Modelling Baryons within the NJL Framework

From Quarks to Baryons

Dominic Kraatz

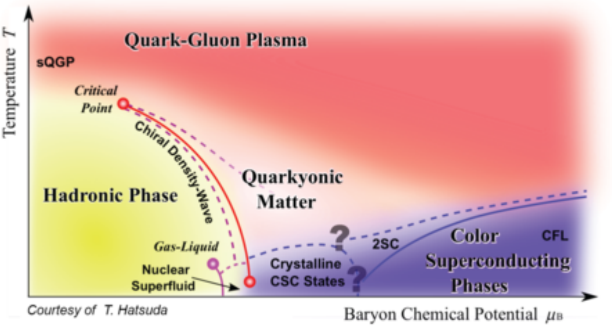
Lunch-Club Seminar, Gießen, July 17th, 2019

July, 17th 2019

Outline

1. Motivation and NJL model framework
   - NJL interaction Lagrangian
2. Particle-particle channel
   - Colour & flavour structure
   - Vacuum results (two flavour)
   - Medium results (two flavour)
   - Colour Superconducting and CFL Phase
3. Three particle interaction
   - Basic idea and assumptions
   - From the Dyson equation to the T-matrix
   - Baryon propagator in the medium
4. Summary
5. Current work & Outlook

Why? ⇒ QCD Phase Diagramm



[https://fair-center.de/de/fuer-nutzer/experimente/cbm/cbm/introduction.html]

Basics of the NJL model

- 1961 originally invented for nucleon-nucleon interaction by Giovanni Jona-Lasinio and Yoichiro Nambu
- later reinterpreted for QCD calculations
- can be motivated directly from QCD Lagrangian (colour-current interaction)
- basic properties
  - shares global symmetries of QCD
  - local four-point interactions
  - description of (spontaneous) chiral symmetry breaking, colour superconductivity, etc.
  - non-renormalizable
  - no confinement
- using Pauli-Villars regularisation method with two regulators

NJL interaction Lagrangian

Particle-Anitparticle channel

$$\mathcal{L}_{\bar{\psi}\psi} = g_s \sum_{a=0}^{N_f^2-1} \left[(\bar{\psi}\tau_a\psi)^2 + (\bar{\psi}i\gamma_5\tau_a\psi)^2 \right] + \dots$$

6-point interaction

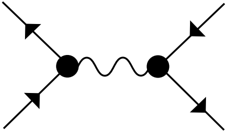
$$\mathcal{L}_I^{(6)} = K \left[\det{}_f \left(\bar{\psi}(1+\gamma_5)\psi \right) + \det{}_f \left(\bar{\psi}(1-\gamma_5)\psi \right) \right]$$

Fierz-Transformation - General Idea

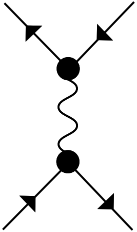
- local four point interaction:

$$\begin{aligned}
\mathcal{L}_{\text{int}} &= g_I(\bar{\psi}\Gamma^I\psi)^2 && (1)\\
&= g_I\Gamma^I_{\bar{i}j}\Gamma^I_{\bar{k}l}\bar{\psi}_{\bar{i}}\psi_j\bar{\psi}_{\bar{k}}\psi_l && (2)\\
&= -g_I\Gamma^I_{\bar{i}j}\Gamma^I_{\bar{k}l}\bar{\psi}_{\bar{i}}\psi_l\bar{\psi}_{\bar{k}}\psi_j && (3)\\
&= g_I\Gamma^I_{\bar{i}j}\Gamma^I_{\bar{k}l}\bar{\psi}_{\bar{i}}\bar{\psi}_{\bar{k}}\psi_l\psi_j && (4)
\end{aligned}$$

eq. (2) $\sim$ direct term:



eq. (3) $\sim$ exchange term:



- Fierz transformation: same interaction, different representations
- Hartree-approximation, neglect non-local contributions
- eq. (4) is related to the particle-particle interaction

NJL interaction Lagrangian

CRC-TR 211

Particle-Anitparticle channel

$$\mathcal{L}_{\bar{\psi}\psi} = g_s \sum_{a=0}^{N_f^2-1} \left[(\bar{\psi}\tau_a\psi)^2 + (\bar{\psi} i\gamma_5\tau_a\psi)^2 \right] + \dots$$

6-point interaction

$$\mathcal{L}_I^{(6)} = K \left[\det_f \left(\bar{\psi}(1+\gamma_5)\psi \right) + \det_f \left(\bar{\psi}(1-\gamma_5)\psi \right) \right]$$

Particle-Particle channel

$$\mathcal{L}_{\psi\psi} = h_s \Big[(\bar{\psi} C \tau_A \lambda_{A'} \bar{\psi}^T)(\psi^T C \tau_A \lambda_{A'} \psi) + (\bar{\psi} i\gamma_5 C \tau_A \lambda_{A'} \bar{\psi}^T)(\psi^T C i\gamma_5 \tau_A \lambda_{A'} \psi) \Big] + \dots$$

Before we can start...

... we have to fix the free parameters of the model ...

- particle-antiparticle channel four-point coupling g_s
- particle-antiparticle channel six-point coupling K
- particle-particle channel four-point coupling h_s
- cutoff parameter Λ

to the physical observables

- pion decay constant $f_\pi = 92.2$ MeV
- meson masses
  $m_\pi = 140$ MeV, $m_K = 495$ MeV, $m_{\eta,\eta'} = 550, 950$ MeV
- nucleon mass $m_N = 950$ MeV

Particle-particle channel
Colour & flavour structure

CRC-TR 211

Particle-Particle channel

$$\mathcal{L}_{\psi\psi} = h_s \Big[(\bar{\psi} C \tau_A \lambda_{A'} \bar{\psi}^T)(\psi^T C \tau_A \lambda_{A'} \psi) + (\bar{\psi} i\gamma_5 C \tau_A \lambda_{A'} \bar{\psi}^T)(\psi^T C i\gamma_5 \tau_A \lambda_{A'} \psi) \Big] + \dots$$

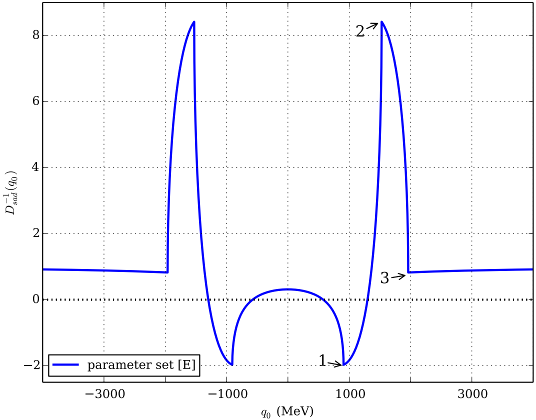
- flavour: $3 \otimes 3 = 6 \oplus \bar{3}$
- colour: $3 \otimes 3 = 6 \oplus \bar{3}$
- possible combinations respecting Pauli's exclusion principle:

	flavour	colour	$\Delta^D = \Gamma^D C$	J^P
scalar	3_A	3_A	$i\gamma_5$	0^+
pseudoscalar	3_A	3_A	$\mathbb{1}$	0^-
axial	6_S	3_A	γ^μ	1^+
vector	3_A	3_A	$\gamma_5\gamma^\mu$	1^-

Particle-particle channel
Vacuum results

- (inverse) diquark propagator $D_{sad}^{-1}(q_0) = 1 - 2h_s\Pi^{sad}(q_0)$ in rest-frame ($h_s = 1.05 g_s$)

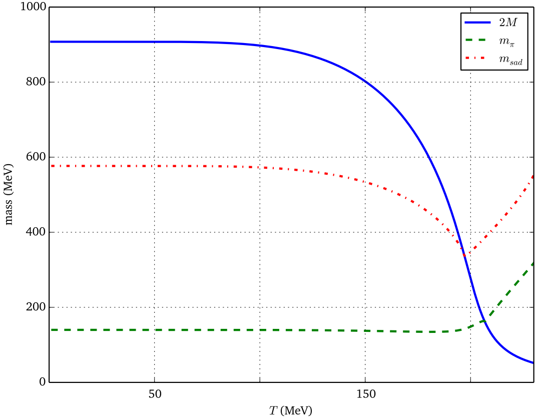
1. non-vanishing imaginary part $q_0 > 2M$ $\Rightarrow$ decay of diquark into quark-quark pair
2. first regulator
3. second regulator



Particle-particle channel
Medium results

- temperature behaviour of the scalar diquarks ($\mu = 0$ MeV)

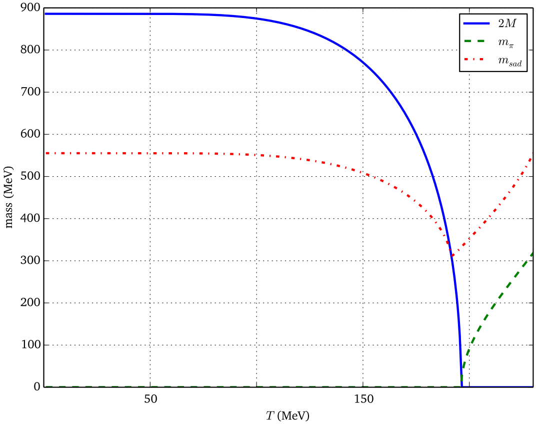
- melting temperature for scalar diquarks $T_m^D = 197$ MeV
- melting temperature for pions $T_m^\pi = 220$ MeV
- chiral limit ($m_u = m_d = 0$)



Particle-particle channel
Medium results

- temperature behaviour of the scalar diquarks ($\mu = 0$ MeV)

- melting temperature for scalar diquarks $T_m^D = 197$ MeV
- melting temperature for pions $T_m^\pi = 220$ MeV
- chiral limit ($m_u = m_d = 0$)



Colour Superconducting and CFL Phase

- **Cooper Theorem:** Fermionic systems with an attractive interaction can form pairs of fermions at no free energy cost
- assuming a non-vanishing diquark condensate $s_{AA'} = \langle \psi^T C \gamma_5 \tau_A \lambda_{A'} \psi \rangle$

  two flavours
  - $\tau_A = \sigma_2$ only

  $\Rightarrow$ flavour- and colour-space are independent (separate rotation within colour-space possible)

  three flavours
  - $\tau_A = \{\tau_2, \tau_5, \tau_7\}$ antisymmetric Gellmann-matrices (analogous to colour-space)

  $\Rightarrow$ flavour- and colour-space are not independent anymore

  $\Rightarrow$ flavour and colour state are locked for a certain condensate (CFL-phase)

Treatment of Colour Superconductivity - Some technical explanation

CRC-TR 211

0. starting point: Three-flavour NJL model with particle-particle interaction Lagrangian
1. define new spinors $\Psi^T = (\psi, \psi_C)^T$ (Nambu-Gorkov spinors)
   with $\psi_C = C\bar{\psi}^T$ and $\bar{\psi}_C = \psi^T C$
2. add chemical potential $(+\gamma_0 \mu)$
3. apply appropriate linearization method (MFA, Hubbard-Stratonovich)
4. rewrite original Lagrangian in terms of new spinors

$$\mathcal{L}(\Psi, \bar{\Psi}) = \bar{\Psi} S^{-1} \Psi - \mathcal{V}$$

Note: S^{-1} is a 72×72 matrix

Three particle interaction
Basic idea and assumptions (2 flavour case)

- quarks are the degrees of freedom within the NJL modelse
- description of baryons $\Leftrightarrow$ three-body problem

$\Rightarrow$ very complicated and computationally relatively expensive

$\rightarrow$ appropriate simplifications were applied:

1. two-flavour NJL model within isospin limit

$\Rightarrow$ proton and neutron are degenerate $\Rightarrow$ description of nucleon

2. no three-body irreducible diagrams (no six-point vertex)
3. consider only scalar diquarks $\Rightarrow$ neglect axialvector diquark contribution

- <u>for the moment</u>: consider quarks as distinguishable

Baryon modelling
Dyson equation for three-particle scattering

Dyson equation

$$D_B = D_0 + D_0 K D_0 + D_0 K D_0 K D_0 + \dots$$

$$D_B = D_0 + D_0 K D_B$$

- baryon propagator D_B
- non interacting three-particle propagator D_0
- without three-body interactions:

  Scattering kernel $K = k_1 + k_2 + k_3$

$\Rightarrow$ $D_B = D_0 + \sum D_i$ with $D_i := D_0 k_i D_B$ (Faddeev-components)

- Dyson equation for two-particle correlations (diquarks) in three-particle Hilbert space

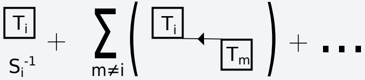
$$d_i = D_0 + D_0 k_i d_i \Longleftrightarrow d_i = D_0 + D_0 \tilde{T}_i D_0$$

Baryon modelling
Dyson equation for three-particle scattering

CRC-TR 211

T-matrix for Faddeev components

$$\tau_i = \tilde{T}_i + \tilde{T}_i D_0(\tau_j + \tau_k) =$$



- first term: free propagation of a diquark and a quark
- following terms: T-matrix in the 2+1 Hilbert space

⇒ T-matrix in three-particle Hilbert space becomes the S-matrix in the 2+1 particle Hilbert space

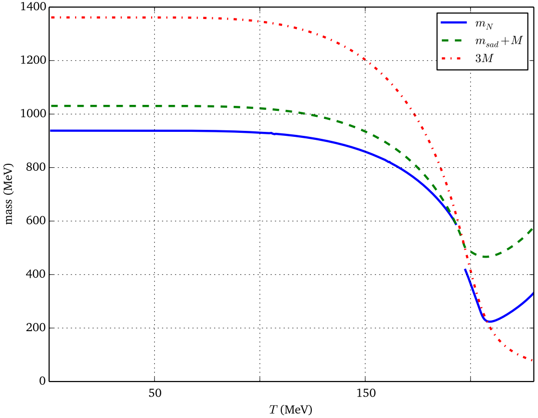
- overview of the following steps:
  1. obtain momentum space representation of $\tau = \sum_{i=1,2,3} \tau_i$
  2. identify BS-like equation for diquark-quark scattering process

  ⇒ identify effective (baryon) scattering kernel and "polarisation loop"

  3. transform results to indistinguishable quarks

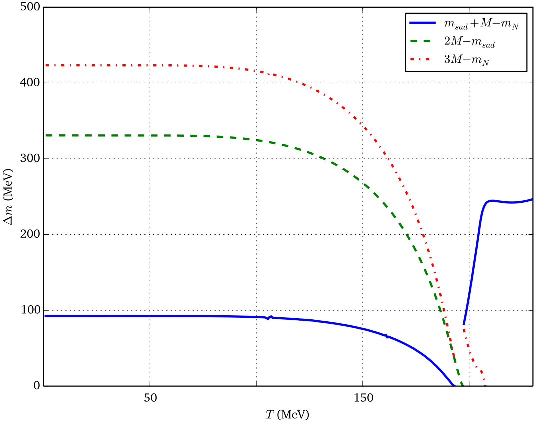
Nucleon in hot matter ($\mu = 0$)

- diquarks melt $\overset{?}{\Rightarrow}$ nucleon melts
- nucleon melts BUT become stable again
- binding energy of nucleon:
- indeed nucleon stable with unstable diquark $\Rightarrow$ Borromean-state
- CAREFUL: methods are not fully reliable for unstable particles



Nucleon in hot matter ($\mu = 0$)

- diquarks melt $\overset{?}{\Rightarrow}$ nucleon melts
- nucleon melts BUT become stable again
- binding energy of nucleon:
- indeed nucleon stable with unstable diquark $\Rightarrow$ Borromean-state
- CAREFUL: methods are not fully reliable for unstable particles



Summary

- NJL Lagrangian can be motivated through the colour-current interaction $\Rightarrow$ effective model of QCD to studie vacuum and medium behaviours of certain particles
  - Fierz-transformation to obtain mesonic and diquark channels
  - 3 flavours: explicit breaking of $U_A(1)$ by 6-point interaction Lagrangian
- diquarks in the (colour) antitriplet configuration are of particular interest for baryon modelling
- Baryon as a bound state of a diquark-quark scattering process
  - starting point: Dyson equation in three-particle Hilbert-space
  - separability of the two particle scattering
    $\Longrightarrow$ T-matrix in three particle Hilbert-space becomes S-matrix in $2+1$ particle Hilbert-space
  - static approximation: exchanged momentum much smaller than the typical quark-mass
    $\Longrightarrow$ point-like interaction of diquark-quark interaction
  - behaviour of inverse nucleon propagator indicates that in hot matter the nucleon becomes a Borromean-like state

Main Goals and Questions for PhD Studies

- modeling baryons in the three-flavour NJL model
- investigate three-body states within this framework
  - understand Borromean-like states $\Rightarrow$ allow diquark decay explicitly
  - what happens in the CFL/2SC phase with three-body bound states?

Thank you for your attention

References I

R. Alkofer and H. Reinhardt
Chiral quark dynamics
Springer, Lecture Notes in Physics Monographs, 1995

U. Vogl and W. Weise
The Nambu and Jona-Lasinio Model: Its Implications of Hadrons and Nuclei
Progress in Particle and Nuclear Physics, 27:195-272, 1991.

M. Buballa
NJL-model analysis of dense quark matter
Phys. Rept., 407:205-376, 2005

J. M. Torres-Rincon, B. Sintes, J. Aichelin
Flavor dependence of baryon melting temperature in effective models of QCD
Phys. Rev., C91(6):065206, 2015

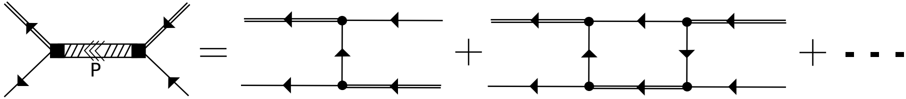
E. Blanquier
The Polyakov, Nambu and Jona-Lasinio model and its applications to describe the sub-nuclear particles
hep-ph, arXiv:1510.07736, 2015

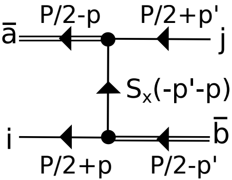
N. Ishii, W. Bentz, K. Yazaki
Baryons in the NJL model as solutions of the relativistic Faddeev equation
Nucl. Phys., A587:617-656, 1995

H. Basler
Goldstone Boson Condensation and Effects of the Axial Anomaly in Color Superconductivity
http://tuprints.ulb.tu-darmstadt.de/2399/, 2011

D. Kraatz (kraatz@itp.uni-frankfurt.de)

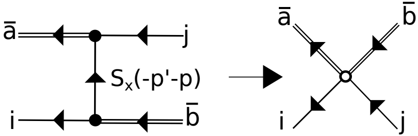
Baryon modelling
BS equation for diquark-quark scattering





- combination of two diquark scattering matrices
- static approximation: infinite heavy quark exchange

⇒ reduction to point-like interaction



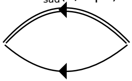
Baryon modelling

Roots of the inverse nucleon propagator

$$D_N^{-1}(P) = \mathbb{1} - \frac{2}{M} \int \frac{d^4 p''}{(2\pi)^4} S(P - p'') i t_{sad}(p'')$$

- nucleon "polarisation loop"

$$J^N(P) :=$$



$$= \int \frac{d^4 p''}{(2\pi)^4} S(P - p'') i t_{sad}(p'')$$

- no closed fermion loop ⇔ diquarks are bosons

⇒ complex structure, non-scalar function

- replace $t_{sad}(p'')$ with diquark propagator in pole-approximation

$$t_{sad} = -\frac{g_{Dqq}^2}{(p'')^2 - m_{sad}^2}$$

with effective diquark-quark coupling g_{Dqq}

Baryon modelling
Dyson equation for three-particle scattering

CRC-TR 211

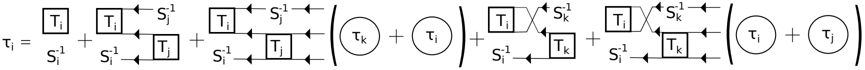
$$D_B = D_0 + D_0 K D_B$$
$$d_i = D_0 + D_0 k_i d_i \Leftrightarrow d_i = D_0 + D_0 T_i D_0$$

$\Rightarrow$ $\boxed{D_B = d_i + d_i(k_j + k_k) D_B}$

$\Rightarrow$ $\boxed{D_i = D_0 T_i D_0 + D_0 T_i (D_j + D_k)}$ for Faddeev components

- T-matrix for three-particle Hilbert space: $\tau = \sum \tau_i$

$$\tau_i = T_i + T_i D_0 (\tau_j + \tau_k)$$

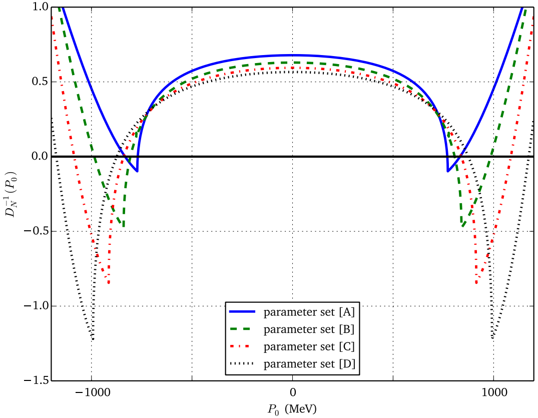


Vacuum investigation of $D_N^{-1}(P_0, \vec{P} = 0)$

Variation of parameter sets ($h_s = 1.1 g_s$)

- diquark mass is 500 ± 40 MeV for the different sets
- non-vanishing imaginary part for $P_0 > (m_{sad} + M)$
- inverse propagator shifts in the vertical axis
- region of stability stays more or less constant
- effective diquark-quark coupling enters quadratic

$\Rightarrow$ compensates the variation of M



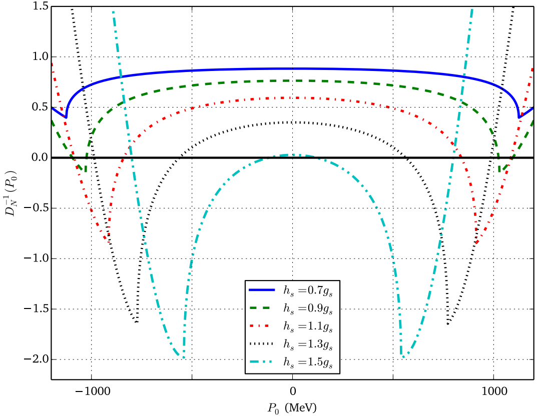
Vacuum investigation of $D_N^{-1}(P_0, \vec{P} = 0)$

CRC-TR 211

Variation of particle-particle coupling h_s (parameter set [D])

- diquarks near the point of instability ($m_{sad} \approx 2M$)

$\Rightarrow$ nucleon region of stability $\pm 3M$



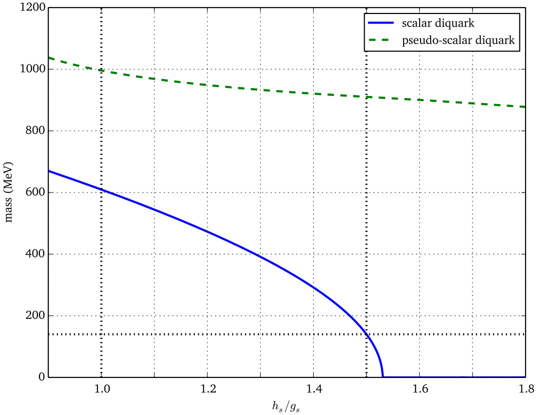
result: physical nucleon mass can not be found for all parameter sets.

Particle-particle channel
results for scalar and pseudoscalar diquarks

- masses of pseudoscalar and scalar diquarks in vacuum

- pseudoscalar diquarks are unstable and too heavy for baryon modelling
- scalar diquarks are much lighter ⇒ more valuable
- scalar diquarks obtain pion mass at $h_s = 1.5 g_s$

⇔ pseudo-scalar mesons and scalar diquarks are degenerated



Baryon modelling
Projectors

- only scalar Diquarks are considered
- flavour and colour space are antisymmetric

- <u>colour structure</u> in BS equation: $\lambda^{\bar{b}'}_{ix}\lambda^{\bar{a}}_{xj'}$

$\Rightarrow$ nucleon observable in nature $\Leftrightarrow$ colour singlet required

- $\lambda^{\bar{b}'}_{ix}\lambda^{\bar{a}}_{xj'} = i\epsilon^{\bar{a}}_{ix} i\epsilon^{\bar{b}'}_{xj'} = -\delta^{\bar{a}}_{i}\delta^{\bar{b}'}_{j'} + \delta^{\bar{a}\bar{b}'}\delta_{ij'}$

$\Rightarrow$ $\delta^{\bar{a}}_{i}\delta^{\bar{b}'}_{j'} \propto$ singlet state

$\Rightarrow$ $\delta^{\bar{a}\bar{b}'}\delta_{ij'} \propto$ octet state

- different pre-signs indicate character of interaction for singlet and octet state

- <u>flavour structure:</u> $\tau^{\bar{b}'}_{ix}\tau^{\bar{a}}_{xj'} = \delta_{ij'}$

- <u>Dirac structure:</u> $(\Gamma^D C)^{\bar{b}'}_{ix}(C\Gamma^D)^{\bar{a}}_{xj'} = (i\gamma_5 C)_{ix}(Ci\gamma_5)_{xj'} = \delta_{ij'}$

Dirac-like equation

$$\left(\not{P} A(P) - B(P)\right) X^{\text{nucl}}(P)\Big|_{P^2=m_B^2} = 0$$

$$A(P) := -i g_{Dqq}^2 \frac{2}{M}\left(I^0(P) - I^1(P)\right),$$

$$B(P) := i g_{Dqq}^2 M I^0(P) - \mathbb{1}$$

$$I^n(P) := \int \frac{\mathrm{d}^4 p''}{(2\pi)^4} \left(\frac{P p''}{P^2}\right)^n \frac{1}{(P-p'')^2 - M^2} \frac{1}{p''^2 - m_{sad}^2}, \qquad n = 0, 1$$

- closely related to BS equation

Particle-particle channel
Medium results ($T = 1$ MeV)

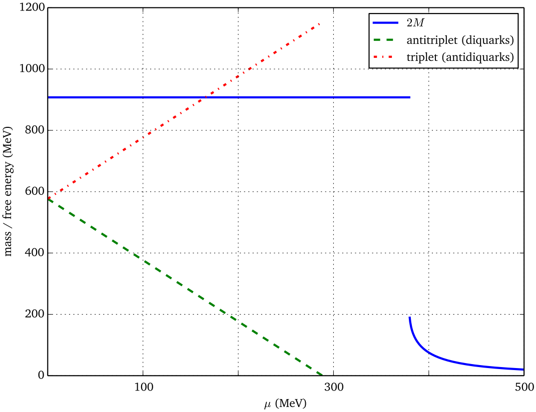
CRC-TR 211

$$D_D(q_0, \vec{q}, \mu) = \frac{1}{(q_0 + 2\mu)^2 - E_D^2}$$

$\Rightarrow$ poles at $q^{\pm}_{\text{Pole}} = \pm E_D - 2\mu$

$\Leftrightarrow$ poles are no longer the diquark mass

- $q^{\pm}_{\text{Pole}}$: excitation energy of the dense medium (free energy of the (anti)diquark)



silver-blaze property: diquark mass does not change until $\mu = m_{sad}/2$