Resonances in $\bar{b}\bar{b}ud$-tetraquark-systems based on static-light lattice-QCD-four-quark-potentials

Martin Pflaumer

Institut für Theoretische Physik
Goethe-Universität Frankfurt am Main

in collaboration with Marc Wagner, Pedro Bicudo and Antje Peters

November 16, 2016

Lunch Club Seminar,
Justus-Liebig-Universität Gießen

$\bar{b}\bar{b}ud$-tetraquark-system in scalar and vector configuration

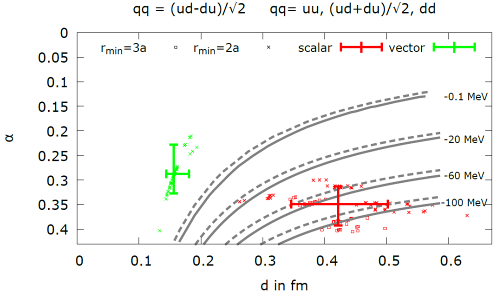

Abbildung: Binding energy isolines in the α-d-plane

- scalar Iso-singlet: bound state
- vector Iso-triplet: no bound state, but close to bound region
  $\Rightarrow$ possible candidate for hadronic resonance

Using quantum mechanical methods

- results: based on Born-Oppenheimer approximation
  $\rightarrow$ quantum mechanical approach
  $\rightarrow$ investigate resonances with a quantum mechanical proceeding
- looking at: Scattering of two B-mesons $\Rightarrow$ examine with quantum mechanical scattering theory

resonance: analogy to quantum mechanics

- *Resonance:* temporarily bound state, decaying after a finite period of time
- in QM: appear as solutions of the Schrödinger-equation for energies greater zero
- simplified consideration of particles as wave and potential respectively

  $\Rightarrow$ Scattering of two B-mesons $\longleftrightarrow$ scattering of wave at potential

described by effective $\bar{b}\bar{b}$-potential:

$$V(r) = -\frac{\alpha}{r} e^{-\frac{r^2}{d^2}} \tag{1.1}$$

- classified by resonance energy E_R and lifetime respectively full width at half maximum Γ

theoretical background

- determinate resonances with quantum mechanical scattering theory
- particle as wave packet $\Rightarrow$ consider even waves

solve Schrödinger-equation for two cases:

- $E < 0 \rightarrow$ bound states
- $E > 0 \rightarrow$ scattered state, for maximum of the scattering amplitude $\rightarrow$ *resonance scattering*

theoretical background

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x})\right]\Psi_k(\mathbf{x}) = E_k\Psi_k(\mathbf{x}), \qquad \text{with: } E_k = \frac{\hbar^2\, k^2}{2m} \geq 0 \tag{2.1}$$

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Solution of the Schrödinger-equation for the eigenstates:

$$\Psi_k(\mathbf{x}) = \mathrm{e}^{i\,\mathbf{kx}} + \frac{\mathrm{e}^{ikr}}{r}\cdot f_k(\vartheta,\varphi) \tag{2.2}$$

$$\text{with:} \quad f_k(\vartheta,\varphi) = -\frac{m}{2\pi\hbar^2}\int d^3x'\,\mathrm{e}^{i\,\mathbf{k'x'}}\,V(\mathbf{x'})\,\Psi_k(\mathbf{x'}) \tag{2.3}$$

$$\text{and:} \quad \mathbf{k}' = k\cdot\frac{\mathbf{x}}{r} \tag{2.4}$$

→ Maximum of the scattering amplitude $f_k(\vartheta,\varphi)$ characterises position of the resonance

theoretical background

- radial part of a free even wave is given by:

$$R_l(r) = \frac{1}{2}\left(h_l^*(kr) + h_l(kr)\right) \quad (2.5)$$

- interacting theory
  assumption: potential vanishes for large distances
  radial part can be written as:

$$R_l(r) = \frac{1}{2}\left(h_l^*(kr) + e^{2i\delta_l(E)} h_l(kr)\right) \quad (2.6)$$

$\Rightarrow$ Difference between free and interacting solution
only in phase term δ_l

theoretical background

Cross section:

$$\sigma = \int d\Omega \, |f_k(\vartheta, \varphi)|^2 \tag{2.7}$$

$$\sigma_l = \frac{4\pi}{k^2}(2l+1)\sin^2(\delta_l) \tag{2.8}$$

$\Rightarrow$ Resonances appear for a maximum of the cross section and are characterised by a phase shift

Physical justification for resonances

- relevant parameter: centrifugal potential $V_Z^L(r) = \frac{\hbar^2 L(L+1)}{2mr^2}$
  $\Rightarrow$ barrier for incoming wave
- combine with an attractive potential: classically permitted and forbidden regions

Physical justification for resonances

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- combine with an attractive potential: classically permitted and forbidden regions
- for $L = 0$: no centrifugal potential, all energies are transmitted $\rightarrow$ no resonances
- for $L \geq 1$: Transmission coefficient very small an no penetration in repulsive potential region
- for resonance energies: entering into inner region possible $\Rightarrow$ meta stable state, until potential barrier is tunnelled again

$\bar{b}\bar{b}$-Potential

- Investigation of an $\bar{b}\bar{b}ud$-tetraquark-system
- $V(r) = -\frac{\alpha}{r}\,\mathrm{e}^{-\frac{r^2}{d^2}}$

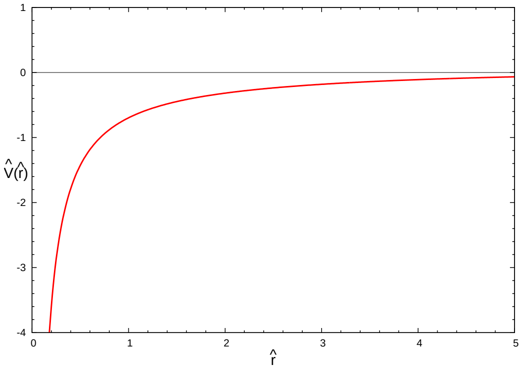

Abbildung: $\bar{b}\bar{b}$-potential for $\alpha = 0.34$ and $d = 0.45$ fm

Possible Quantum numbers

Wave function

$$|\Psi\rangle = |space\rangle \otimes |colour\rangle \otimes |spin\rangle \otimes |flavour\rangle \tag{3.1}$$

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- separate consideration of Diquark ud and Anti-Diquark $\bar{b}\bar{b}$
- both: antisymmetrical wave function
- Colour-Triplet and Antitriplet with antisymmetrical wave function
- realised by skalar B-mesons B^+ and B^0 or vector B-mesons B^{*+} and B^{*0}

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Parity

$$P = (-1)^{l_b} \cdot (-1) \cdot (-1) = (-1)^{l_b} \quad (3.2)$$

Light Diquarks *ud*

- Combination of total spin- and flavour wave function have to be symmetrically
- two possible realisations for total spin and flavour:

- both antisymmetric → *scalar Iso-Singlet*
  with: Isospin $i_l = 0$, total spin $j_l = 0$.
- both symmetric →*vector Iso-Triplet*
  with: Isospin $i_l = 1$, total spin $j_l = 1$.

Heavy Anti-Diquarks $\bar{b}\bar{b}$

- Schrödinger-equation only contains angular momentum of heavy anti-Diquarks
- symmetric in flavour
- Combination of spin- and space wave function have to be symmetrically
- coupling to $j_b = s_b + l_b$
- $|j_l - j_b| \leq J \leq |j_l + j_b|$

l_b	s_b	j_b	P	$I(J^P)$
0	1	1	+	$0(1^+)$
1	0	1	-	$0(1^-)$
2	1	1, 2, 3	+	$0(1^+)$, $0(2^+)$, $0(3^+)$

Tabelle: possible quantum numbers for scalar Iso-Singlet with different angular momentum $l = l_b$. It is given: $I = i_l = 0$; $j_l = 0$; $i_b = 0$

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1	0	1	-	$1(0^-)$, $1(1^-)$, $1(2^-)$
2	1	1, 2, 3	+	$1(0^+)$, $1(1^+)$, $1(2^+)$, $1(3^+)$, $1(4^+)$

Tabelle: possible quantum numbers for vector Iso-Triplet with different angular momentum $l = l_b$. It is given: $I = i_l = 1$; $j_l = 1$; $i_b = 0$

Bound states

- just for scalar Iso-singlet ud with $I = 0$ a bound state appears
- for other states, there exist no bound states

qq	Spin	$\hat{E}_B$	E_B [MeV]
$(ud - du)/\sqrt{2}$	Scalar	$-0.077\,^{+0.037}_{-0.037}$	$-91.48\,^{+43.77}_{-43.69}$

Tabelle: Binding energy of the scalar Iso-singlets $(ud - du)/\sqrt{2}$ for angular momentum $L = 0$

Resonances

Investigate resonance phenomena for:

- Scalar Iso-singlet $(ud - du)/\sqrt{2}$
- Vector Iso-triplet uu, $(ud + du)/\sqrt{2}$, dd

- for $L = 0 \rightarrow$ no resonances

Resonances

Determining parameters E_R and Γ:

- read out from the given data
- performing a Breit-Wigner-Fit
  $\rightarrow$ comparable to experimental data

Tetraquark mass is given by:

$$m_{\bar{b}\bar{b}ub} = 2 \cdot m_B + E_R \quad (3.3)$$

Scalar Iso-Singlet $(ud - du)/\sqrt{2}$, $L = 1$

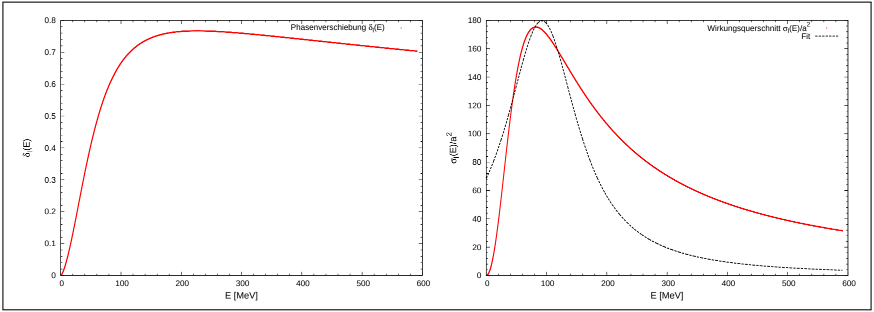

Abbildung: scalar Iso-singlet $(ud - du)/\sqrt{2}$ for $L = 1$; left: Phase shift $\delta_L(E)$; right: cross section $\sigma_L(E)$

Scalar Iso-Singlet $(ud - du)/\sqrt{2}$, $L = 2$

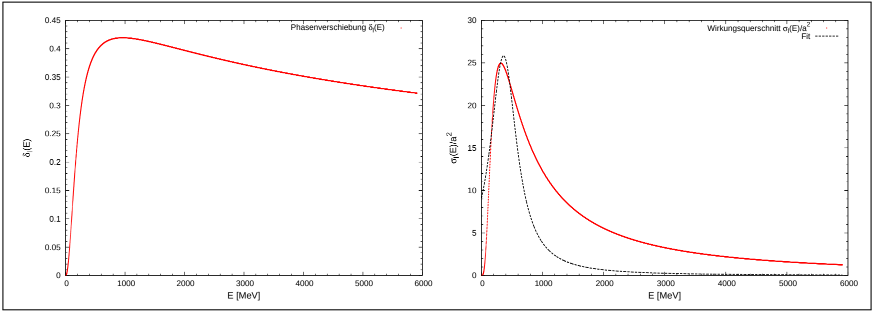

Abbildung: scalar Iso-Singlet $(ud - du)/\sqrt{2}$ for $L = 2$; left: Phase shift $\delta_L(E)$; right: cross section $\sigma_L(E)$

	value (read out)	value (fitted)
E_R [MeV]	$78.21\,^{+69.32}_{-63.54}$	$87.98\,^{+71.87}_{-64.66}$
Γ [MeV]	$195.45\,^{+190.92}_{-169.79}$	$135.14\,^{+122.69}_{-107.53}$

Tabelle: Resonance energy and full width at half maximum of the scalar Iso-Singlets $(ud - du)/\sqrt{2}$ for angular momentum $L = 1$

	value (read out)	value (fitted)
E_R [MeV]	$323.0\,^{+179.2}_{-181.1}$	$368.0\,^{+205.4}_{-207.1}$
Γ [MeV]	$857.1\,^{+497.1}_{-493.7}$	$542.2\,^{+315.4}_{-313.7}$

Tabelle: Resonance energy and full width at half maximum of the scalar Iso-Singlets $(ud - du)/\sqrt{2}$ for angular momentum $L = 2$

Scalar Iso-Singlet $(ud - du)/\sqrt{2}$, $L = 3$, $L = 4$

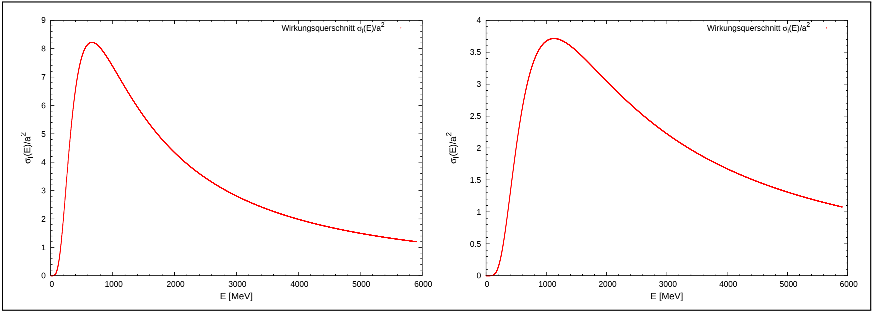

Abbildung: scalar Iso-singlet $(ud - du)/\sqrt{2}$; Corss section $\sigma_L(E)$ for: left $L = 3$; right $L = 4$

Vector Iso-Triplet uu, $(ud + du)/\sqrt{2}$, dd, $L = 1$

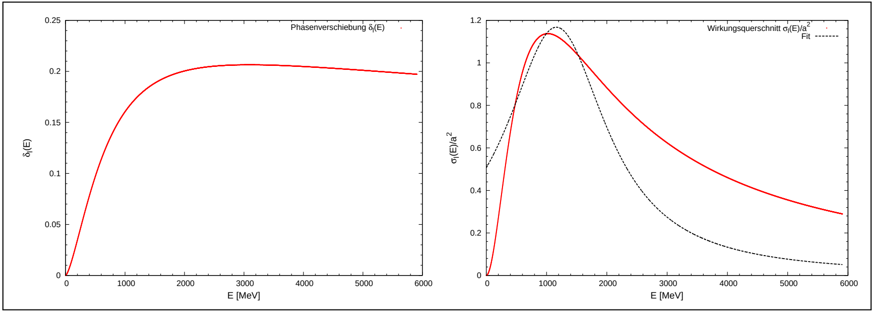

Abbildung: vector Iso-triplet uu, $(ud + du)/\sqrt{2}$, dd for $L = 1$; left: Phase shift $\delta_L(E)$; right: cross section $\sigma_L(E)$

	value (read out)	value (fitted)
E_R [MeV]	$1081.7\,^{+196.7}_{-462.4}$	$1231.9\,^{+222.9}_{-526.5}$
Γ [MeV]	$3120.0\,^{+552.2}_{-1332.5}$	$2156.5\,^{+387.6}_{-927.2}$

Tabelle: Resonance energy and full width at half maximum of the vector Iso-triplet uu, $(ud + du)/\sqrt{2}$, dd for angular momentum $L = 1$

Vector Iso-Triplet uu, $(ud + du)/\sqrt{2}$, dd, $L = 2$

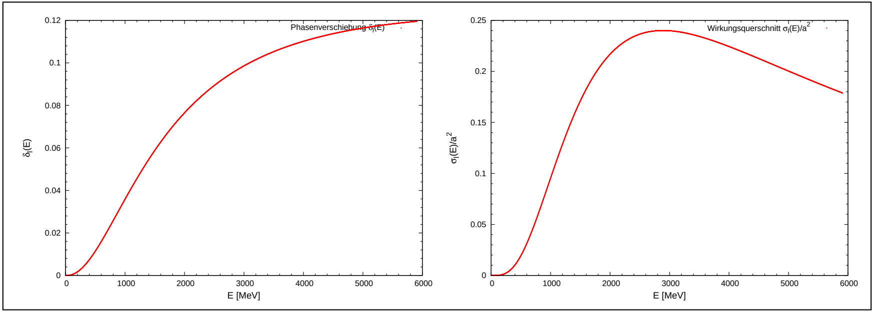

Abbildung: vector Iso-triplet uu, $(ud + du)/\sqrt{2}$, dd für $L = 2$; left: Phase shift $\delta_L(E)$; right: cross section $\sigma_L(E)$

Conclusion

- for scalar iso-singlet: bound state with $E_B = -91.48^{+43.77}_{-43.69}$ has been verified
- scalar iso-singlet and vector iso-triplet has been investigated for resonances
- as predicted: no resonances for angular momentum $L = 0$
- Existence of resonances for higher L for scalar Iso-Singlet is presumably
- for increasing L: resonance becomes less stable

Thank You for Your Attention!

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