Complex poles and spectral function of Yang-Mills theory

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in collaboration with K.-I. Kondo: based on
Y. H. and K.-I. Kondo, arXiv:1812.03116 (to appear in PRD),

—— see also

K.-I. Kondo, M. Watanabe, Y.H., R. Matsudo, Y. Suda, arXiv:1902.08894

Introduction

Color Confinement: the central feature of the strong interactions
Colored particles, especially quarks and gluons, are absent in the observed spectrum.

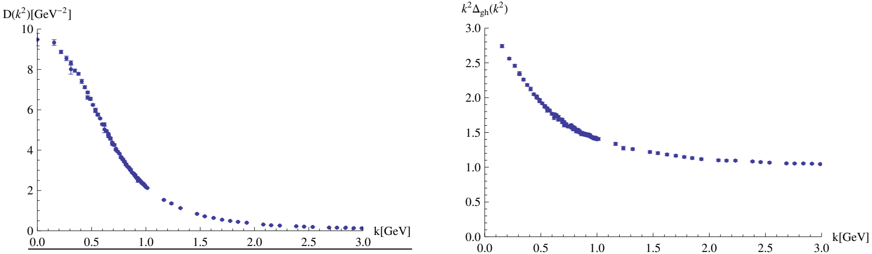
To investigate the gluon confinement, we consider the Landau-gauge gluon propagator.

Introduction: decoupling solution

In the Landau gauge, the gluon and ghost propagators $D(k^2), \Delta_{gh}(k^2)$ take the forms,

$$D^{AB}_{\mu\nu}(k) = \delta^{AB} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) D(k^2), \ \Delta^{AB}_{gh}(k) = \delta^{AB} \Delta_{gh}(k^2)$$

Recent lattice studies support the decoupling solution: massive gluon and massless ghost.[1]



[1]Figures: $SU(3)$ Yang-Mills, $\beta = 6.3$, $L = 128$, A.G. Duarte, O. Oliveira, and P.J. Silva, Phys. Rev. D**94** (2016) 014502. arXiv:1605.00594 [hep-lat]

Introduction: spectral representation

A physical particle: Källén-Lehmann spectral representation, having singularities only on the time-like momentum (if analytically continued to k^2 complex plane),

$$D(k^2) = \int_0^\infty d\sigma^2 \frac{\rho(\sigma^2)}{\sigma^2 - k^2},$$

follows from

- Poincaré invariance
- spectral condition : positive-definiteness of $P^\mu P_\mu$ and P^0
- completeness of the state space: $1 = \sum_n |n\rangle \langle n|$

The spectral function

$$\rho(\sigma^2) = \frac{1}{\pi} \operatorname{Im} D(\sigma^2 + i\epsilon).$$

contains "kinematic information".

Introduction: complex poles

A confined particle can have other analytic structure
e.g. Gribov-Zwanziger model predicts the gluon propagator with a pair of complex conjugate poles

$$D(k^2) = -\frac{k^2}{k^4 + \gamma^4}$$

In contrary, such “one-gluon state” should be excluded from a physical subspace via *some* confinement mechanism, since existence of complex poles invalidates the Källén-Lehmann spectral representation. The existence of complex poles can be a signal of confinement.
$\rightarrow$ Consider the possibility of complex poles!

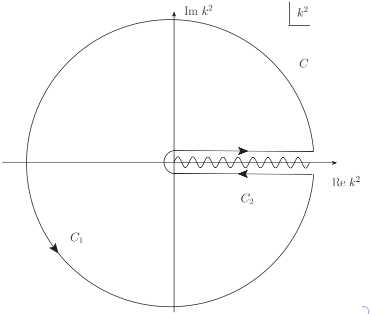
Plan

Spectral representation from analyticity

The Källén-Lehmann spectral representation can be rederived by the following assumptions

1. $D(z)$ is holomorphic except singularities on the positive real axis.
2. $D(z) \to 0$ as $|z| \to \infty$.
3. $D(z)$ is real on the negative real axis.

$$D(k^2) = \frac{1}{2\pi i} \oint_C d\zeta \frac{D(\zeta)}{\zeta - k^2}$$
$$= \int_0^\infty d\sigma^2 \frac{\rho(\sigma^2)}{\sigma^2 - k^2},$$
$$\rho(\sigma^2) := \frac{1}{\pi} \mathrm{Im}\, D(\sigma^2 + i\epsilon).$$



Generalization

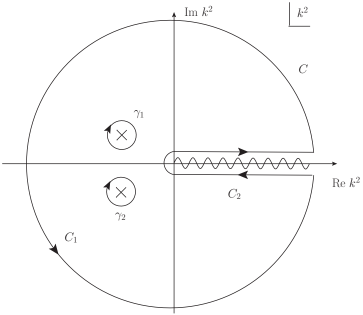
Generalization to the case in the presence of complex simple poles[2]

1. $D(z)$ is holomorphic except singularities on the positive real axis and a finite number of simple poles.
2. $D(z) \to 0$ as $|z| \to \infty$.
3. $D(z)$ is real on the negative real axis.

$$D(k^2) = \int_0^\infty d\sigma^2 \frac{\rho(\sigma^2)}{\sigma^2 - k^2} + \sum_{\ell=1}^{n} \frac{Z_\ell}{z_\ell - k^2},$$

$$\rho(\sigma^2) := \frac{1}{\pi} \mathrm{Im}\ D(\sigma^2 + i\epsilon),$$

$$Z_\ell := \oint_{\gamma_\ell} \frac{d\zeta}{2\pi i} D(\zeta).$$



[2]F. Siringo, EPJ Web Conf. **137**, 13017 (2017) arXiv:1606.03769 [hep-ph]

Note: a generalized sum rule for the gluon propagator with complex poles

In the presence of complex poles, the superconvergence relation[3]

$$\int_0^\infty d\sigma^2 \rho(\sigma^2) = 0$$

does not hold generically. Instead, the RG analysis from the asymptotic freedom and the negativity of the gluon anomalous dimension yields

$$\lim_{|k^2| \to \infty} k^2 D(k^2) = 0,$$

and therefore,

$$\sum_{\ell=1}^{n} Z_\ell + \int_0^\infty d\sigma^2 \rho(\sigma^2) = 0.$$

[3] R. Oehme and W. Zimmermann, Phys. Rev. D **21**, 471–484 (1980).

Note: a generalized sum rule for the gluon propagator with complex poles

In particular, for the gluon propagator with one pair of complex conjugate poles,

$$D(k^2) = \int_0^\infty d\sigma^2 \frac{\rho(\sigma^2)}{\sigma^2 - k^2} + \frac{Z}{(v+iw) - k^2} + \frac{Z^*}{(v-iw) - k^2},$$

we obtain the modified sum rule,

$$2 \operatorname{Re} Z + \int_0^\infty d\sigma^2 \rho(\sigma^2) = 0.$$

Conversely, the violation of the Oehme-Zimmermann superconvergence relation indicates the existence of complex poles or other singularities.

Spectral function and the number of complex poles

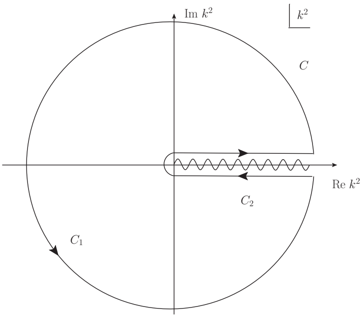
Argument principle for a propagator $D(k^2)$:

$$N_W(C) := \frac{1}{2\pi i} \oint_C dk^2 \frac{D'(k^2)}{D(k^2)} = N_Z - N_P.$$

$\rightarrow$ relations between the spectral function, $\rho(\sigma^2) = \frac{1}{\pi} \operatorname{Im} D(\sigma^2 + i\epsilon)$, and the number of complex poles.

(with suitable assumptions)

- (I) Positive spectral function $\rightarrow N_P = N_Z$,
- (II) Negative spectral function $\rightarrow N_P = 2 + N_Z$: $2 + N_Z$ complex poles



etc.

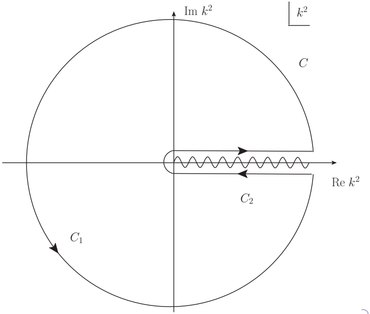
Case (I) Positive spectral function

Suppose that a propagator exhibits the following behaviors.

1. The propagator has the leading asymptotic behavior: $D(z) \sim -\frac{1}{z}\tilde{D}(z)$ as $|z| \to \infty$, where $\tilde{D}(z)$ is a real and positive function $\tilde{D}(z) > 0$ for large $|z|$.
2. $\rho(\sigma^2) > 0$, *i.e.*, $\text{Im}\, D(\sigma^2 + i\epsilon) > 0$ for $\sigma^2 > 0$.
3. $D(k^2 = 0) > 0$.

Assumption (1) $\rightarrow$

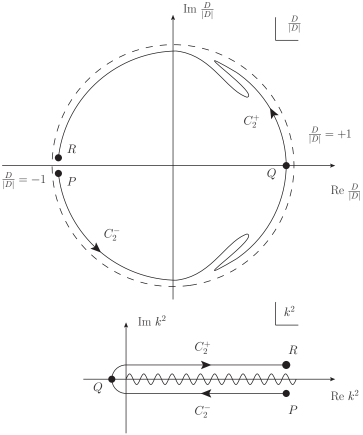
$$N_W(C_1) := \frac{1}{2\pi i}\int_{C_1} dk^2 \frac{D'(k^2)}{D(k^2)} = -1.$$



Case (I) Positive spectral function

Assumption (1) :
$D(k^2 \to +\infty) \to -0$,
Assumption (3) :
$D(k^2 = 0) > 0$, and
Assumption (2) :
$\rho(\sigma^2) = \mathrm{Im}\, D(\sigma^2 + i\epsilon) > 0$
Then, the winding number of the phase reads

$$N_W(C_2) := \frac{1}{2\pi i} \int_{C_2} dk^2 \frac{D'(k^2)}{D(k^2)}$$
$$= +1.$$



Case (I) Positive spectral function

1. The propagator has the leading asymptotic behavior: $D(z) \sim -\frac{1}{z}\tilde{D}(z)$ as $|z| \to \infty$, where $\tilde{D}(z)$ is a real and positive function $\tilde{D}(z) > 0$ for large $|z|$.
2. $\rho(\sigma^2) > 0$, *i.e.*, $\mathrm{Im}\, D(\sigma^2 + i\epsilon) > 0$ for $\sigma^2 > 0$.
3. $D(k^2 = 0) > 0$.

$$\begin{aligned} \Longrightarrow N_Z - N_P &= N_W(C) \\ &= N_W(C_1) + N_W(C_2) \\ &= -1 + 1 = 0 \end{aligned}$$

It is consitent with the physical case, where $N_Z = N_P = 0$.

Case (II) Negative spectral function

Suppose

1. The propagator has the leading asymptotic behavior: $D(z) \sim -\frac{1}{z}\tilde{D}(z)$ as $|z| \to \infty$, where $\tilde{D}(z)$ is a real and positive function $\tilde{D}(z) > 0$ for large $|z|$.
2. $\rho(\sigma^2) < 0$, *i.e.*, $\mathrm{Im}\, D(\sigma^2 + i\epsilon) < 0$ for $\sigma^2 > 0$.
3. $D(k^2 = 0) > 0$.

Then, as before,

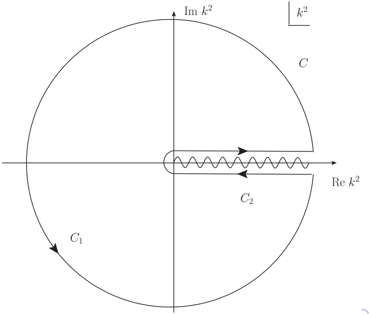
$$N_W(C_1) = -1,$$

but, due to $\rho(\sigma^2) < 0$,

$$N_W(C_2) = -1,$$

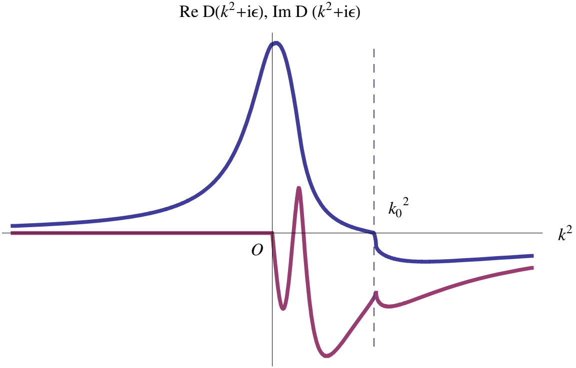
leads to

$$N_Z - N_P = N_W(C) = -2$$



Note: "negativity in a weak sense"

To have $N_W(C) = -2$, the following "negativity in a weak sense" is enough instead of the assumption (2) $\rho(\sigma^2) < 0$ for $\sigma^2 > 0$:
$\rho(k_0^2) < 0$ for all real positive zeros $k_0^2 > 0$ of $\mathrm{Re}\, D(k^2)$,
$\mathrm{Re}\, D(k_0^2) = 0$



Generalization

We define "positivity and negativity in a weak sense" as follows.

- quasi-positive
  A spectral function $\rho(\sigma^2)$ is quasi-positive if and only if $k^2 > 0 \wedge \mathrm{Re}\, D(k^2) = 0 \Rightarrow \rho(k^2) > 0$, i.e., the spectral function is positive at all time-like zeros of $\mathrm{Re}\, D$.
- quasi-negative
  A spectral function $\rho(\sigma^2)$ is quasi-negative if and only if $k^2 > 0 \wedge \mathrm{Re}\, D(k^2) = 0 \Rightarrow \rho(k^2) < 0$, i.e., the spectral function is negative at all time-like zeros of $\mathrm{Re}\, D$.

Although they are sufficient, not necessary, assumption to establish the generalized claim, the classification covers many spectral functions.
e.g. If $\mathrm{Re}\, D(k^2)$ has one zero on $k^2 > 0$, a spectral function is quasi-positive or quasi-negative.

Case (I') Quasi-positive spectral function

1. The propagator has the leading asymptotic behavior: $D(z) \sim -\frac{1}{z}\tilde{D}(z)$ as $|z| \to \infty$, where $\tilde{D}(z)$ is a real and positive function $\tilde{D}(z) > 0$ for large $|z|$.
2. The spectral function $\rho(\sigma^2)$ is quasi-positive, *i.e.*, $\mathrm{Im}\, D(\sigma^2 + i\epsilon) > 0$ for any $\sigma^2 > 0$ satisfying $\mathrm{Re}\, D(\sigma^2) = 0$.
3. $D(k^2 = 0) > 0$.

- $N_W(C_1) = -1$ as before,
- $N_W(C_2) = 1$, because the trajectory on the phase $S^1: \ D(k^2)/|D(k^2)|$ can be continuously deformed into one of the positive case.

$$N_Z - N_P = N_W(C) = 0.$$

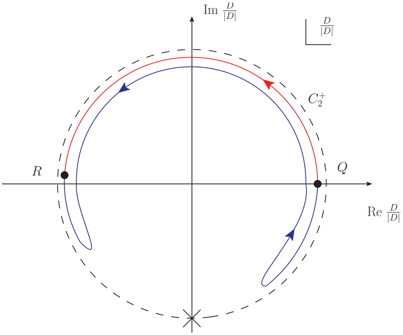
~~Example?: the numerical solution~~ of DSE on complex k^2 plane[4]

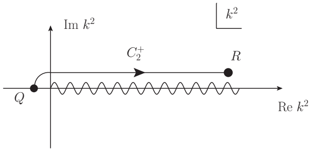
[4] S. Strauss, C. S. Fischer, and C. Kellermann, Phys. Rev. Lett. **109**, 252001 (2012)

Case (I') Quasi-positive spectral function: $N_W(C_2) = 1$

Let us look into the upper part of C_2, from Q ($k^2 = 0$, $D/|D| = +1$) to R ($k^2 = +\infty + i\epsilon$, $D/|D| \to -1$). Since the spectral function is quasi-positive, the trajectory of $D/|D|$ never passes through $D/|D| = -i$. The trajectory can then be continuously deformed into one of the positive case.

$$\implies N_W(C_2) = 1.$$





Case (II') Quasi-negative spectral function

1. The propagator has the leading asymptotic behavior: $D(z) \sim -\frac{1}{z}\tilde{D}(z)$ as $|z| \to \infty$, where $\tilde{D}(z)$ is a real and positive function $\tilde{D}(z) > 0$ for large $|z|$.
2. The spectral function $\rho(\sigma^2)$ is quasi-negative, *i.e.*, $\operatorname{Im} D(\sigma^2 + i\epsilon) < 0$ for any $\sigma^2 > 0$ satisfying $\operatorname{Re} D(\sigma^2) = 0$.
3. $D(k^2 = 0) > 0$.

- $N_W(C_1) = -1$ as before,
- $N_W(C_2) = -1$, because the trajectory $D(k^2)/|D(k^2)|$ can be continuously deformed into one of the negative case.

$$N_Z - N_P = N_W(C) = -2.$$

Example: massive Yang-Mills model and its related models[5]

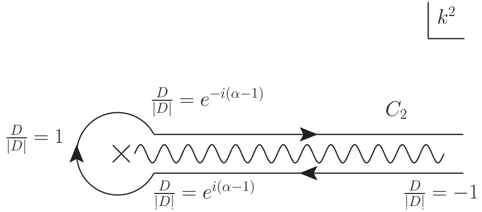
[5] M. Tissier and N. Wschebor, Phys.Rev. D**84**, 045018 (2011);M. Peláez, M. Tissier, and N. Wschebor, Phys. Rev. D**90**, 065031 (2014); F. Siringo, Nucl.Phys. B **907**, 572 (2016)

Note: scaling behavior yields negative contribution to $N_W(C)$

If the propagator has the scaling behavior $D(k^2) \to Z(-k^2)^{\alpha-1}$ $(1 > \alpha - 1 > 0)$, the integration around $k^2 = 0$ gives **negative** contribution to the winding number $N_W(C) = N_Z - N_P$.
C encloses $k^2 = 0$ **clockwise**

$\to$ the zero at $k^2 = 0$ gives negative contribution to the winding number

$$N_W(C) = \frac{1}{2\pi i} \oint_C dk^2 \frac{D'(k^2)}{D(k^2)}$$



Scaling behavior

For example, we can obtain the following proposition

1. The propagator has the leading asymptotic behavior: $D(z) \sim -\frac{1}{z}\tilde{D}(z)$ as $|z| \to \infty$, where $\tilde{D}(z)$ is a real and positive function $\tilde{D}(z) > 0$ for large $|z|$.
2. The spectral function $\rho(\sigma^2)$ is quasi-negative or quasi-positive.
3. $D(k^2) \to Z(-k^2)^{\alpha-1}$, where $Z > 0$ and $2 > \alpha > 1$.

Then,

$$N_Z - N_P = \begin{cases} 0 & (1 < \alpha < 1.5,\ \rho \text{ is quasi-positive}) \\ -2 & (1 < \alpha < 1.5,\ \rho \text{ is quasi-negative}) \\ -2 & (1.5 < \alpha < 2,\ \rho \text{ is quasi-positive}) \\ -2 & (1.5 < \alpha < 2,\ \rho \text{ is quasi-negative}). \end{cases}$$

Example: Gribov propagator

"Negative" spectral function $\to$ complex poles

Argument principle for a propagator $D(k^2)$:

$$N_W(C) := \frac{1}{2\pi i} \oint_C dk^2 \frac{D'(k^2)}{D(k^2)} = N_Z - N_P.$$

- Negative spectral function $\to N_W(C) = -2$, $N_P = 2 + N_Z$
- "Negativity of the spectral function in a weak sense": $N_W(C) = -2$ yields the existence of complex poles in $D(k^2)$.
- The Landau-gauge gluon propagator has a negative spectral function in UV: $\rho(\sigma^2) \to \rho_{UV}(\sigma^2) < 0, \sigma^2 \to \infty$.[6]
  $\to$ This relation, arising from the argument principle, implies the existence of complex poles in the Landau-gauge gluon propagator.

[6] R. Oehme and W. Zimmermann, Phys. Rev. D **21**, 471–484 (1980).

Massive Yang-Mills model: an effective theory of the Landau-gauge pure Yang-Mills theory

To capture the decoupling feature, we add the naive mass term to the Landau-gauge Yang-Mills theory ($\alpha \to 0$),

$$\mathcal{L}_{mYM} = -\frac{1}{4} F^A_{\mu\nu} F^{A\mu\nu} - \frac{1}{2\alpha}(\partial^\mu A^A_\mu)^2 + i\bar{c}^A \partial^\mu \mathcal{D}_\mu[A]^{AB} c^B + \frac{1}{2} M^2 A^A_\mu A^{A\mu}$$

The origin of the effective mass term can be related with Gribov ambiguity [7] , $A_\mu A_\mu$ condensation[8], or other non-perturbative effects.

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[7] In the absolute Landau gauge, the functional along a gauge orbit

$$F_A[U] := \int d^D x \, \mathrm{tr}(A^U_\mu A^U_\mu)$$

is minimized to pick up one configuration from a gauge orbit.

[8] H. Verschelde, K. Knecht, K. Van Acoleyen and M. Vanderkelen, Phys. Lett. B **516** 307 (2001) [arXiv:hep-th/0105018].

Massive Yang-Mills model: an effective theory of the Landau-gauge pure Yang-Mills theory

- Good accordance with the lattice results
- For some renormalization conditions and parameters, the running coupling has **no Landau pole in all scales**[9]
- Finite-temperature applications[10]
- It can reproduce both decoupling and (Gribov-type) scaling solution[11]
- also can be a probe to the radially-fixed gauge-scalar model[12]

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[9] M. Tissier and N. Wschebor, Phys.Rev. D**84**, 045018 (2011). arXiv:1105.2475 [hep-th]

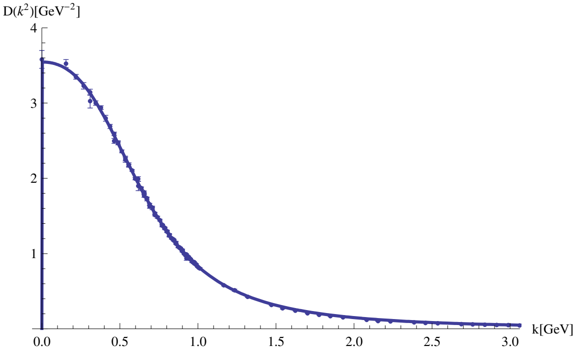
[10] U. Reinosa, J. Serreau, M. Tissier and N. Wschebor, Phys. Rev. D **89** 105016 (2014); Phys. Lett. B **742** (2015) 61; Phys. Rev. D **91** 045035 (2015); Phys. Rev. D **93** 105002 (2016).

[11] U. Reinosa, J. Serreau, M. Tissier, and N. Wschebor, Phys. Rev. D**96**, 014005 (2017). arXiv:1703.04041 [hep-th]

[12] K.-I. Kondo, Eur. Phys. J. C **78**, 577 (2018). arXiv:1804.03279 [hep-th]

Massive Yang-Mills model: an effective theory of the Landau-gauge pure Yang-Mills theory

Good accordance with the lattice results in the strict one-loop level:[13]



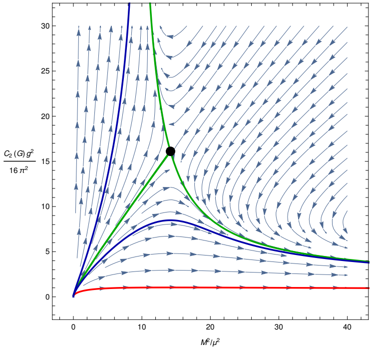
[13] $SU(3)$ Yang-Mills, $\beta = 6.3$, $L = 128$, rescaled due to some renormalization factor, A.G. Duarte, O. Oliveira, and P.J. Silva, Phys. Rev. D**94** (2016) 014502. arXiv:1605.00594 [hep-lat]; fitting parameters: $g = 4.1$, $M = 0.45$ GeV

Massive Yang-Mills model: Infrared-safety

In the renormalization condition respecting the non-renormalization theorems,[14]

$$\begin{cases} Z_A Z_C Z_{M^2} = 1 \\ \Gamma_A^{(2)}(k_E = \mu) = \mu^2 + M^2 \\ \Gamma_{gh}^{(2)}(k_E = \mu) = \mu^2 \\ Z_g \sqrt{Z_A} Z_C = 1 \end{cases}$$

this model has trajectories without Landau pole in all scale, especially the flow of the fitting parameters (red line).[15]

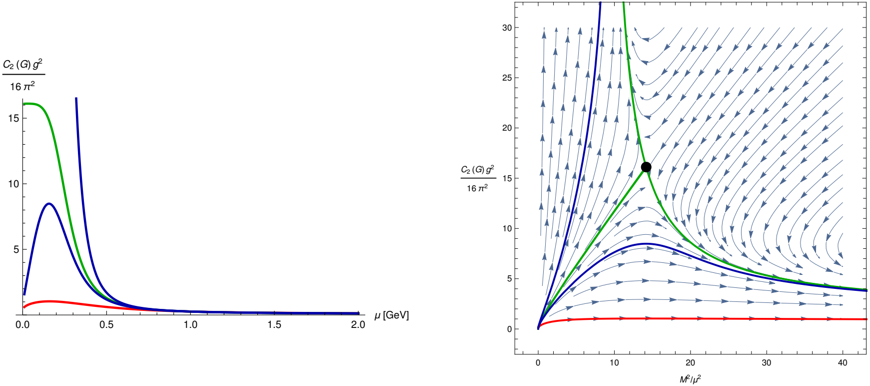


[1] D. Dudal, H. Verschelde and S. P. Sorella, Phys. Lett. B 555 126 (2003). N. Wschebor, Int. J. Mod. Phys. A **23** 2961 (2008)J. C. Taylor, Nucl. Phys. B **33** 436 (1971)

[2] M. Tissier and N. Wschebor, Phys.Rev. D**84**, 045018 (2011). arXiv:1105.2475 [hep-th]

Massive Yang-Mills model: decoupling and scaling

The infrared-safe trajectories exhibits the decoupling features. On the separatrix(green), the flow has non-trivial IR fixed point, and behave as Gribov-type scaling solution.[16]



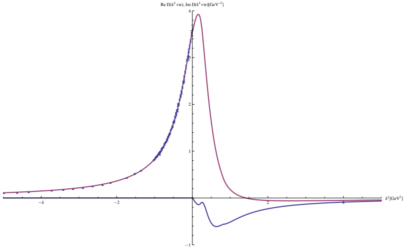
[16] U. Reinosa, J. Serreau, M. Tissier, and N. Wschebor, Phys. Rev. D**96**, 014005 (2017). arXiv:1703.04041 [hep-th]

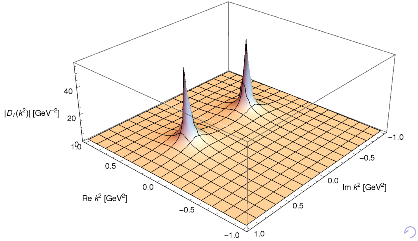
Analytic structure of the propagators in massive Yang-Mills model

In this model, for any parameters (g^2, M^2),

- the gluon propagator has a **negative spectral function**, and therefore **one pair of complex conjugate poles**, or "tachyonic" (real negative) poles with multiplicity two,
- the ghost propagator has no complex poles.

The RG improvement for a flow without Landau pole does not change these consequences.





Negative spectral function and one pair of complex conjugate poles of the transverse gluon propagator

At one-loop, we find for any (g^2, M^2),

$$\rho(\sigma^2) = \frac{1}{\pi} \operatorname{Im} D(\sigma^2 + i\epsilon) = -\frac{1}{\pi} \frac{\operatorname{Im} \Pi(\sigma^2 + i\epsilon)}{(M^2 - \sigma^2 + \operatorname{Re} \Pi)^2 + (\operatorname{Im} \Pi)^2} < 0.$$

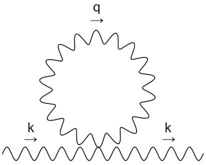
and $\Gamma_A^{(2)}(k^2) = \frac{1}{D(k^2)}$ is regular on the complex k^2 plane. Then $N_P - N_Z = 2$ and $N_Z = 0$, thus $N_P = 2$: one pair of complex conjugate poles, or "tachyonic" (real negative) poles with multiplicity two.

The two properties of $\operatorname{Im} D(\sigma^2 + i\epsilon) < 0$ and regularity of $\Gamma_A^{(2)}(k^2)$ still hold after the RG improvement in one-loop level, if the flow has no Landau pole.

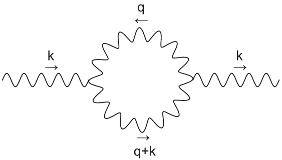
Origin of the negativity

Why $\rho(\sigma^2) < 0$ in this model?

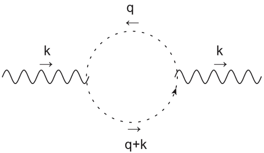
- In UV region: the negativity arises from the negativity of the gluon anomalous dimension, as in the pure Yang-Mills theory.
- In IR region: from the ghost-loop contribution.



(a)



(b)



(c)

Related topic: Violation of Reflection Positivity

If we consider Euclidean formulation as a starting point, the reconstruction of QFT requires the **Osterwalder-Schrader axioms**. One of the axioms is **reflection positivity**. The reflection positivity is violated in this model even in the transverse gluon sector, which is a signal of gluon confinement.
The reflection positivity for a 2-point Green function reads,

$$\int d^D x \int d^D y \; f^*(\vec{x}, -x_D) D(x-y) f(\vec{y}, y_D) \geq 0,$$

for any f, where $f(\vec{x}, x_D)$ is a complex valued test function with support in $\{(\vec{x}, x_D); x_D > 0\}$. Some limit of this inequality requires the positivity of the **Schwinger function** $\Delta(t)$,

$$\Delta(t) \geq 0,$$

$$\Delta(t) := \int d^{D-1}\vec{x} \; D(\vec{x}, t) = \int \frac{dk^D}{2\pi} e^{ik^D t} D(\vec{k} = 0, k^D)$$

Related topic: Violation of Reflection Positivity

The gluon propagator in the massive Yang-Mills model has a negative spectral function $\rho(\sigma^2) < 0$ and one pair of complex poles,

$$D(k^2) = \int_0^\infty d\sigma^2 \frac{\rho(\sigma^2)}{\sigma^2 - k^2} + \frac{Z}{(v+iw)-k^2} + \frac{Z^*}{(v-iw)-k^2},$$

This leads to violation of the reflection positivity, by simply evaluating the Schwinger function.[17]

$$\Delta(t) = \int \frac{dk_D}{2\pi} e^{ik_D t} D(\vec{k}=0, k_D)$$

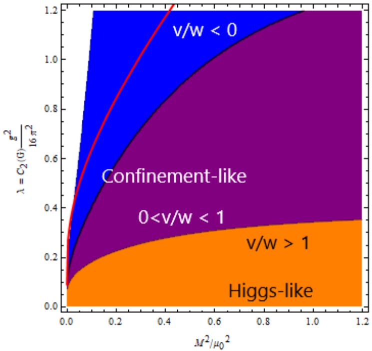
Since the contribution from the spectral function is negative for any t, and the one from the poles oscillates, the Schwinger function $\Delta(t) < 0$ for some $t > 0$.

[17] K.-I. Kondo, M. Watanabe, Y.H., R. Matsudo, Y. Suda, arXiv:1902.08894 [hep-th]

Massive Yang-Mills model in the parameter space

Since the gluon propagator has one pair of complex conjugate poles, we focus on the position of the complex conjugate poles $k^2 = v \pm iw, \ w \geq 0$ in the parameter space (g^2, M^2).

- No physical poles : gluons are always confined in this model
- Confinement-Higgs crossover
  $v \gg w$: particle-like gluon ("Higgs-like");
  otherwise no particle picture ("Confinement-like").
- Similar structure can appear in the radially-fixed (strong scalar self-coupling) gauge-scalar model
  cf.) Fradkin-Shenker continuity



Summary

- We have found the general relationships between the number of complex poles of a propagator and the sign of the spectral function by applying the **argument principle** to the propagator.
- In particular, "**Negativity** of the spectral function in a weak sense" yields the existence of **complex poles**.
  It is well-known that the Landau-gauge gluon propagator has a negative spectral function in UV.
- In the effective (massive Yang-Mills) model of the Landau-gauge Yang-Mills theory, the gluon propagator has a **negative** spectral function and **one pair of complex conjugate poles**.
  → Our results implies the existence of complex poles in the Landau-gauge gluon propagator, signaling gluon confinement.

Related topics and future works

- Quark loop contribution seems not to affect the conclusion $N_P = 2$ in the gluon propagator if $N_f < \frac{13}{4} C_2(G)$. Detailed analyses including the quark propagator: future work.
- Thermal contribution also seems not to affect the conclusion $N_P = 2$. Some analytic feature from the gluon propagator near the deconfinement temperature?
- Relation with center vortices?
- Kinematic origin of complex poles, the violation of the Källén-Lehmann spectral representation?
- Confinement mechanism eliminating states yielding complex poles?