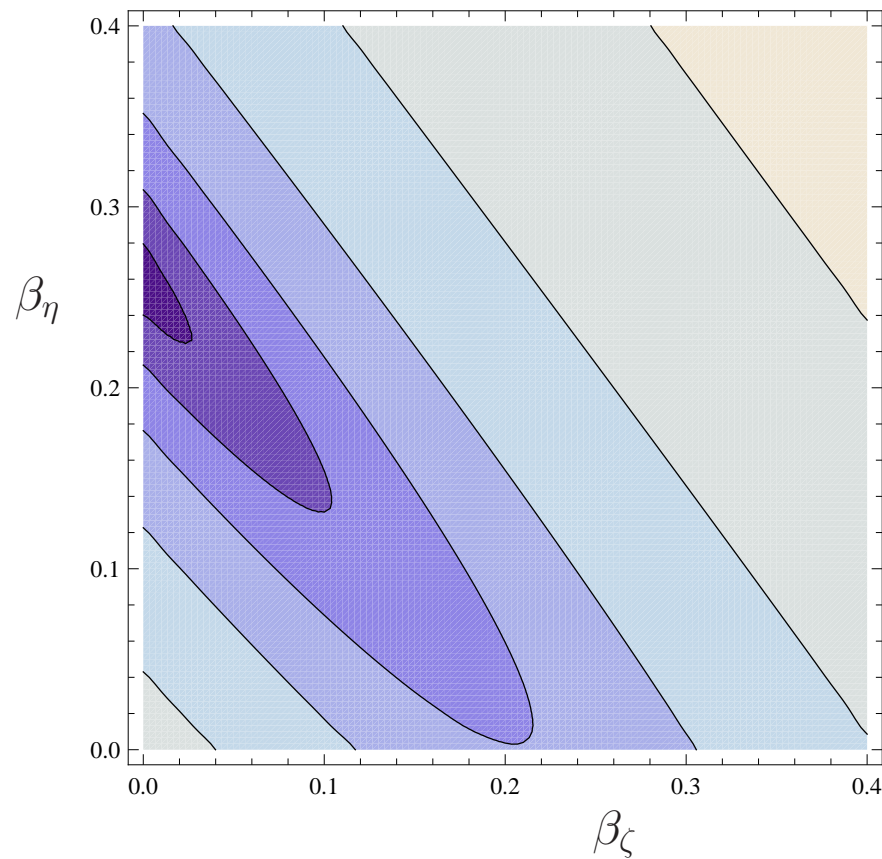
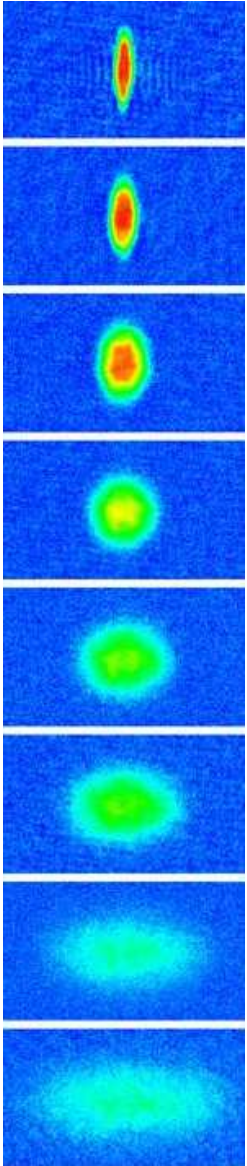


no freezeout seen in the data

Elliptic flow: Shear vs bulk viscosity

Dissipative hydro with both η, ζ

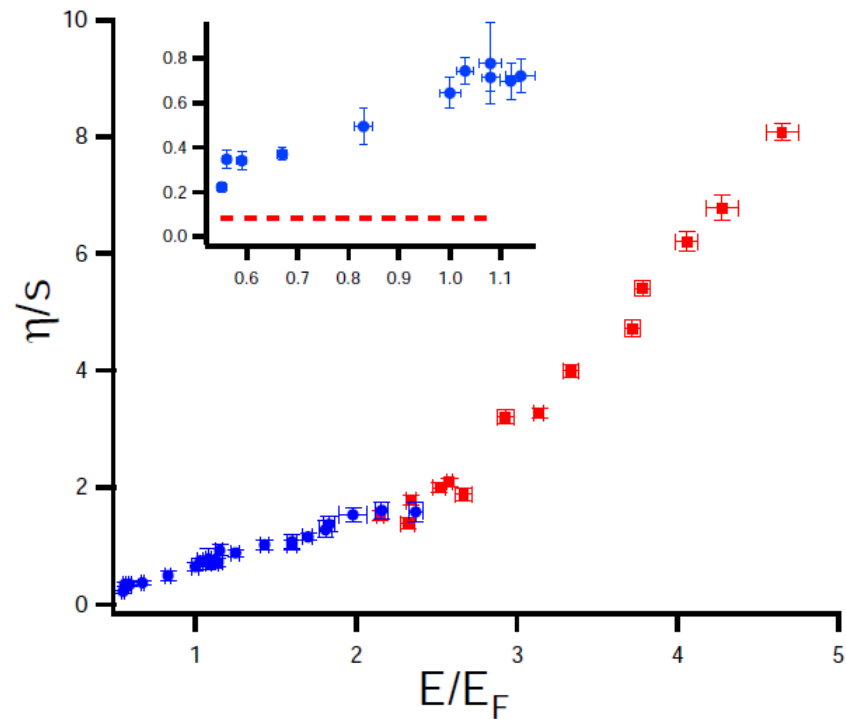
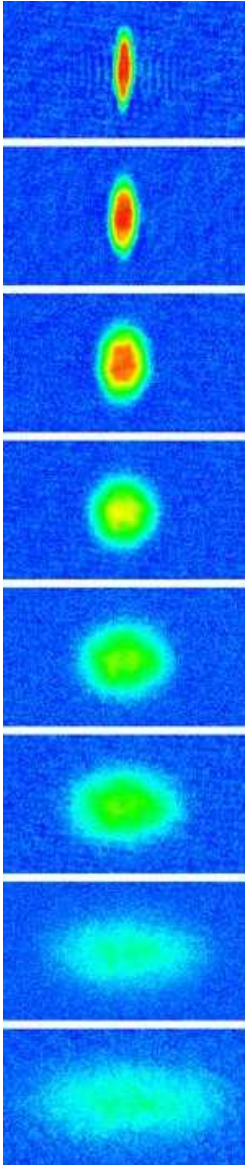
$$\beta_{\eta, \zeta} = (\eta, \zeta) \frac{E_F}{E} \frac{1}{(3\lambda N)^{1/3}}$$



$\eta \gg \zeta$

Viscosity to entropy density ratio

consider both collective modes (low T)
and elliptic flow (high T)

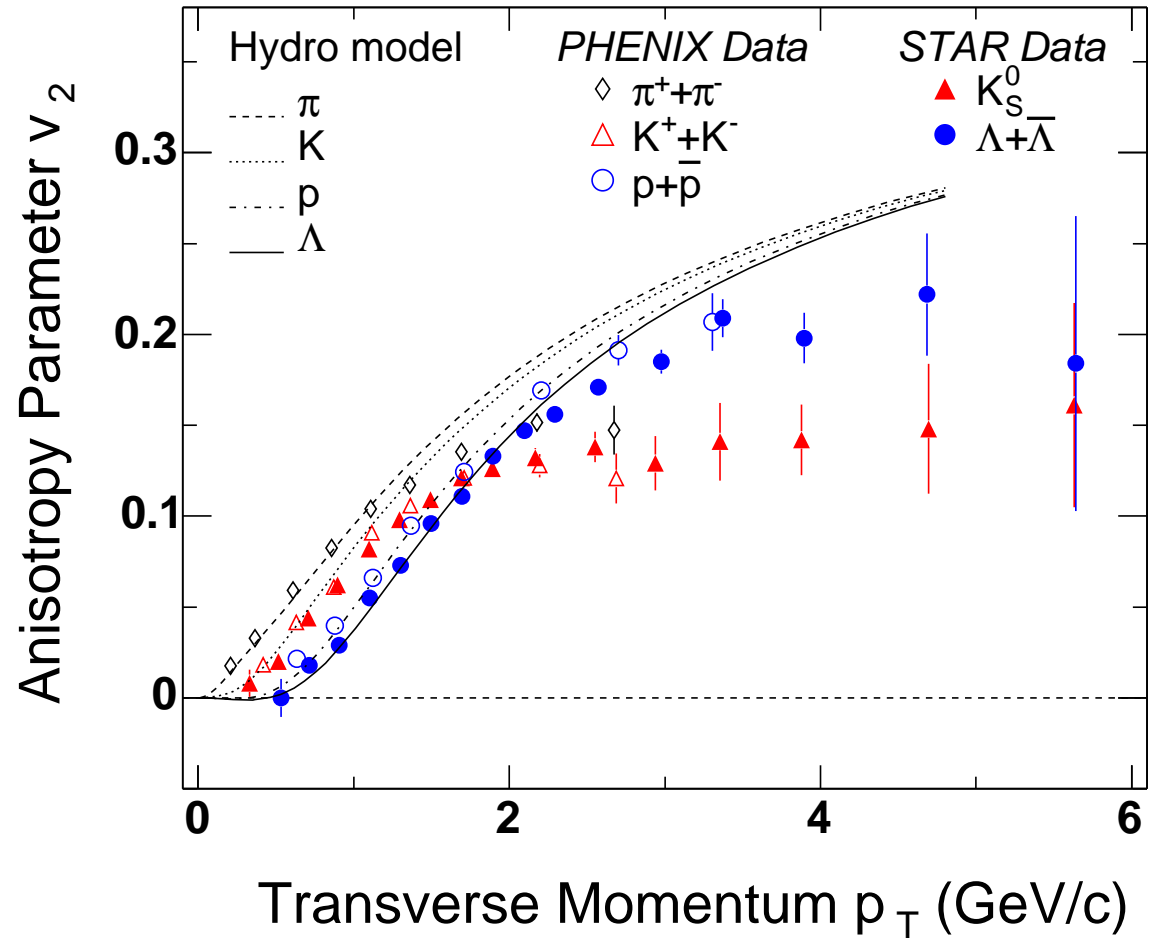
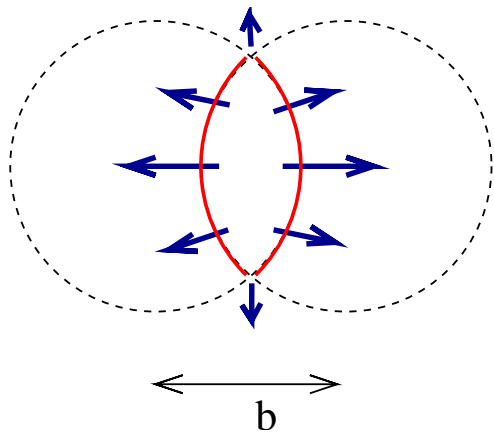


Cao et al., Science (2010)

$$\eta/s \leq 0.4$$

III. Elliptic Flow (QGP)

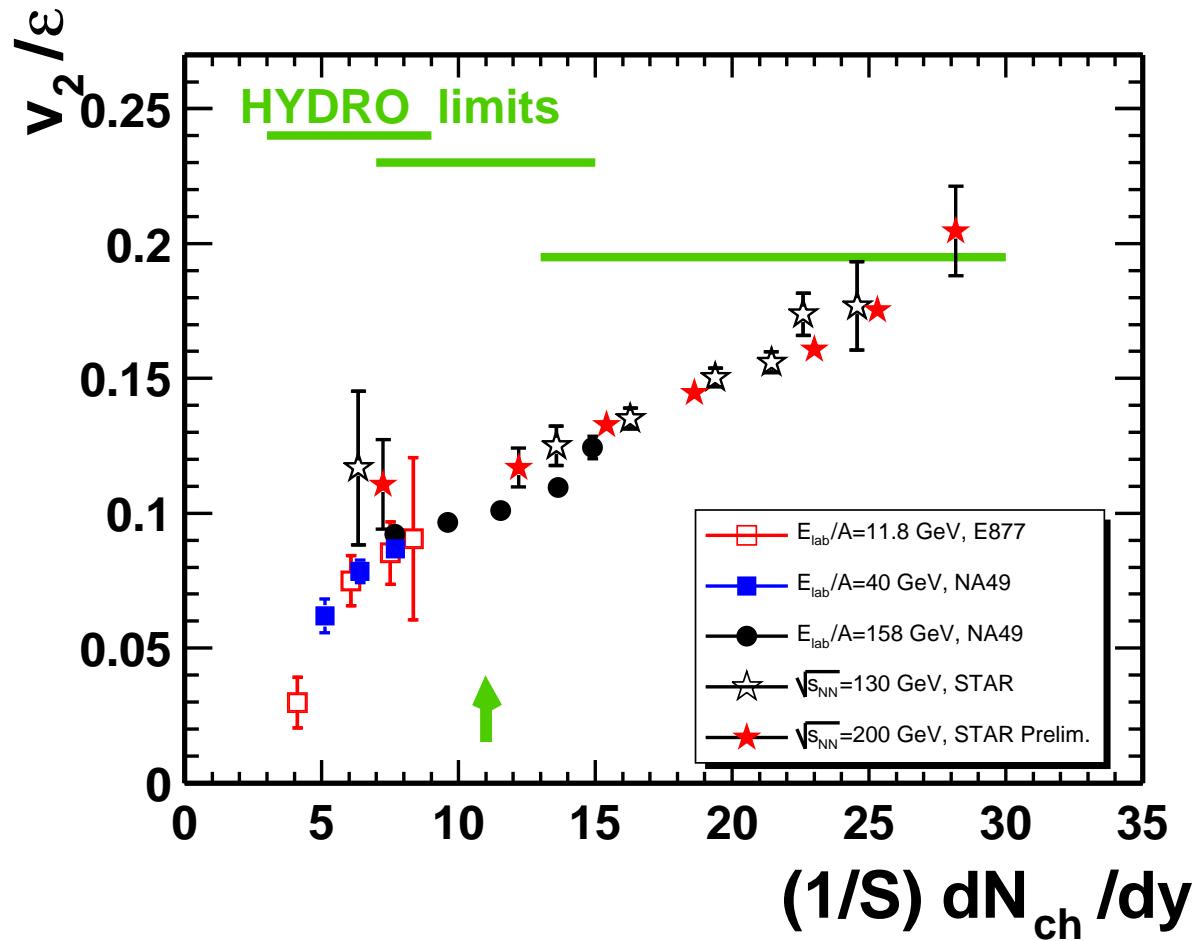
Hydrodynamic expansion converts
 coordinate space
 anisotropy
 to momentum space
 anisotropy



source: U. Heinz (2005)

$$p_0 \left. \frac{dN}{d^3p} \right|_{p_z=0} = v_0(p_\perp) (1 + 2v_2(p_\perp) \cos(2\phi) + \dots)$$

Elliptic flow: initial entropy scaling



source: U. Heinz (2005)

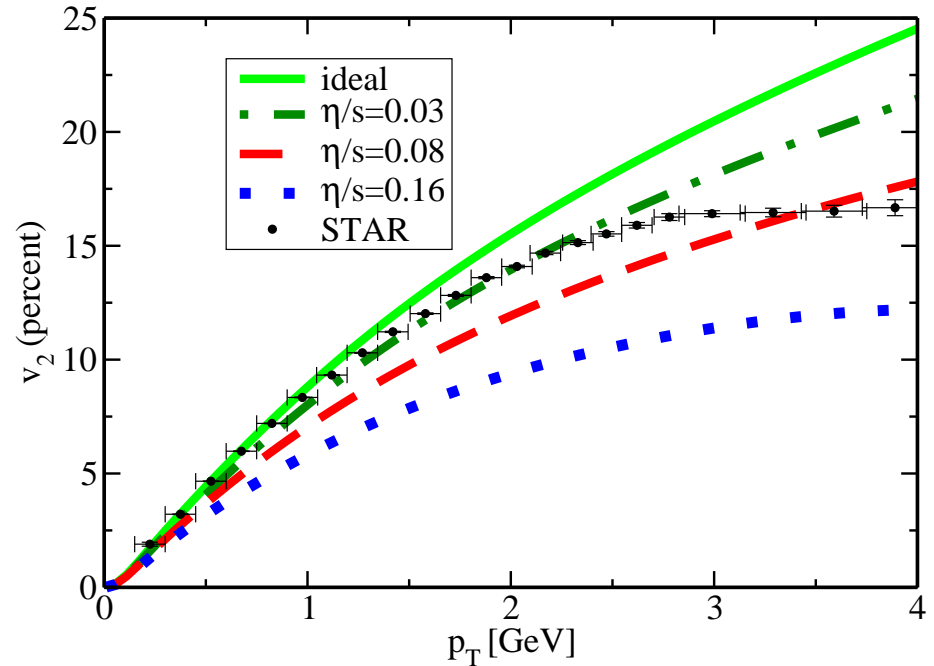
Viscosity and Elliptic Flow

Consistency condition $T_{\mu\nu} \gg \delta T_{\mu\nu}$
(applicability of Navier-Stokes)

$$\frac{\eta + \frac{4}{3}\zeta}{s} \ll \frac{3}{4}(\tau T)$$

Danielewicz, Gyulassy (1985)

Very restrictive for $\tau < 1$ fm



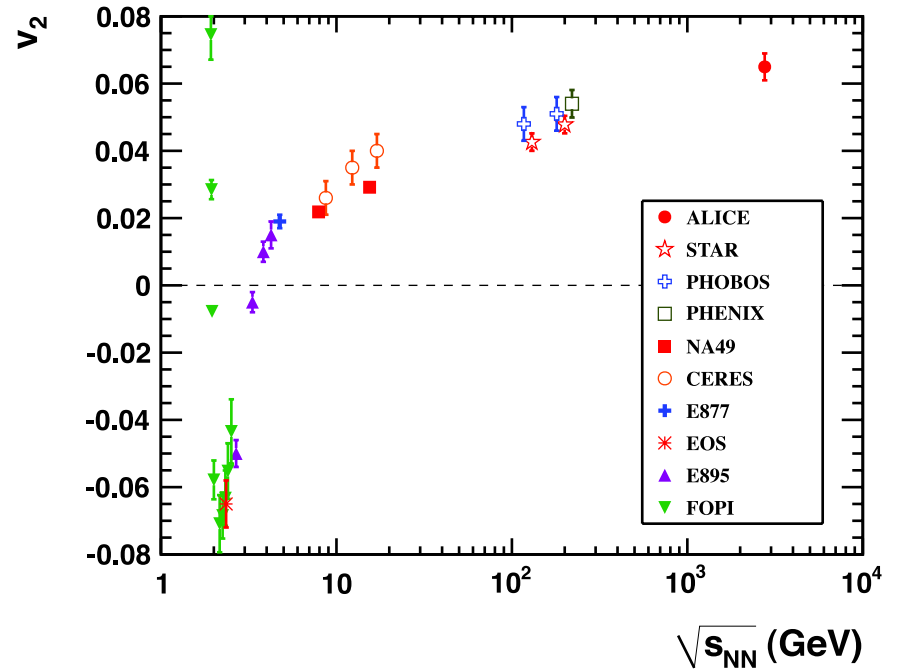
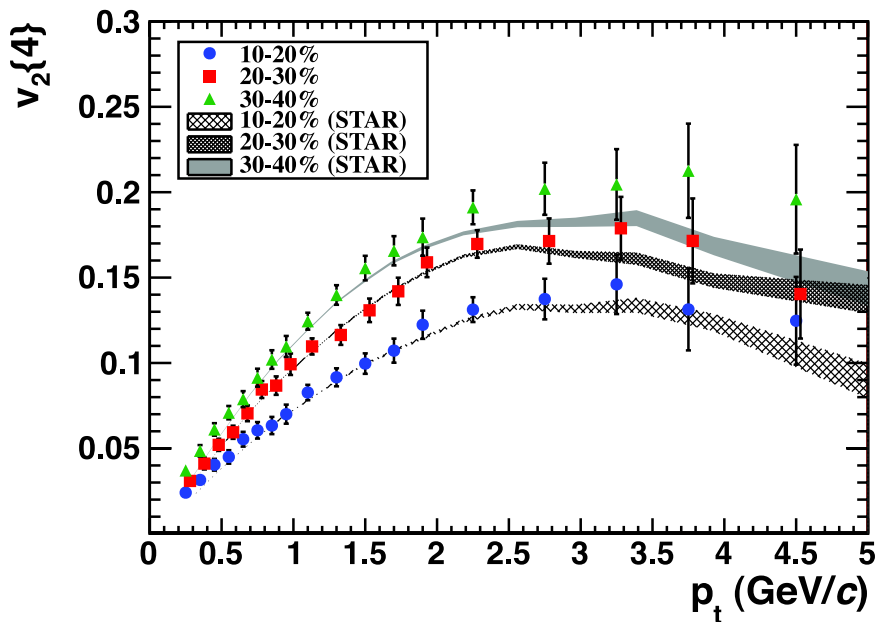
Romatschke (2007), Teaney (2003)

Many questions: Dependence on initial conditions, freeze out, etc.

conservative bound

$$\frac{\eta}{s} < 0.4$$

Alice flow: Nearly perfect fluidity at the LHC?



Differential v_2 equal to RHIC

Integrated v_2 somewhat high

accidental cancellation? freezeout?

mean p_T increase? acceptance?

The bottom-line

Remarkably, the best fluids that have been observed are the *coldest* and the *hottest* fluid ever created in the laboratory, cold atomic gases (10^{-6}K) and the quark gluon plasma (10^{12}K) at RHIC.

Both of these fluids come close to a bound on the shear viscosity that was first proposed based on calculations in string theory, involving non-equilibrium evolution of black holes in 5 (and more) dimensions.