Chiral magnetic effect & anomalous transport from real-time lattice simulations

Soeren Schlichting

Based on:
N. Mueller, S. Schlichting and S. Sharma, PRL 117 (2016) no.14, 142301
M. Mace, N. Mueller, S. Schlichting and S. Sharma, 1612.02477

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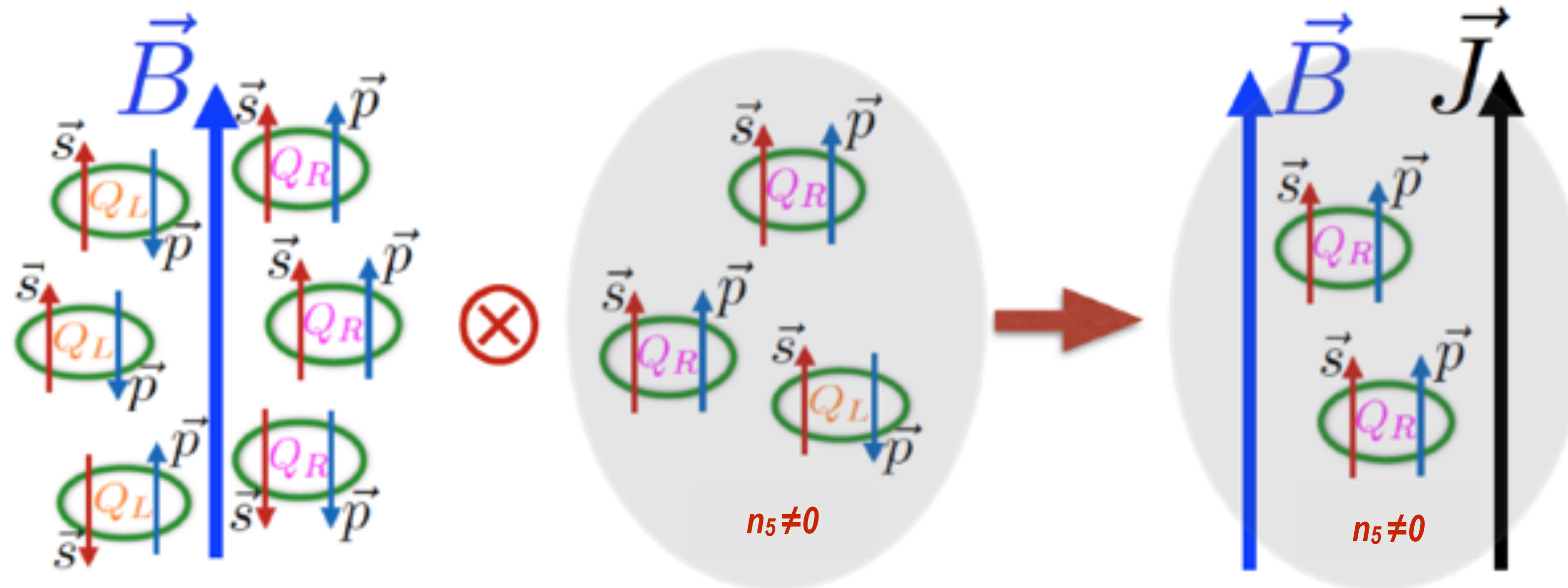
Outline

1. Introduction

2. Chiral magnetic effect & anomalous transport from real-time lattice simulations

3. Conclusions & Outlook

1



Introduction

Chiral Magnetic Effect (CME) & anomalous transport

Discovery of new kinds of conductivity for systems with (approx.) chiral fermions and chirality imbalanced

(Fukushima, Kharzeev, Warringa PRD 78 (2008) 074033)

Chiral Magnetic Effect: $\vec{j}_v \propto j_a^0 \vec{B}$

axial charge density *magnetic field*

Several interesting effects due to interplay of axial and vector charges

Chiral Separation Effect (CSE), Chiral Magnetic Wave (CMW), …

Manifestations from high-energy QCD to Dirac/Weyl semi-metals

Observation in $ZrTe_5$: Kharzeev et al. Nature Physics (2016)

CME in Heavy-Ion Collisions

Chiral Magnetic Effect: $\vec{j}_v \propto j_a^0 \vec{B}$

axial charge density — *magnetic field*

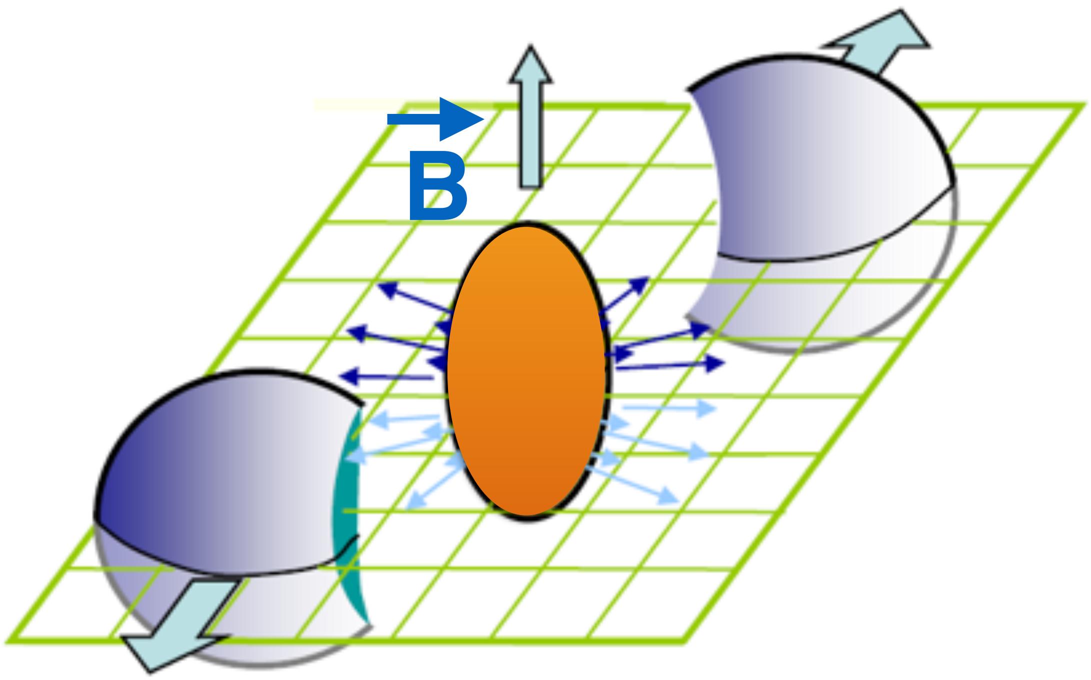
High-energy heavy-ion collisions provide an exciting environment to explore anomalous transport phenomena

— *strong magnetic field* $eB \sim m_\pi^2$ *present over the first ~1 fm/c*

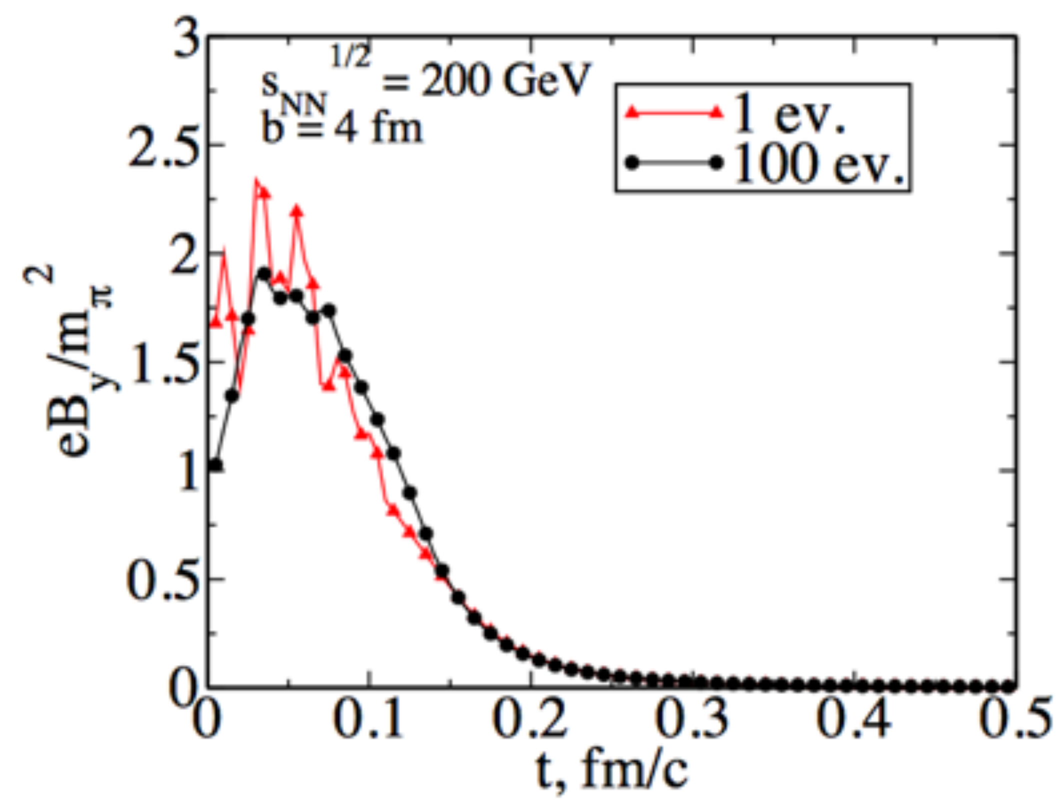
— *expect axial charge fluctuations e.g. due to sphaleron transitions*

CME in Heavy-Ion collisions

Magnetic field: Spectators in off-central collisions create a strong magnetic field $eB \sim m_\pi^2$ (unit conversion $m_\pi^2 \sim 10^{14}$ T)



STAR PRC 81 (2010) 054908



Skokov,Illarionov,Toneev
Int.J.Mod.Phys. A24 (2009) 5925-5932

Expected life-time of magnetic field is short < 1 fm/c, so the effect should take place during the pre-equilibrium stage

CME in Heavy-Ion collisions

Axial charge imbalance (n_5): Sourced by fluctuations of the non-abelian field strength tensor due to the axial anomaly

$$\partial_\mu j^\mu_{5,f} = 2m_f\bar{q}\gamma_5 q - \frac{g^2}{16\pi^2}F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$$

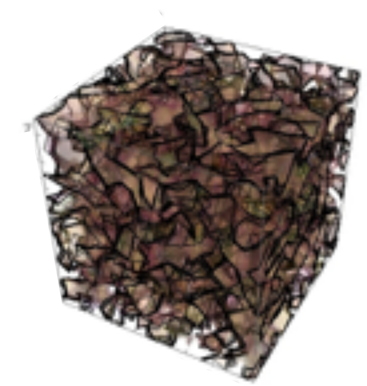
axial current $j_5^\mu = (n_5, \vec{j}_5)$

quark mass

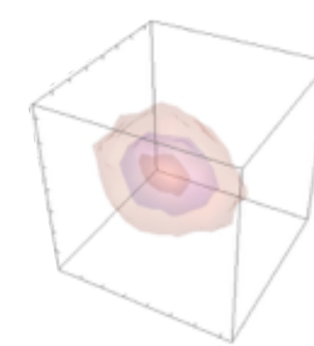
$\propto \vec{E}\cdot\vec{B}$ field-strength fluctuations

Considering the contribution to local imbalances of the axial charge $n_5(x,t)$ one can distinguish contributions due to

- space-time dependent fluctuations of $\vec{E}\cdot\vec{B}$
- topological sphaleron transitions



$\vec{E}\cdot\vec{B}$

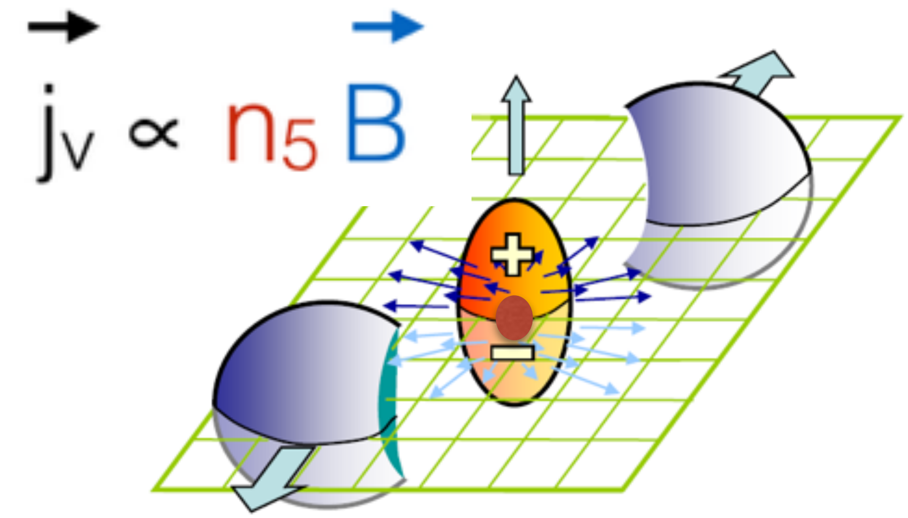


Sphaleron transition rate $\Gamma_{sph} \sim Q_s^4$ enhanced at early times

(Mace,SS, Venugopalan PRD 93 (2016) no.7, 074036)

Experimental observation of CME current would provide macroscopic manifestation of topological transitions via quantum anomaly

CME in Heavy-Ion collisions

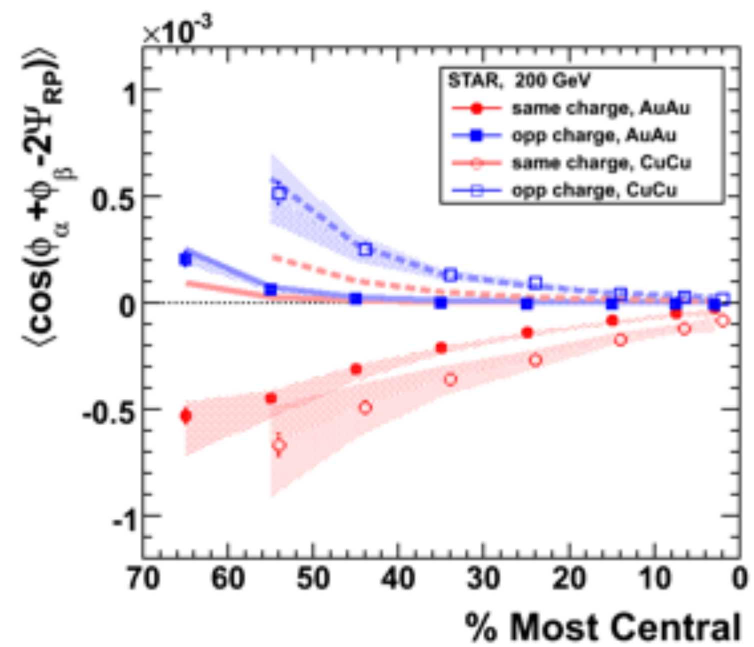


Experimental status:

Since axial charge fluctuates from event to event on average $\langle j_V \rangle = 0$, so one can only measure fluctuations

Basic idea is to look for back-to-back correlations of opposite charge particles with respect to the reaction plane

While intriguing hints of CME and associated phenomena have been observed at RHIC and LHC, measurements are also subject to potentially large backgrounds



STAR PRC 81 (2010) 054908

``Chiral Magnetic Effect Task Force Report,'' arXiv:1608.00982 [nucl-th]

Isobar scan envisioned as decisive test to isolate signal vs. background

CME in Heavy-Ion collisions

Quantitative theoretical understanding of anomaly induced transport phenomena (CME,CMW,…) in heavy-ion collisions important experimental searches for these effects

Theoretical challenges:

Since life time of magnetic field is presumably very short (~0.1-1 fm/c) system is out-of-equilibrium during the time scales relevant for CME & Co.

Need to understand non-equilibrium dynamics of axial and vector charges during the early-time pre-equilibrium phase

Existing theoretical approaches such as anomalous hydro or chiral kinetic theory effectively treat axial charge as a conserved quantity

In order to correctly describe generation of axial charge imbalance (e.g. due to sphalerons) field theoretical description is required

-> Develop field theoretical approach to describe early time dynamics and possibly devise improved macroscopic description of anomalous transport

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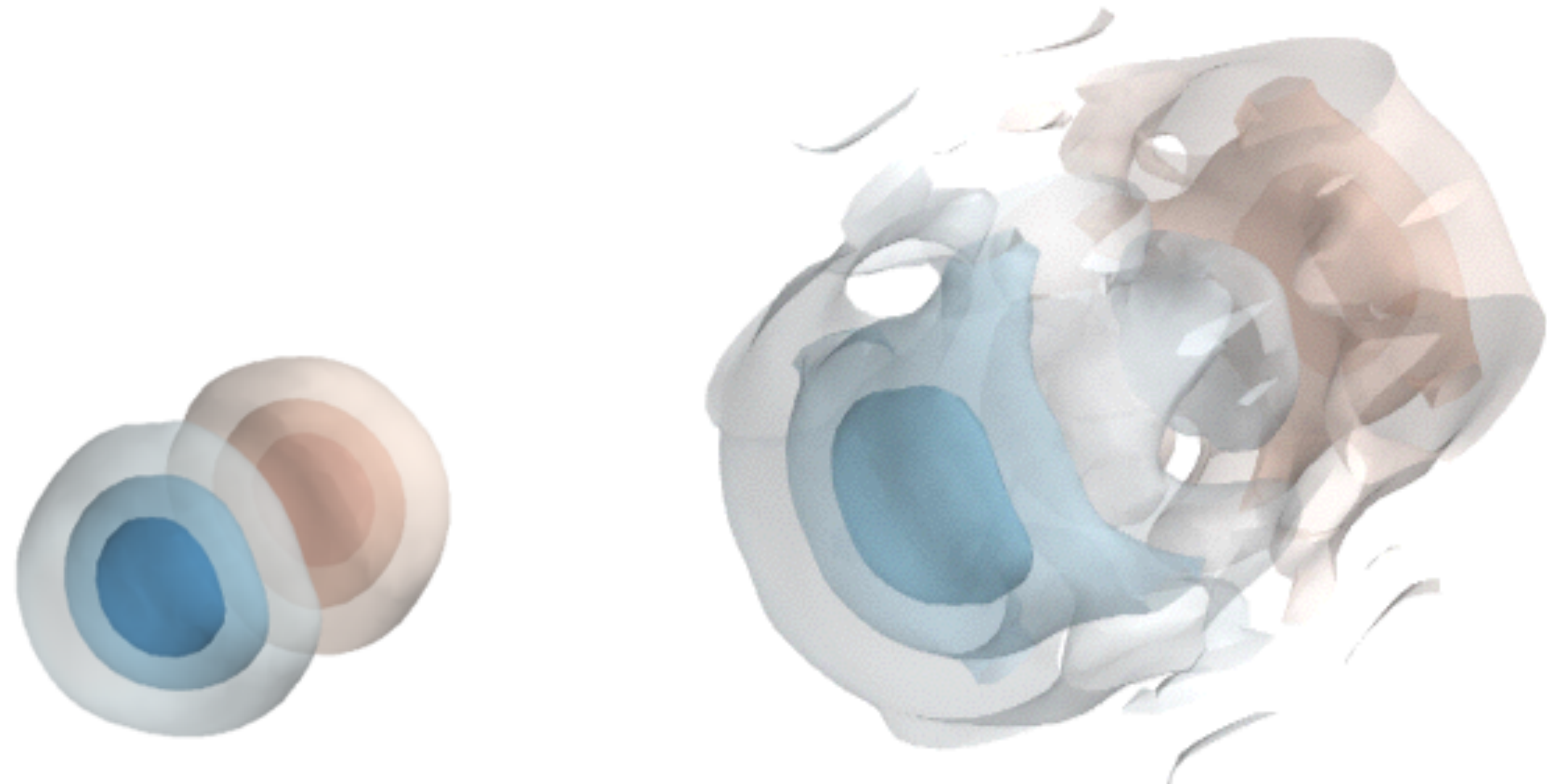
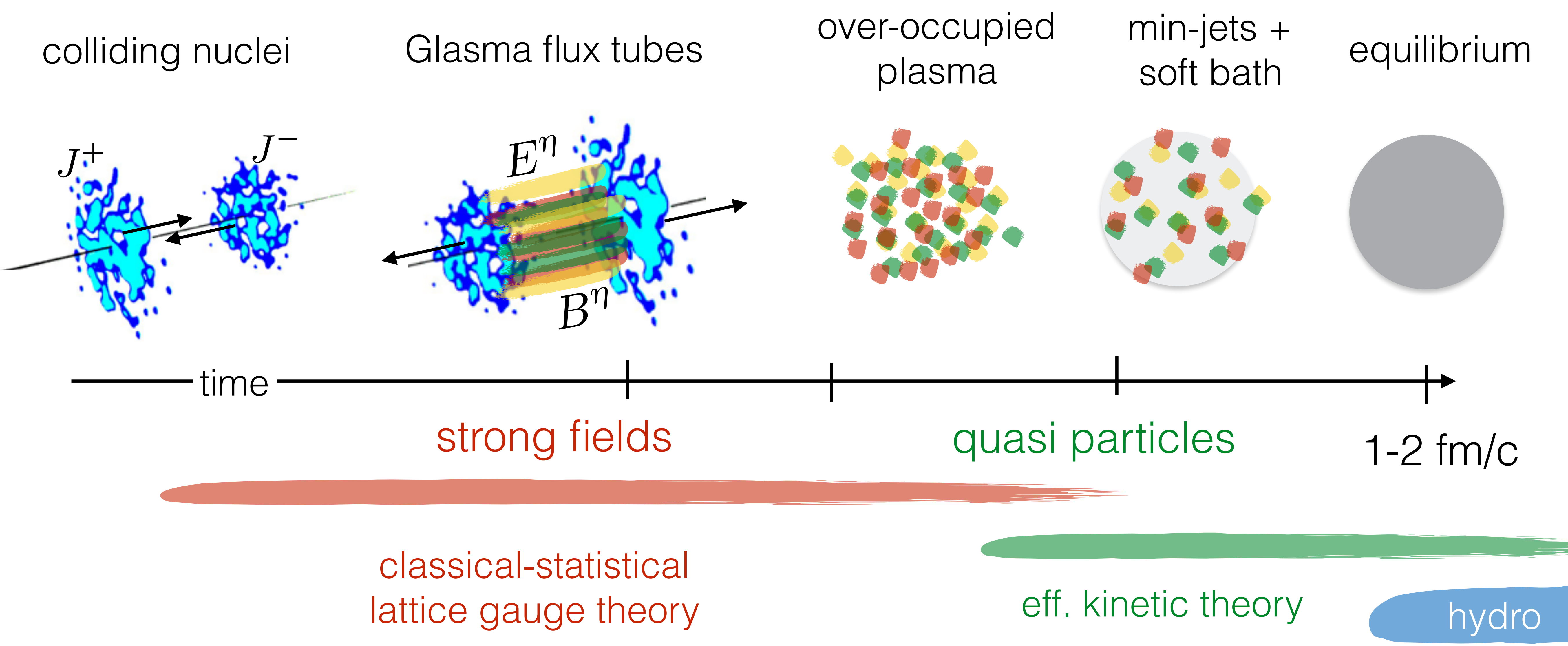


Chiral magnetic effect
& anomaly induced transport
from real-time lattice simulations

Early-time dynamics of HIC



Early time dynamics described in terms of classical field dynamics amenable to non-perturbative real-time lattice simulations

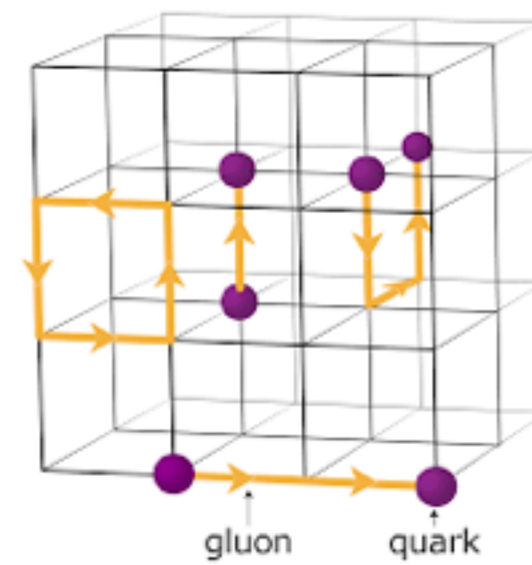
-> Include dynamical fermions to study anomalous transport

Simulation technique

Classical-statistical lattice simulation with dynamical fermions

(Aarts, Smit; Berges, Hebenstreit,Kasper, Mueller; Tranberg, Saffin; …)

- Discretize theory on 3D spatial lattice using the Hamiltonian lattice formalism



- Solve operator Dirac equation in the presence of SU(N) and U(1) gauge fields

$$i\gamma^0\partial_t\hat{\psi} = (-i\not{D}^s_W + m)\hat{\psi}$$

- Compute expectation values of vector and axial currents to study anomalous transport processes

$$j^\mu_v(x) = \langle\hat{\bar{\psi}}(x)\gamma^\mu\hat{\psi}(x)\rangle \quad j^\mu_a(x) = \langle\hat{\bar{\psi}}(x)\gamma^\mu\gamma^5\hat{\psi}(x)\rangle$$

Dynamical fermions

Solving the operator Dirac equation can be achieved by expanding the fermion field in operator basis at initial time

$$\hat{\psi}(x,t) = \sum_{p,\lambda} \hat{b}_{p,\lambda}(t=0)\phi_u^{p,\lambda}(x,t) + \hat{d}^\dagger_{p,\lambda}(t=0)\phi_v^{p,\lambda}(x,t)$$

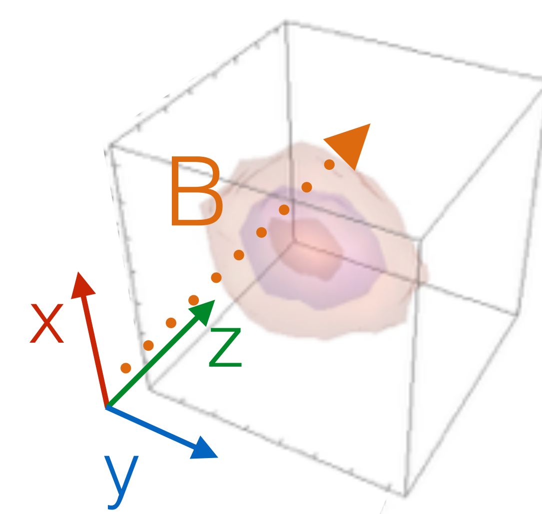
and solving the Dirac equation for evolution of $4N_cN^3$ wave-functions

Not clear to what extent stochastic estimators are useful to reduce problem size
Computationally extremely demanding (~TB memory, ~M CPU hours)

So far first results on small lattices 24 x 24 x 64 in a clean theoretical setup

SU(N): Single sphaleron transition
U(1): constant magnetic field

Back-reaction of fermions on gauge field evolution not considered



Axial anomaly in real-time

Definition of chiral properties (axial charge) of fermions on the lattice generally a tricky issue

Naive fermion discretization: Cancellation of axial anomaly due to Fermion doublers

$$\partial_\mu j_5^\mu(x) = 2m < \bar{\psi}(x) i\gamma_5 \psi(x) >$$

Exploit knowledge from Euclidean lattice simulations

Wilson fermions: Explicit symmetry breaking term added to the Hamiltonian to decouple doublers (c.f. Aarts,Smit)

cont. limit

$$\partial_\mu j_5^\mu(x) = 2m < \bar{\psi}(x) i\gamma_5 \psi(x) > + r_W < W(x) > \rightarrow -\frac{g^2}{8\pi^2} \mathrm{Tr} F_{\mu\nu} F^{\mu\nu}$$

Overlap fermions: Non-local derivative operator with exact chiral properties on the lattice

Axial anomaly in real-time

Non-trivial cross check of axial charge production (B=0)

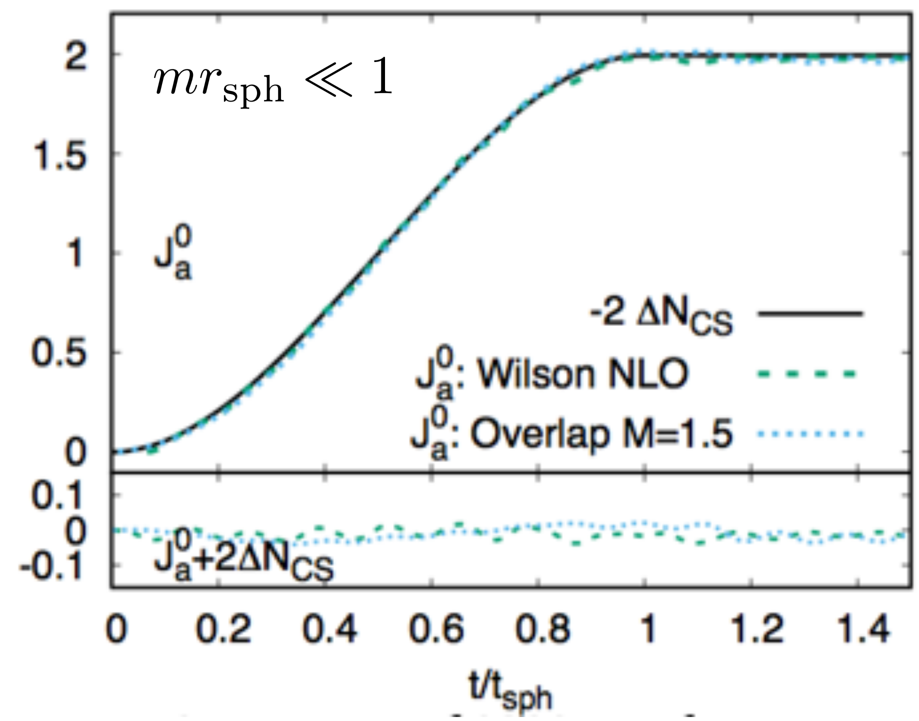
Over the coarse of the sphaleron transition Chern-Simons number

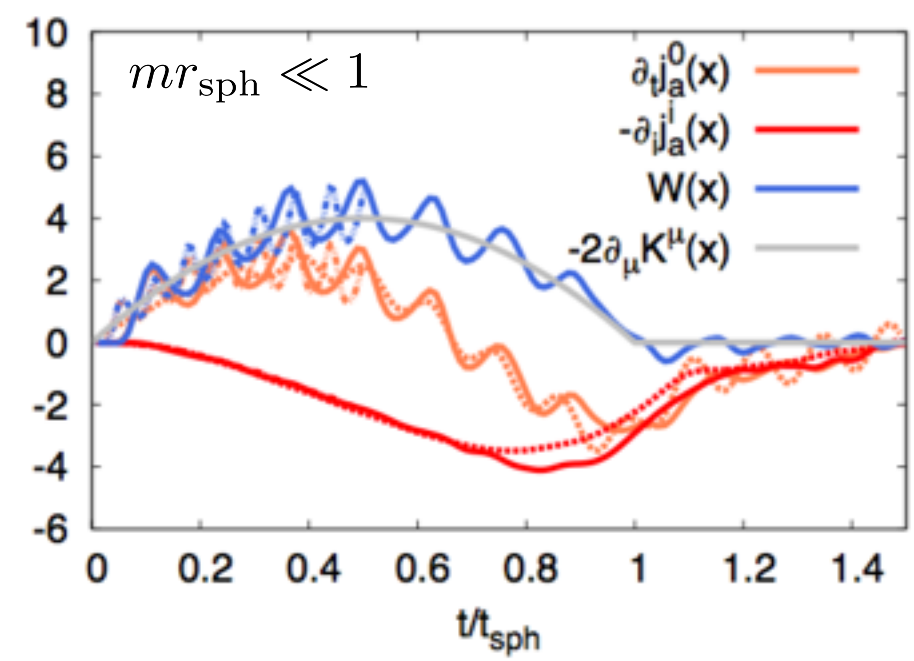
$$\Delta N_{CS} = \frac{g^2}{8\pi^2} \int d^4x \; \vec{E}_a \vec{B}_a$$

changes by an integer amount leading to an imbalance of axial charge

$$\Delta J_5^0 = -2\Delta N_{CS} + 2m_f \int d^4x \langle \bar{\psi} i \gamma_5 \psi \rangle$$

Excellent agreement for (almost) massless fermions from simulations with improved Wilson fermions and Overlap fermions

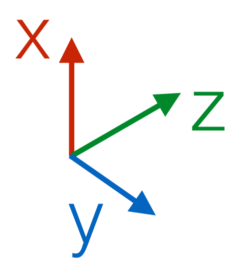




(Mace,Mueller,SS, Sharma, 1612.02477)

CME Dynamics

Axial charge j_5^0



Sphaleron transition induces local imbalance of axial charge density

Vector current j_V^z

Non-zero magnetic field B_z leads to vector current j_V^z in z-direction

Vector charge j_V^0

Vector current j_V^z leads to separation of electric charges j_V^0 along the z-direction

(N.Mueller,SS, S. Sharma PRL 117 (2016) no.14, 142301)

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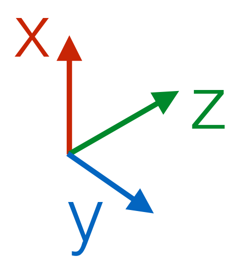

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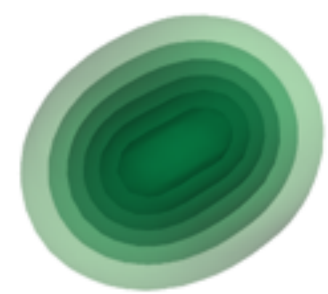

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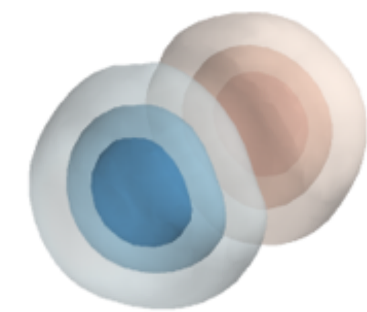


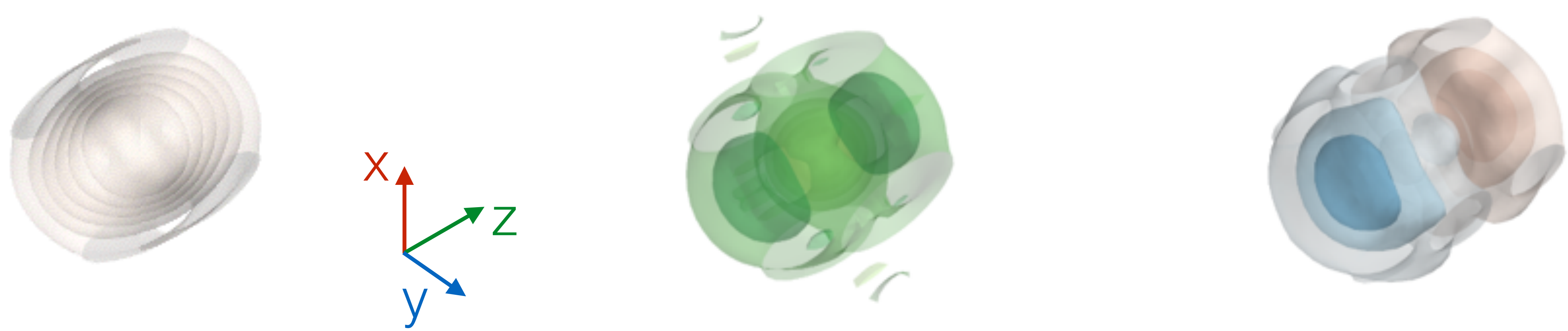
Vector current j_V^z leads to separation of electric charges j_V^0 along the z-direction

(N.Mueller,SS, S. Sharma PRL 117 (2016) no.14, 142301)

CMW Dynamics

Vector charge imbalance j_V^0 generates an axial current j_5^z so that axial charge also flows along the B-field direction

Axial charge j_5^0 Vector current j_V^z Vector charge j_V^0



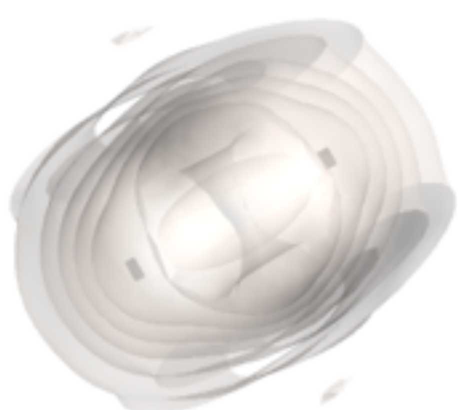
Emergence of a Chiral Magnetic Shock-wave of vector charge and axial charge propagating along B-field direction

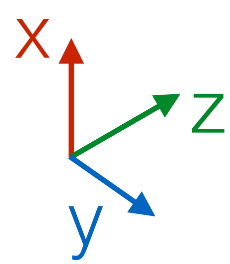
(N.Mueller,SS, S. Sharma PRL 117 (2016) no.14, 142301)

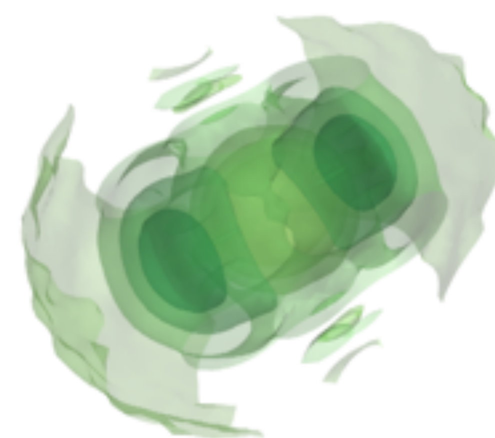
CMW Dynamics

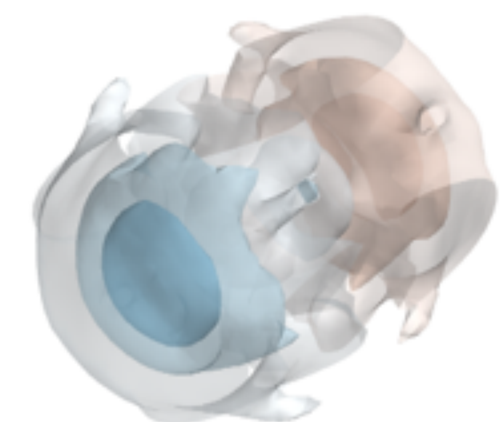
Vector charge imbalance j_V^0 generates an axial current j_5^z so that axial charge also flows along the B-field direction

Axial charge j_5^0 | Vector current j_V^z | Vector charge j_V^0









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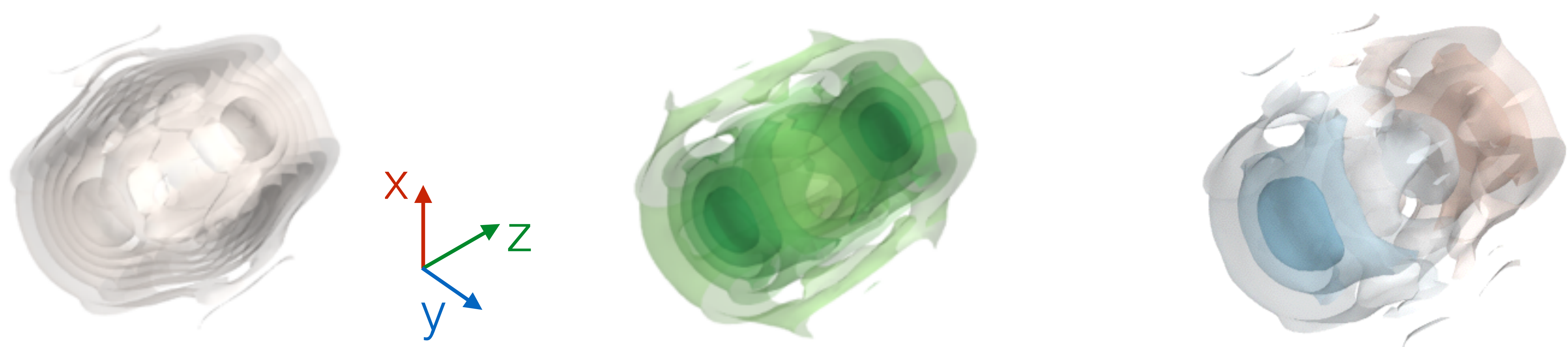
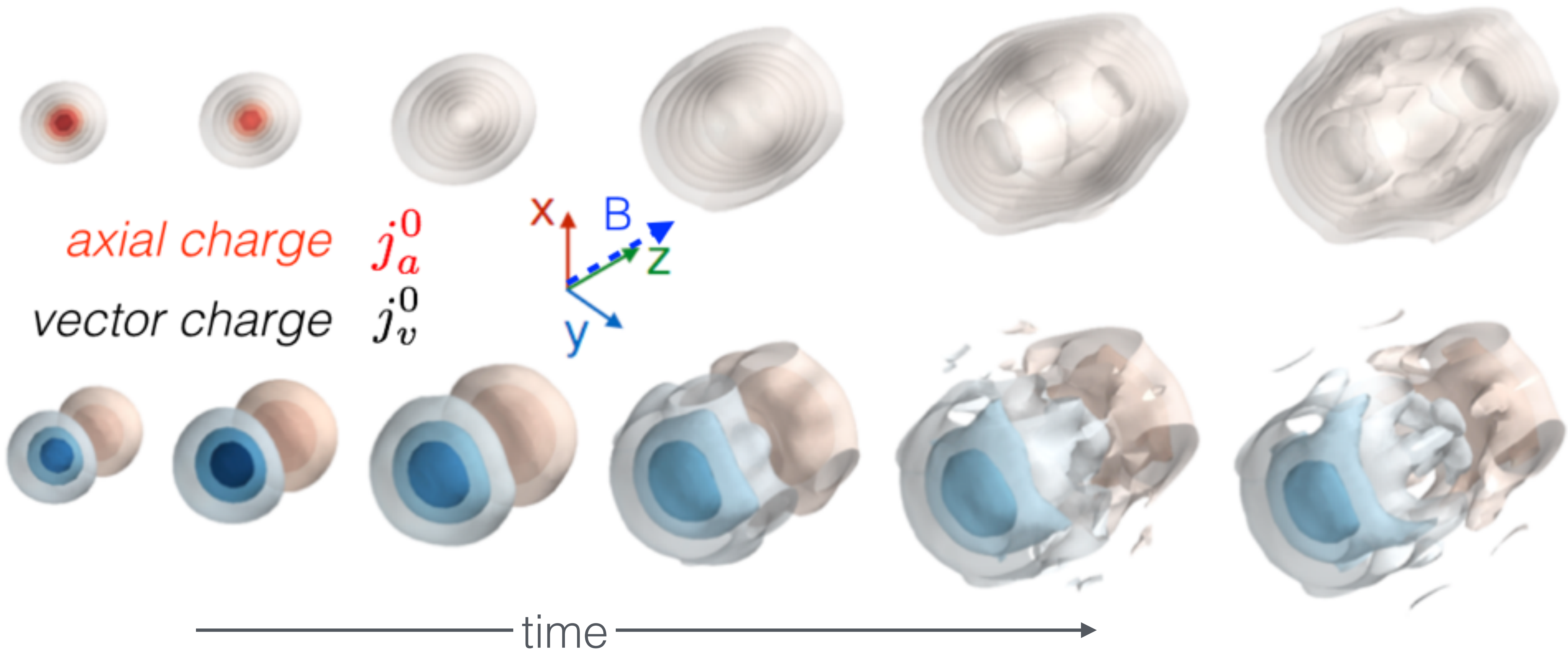
Axial charge j_5^0 | Vector current j_V^z | Vector charge j_V^0



Emergence of a Chiral Magnetic Shock-wave of vector charge and axial charge propagating along B-field direction

(N.Mueller,SS, S. Sharma PRL 117 (2016) no.14, 142301)

Non-equilibrium dynamics of vector and axial charges



Clear separation of electric charge j_V^0 along the B-field direction

First time anomalous transport phenomena have been confirmed from non-perturbative real-time simulations

Non-equilibrium dynamics of vector and axial charges

Comparison with anomalous hydro
(light quarks $mr_{\text{sph}} \ll 1$)

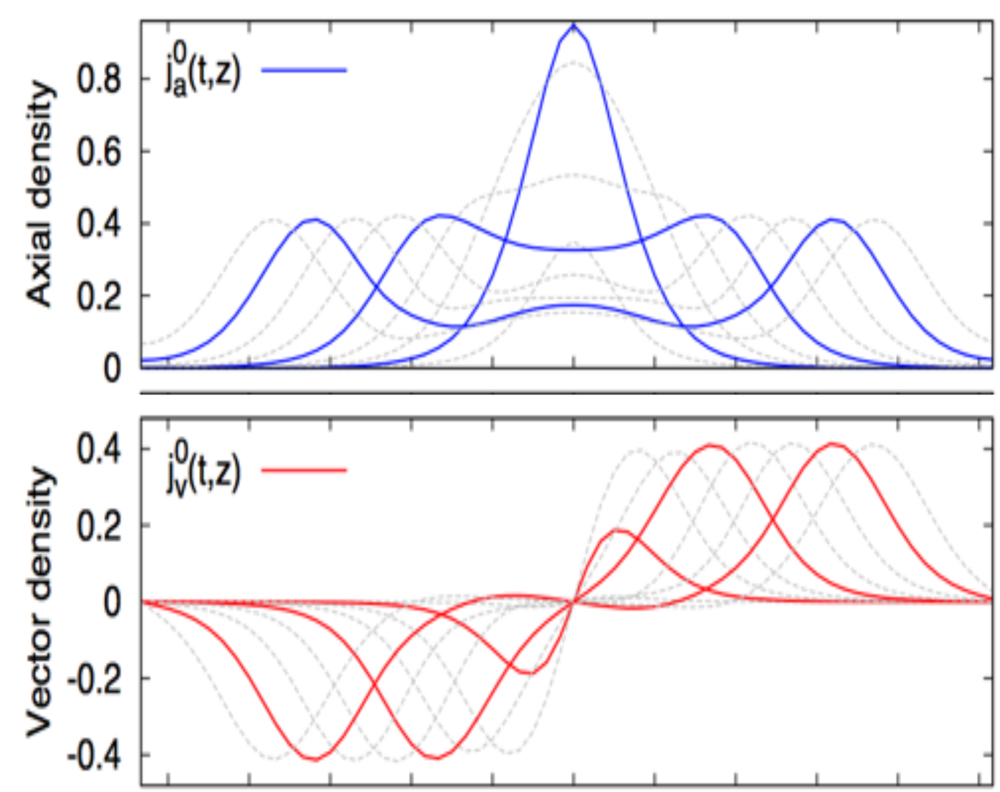
$$\partial_\mu j_a^\mu = S(x)\ , \qquad \partial_\mu j_v^\mu = 0$$

$$j^\mu_{v/a} = n_{v/a} u^\mu + \sigma^B_{v/a} B^\mu$$

Strong field limit ($B \gg r_{\text{sph}}^{-2}, m^2$)

$$\partial_t \begin{pmatrix} j_v^0(t,z) \\ j_a^0(t,z) \end{pmatrix} = -\partial_z \begin{pmatrix} j_a^0(t,z) \\ j_v^0(t,z) \end{pmatrix} + \begin{pmatrix} 0 \\ S(t,z) \end{pmatrix}$$

Simulation results for light quarks

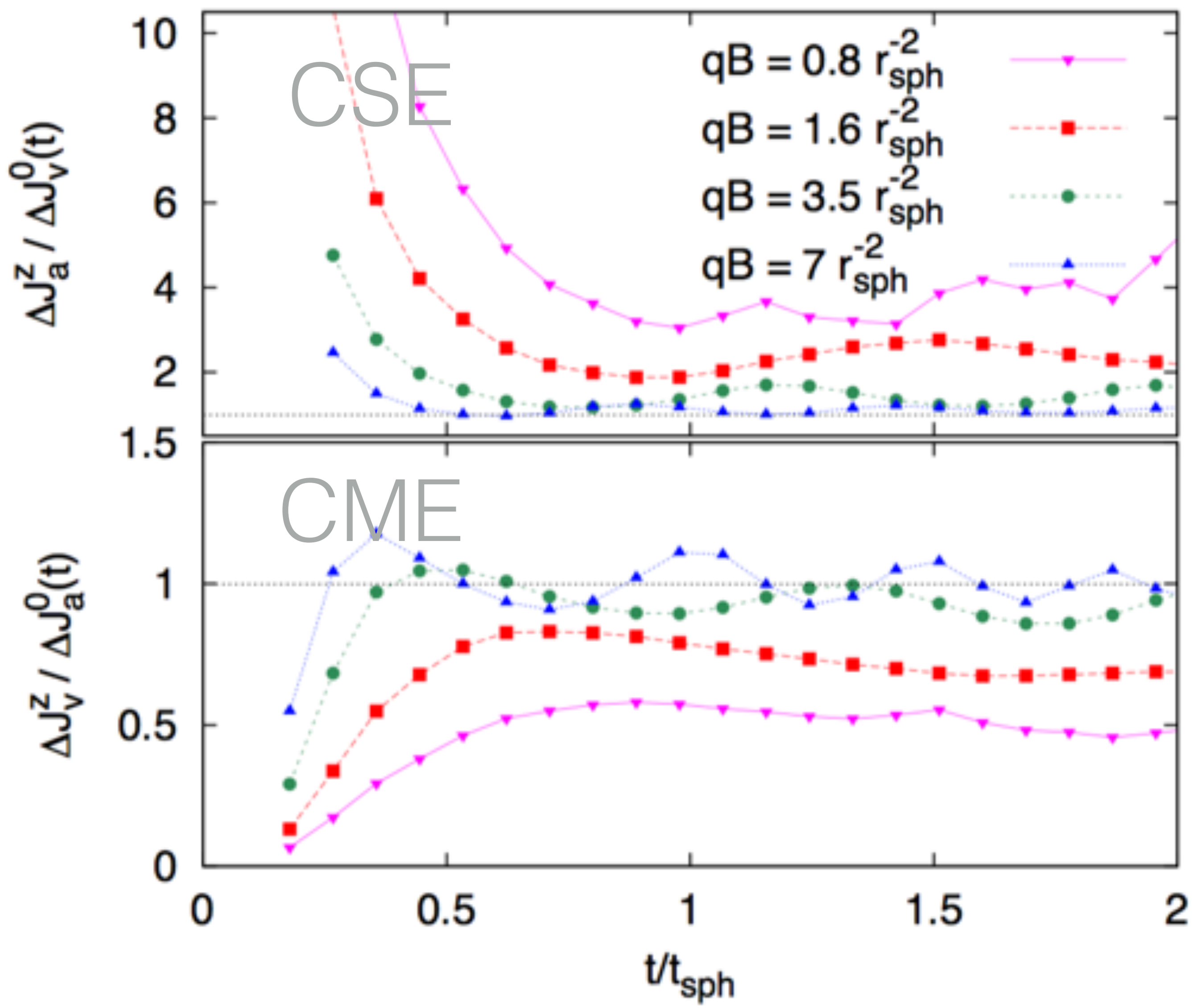


Chiral magnetic shock-wave

$$j^0_{v/a}(t > t_{\text{sph}}, z) = \frac{1}{2}\int_0^{t_{\text{sph}}} dt' \left[S(t', z - c(t-t')) \mp S(t', z + c(t-t')) \right]$$

-> Evolution for light quarks and strong magnetic fields well described by anomalous hydrodynamics at late times

Validity of constitutive relations



(Mace,Mueller,SS, Sharma, 1612.02477)

Verify ratios vector/axial currents and axial/vector charge

$$C_{\mathrm{CME}}(t) = \frac{\Delta J_v^z(t)}{\Delta J_a^0(t)}\,, \qquad C_{\mathrm{CSE}}(t) = \frac{\Delta J_a^z(t)}{\Delta J_v^0(t)}\,.$$

In the strong field limit related to thermodynamic constitutive relations

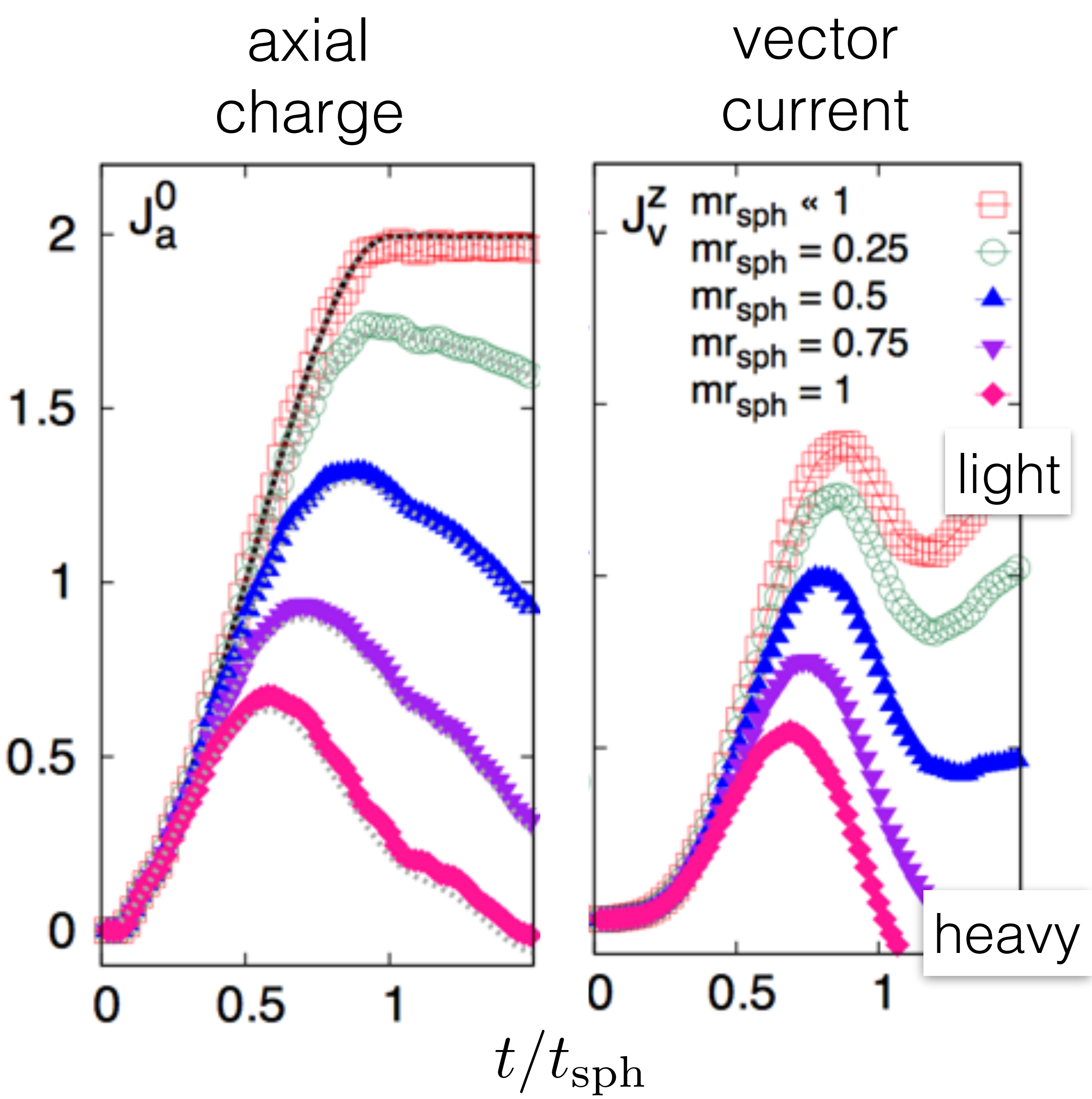
$$C_{CME} = 1\,, \qquad C_{CSE} = 1\,.$$

equal to time independent constants.

Simulation results indicate approach towards constant value with a finite relaxation time

Since lifetime of magnetic field in HIC is short this effect should also be incorporated in phenomenological approaches

Quark mass dependence



Explicit violation of axial charge conservation for finite quark mass

$$\partial_\mu j_a^\mu(x) = 2m\langle\hat{\bar{\psi}}(x) i\gamma_5 \hat{\psi}(x)\rangle + S(x)$$

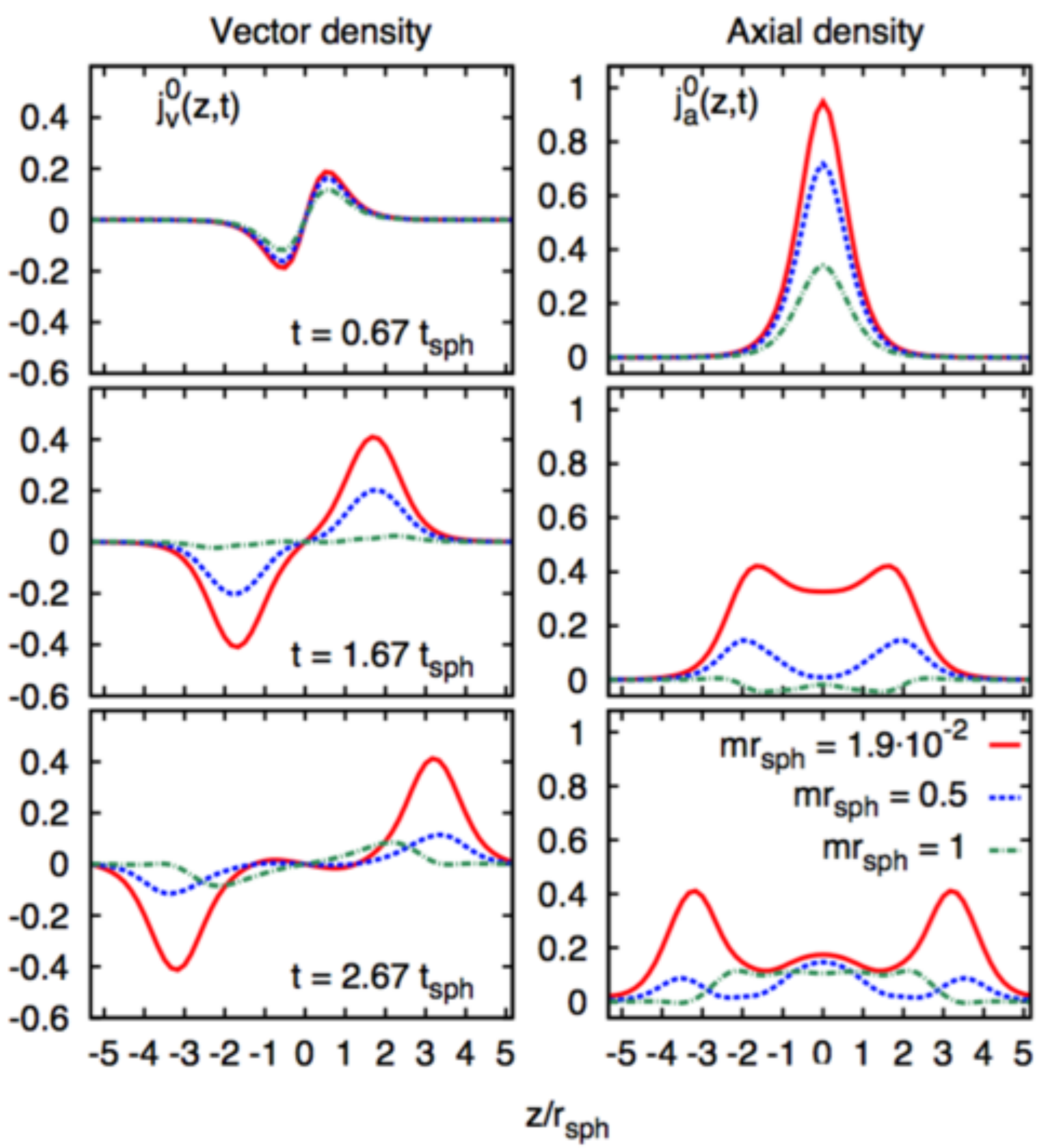
leads to damping of axial charge

Since chiral magnetic effect current is proportional to axial charge density it will also be reduced

$$\vec{j}_v \propto j_a^0 \vec{B}$$

(N.Mueller,SS, S. Sharma PRL 117 (2016) no.14, 142301)

Quark mass dependence



(Mace,Mueller,SS, Sharma, 1612.02477)

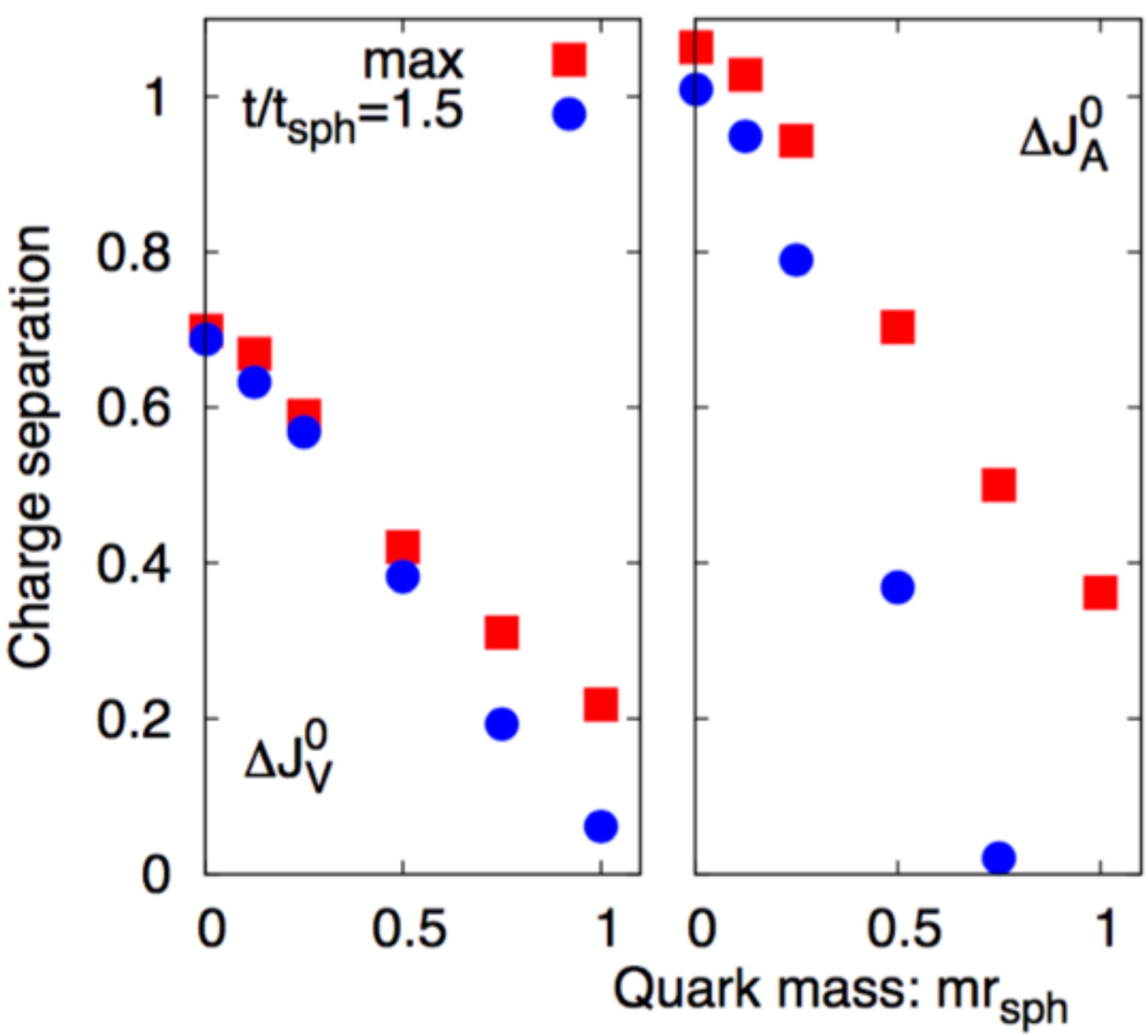
Light quarks **(** $\mathbf{mt_{sph} \ll 1}$ **)**

Chiral magnetic wave leads to non-dissipative transport of axial and vector charges

Heavy quarks **(** $\mathbf{mt_{sph} \sim 1}$ **)**

Dissipation of axial charge leads to significant reduction of charge separation

Quark mass dependence



(Mace,Mueller,SS, Sharma, 1612.02477)

Significant reduction of the charge separation signal by factor ~5 already for moderate quark masses

Phenomenological consequences

Desirable to include dissipative effects
in macroscopic description

Unlikely that strange quarks participate in CME

Expect backreaction (not included so far) to suppress the signal even further

Conclusions & Outlook

Development of first-principle techniques to study dynamics of vector and axial charges out-of-equilibrium

Successful microscopic description of CME & CMW

Dissipative effects important already at moderate quark mass

Should be included in macroscopic descriptions

Next step is to include back-reaction of fermions perform simulations for a realistic heavy-ion environment

- Chiral magnetic effect & anomalous transport in HIC
- Quark production & electro-magnetic response

Expect several applications beyond high-energy QCD

Dirac semi-metals, strong field QED, Cosmology