JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Spectrum and structure of positive and negative parity baryons

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Seminar Theoretical Hadron Physics

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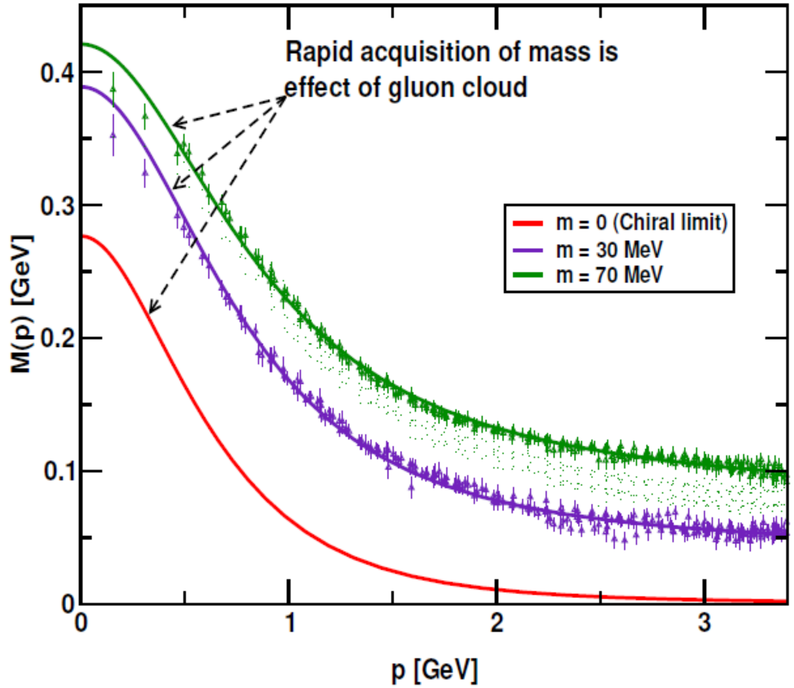
Non-Perturbative QCD:

- **Hadrons, as bound states, are dominated by non-perturbative QCD dynamics –** ***Two emergent phenomena***
  - **Confinement**: Colored particles have never been seen isolated
  - Explain how quarks and gluons bind together
  - **DCSB**: Hadrons do not follow the chiral symmetry pattern
  - Explain the most important mass generating mechanism for visible matter in the Universe
  - Neither of these phenomena is apparent in QCD 's Lagrangian, HOWEVER, They play a dominant role in determining the characteristics of real-world QCD!

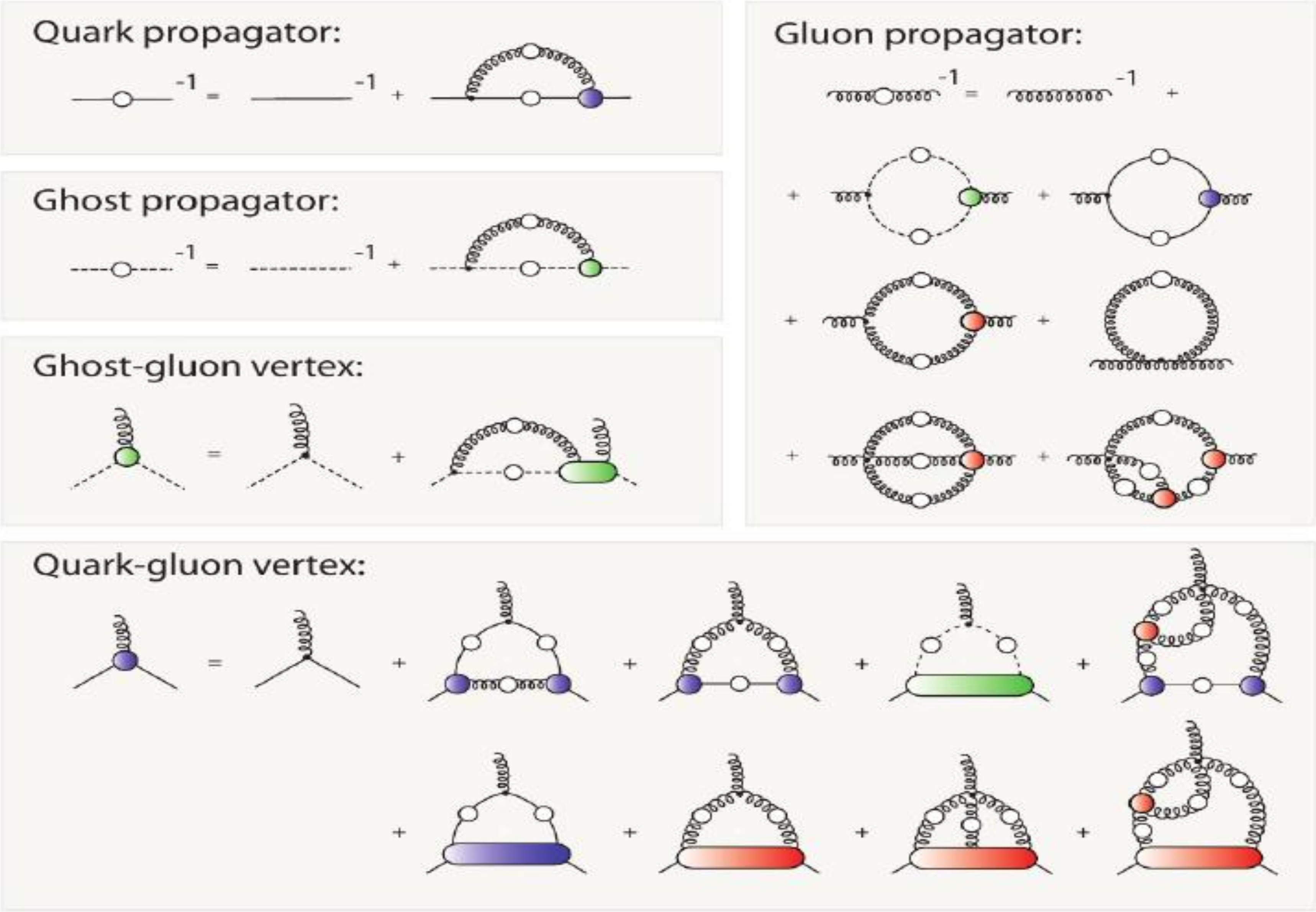
Non-Perturbative QCD:

- **From a quantum field theoretical point of view**, these emergent phenomena could be associated with dramatic, dynamically driven changes in the analytic structure of QCD 's Schwinger functions (propagators and vertices). The Schwinger functions are solutions of the quantum equations of motion (**Dyson-Schwinger equations**).
- **Dressed-quark propagator:**



  - Mass generated from the interaction of quarks with the gluon.
  - Light quarks acquire a **HUGE** constituent mass.
  - Responsible of the 98% of the mass of the proton and the large splitting between parity partners.

Dyson-Schwinger equations (DSEs)



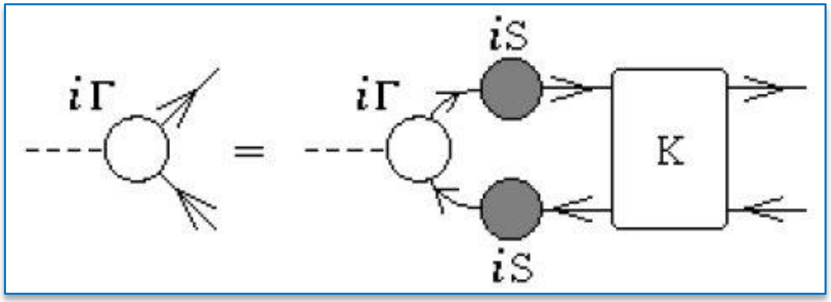
Dyson-Schwinger equations (DSEs)

Dyson-Schwinger equations

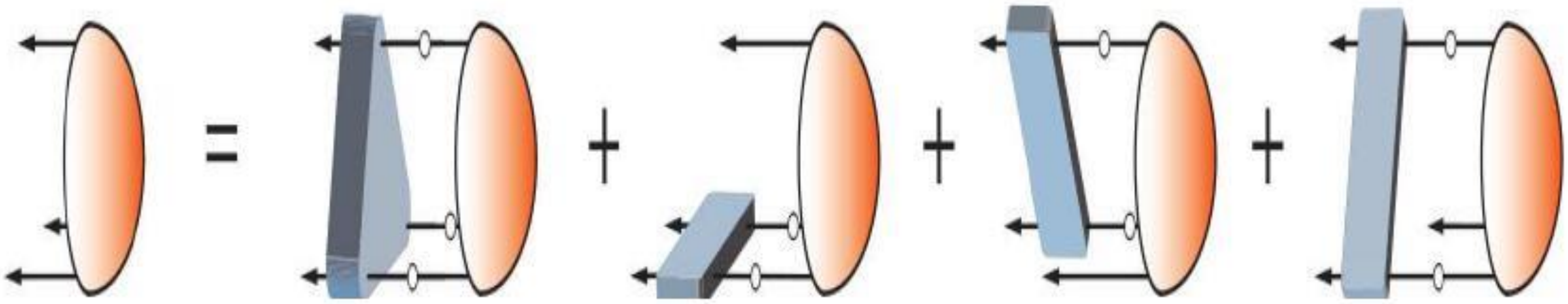
- A Nonperturbative symmetry-preserving tool for the study of Continuum-QCD
- Well suited to Relativistic Quantum Field Theory
- A method connects observables with long-range behaviour of the running coupling
- Experiment ↔ Theory comparison leads to an understanding of long-range behaviour of strong running-coupling

Hadrons: Bound-states in QFT

- **Mesons**: a 2-body bound state problem in QFT
  - ***Bethe-Salpeter Equation***
  - **K - fully amputated, two-particle irreducible, quark-antiquark scattering kernel**



- **Baryons**: a 3-body bound state problem in QFT.
- ***Faddeev equation***: **sums all possible quantum field theoretical exchanges and interactions that can take place between the three dressed-quarks that define its valence quark content.**



Hadrons: Bound-states in QFT

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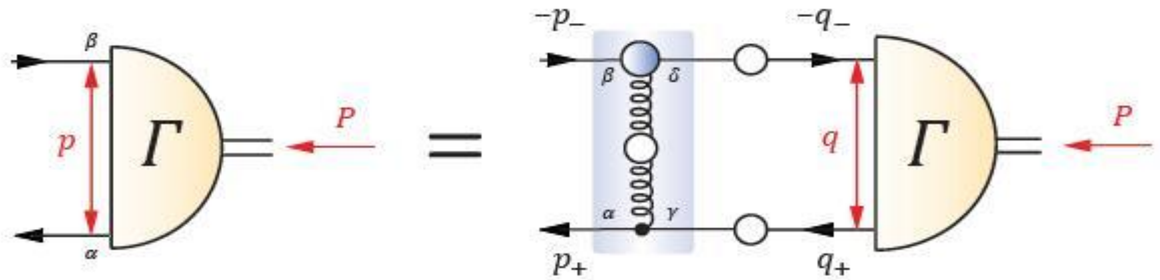
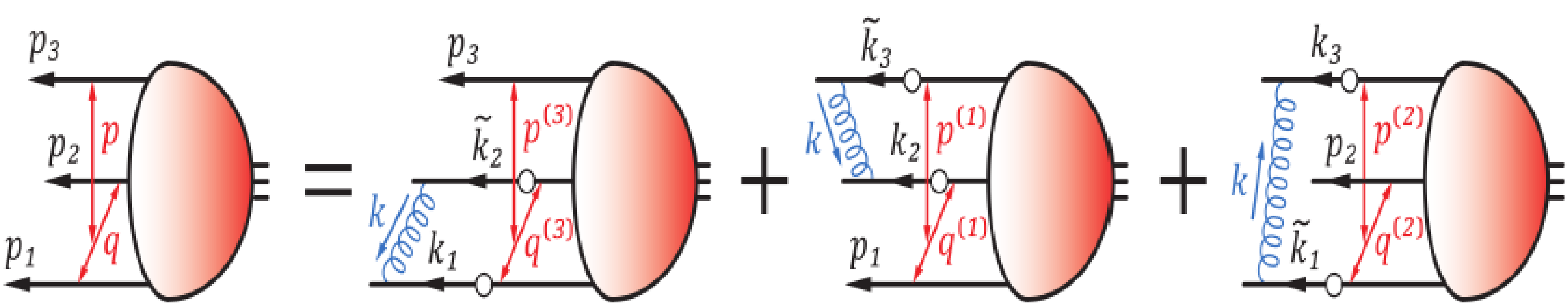


- **Baryons: a 3-body bound state problem in QFT.**
- ***Faddeev equation*: sums all possible quantum field theoretical exchanges and interactions that can take place between the three dressed-quarks that define its valence quark content.**

Faddeev equation in rainbow-ladder truncation



2-body correlations

- **Mesons: quark-antiquark correlations -- color-singlet**
- **Diquarks: quark-quark correlations within a color-singlet baryon.**

- **Diquark correlations:**
  - In our approach: non-pointlike color-antitriplet and fully interacting.
  - Diquark correlations are soft, they possess an electromagnetic size.
  - Owing to properties of charge-conjugation, a diquark with spin-parity J^P may be viewed as a partner to the analogous J^{-P} meson.

$$\Gamma_{q\bar{q}}(p;P) = - \int \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu\, S(q+P)\Gamma_{q\bar{q}}(q;P)S(q)\,\frac{\lambda^a}{2}\,\gamma_\nu$$

$$\Gamma_{qq}(p;P)C^\dagger = -\frac{1}{2}\int \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu\, S(q+P)\Gamma_{qq}(q;P)C^\dagger S(q)\,\frac{\lambda^a}{2}\,\gamma_\nu$$

2-body correlations

- Quantum numbers:
  - (I=0, J^P=0^+): isoscalar-scalar diquark
  - (I=1, J^P=1^+): isovector-pseudovector diquark
  - (I=0, J^P=0^-): isoscalar-pesudoscalar diquark
  - (I=0, J^P=1^-): isoscalar-vector diquark
  - (I=1, J^P=1^-): isovector-vector diquark
  - Tensor diquarks
- Three-body bound states ➡

  Quark-Diquark two-body bound states

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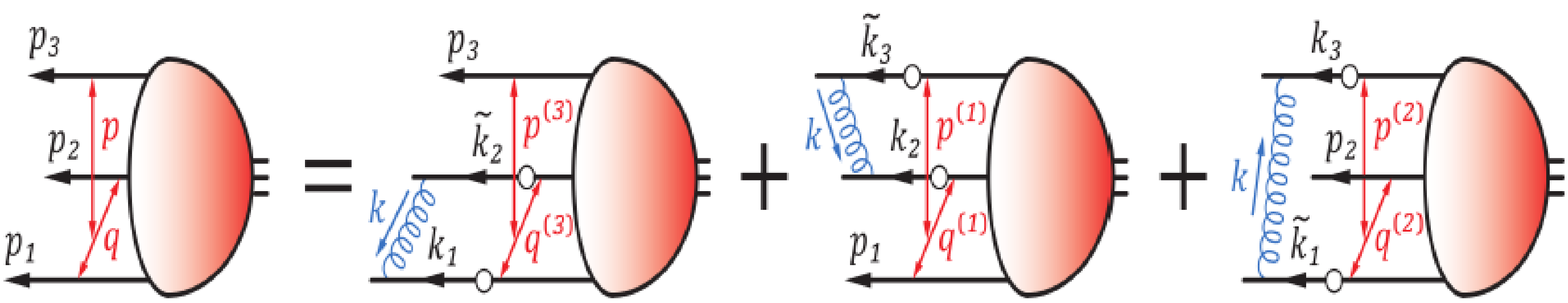

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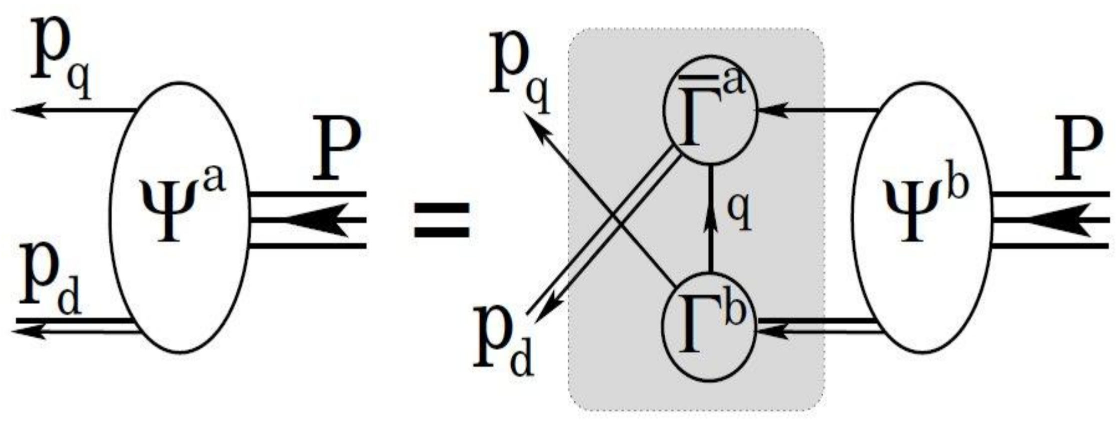
- ✓ M. Oettel, G. Hellstern, Reinhard Alkofer, H. Reinhardt , Phys.Rev. C58 (1998) 2459-2477
- ✓ Martin Oettel, Mike Pichowsky, Lorenz von Smekal, Eur.Phys.J. A8 (2000) 251-281
- ✓ Jorge Segovia, Bruno El-Bennich, Eduardo Rojas, Ian C. Cloet, Craig D. Roberts, Shu-Sheng Xu, Hong-Shi Zong, Phys.Rev.Lett. 115 (2015) no.17, 171801
- ✓ G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C. S. Fischer, Prog.Part.Nucl.Phys. 91 (2016) 1-100



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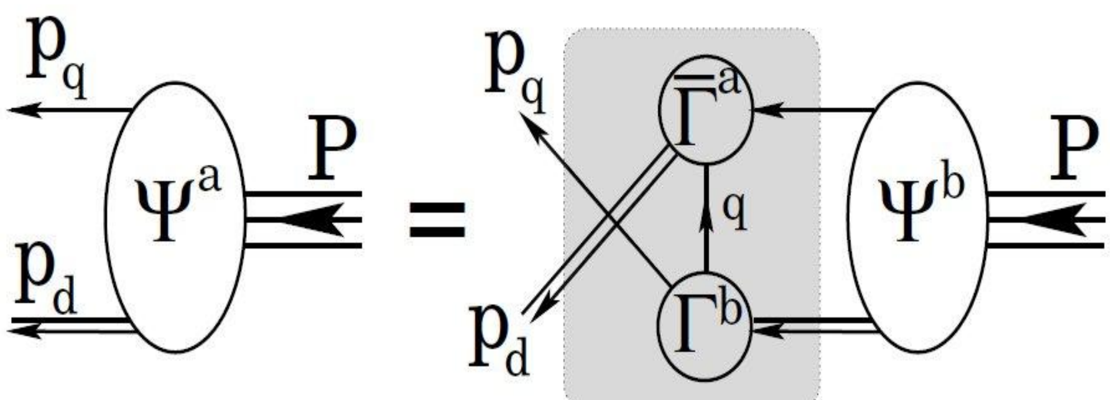
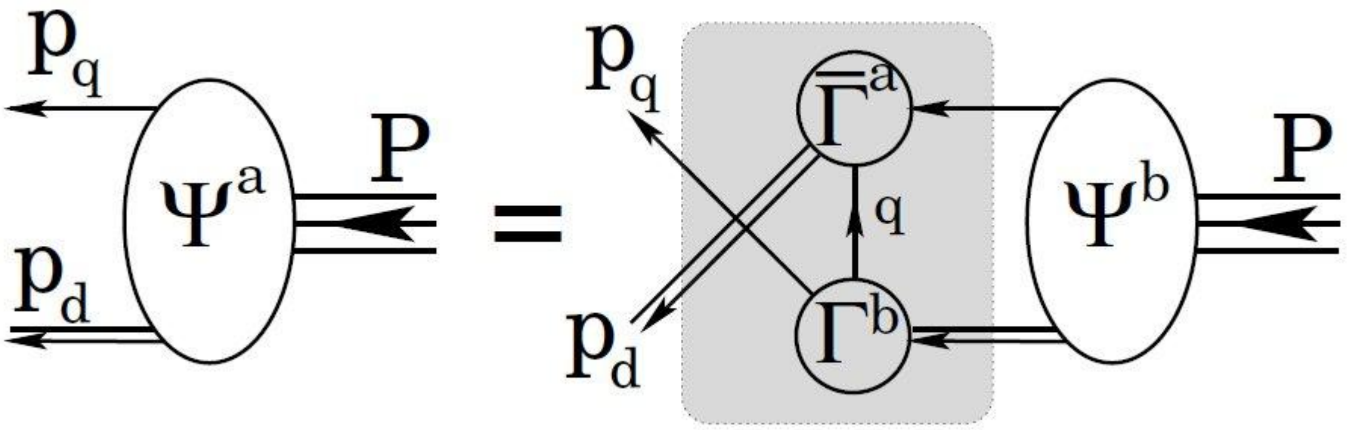
✓ CC, B. El-Bennich, C. D. Roberts, S. M. Schmidt, J. Segovia, S-L. Wan, Phys.Rev. D97 (2018) no.3, 034016



QCD-kindred model

- The dressed-quark propagator
- Diquark amplitudes
- Diquark propagators
- Faddeev amplitudes



QCD-kindred model

- **Diquark masses (in GeV):**

  $$m_{0^+} = 0.8, \quad m_{1^+} = 0.9, \quad m_{0^-} = 1.2, \quad m_{1^-} = 1.3,$$

- **The first two values (positive-parity) provide for a good description of numerous dynamical properties of the nucleon, Δ-baryon and Roper resonance.**
- **Masses of the odd-parity correlations are based on those computed from a contact interaction.**

PHYSICAL REVIEW C **96**, 015208 (2017)

Parity partners in the baryon resonance spectrum

Ya Lu,[1,*] Chen Chen,[2,†] Craig D. Roberts,[3,‡] Jorge Segovia,[4,§] Shu-Sheng Xu,[1,‖] and Hong-Shi Zong[1,5,¶]

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(Received 10 May 2017; published 28 July 2017)

QCD-kindred model

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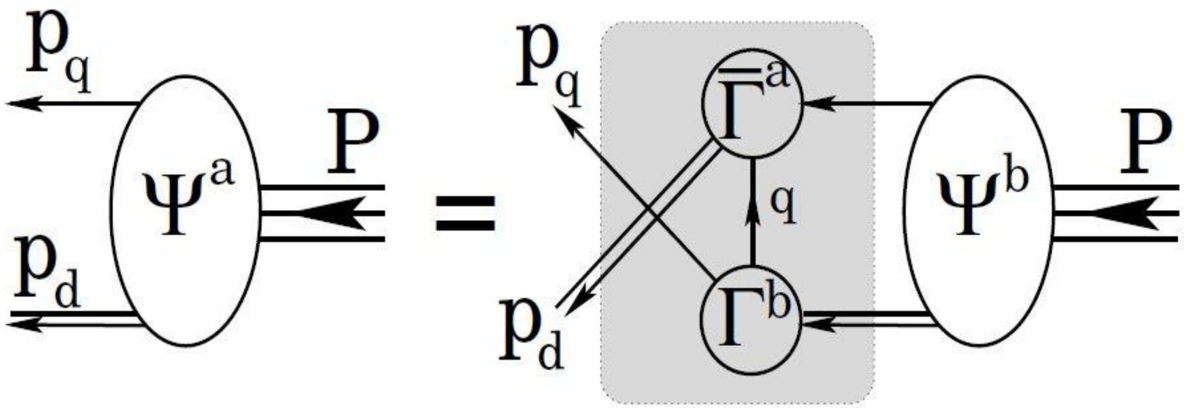
- **The first two values (positive-parity) provide for a good description of numerous dynamical properties of the nucleon, Δ-baryon and Roper resonance.**
- **Masses of the odd-parity correlations are based on those computed from a contact interaction.**
- **Such values are typical; and in truncations of the two-body scattering problem that are most widely used (RL), isoscalar-vector and isovector-vector correlations are degenerate.**
- **Normalization condition → couplings:**

$$g_{0^+} = 14.8, \qquad g_{1^+} = 12.7,$$
$$g_{0^-} = 12.8, \qquad g_{1^-} = 5.4, \qquad \cancel{g_{\bar{1}^-} = 2.5}.$$

- **Faddeev kernels: 22 × 22 matrices are reduced to 16 × 16 !**

QCD-kindred model



- There is an absence of spin-orbit repulsion owing to an oversimplification of the gluon-quark vertex when formulating the RL bound-state equations. We therefore employ a simple artifice in order to implement the missing interactions.

  - ✓ We introduce a single parameter into the Faddeev equation for J^P=1/2^{+-} baryons: gDB, a linear multiplicative factor attached to each opposite-parity (-P) diquark amplitude in the baryon's Faddeev equation kernel.
  - ✓ gDB is the single free parameter in our study.

QCD-kindred model

- **Solution to the 50 year puzzle -- Roper resonance: Discovered in 1963, the Roper resonance appears to be an exact copy of the proton except that its mass is *50%* greater and it is unstable…**

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PRL 115, 171801 (2015) PHYSICAL REVIEW LETTERS week ending 23 OCTOBER 2015

Completing the Picture of the Roper Resonance

Jorge Segovia,[1] Bruno El-Bennich,[2,3] Eduardo Rojas,[2,4] Ian C. Cloët,[5] Craig D. Roberts,[5] Shu-Sheng Xu,[6] and Hong-Shi Zong[6]

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We employ a continuum approach to the three valence-quark bound-state problem in relativistic quantum field theory to predict a range of properties of the proton's radial excitation and thereby unify them with those of numerous other hadrons. Our analysis indicates that the nucleon's first radial excitation is the Roper resonance. It consists of a core of three dressed quarks, which expresses its valence-quark content and whose charge radius is 80% larger than the proton analogue. That core is complemented by a meson cloud, which reduces the observed Roper mass by roughly 20%. The meson cloud materially affects long-wavelength characteristics of the Roper electroproduction amplitudes but the quark core is revealed to probes with $Q^2 \gtrsim 3m_N^2$.

DOI: 10.1103/PhysRevLett.115.171801 PACS numbers: 13.40.Gp, 14.20.Dh, 14.20.Gk, 11.15.Tk

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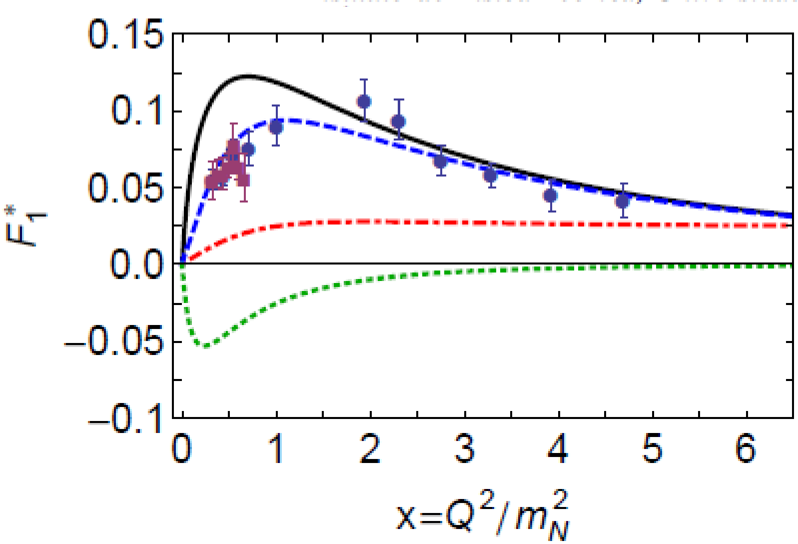

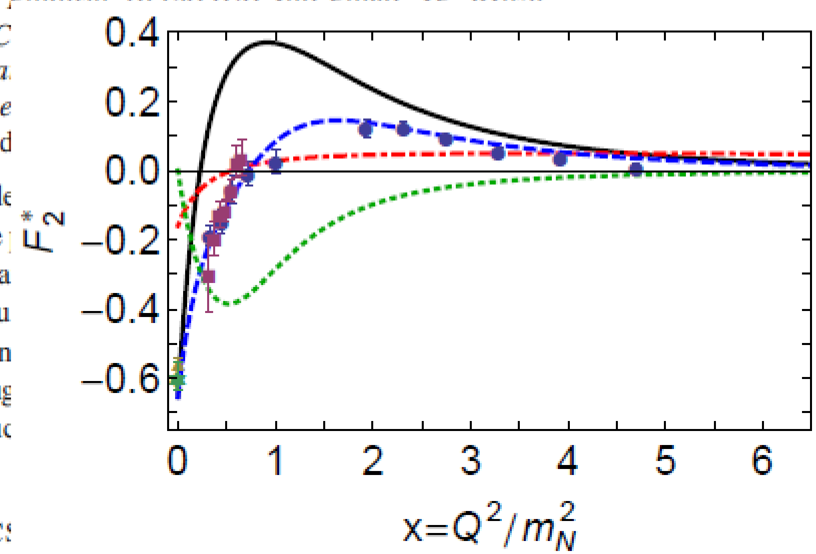

Part A

PHYSICAL REVIEW D **97**, 034016 (2018)

Structure of the nucleon's low-lying excitations

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Part A

- The four lightest baryon (I=1/2, J^P=1/2^{+-}) isospin doublets: nucleon, roper, N(1535), N(1650)
- **Masses**
- **Rest-frame orbital angular momentum**
- **Diquark content**
- **Pointwise structure**

Masses

- We choose gDB=0.43 so as to produce a mass splitting of 0.1 GeV (the empirical value) between the lowest-mass P=- state (N(1535)) and the first excited P=+ state (Roper).
- Our computed values for the masses of the four lightest 1/2^{+-} baryon doublets are listed here, in GeV:

g_{DB}	m_N	$m_{N(1440)}^{1/2^+}$	$m_{N(1535)}^{1/2^-}$	$m_{N(1650)}^{1/2^-}$
0.43	1.19	1.73	1.83	1.91
1.0	1.19	1.73	1.43	1.61

- Pseudoscalar and vector diquarks have no impact on the mass of the two positive-parity baryons, whereas scalar and pseudovector diquarks are important to the negative parity systems.

Masses

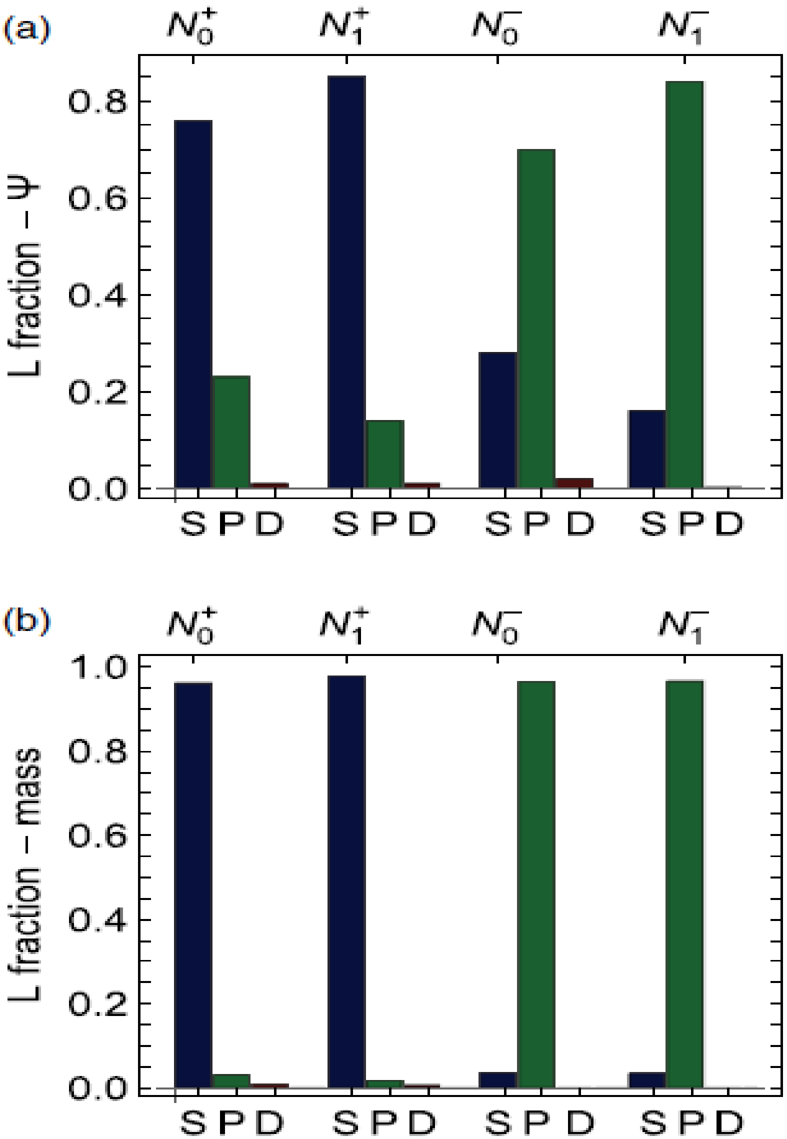
- The quark-diquark kernel omits all those resonant contributions which may be associated with meson-baryon final-state interactions that are resummed in dynamical coupled channels models in order to transform a bare baryon into the observed state.
- The Faddeev equations analyzed to produce the results should therefore be understood as producing the dressed-quark core of the bound state, not the completely dressed and hence observable object.
- In consequence, a comparison between the empirical values of the resonance pole positions and the computed masses is not pertinent. Instead, one should compare the masses of the quark core with values determined for the meson-undressed bare excitations, e.g.,

	m_N	$m_{N(1440)}^{1/2^+}$	$m_{N(1535)}^{1/2^-}$	$m_{N(1650)}^{1/2^-}$
herein	1.19	1.73	1.83	1.91
M_B^0 [53]		1.76	1.80	1.88

where M^0_B is the relevant bare mass inferred in the associated dynamical coupled-channels analysis.

- The relative difference is just 1.7%. We consider this to be a success of our calculation.

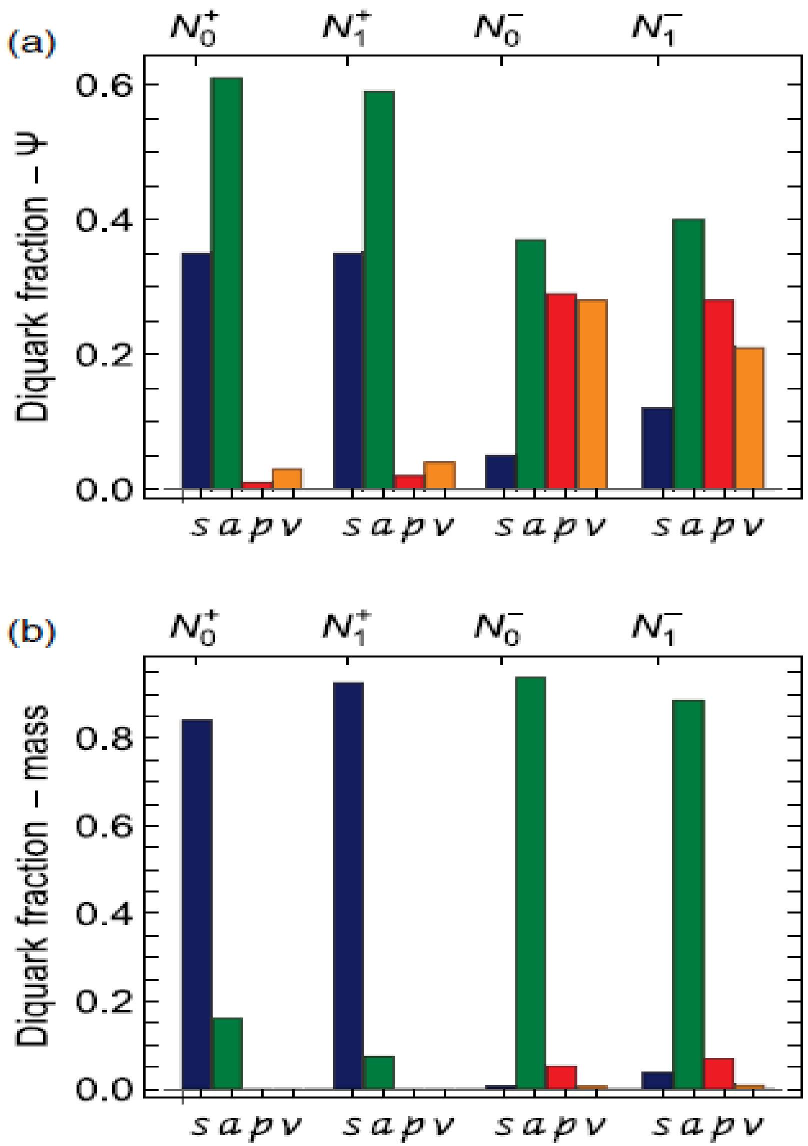
Rest-frame orbital angular momentum



- (a) Computed from the wave functions directly.

  (b) Computed from the relative contributions to the masses.
- (b) delivers the same qualitative picture as that presented in (a). Therefore, there is **little** mixing between partial waves in the computation of a baryon's mass.
- The nucleon and Roper are primarily **S-wave** in nature. On the other hand, the N(1535)1/2^-,N(1650)1/2^- are essentially **P-wave** in character.
- These observations provide support in quantum field theory for the constituent-quark model classifications of these systems.

Diquark content

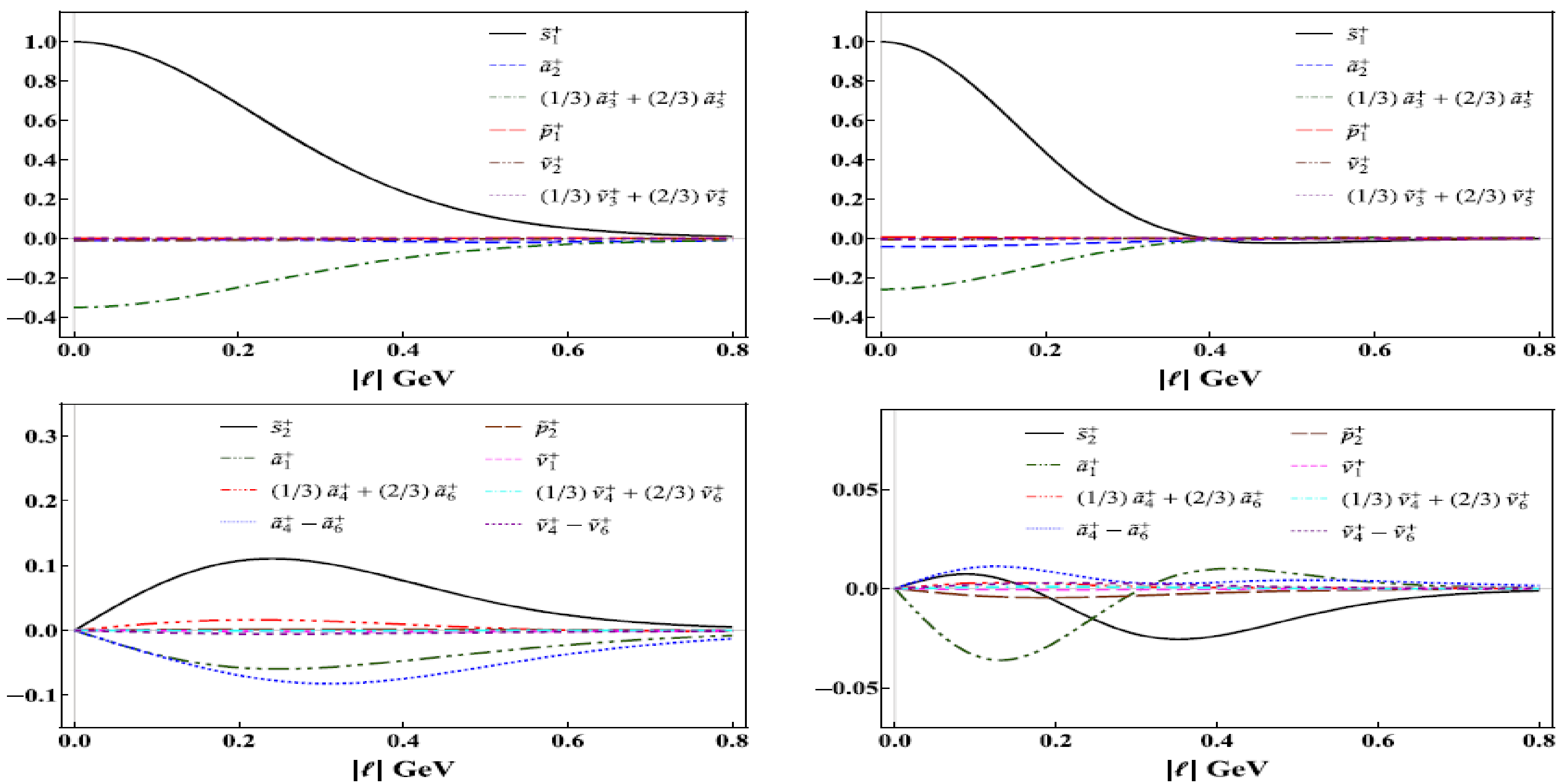


- (a) Computed from the amplitudes directly.

  (b) Computed from the relative contributions to the masses.
- From (a): The amplitudes associated with these negative-parity states contain roughly ***equal fractions*** of even and odd parity diquarks. Positive-parity states: negative-parity diquarks are almost ZERO.
- From (b): In each, there is a single dominant diquark component. There are **significant interferences** between different diquarks.

Pointwise structure

- We consider the zeroth Chebyshev moment of all S- and P-wave components in a given baryon's Faddeev wave function.
- Nucleon's first positive-parity excitation: all S-wave components exhibit a single zero; and four of the P-wave projections also possess a zero. This pattern of behavior for the first excited state indicates that it may be interpreted as a radial excitation.

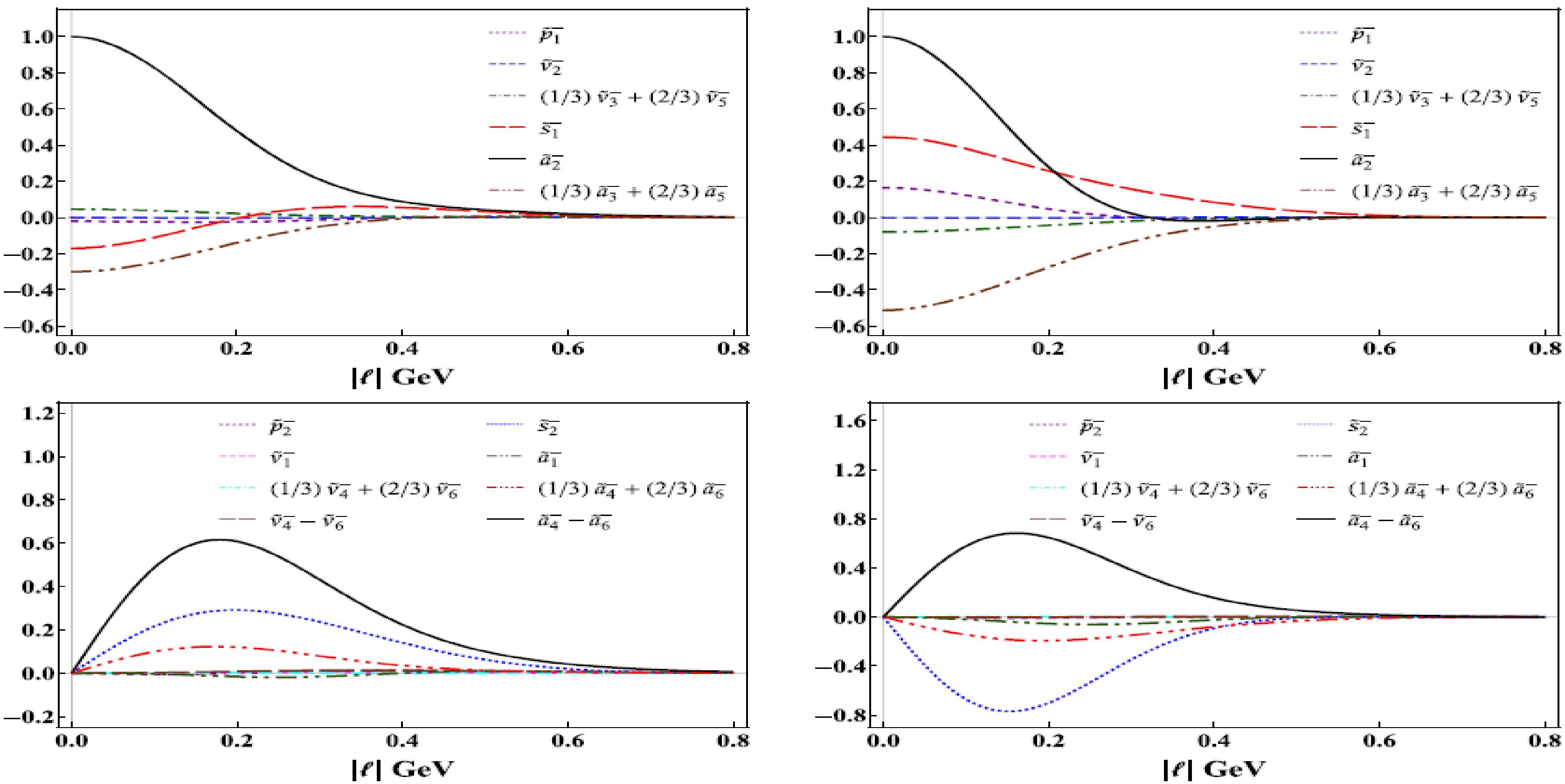


Pointwise structure

- For N(1535)1/2^-,N(1650)1/2^- : the contrast with the positive-parity states is STARK.

  In particular, there is no simple pattern of zeros, with all panels containing at least one function that possesses a zero.

- In their rest frames, these systems are predominantly P-wave in nature, but possess material S-wave components; and the first excited state in this negative parity channel—*N(1650)1/2^−*—has little of the appearance of a radial excitation, since most of the functions depicted in the right panels of the figure do not possess a zero.



Part A - Summary & Outlook

- By including all kinds of diquarks, we performed a comparative study of the four lightest baryon (I=1/2, J^P=1/2^{+-}) isospin doublets in order to both elucidate their structural similarities and differences.
- The two lightest (I=1/2, J^P=1/2^+) doublets are dominated by scalar and pseudovector diquarks; the associated rest-frame Faddeev wave functions are primarily S-wave in nature; and the first excited state in this 1/2^+ channel has very much the appearance of a radial excitation of the ground state.
- In the two lightest (I=1/2, J^P=1/2^-) systems, **TOO**, scalar and pseudovector diquarks play a material role. In their rest frames, the Faddeev amplitudes describing the dressed-quark cores of these negative-parity states contain roughly equal fractions of even and odd parity diquarks; the associated wave functions of these negative-parity systems are predominantly P-wave in nature, but possess measurable S-wave components; and, the first excited state in this negative parity channel has little of the appearance of a radial excitation.
- NEXT: N(1720)3/2^+, N(1520)3/2^-, form factors, PDAs, PDFs, …

Part B

arXiv.org > nucl-th > arXiv:1901.04305

Spectrum and structure of octet and decuplet baryons and their positive-parity excitations

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(Dated: 10 January 2019)

QCD-kindred model

- **Spin-flavor structure:**

$$u_p = \begin{bmatrix} u[ud]_{0^+} \\ d\{uu\}_{1^+} \\ u\{ud\}_{1^+} \end{bmatrix} \leftrightarrow \begin{bmatrix} s_p^1 \\ a_p^4 \\ a_p^5 \end{bmatrix},$$

$$u_\Lambda = \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{2}\, s[ud]_{0^+} \\ d[us]_{0^+} - u[ds]_{0^+} \\ d\{us\}_{1^+} - u\{ds\}_{1^+} \end{bmatrix} \leftrightarrow \begin{bmatrix} s_\Lambda^1 \\ s_\Lambda^{[2,3]} \\ a_\Lambda^{[6,8]} \end{bmatrix},$$

$$u_\Sigma = \begin{bmatrix} u[us]_{0^+} \\ s\{uu\}_{1^+} \\ u\{us\}_{1^+} \end{bmatrix} \leftrightarrow \begin{bmatrix} s_\Sigma^2 \\ a_\Sigma^4 \\ a_\Sigma^6 \end{bmatrix},$$

$$u_\Xi = \begin{bmatrix} s[us]_{0^+} \\ s\{us\}_{1^+} \\ u\{ss\}_{1^+} \end{bmatrix} \leftrightarrow \begin{bmatrix} s_\Sigma^2 \\ a_\Sigma^6 \\ a_\Sigma^9 \end{bmatrix},$$

$$u_\Delta = [u\{uu\}_{1^+}] \leftrightarrow [a_\Delta^4],$$

$$u_{\Sigma^*} = \begin{bmatrix} s\{uu\}_{1^+} \\ u\{us\}_{1^+} \end{bmatrix} \leftrightarrow \begin{bmatrix} a_{\Sigma^*}^4 \\ a_{\Sigma^*}^6 \end{bmatrix},$$

$$u_{\Xi^*} = \begin{bmatrix} s\{us\}_{1^+} \\ u\{ss\}_{1^+} \end{bmatrix} \leftrightarrow \begin{bmatrix} a_{\Xi^*}^6 \\ a_{\Xi^*}^9 \end{bmatrix},$$

$$u_\Omega = [s\{ss\}_{1^+}] \leftrightarrow [a_\Omega^9].$$

- **Diquark masses (in GeV):**

$$m_{[ud]_{0^+}} = 0.80,$$
$$m_{[us]_{0^+}} = m_{[ds]_{0^+}} = 0.95,$$
$$m_{\{uu\}_{1^+}} = m_{\{ud\}_{1^+}} = m_{\{dd\}_{1^+}} = 0.90,$$
$$m_{\{us\}_{1^+}} = m_{\{ds\}_{1^+}} = 1.05,$$
$$m_{\{ss\}_{1^+}} = 1.20.$$

- **The values of m_[ud] & m_{uu/ud/dd} provide for a good description of numerous dynamical properties of the nucleon, Δ-baryon and Roper resonance.**

- **Other masses are derived therefrom via an equal-spacing rule: *viz*. replacing by a *s-quark* bring an extra 0.15 GeV (~ *Ms - Mu*).**

Part B

- Spectrum and structure of *octet & decuplet* baryons and their positive-parity excitations
- **Masses**
- **Rest-frame orbital angular momentum**
- **Diquark content**
- **Pointwise structure**

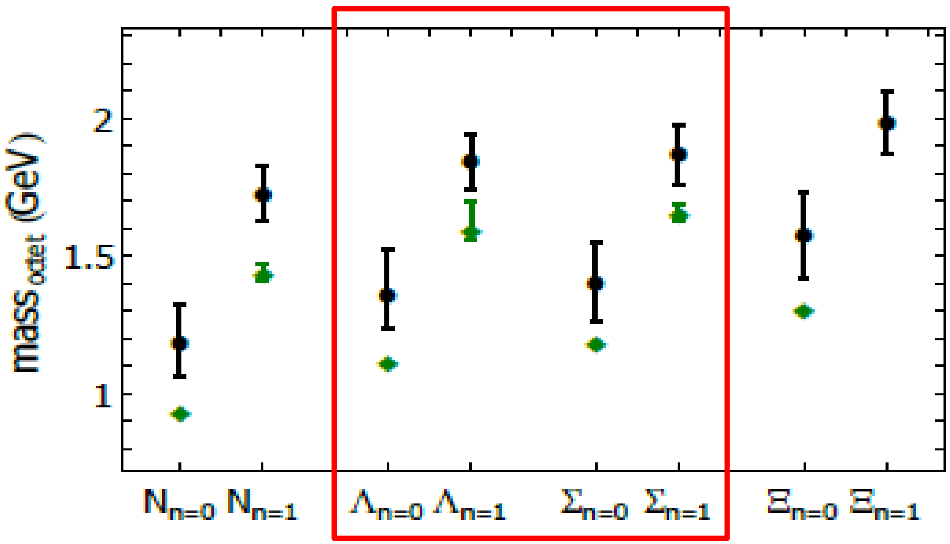
Masses

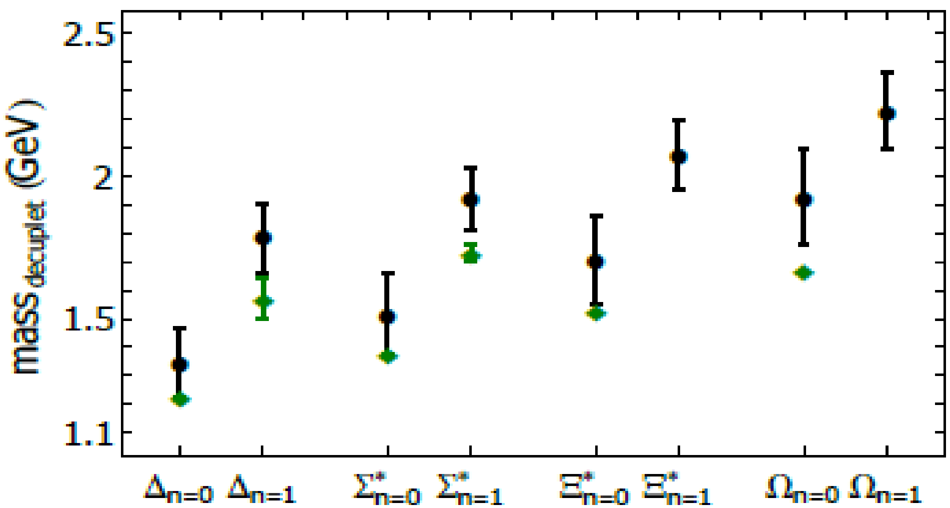
	Row		N	Λ	Σ	Ξ	Δ	Σ^*	Ξ^*	Ω
n=0	1	DSE	1.19(13)	1.37(14)	1.41(14)	1.58(15)	1.35(12)	1.52(14)	1.71(15)	1.93(17)
	3	expt.	0.94	1.12	1.19	1.31	1.23	1.38	1.53	1.67
n=1	5	DSE	1.73(10)	1.85(09)	1.88(11)	1.99(11)	1.79(12)	1.93(11)	2.08(12)	2.23(13)
	7	expt.	1.44(03)	$1.51^{+0.10}_{-0.04}$	1.66(03)	-	1.57(07)	1.73(03)	-	-

- Σ-Λ mass splitting
- While the Σ and Λ are associated with the same combination of valence-quarks, their spin-flavor wave functions are different: the Λ contains more of the (lighter) scalar diquark correlations than the Σ

$$u_\Sigma = \begin{bmatrix} u[us]_{0^+} \\ s\{uu\}_{1^+} \\ u\{us\}_{1^+} \end{bmatrix} \leftrightarrow \begin{bmatrix} s_\Sigma^2 \\ a_\Sigma^4 \\ a_\Sigma^6 \end{bmatrix} \qquad u_\Lambda = \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{2}\, s[ud]_{0^+} \\ d[us]_{0^+} - u[ds]_{0^+} \\ d\{us\}_{1^+} - u\{ds\}_{1^+} \end{bmatrix} \leftrightarrow \begin{bmatrix} s_\Lambda^1 \\ s_\Lambda^{[2,3]} \\ a_\Lambda^{[6,8]} \end{bmatrix}$$

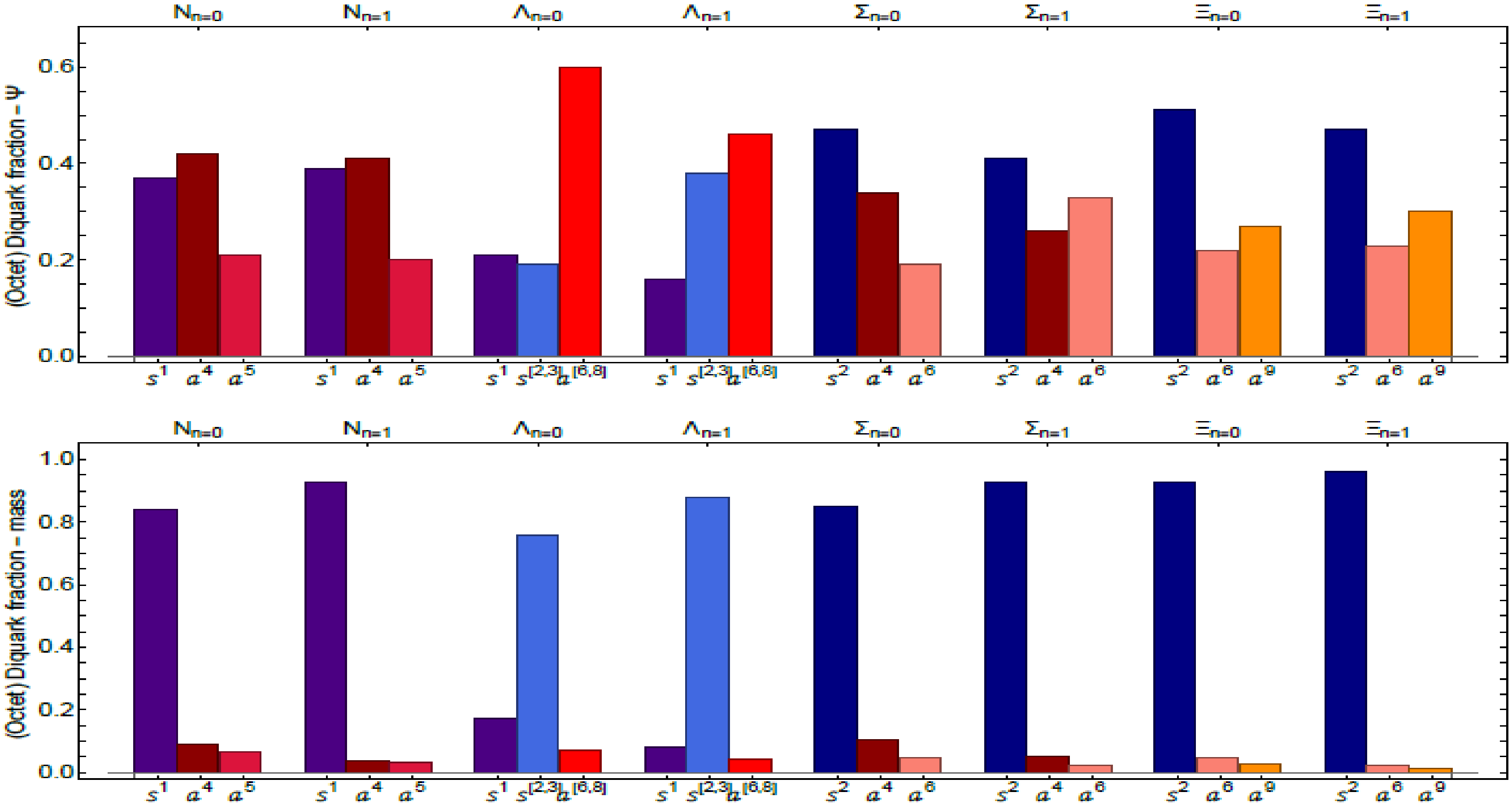
Masses





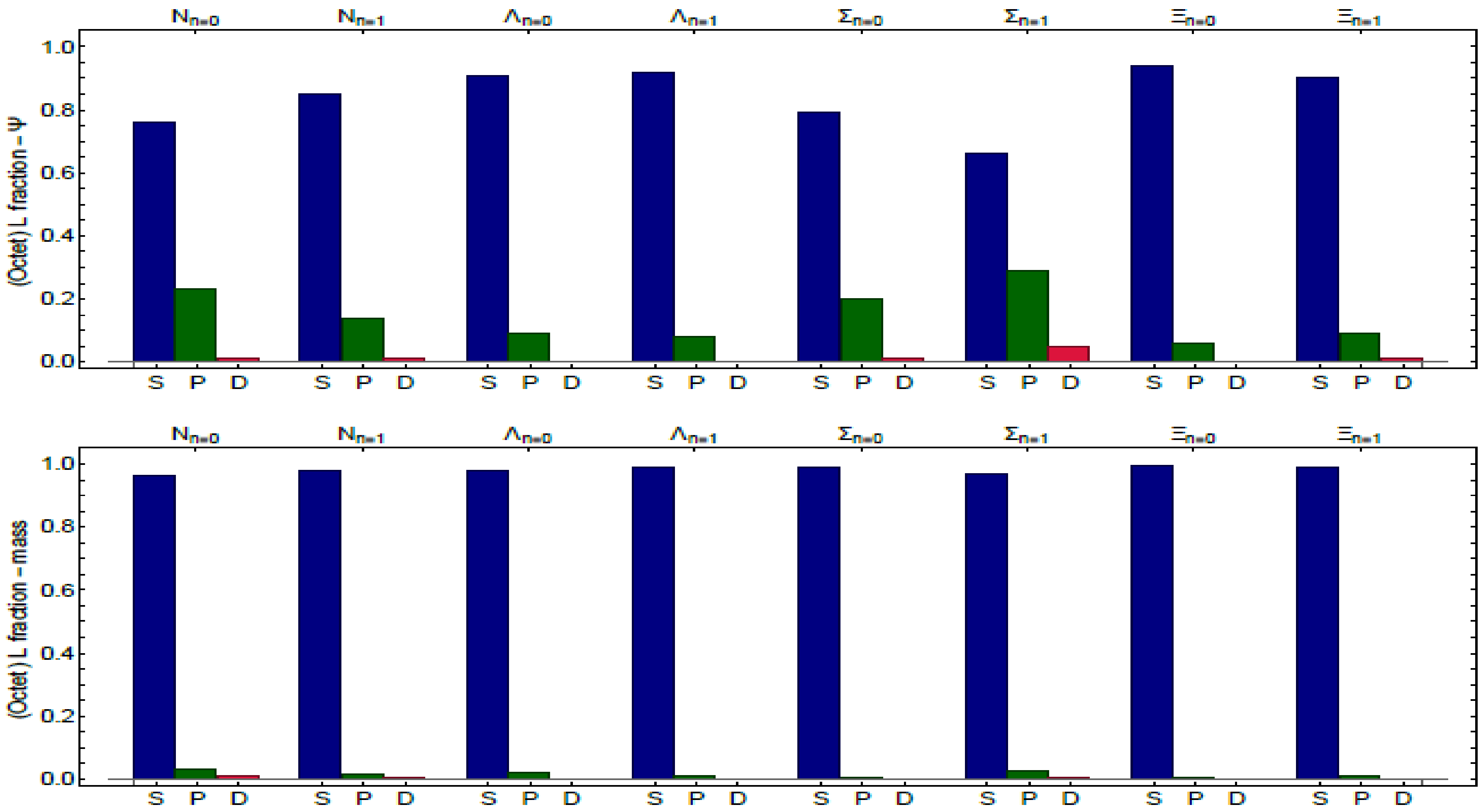
- The computed masses are uniformly larger than the corresponding empirical values.
- The quark-diquark kernel omits all those resonant contributions associated with meson-baryon final-state interactions, which typically generate a measurable reduction.
- The Faddeev equations analyzed to produce the results should be understood as producing the dressed-quark core of the bound state, NOT the completely dressed and hence observable object.

Diquark content (Octet)



- Upper: Computed from the amplitudes directly.
  Lower: Computed from the relative contributions to the masses.
- Lower : In each, there is a single dominant diquark component: scalar diquark
- Difference -> the lack of interference between diquark components

Rest-frame orbital angular momentum (Octet)

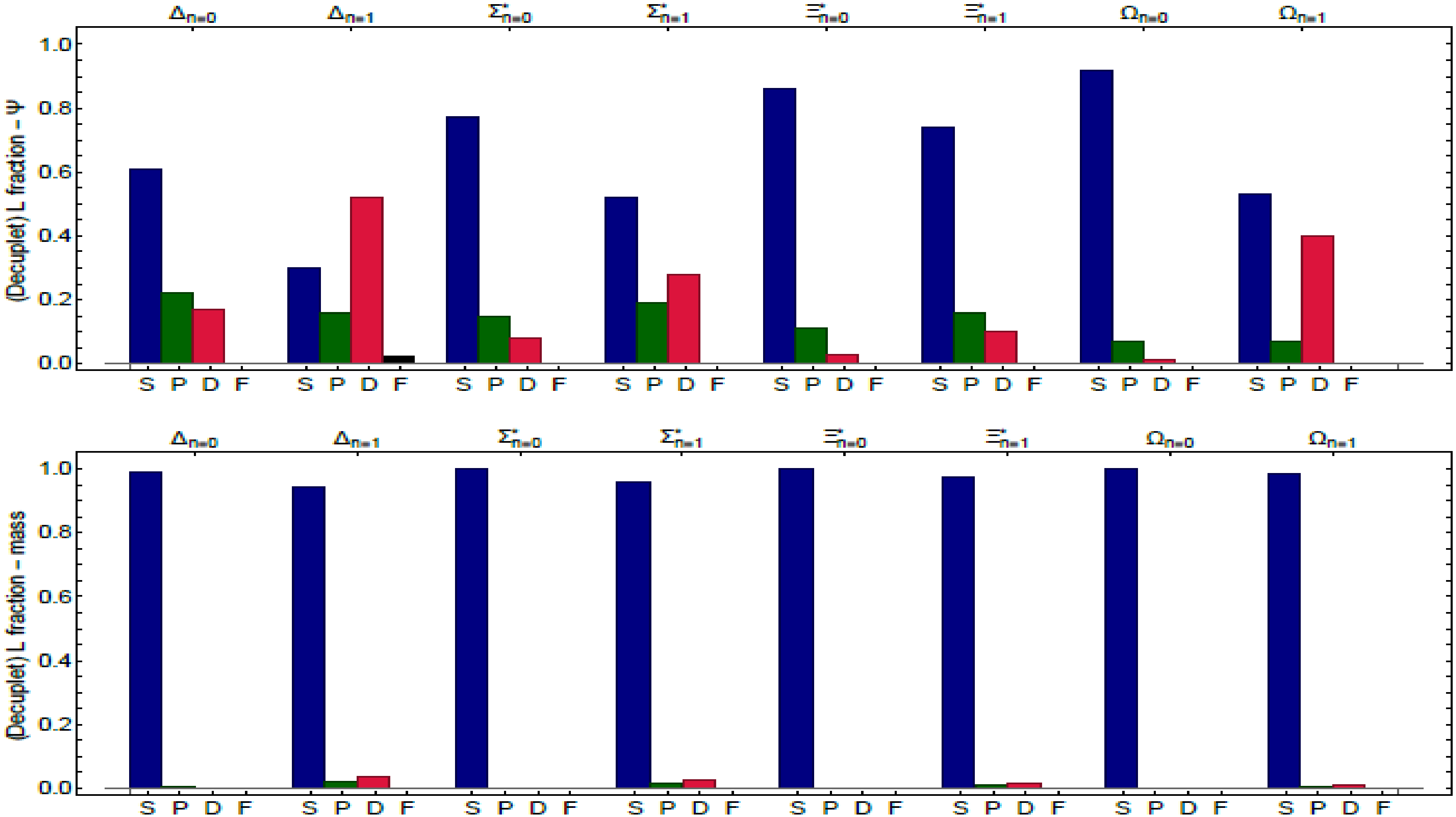


- Upper: Computed from the wave functions directly.

  Lower: Computed from the relative contributions to the masses.

- Both measures deliver the same qualitative picture of each baryon's internal structure. So there is little mixing between partial waves in the computation of a baryon's mass.

Rest-frame orbital angular momentum (Decuplet)



- Upper: Computed from the wave functions directly.

  Lower: Computed from the relative contributions to the masses.

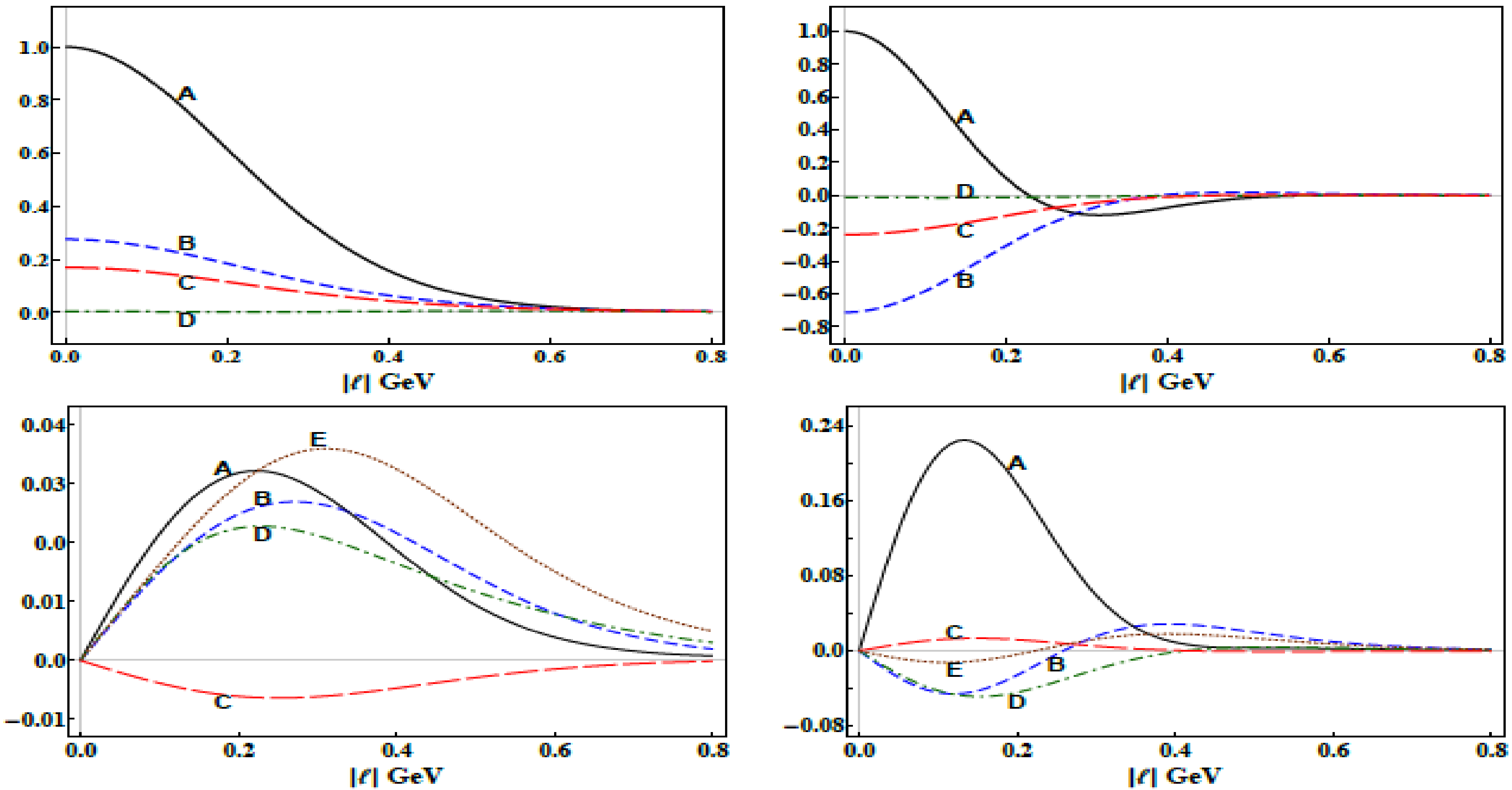
- In both panels that *S-wave* strength is shifted into *D-wave* contributions within decuplet positive-parity excitations.

Rest-frame orbital angular momentum

L content	$N_{n=0}$	$N_{n=1}$	$\Lambda_{n=0}$	$\Lambda_{n=1}$	$\Sigma_{n=0}$	$\Sigma_{n=1}$	$\Xi_{n=0}$	$\Xi_{n=1}$
S, P, D	1.19	1.73	1.37	1.85	1.41	1.88	1.58	1.99
$-, P, D$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$S, -, D$	1.24	1.71	1.40	1.83	1.42	1.84	1.59	1.97
$S, P, -$	1.20	1.74	1.37	1.85	1.41	1.89	1.58	1.99
$S, -, -$	1.24	1.71	1.40	1.83	1.42	1.84	1.59	1.97

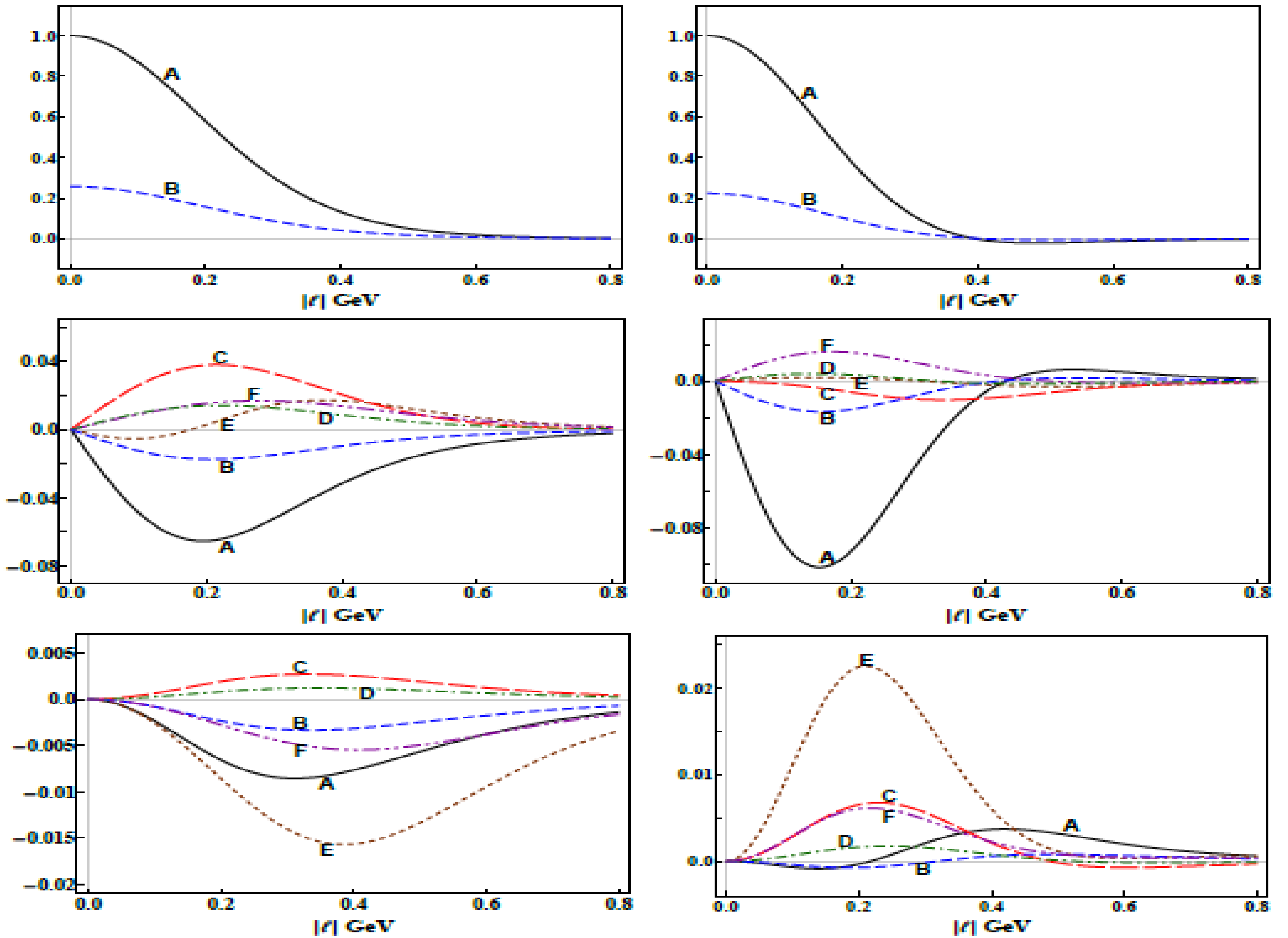
L content	$\Delta_{n=0}$	$\Delta_{n=1}$	$\Sigma^*_{n=0}$	$\Sigma^*_{n=1}$	$\Xi^*_{n=0}$	$\Xi^*_{n=1}$	$\Omega_{n=0}$	$\Omega_{n=1}$
S, P, D, F	1.35	1.79	1.52	1.93	1.71	2.08	1.93	2.23
$-, P, D, F$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$S, -, D, F$	1.36	1.75	1.52	1.90	1.71	2.06	1.93	2.22
$S, P, -, F$	1.35	1.82	1.52	1.95	1.71	2.09	1.93	2.24
$S, P, D, -$	1.35	1.79	1.52	1.93	1.71	2.08	1.93	2.23
$S, -, -, -$	1.35	1.80	1.52	1.93	1.71	2.08	1.93	2.23

Pointwise structure (Octet – Λ baryon)



- The zeroth Chebyshev moment of all *S-* and *P-wave* components in a given baryon's Faddeev wave function.
- Each projection for the ground-state is of a single sign (+ or -).
- First excitation: all *S-* and *P-wave* components exhibit a single zero at some point. It may be interpreted as the simplest *radial excitation* of its ground-state partner.

Pointwise structure (Decuptet – Ξ* baryon)



Part B - Summary & Outlook

- We computed the spectrum and Poincare-covariant wave functions for all favor-SU(3) octet and decuplet baryons and their first positive-parity excitations.
- Negative-parity diquarks are **negligible** in these positive-parity baryons.
- In its rest-frame, every system considered may be judged as primarily *S-wave* in character; and the first positive-parity excitation of each octet or decuplet baryon exhibits the characteristics of a radial excitation of the ground-state.
- Next: Negative-parity partners; Form factors & axial couplings; PDFs, PDAs, GPDs, TMDs...

Part A - Summary & Outlook

- By including all kinds of diquarks, we performed a comparative study of the four lightest baryon $(I=1/2,\ J^P=1/2^{\pm})$ isospin doublets in order to both elucidate their structural similarities and differences.
- The two lightest $(I=1/2,\ J^P=1/2^+)$ doublets are dominated by scalar and pseudovector diquarks; the associated rest-frame Faddeev wave functions are primarily S-wave in nature; and the first excited state in this $1/2^+$ channel has very much the appearance of a radial excitation of the ground state.
- In the two lightest $(I=1/2,\ J^P=1/2^-)$ systems, **TOO**, scalar and pseudovector diquarks play a material role. In their rest frames, the Faddeev amplitudes describing the dressed-quark cores of these negative-parity states contain roughly equal fractions of even and odd parity diquarks; the associated wave functions of these negative-parity systems are predominantly P-wave in nature, but possess measurable S-wave components; and, the first excited state in this negative parity channel has little of the appearance of a radial excitation.
- NEXT: $N(1720)3/2^+$, $N(1520)3/2^-$, form factors, PDAs, PDFs, …

Thank you!

Rest-frame orbital angular momentum

L content	$N_{n=0}$	$N_{n=1}$	$\Lambda_{n=0}$	$\Lambda_{n=1}$	$\Sigma_{n=0}$	$\Sigma_{n=1}$	$\Xi_{n=0}$	$\Xi_{n=1}$
S,P,D	1.19	1.73	1.37	1.85	1.41	1.88	1.58	1.99
$-,P,D$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$S,-,D$	1.24	1.71	1.40	1.83	1.42	1.84	1.59	1.97
$S,P,-$	1.20	1.74	1.37	1.85	1.41	1.89	1.58	1.99
$S,-,-$	1.24	1.71	1.40	1.83	1.42	1.84	1.59	1.97

L content	$\Delta_{n=0}$	$\Delta_{n=1}$	$\Sigma^*_{n=0}$	$\Sigma^*_{n=1}$	$\Xi^*_{n=0}$	$\Xi^*_{n=1}$	$\Omega_{n=0}$	$\Omega_{n=1}$
S,P,D,F	1.35	1.79	1.52	1.93	1.71	2.08	1.93	2.23
$-,P,D,F$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$S,-,D,F$	1.36	1.75	1.52	1.90	1.71	2.06	1.93	2.22
$S,P,-,F$	1.35	1.82	1.52	1.95	1.71	2.09	1.93	2.24
$S,P,D,-$	1.35	1.79	1.52	1.93	1.71	2.08	1.93	2.23
$S,-,-,-$	1.35	1.80	1.52	1.93	1.71	2.08	1.93	2.23

- Upper: octet baryons
  Lower: decuplet baryons
- Every one of the systems considered is primarily *S-wave* in nature.
- *P-wave* components play a measurable role in octet ground-states & first positive-parity excitations: they are attractive in ground-states & repulsive in the excitations
- Decuplet systems: the ground-state masses are almost completely insensitive to non-S-wave components; and in the first positive parity excitations, *P-wave* components generate a little repulsion, some attraction is provided by *D-waves*.

QCD-kindred model

- **The dressed-quark propagator**

$$S(p) = -i\gamma \cdot p\sigma_V(p^2) + \sigma_S(p^2)$$

- **algebraic form:**

$$\bar{\sigma}_S(x) = 2\bar{m}\mathcal{F}(2(x+\bar{m}^2)) + \mathcal{F}(b_1 x)\mathcal{F}(b_3 x)[b_0 + b_2\mathcal{F}(\epsilon x)], \tag{A3a}$$

$$\bar{\sigma}_V(x) = \frac{1}{x+\bar{m}^2}[1-\mathcal{F}(2(x+\bar{m}^2))], \tag{A3b}$$

with $x = p^2/\lambda^2$, $\bar{m} = m/\lambda$,

$$\mathcal{F}(x) = \frac{1-e^{-x}}{x}, \tag{A4}$$

$\bar{\sigma}_S(x) = \lambda\sigma_S(p^2)$ and $\bar{\sigma}_V(x) = \lambda^2\sigma_V(p^2)$. The mass scale, $\lambda = 0.566$ GeV, and parameter values,

$\bar{m}$	b_0	b_1	b_2	b_3
0.00897	0.131	2.90	0.603	0.185

(A5)

associated with Eq. (A3) were fixed in a least-squares fit to light-meson observables [79,80]. [$\epsilon = 10^{-4}$ in Eq. (A3a) acts only to decouple the large- and intermediate-p^2 domains.]

QCD-kindred model

➢ The dressed-quark propagator

$$S(p) = -i\gamma \cdot p\sigma_V(p^2) + \sigma_S(p^2)$$

- Based on solutions to the gap equation that were obtained with a dressed gluon-quark vertex.
- Mass function has a real-world value at p^2 = 0, NOT the highly inflated value typical of ~~RL~~ truncation.
- Propagators are entire functions, consistent with sufficient condition for confinement and completely unlike known results from ~~RL~~ truncation.
- Parameters in quark propagators were fitted to a diverse array of meson observables. **ZERO** parameters changed in study of baryons.
- Compare with that computed using the DCSB-improved gap equation kernel (DB). The parametrization is a sound representation numerical results, although simple and introdu long beforehand.

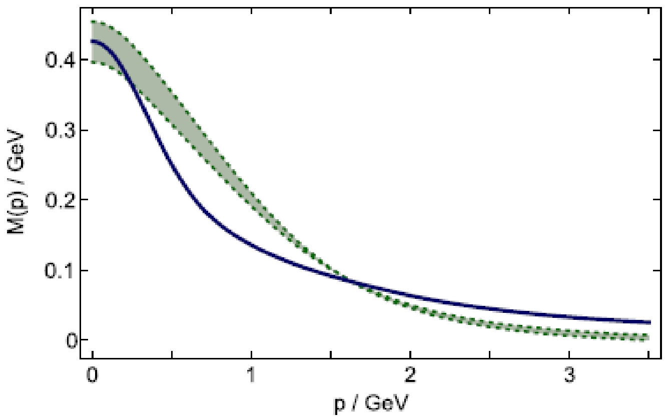


FIG. 6. Solid curve (blue)—quark mass function generated by the parametrization of the dressed-quark propagator specified by Eqs. (A3) and (A4) (A5); and band (green)—exemplary range of numerical results obtained by solving the gap equation with the modern DCSB-improved kernels described and used in Refs. [16,81–83].

QCD-kindred model

- **Diquark amplitudes: five types of correlation are possible in a *J=1/2* bound state: isoscalar scalar(I=0,J^P=0^+), isovector pseudovector, isoscalar pseudoscalar, isoscalar vector, and isovector vector.**
- **The LEADING structures in the correlation amplitudes for each case are, respectively (Dirac-flavor-color),**

$$\Gamma^{0^+}(k;K) = g_{0^+}\gamma_5 C\tau^2 \vec{H}\mathcal{F}(k^2/\omega_{0^+}^2),$$

$$\vec{\Gamma}_\mu^{1^+}(k;K) = i g_{1^+}\gamma_\mu C\vec{\tau}\vec{H}\,\mathcal{F}(k^2/\omega_{1^+}^2),$$

$$\Gamma^{0^-}(k;K) = i g_{0^-} C\tau^2 \vec{H}\mathcal{F}(k^2/\omega_{0^-}^2),$$

$$\Gamma_\mu^{1^-}(k;K) = g_{1^-}\gamma_\mu\gamma_5 C\tau^2 \vec{H}\mathcal{F}(k^2/\omega_{1^-}^2),$$

$$\vec{\Gamma}_\mu^{\bar{1}^-}(k;K) = i g_{\bar{1}^-}[\gamma_\mu, \gamma\cdot K]\gamma_5 C\vec{\tau}\vec{H}\,\mathcal{F}(k^2/\omega_{\bar{1}^-}^2),$$

- **Simple form. Just one parameter: diquark masses.**
- **Match expectations based on solutions of meson and diquark Bethe-Salpeter amplitudes.**

QCD-kindred model

- **The diquark propagators**

$$\Delta^{0^\pm}(K) = \frac{1}{m_{0^\pm}^2}\mathcal{F}(k^2/\omega_{0^\pm}^2),$$

$$\Delta_{\mu\nu}^{1^\pm}(K) = \left[\delta_{\mu\nu} + \frac{K_\mu K_\nu}{m_{1^\pm}^2}\right]\frac{1}{m_{1^\pm}^2}\mathcal{F}(k^2/\omega_{1^\pm}^2).$$

- **The *$\mathcal{F}$-functions*: Simplest possible form that is consistent with infrared and ultraviolet constraints of confinement (IR) and 1/q^2 evolution (UV) of meson propagators.**

- **Diquarks are confined.**
  - **free-particle-like at spacelike momenta**
  - **pole-free on the timelike axis**
  - **This is NOT true of ~~RL~~ studies. It enables us to reach arbitrarily high values of momentum transfer.**

QCD-kindred model

- **The Faddeev amplitudes:**

$$\begin{aligned}\psi^{\pm}(p_i,\alpha_i,\sigma_i) = {} & [\Gamma^{0^+}(k;K)]^{\alpha_1\alpha_2}_{\sigma_1\sigma_2}\Delta^{0^+}(K)[\boxed{\varphi^{\pm}_{0^+}(\ell;P)}u(P)]^{\alpha_3}_{\sigma_3} \\ & + [\Gamma^{1^+j}_{\mu}]\Delta^{1^+}_{\mu\nu}[\varphi^{j\pm}_{1^+\nu}(\ell;P)u(P)] \\ & + [\Gamma^{0^-}]\Delta^{0^-}[\varphi^{\pm}_{0^-}(\ell;P)u(P)] \\ & + [\Gamma^{1^-}_{\mu}]\Delta^{1^-}_{\mu\nu}[\varphi^{\pm}_{1^-\nu}(\ell;P)u(P)], \end{aligned} \tag{9}$$

- **Quark-diquark vertices:**

$$\varphi^{\pm}_{0^+}(\ell;P) = \sum_{i=1}^{2} s^{\pm}_i(\ell^2,\ell\cdot P)\mathcal{S}^i(\ell;P)\mathcal{G}^{\pm},$$

$$\varphi^{j\pm}_{1^+\nu}(\ell;P) = \sum_{i=1}^{6} a^{j\pm}_i(\ell^2,\ell\cdot P)\gamma_5\mathcal{A}^i_{\nu}(\ell;P)\mathcal{G}^{\pm},$$

$$\varphi^{\pm}_{0^-}(\ell;P) = \sum_{i=1}^{2} p^{\pm}_i(\ell^2,\ell\cdot P)\mathcal{S}^i(\ell;P)\mathcal{G}^{\mp},$$

$$\varphi^{\pm}_{1^-\nu}(\ell;P) = \sum_{i=1}^{6} v^{\pm}_i(\ell^2,\ell\cdot P)\gamma_5\mathcal{A}^i_{\nu}(\ell;P)\mathcal{G}^{\mp},$$

where $\mathcal{G}^{+(-)} = \mathbf{I}_{\rm D}(\gamma_5)$ and

$$\mathcal{S}^1 = \mathbf{I}_{\rm D}, \qquad \mathcal{S}^2 = i\gamma\cdot\hat{\ell} - \hat{\ell}\cdot\hat{P}\mathbf{I}_{\rm D}$$

$$\mathcal{A}^1_{\nu} = \gamma\cdot\ell^{\perp}\hat{P}_{\nu}, \qquad \mathcal{A}^2_{\nu} = -i\hat{P}_{\nu}\mathbf{I}_{\rm D}, \qquad \mathcal{A}^3_{\nu} = \gamma\cdot\hat{\ell}^{\perp}\hat{\ell}^{\perp}_{\nu}$$

$$\mathcal{A}^4_{\nu} = i\hat{\ell}^{\perp}_{\nu}\mathbf{I}_{\rm D}, \quad \mathcal{A}^5_{\nu} = \gamma^{\perp}_{\nu} - \mathcal{A}^3_{\nu}, \quad \mathcal{A}^6_{\nu} = i\gamma^{\perp}_{\nu}\gamma\cdot\hat{\ell}^{\perp} - \mathcal{A}^4_{\nu},$$

QCD-kindred model

- **Both the Faddeev amplitude and wave function are Poincare covariant, i.e. they are qualitatively identical in all reference frames.**
- **Each of the scalar functions that appears is frame independent, but the frame chosen determines just how the elements should be combined.**
- **In consequence, the manner by which the dressed quarks' spin, *S*, and orbital angular momentum, *L*, add to form the total momentum *J*, is frame dependent: *L*, *S* are not independently Poincare invariant.**
- **The set of baryon rest-frame quark-diquark angular momentum identifications:**

$$^2S:\ \mathcal{S}^1, \mathcal{A}_\nu^2, (\mathcal{A}_\nu^3 + \mathcal{A}_\nu^5),$$

$$^2P:\ \mathcal{S}^2, \mathcal{A}_\nu^1, (\mathcal{A}_\nu^4 + \mathcal{A}_\nu^6),$$

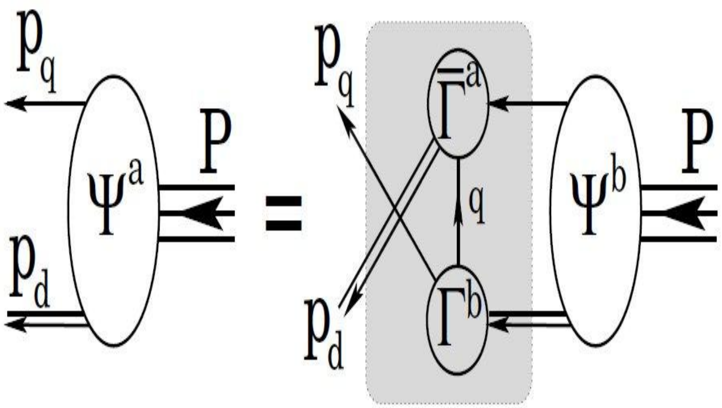
$$^4P:\ (2\mathcal{A}_\nu^4 - \mathcal{A}_\nu^6)/3,$$

$$^4D:\ (2\mathcal{A}_\nu^3 - \mathcal{A}_\nu^5)/3,$$

- **The scalar functions associated with these combinations of Dirac matrices in a Faddeev wave function possess the identified angular momentum correlation between the quark and diquark.**

Quark-diquark picture

- A baryon can be viewed as a Borromean bound-state, the binding within which has two contributions:
  - ✓ Formation of tight diquark correlations.
  - ✓ Quark exchange depicted in the shaded area.



- The exchange ensures that diquark correlations within the baryon are fully dynamical: no quark holds a special place.
- The rearrangement of the quarks guarantees that the baryon's wave function complies with Pauli statistics.
- Modern diquarks are different from the old static, point-like diquarks which featured in early attempts to explain the so-called missing resonance problem.
- The number of states in the spectrum of baryons obtained is similar to that found in the three-constituent quark model, just as it is in today's LQCD calculations.