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Competing phases of **interacting electrons** in **twisted bilayer graphene** and related **moiré heterostructures**

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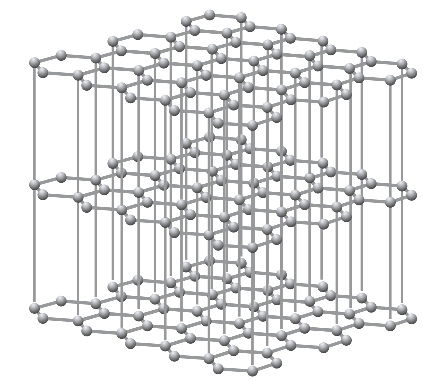
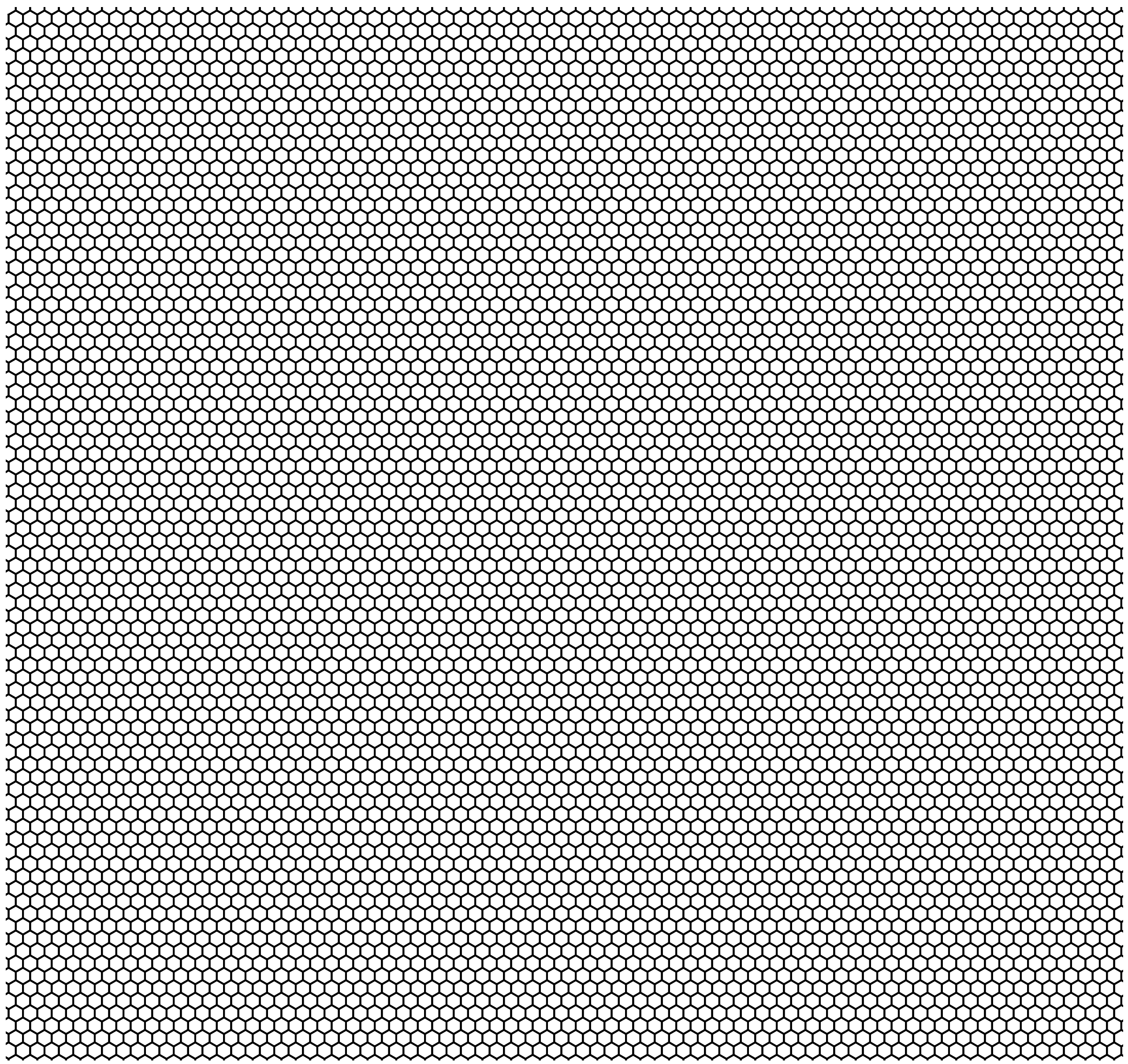
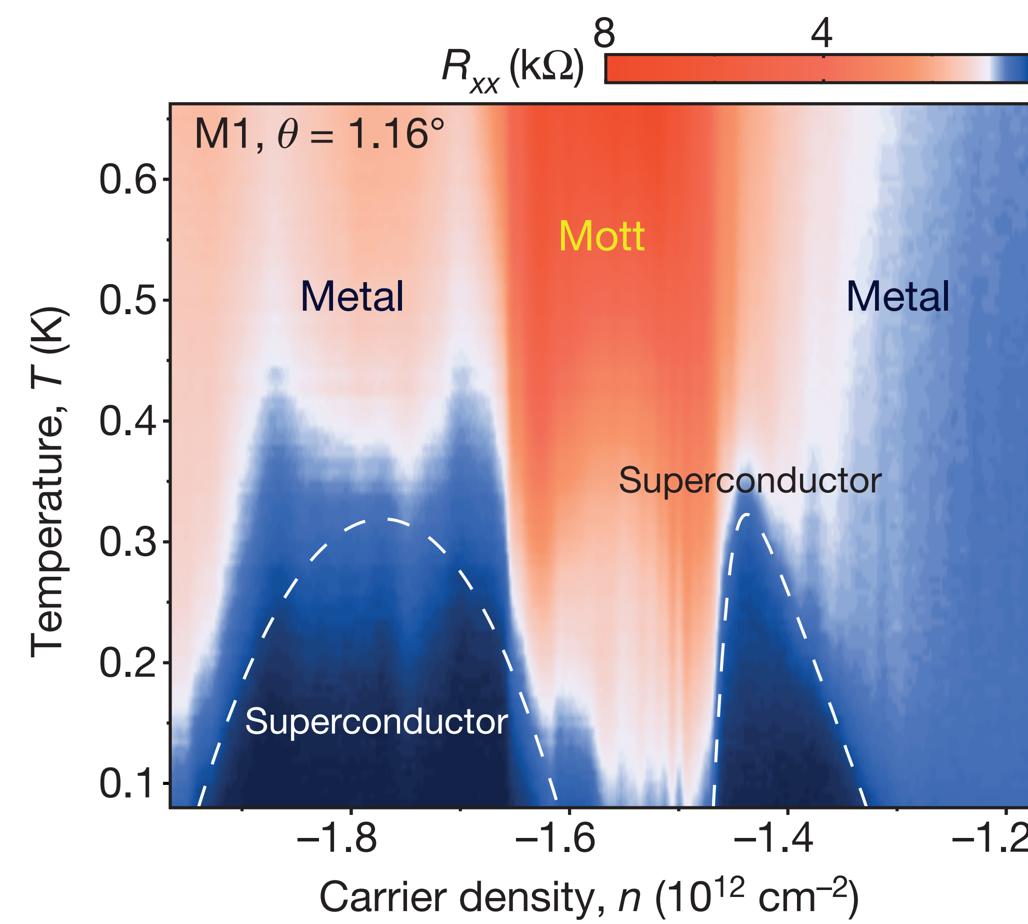
Magic-angle **twisted bilayer graphene**



- two graphene layers with small rotational mismatch θ
  - long-wavelength moiré pattern
  - @ $\theta_M \approx 1.05°, 1.16°$: ~10,000 atoms per moiré cell



- *correlated insulator* and *superconductivity*



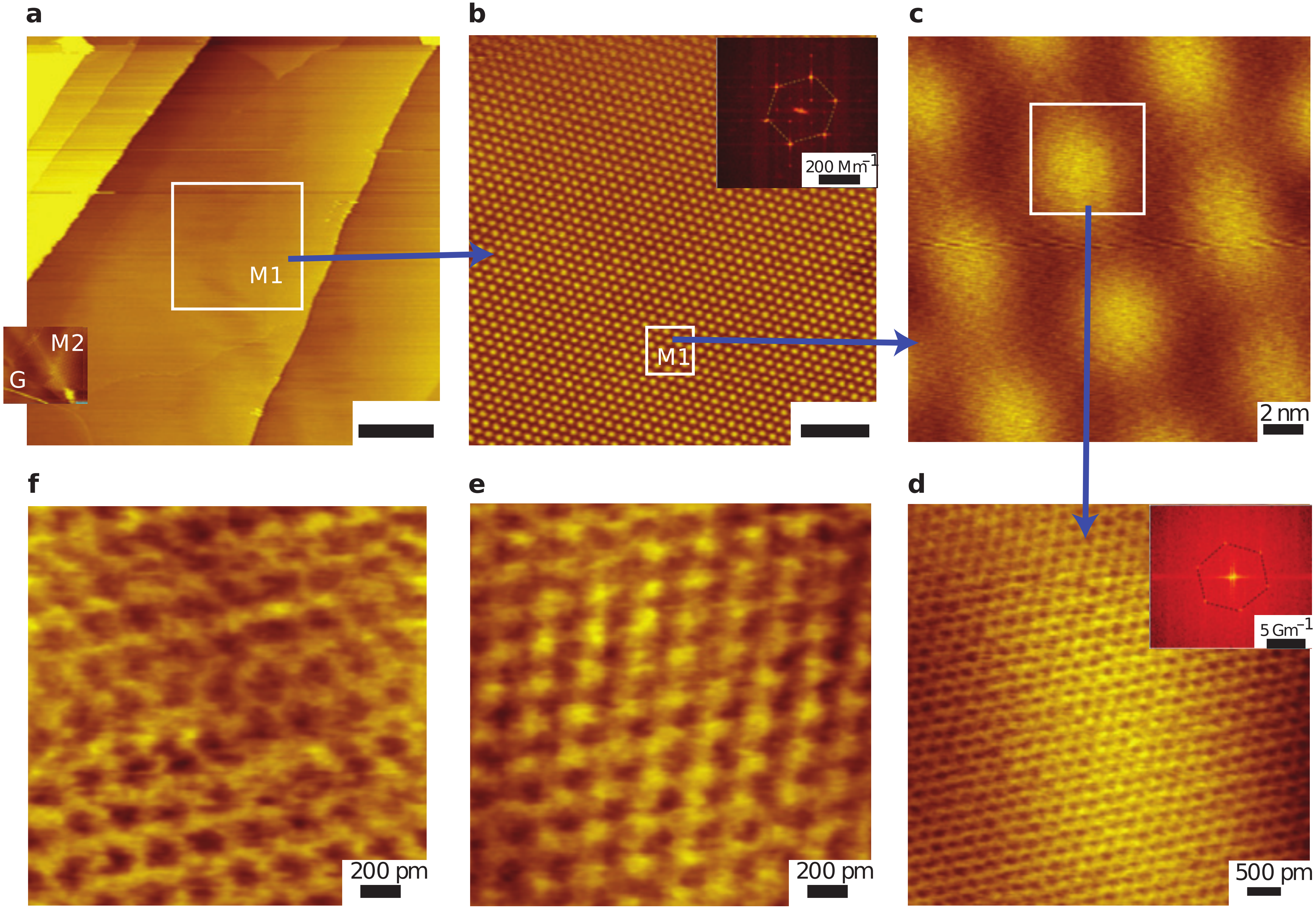
Cao et al., Nature 556, 43 (2018)
Cao et al., Nature 556, 80 (2018)
} >900 citations

Outline

- **Introduction & motivation**
  - twisted bilayer graphene **- basics & experiments**
  - related moiré heterostructures
  - twisted bilayer graphene **- models**
- **Effective lattice models & many-body approaches**
  - **functional renormalization group** approach
  - next-to-minimal model and tentative **phase diagram**
  - conclusions & outlook

Twisted bilayer graphene - **basics & experiments**

Moiré pattern in STM

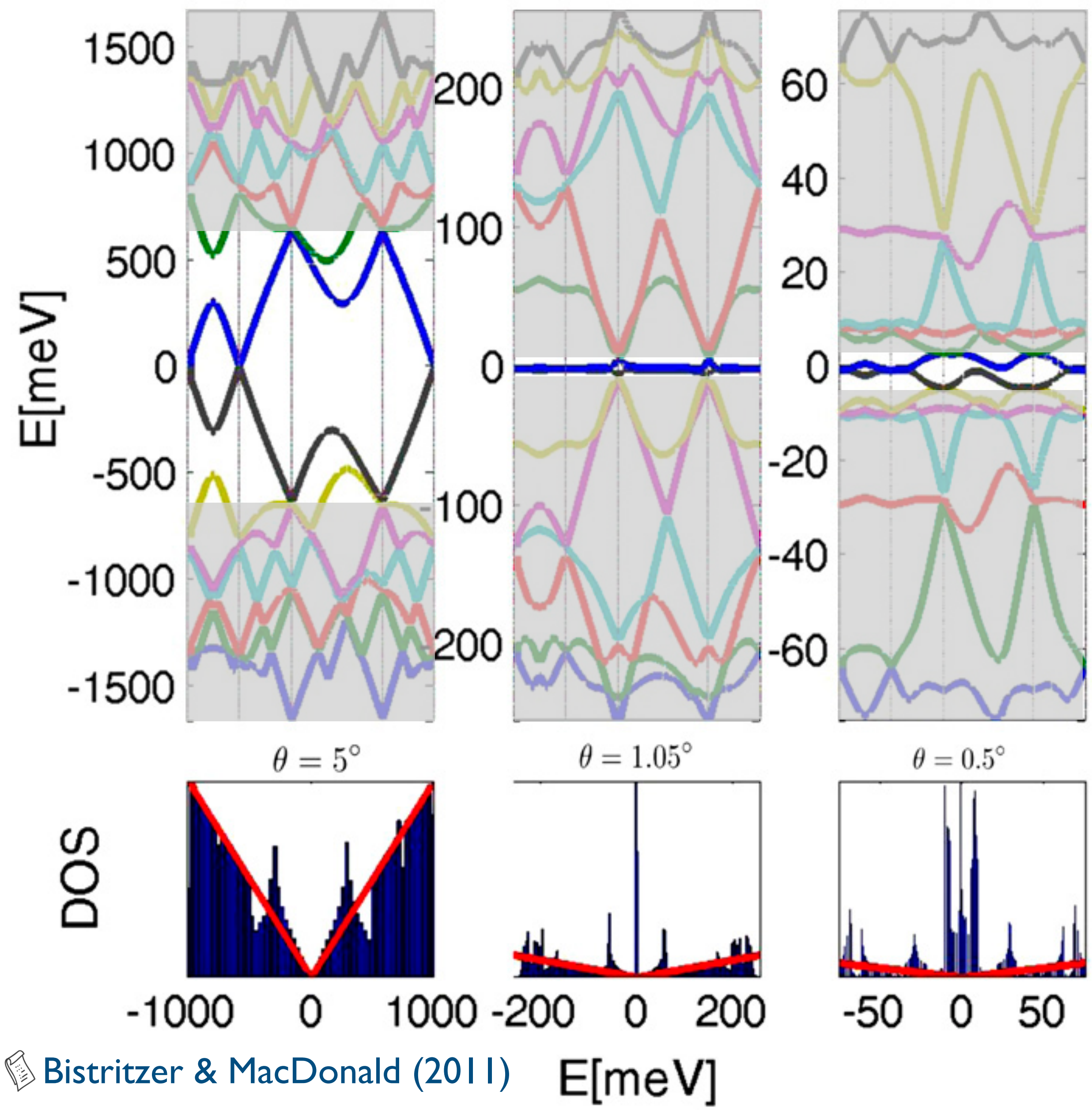


Li et al., Nature Physics 6 (2010)

Correlated magic-angle **tBLG - basic mechanism**

- for small twist angles θ < 10°
  - moiré pattern → superlattice potential → *reconstruction of low-energy band structure:*

θ = 5° θ = 1.05° θ = 0.5°



Bistritzer & MacDonald (2011)

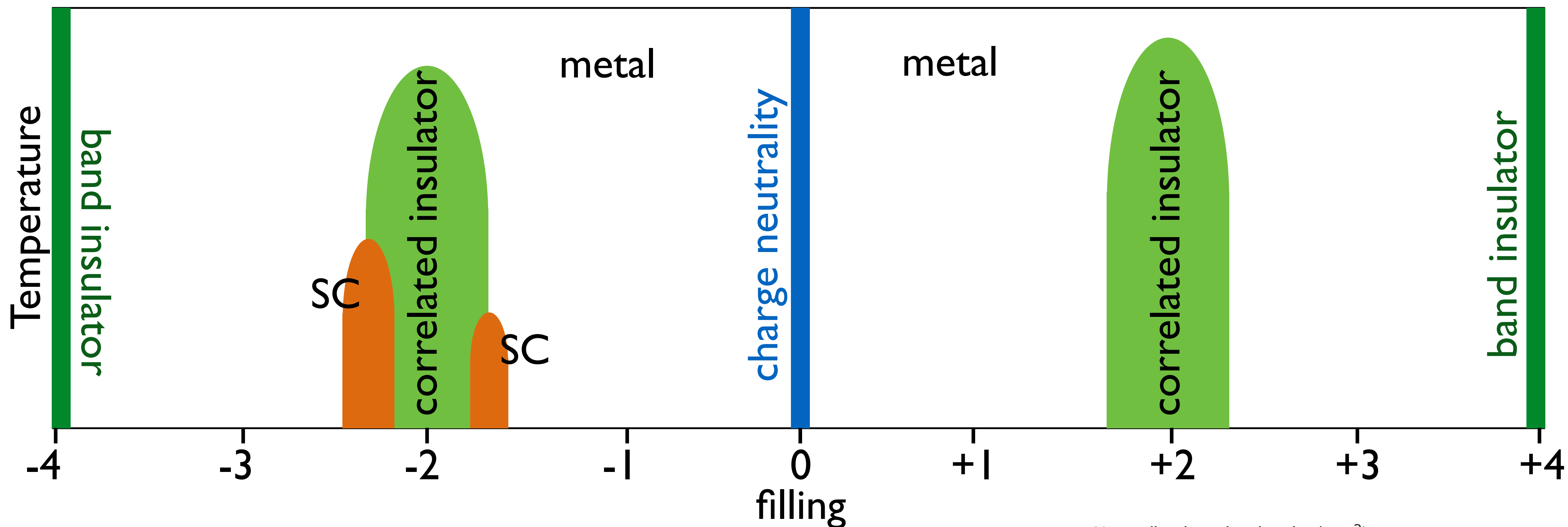
- flat band structure at magic angle θ ≈ 1°:
  - 2 x 2 x 2 = 8 nearly flat bands
    - 2 bands crossing at mini-Dirac points
    - spin (2 x)
    - singlelayer valley (2 x)
  - vary carrier concentration by gate voltage:
    - between $n = \pm 4n_0$

      (relative to charge neutrality)
    - $n_0 \triangleq 1\ e^-$ per unit cell of superlattice

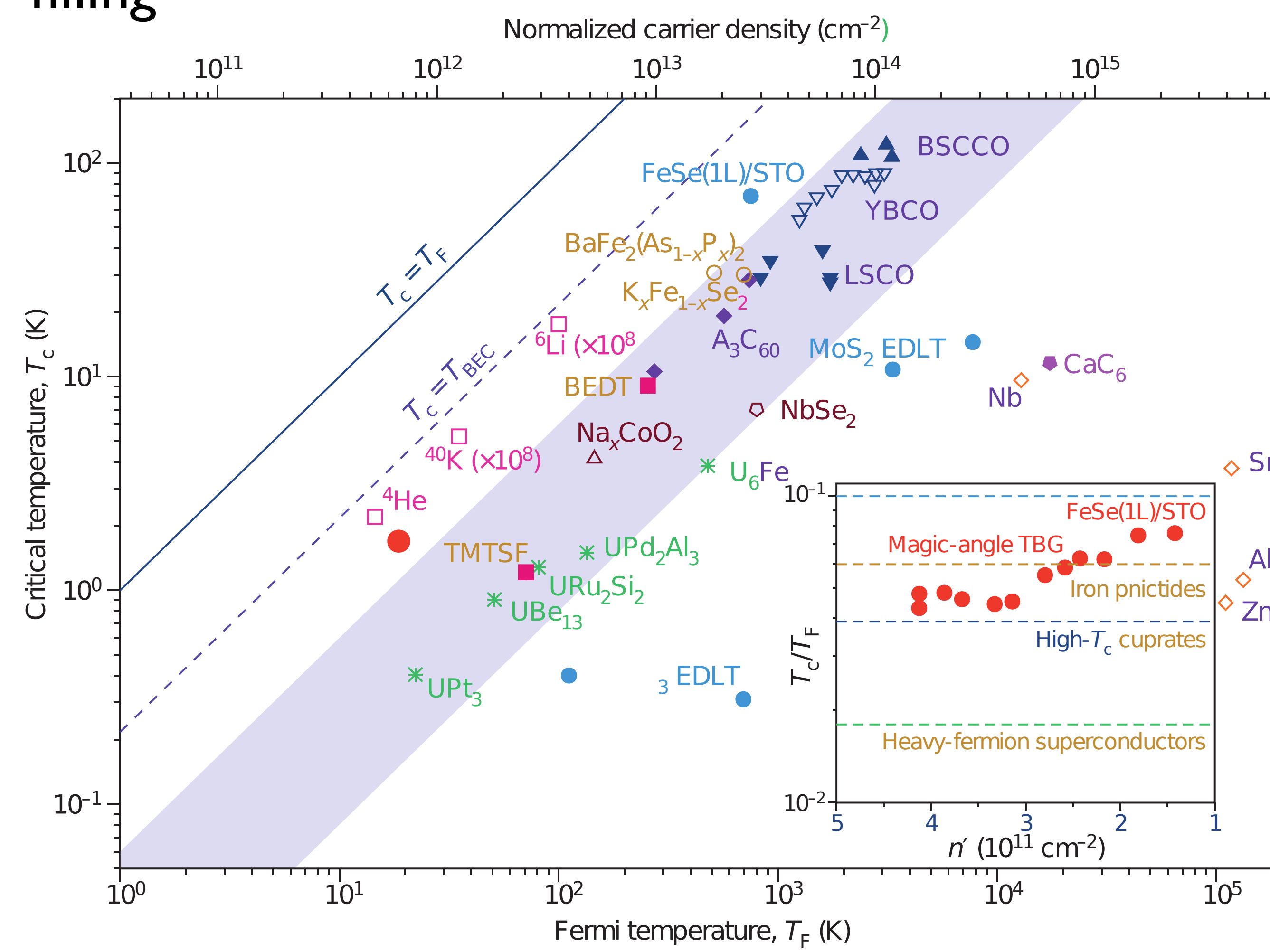
angle θ controls ratio bandwith/interaction: $t/U \sim |\theta - \theta_M|$

strongly-correlated states!

Magic-angle **tBLG - phase diagram 1.0** (2018)



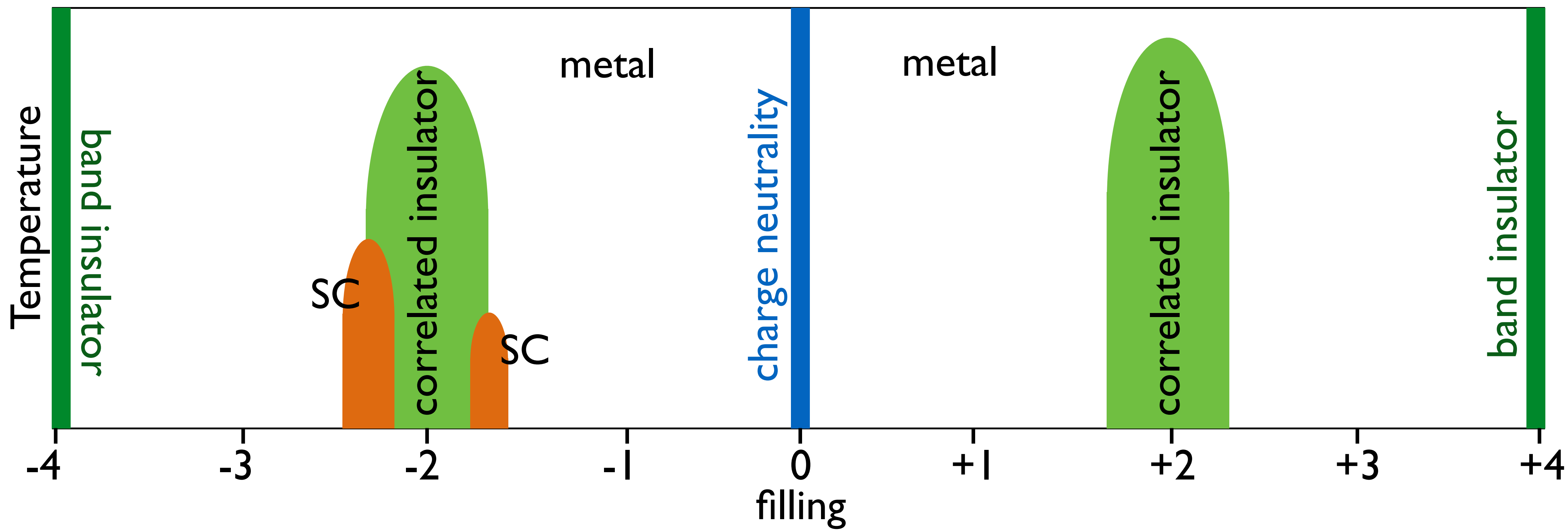
- ***band insulator*** at $n = \pm 4n_0$
- ***correlated insulator*** at $n = \pm 2n_0$
- ***superconducting domes*** at $n/n_0 = -2 \pm \delta$
  - $T_c \sim 1.7\text{K}$
  - T_c/T_F large
- resemblance to cuprate phase diagram!



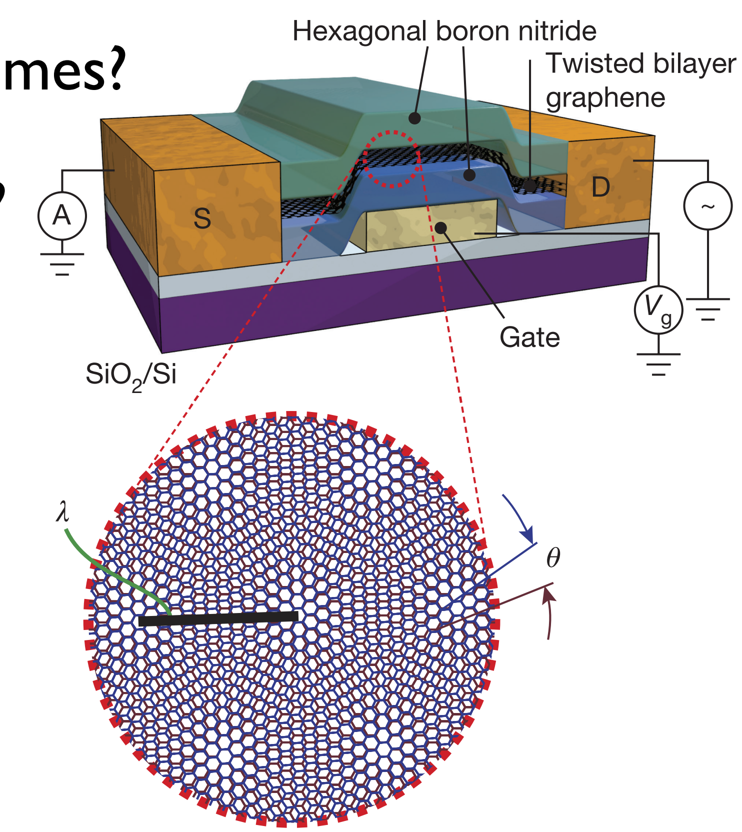
Cao et al., Nature 556, 43 (2018)

Cao et al., Nature 556, 80 (2018)

Questions

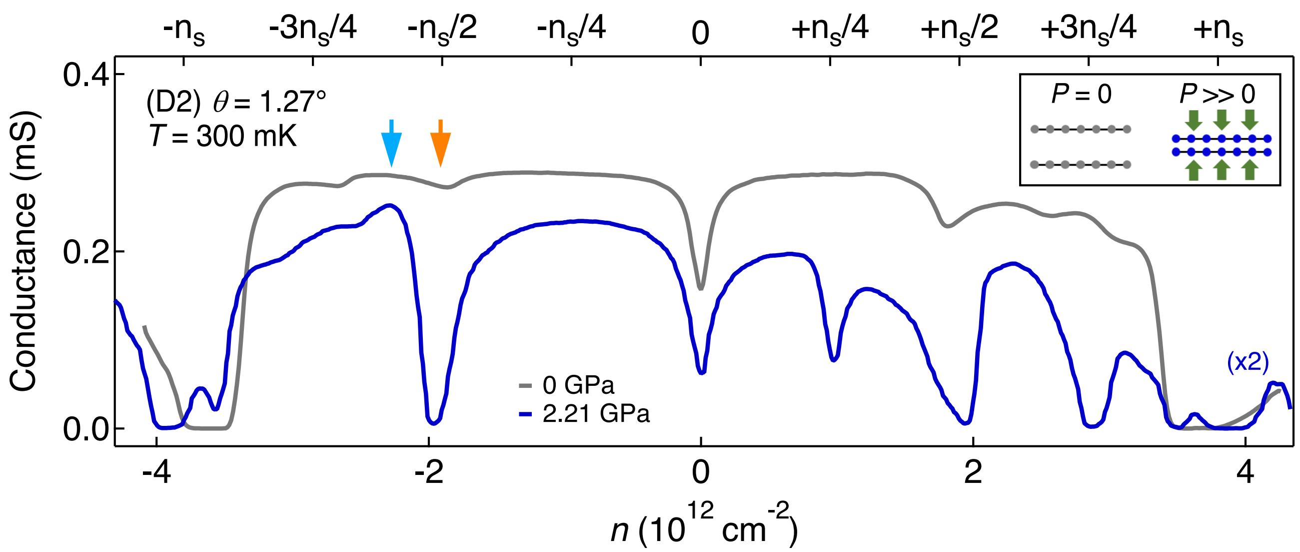


- Are the integer fillings $n/n_0 = \pm 2$ special?
- Are there ***further correlated states*** in different parameter regimes?
- Which ***symmetries*** are broken/preserved in correlated states?
  - spin-rotation / time-reversal / … ?
- What is the ***pairing channel*** of the superconducting state?
- What is the role of ***electron-phonon interactions***?
- How does the phenomenology depend on the ***substrate***?



2^{nd} generation of tBLG experiments (2019)

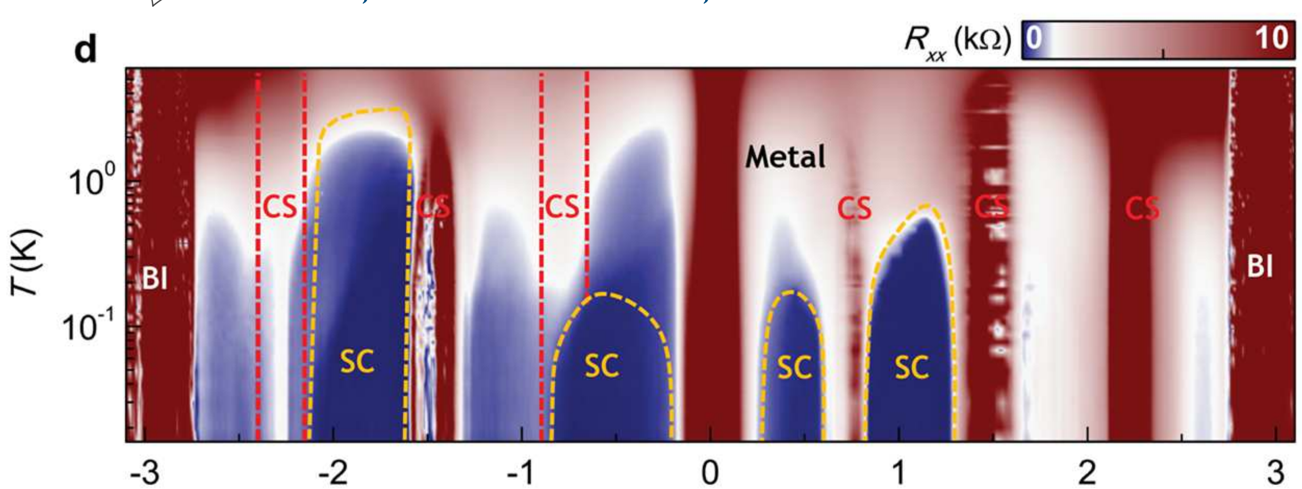
Columbia University: Yankowitz et al., Science 363, 1059 (2019)



- *apply* hydrostatic *pressure P*
  - ➡ variation of interlayer tunneling w
  - ➡ changes magic angle $\theta_M \sim w$
  - ➡ in-situ control of $t/U \sim |\theta - \theta_M|$
- *pressure increases* θ_M
  - ➡ reduce moiré lattice constant
  - ➡ increase energy scale: $T_{c,SC} \sim 3K$

‣ correlated insulator also at $n/n_0 = +3$, superconductor near $n/n_0 = \pm 2$

Efetov Lab, Barcelona: Lu et al., arXiv:1903.06513



- *very low twist-angle disorder*
  - ‣ reduced modulations of θ across device
  - ‣ $\Delta\theta < 0.02°$
  - ‣ insulating gaps at $n/n_0 = -3,-2,-1,$**0**$,+1,+2,+3$
  - ‣ SC domes in between ($T_{c,SC} \sim 0.14$ - 3K)

- more results on finite B⊥, role of topology with aligned hBN substrate, Landau fans

Stanford: Sharpe et al., arXiv:1901.03520

new platform established for study of correlated electrons

✓ high control of twist angle, low level of disorder, pressure/gate tunable band widths/fillings

Related 2D van der Waals **moiré heterostructures**

- *magic-angle twisted bilayer graphene* ✓
- *ABC trilayer graphene on hexagonal boron nitride*
  - **experiments**: tunable insulating and SC behavior

    Chen et al., Nat. Phys. 15, 237 (2019)

    Chen et al., arXiv:1901.04621 (2019)
- *twisted double bilayer graphene*
  - **experiments**: spin-polarized correlated insulating and SC behavior

    Liu et al., arXiv:1903.08130 (2019)

    Cao et al., arXiv:1903.08596 (2019)
- *twisted bilayer boron nitride*
  - **theoretical**: multi-flat bands and strong correlations

    Xian et al., arXiv:1812.08097 (2018)
- *transition metal dichalcogenides*
  - **theoretical**: flat bands and strong correlations with and without twist…

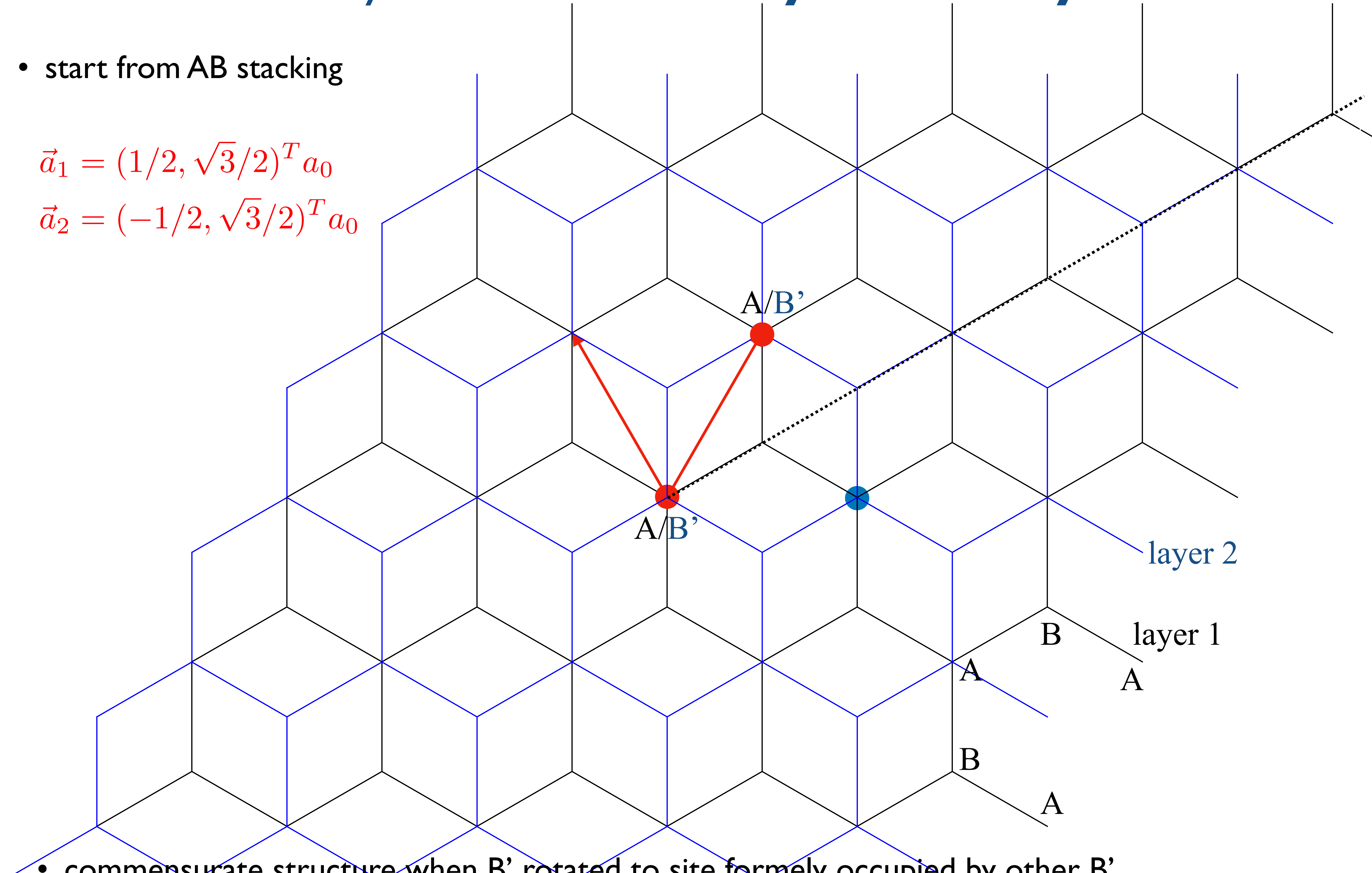
    Wu et al., PRL 121, 026402 (2018)

Twisted bilayer graphene - **models**

Geometry of **twisted honeycomb bilayers**

- start from AB stacking

$$\vec{a}_1 = (1/2, \sqrt{3}/2)^T a_0$$
$$\vec{a}_2 = (-1/2, \sqrt{3}/2)^T a_0$$

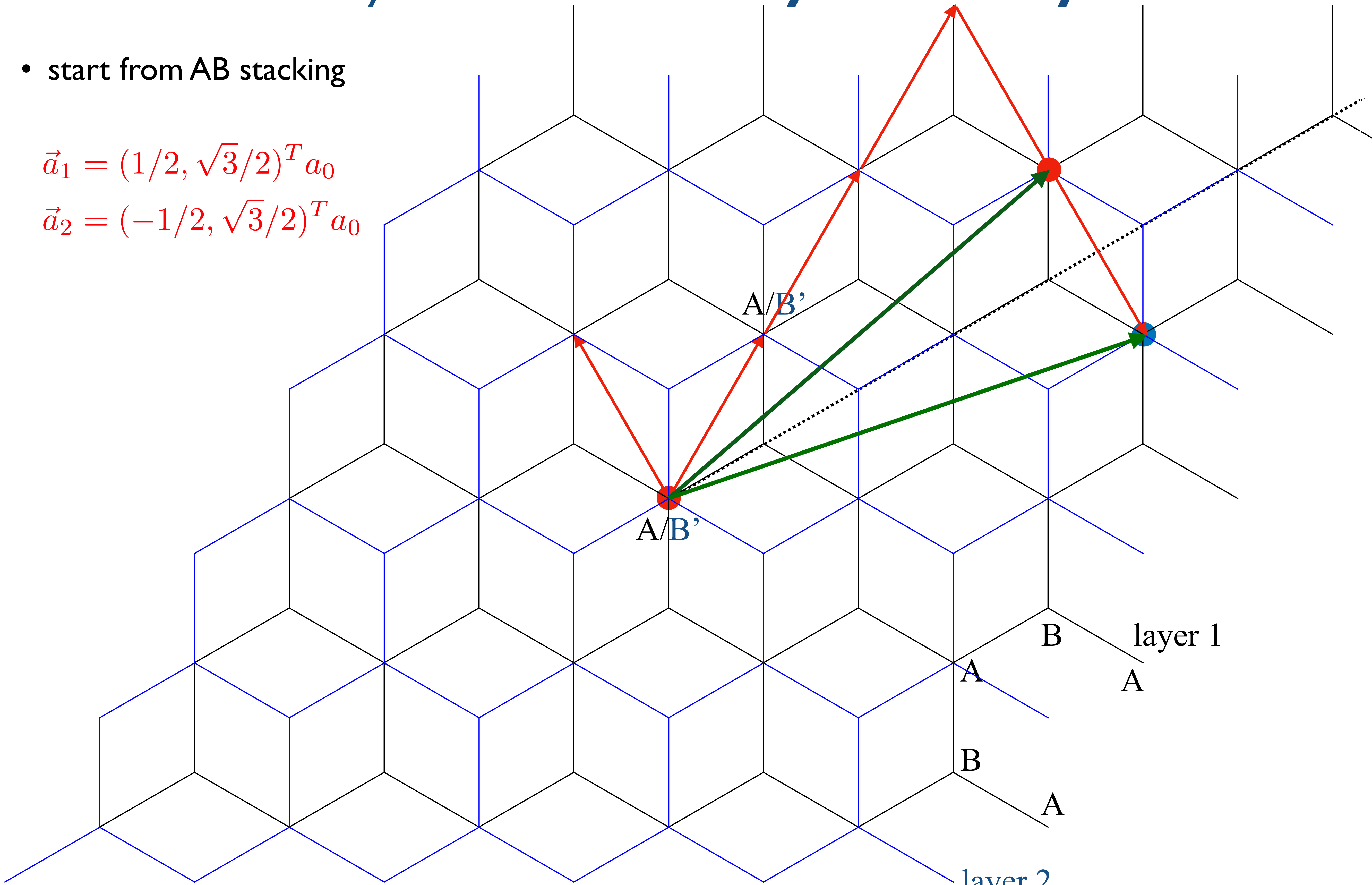


- commensurate structure when B' rotated to site formely occupied by other B'
- 2D bilayer crystal only at discrete set of commensurate rotation angles

Geometry of **twisted honeycomb bilayers**

- start from AB stacking

$$\vec{a}_1 = (1/2, \sqrt{3}/2)^T a_0$$
$$\vec{a}_2 = (-1/2, \sqrt{3}/2)^T a_0$$

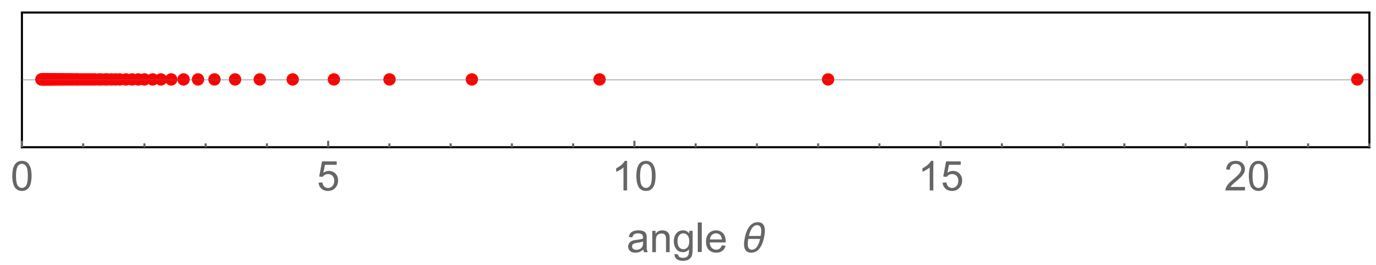


here: θ ≈ 21.8°

Geometry of **twisted honeycomb bilayers**

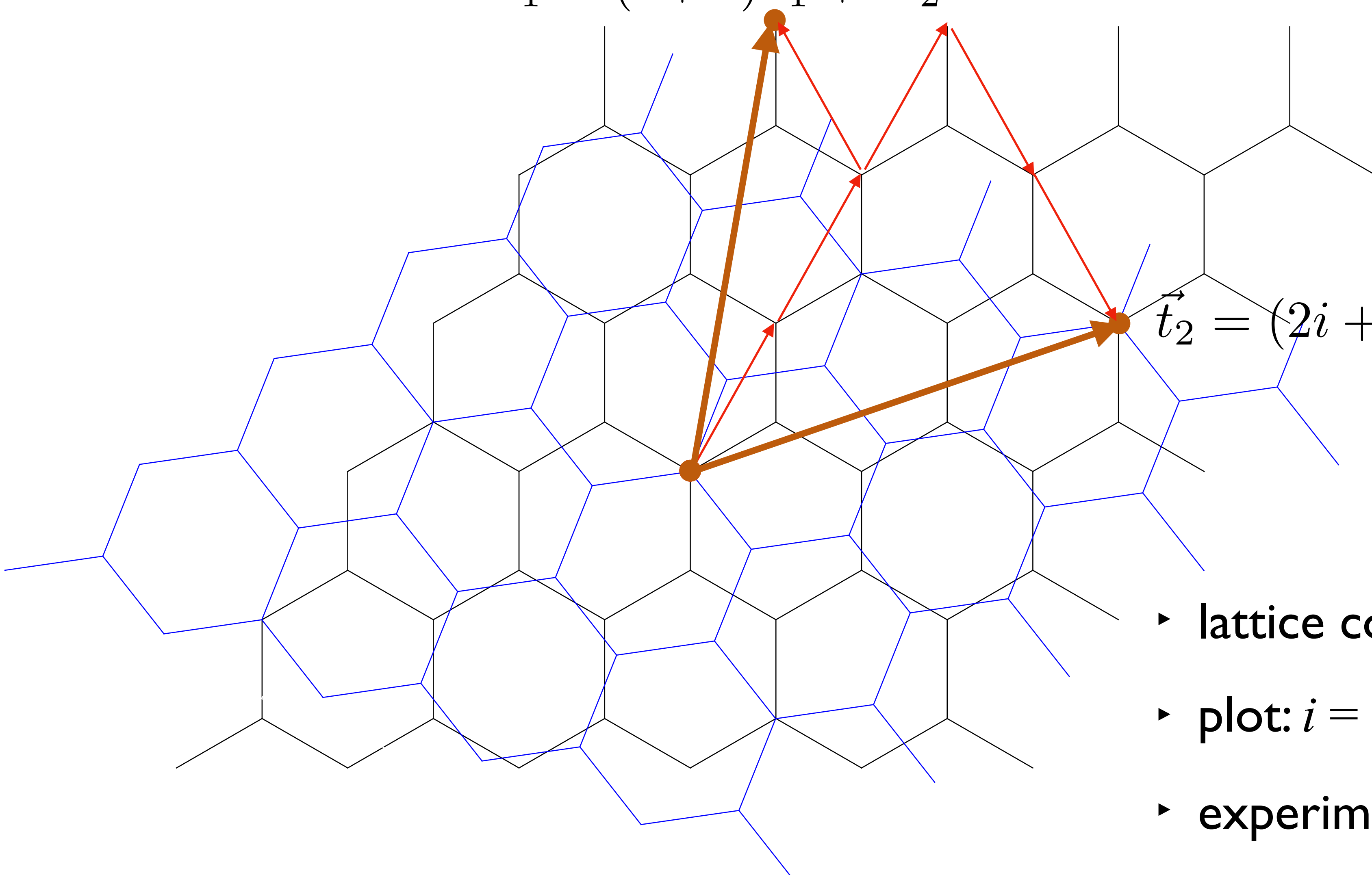
- 2D bilayer crystal only at discrete set of commensurate rotation angles

$$\cos(\theta_i) = \frac{3i^2 + 3i + 1/2}{3i^2 + 3i + 1}, \quad i \in \mathbb{N}_0$$



- emergent moiré superlattice spanned by real space lattice vectors t_1 and t_2

$$\vec{t}_1 = (i+1)\vec{a}_1 + i\vec{a}_2$$

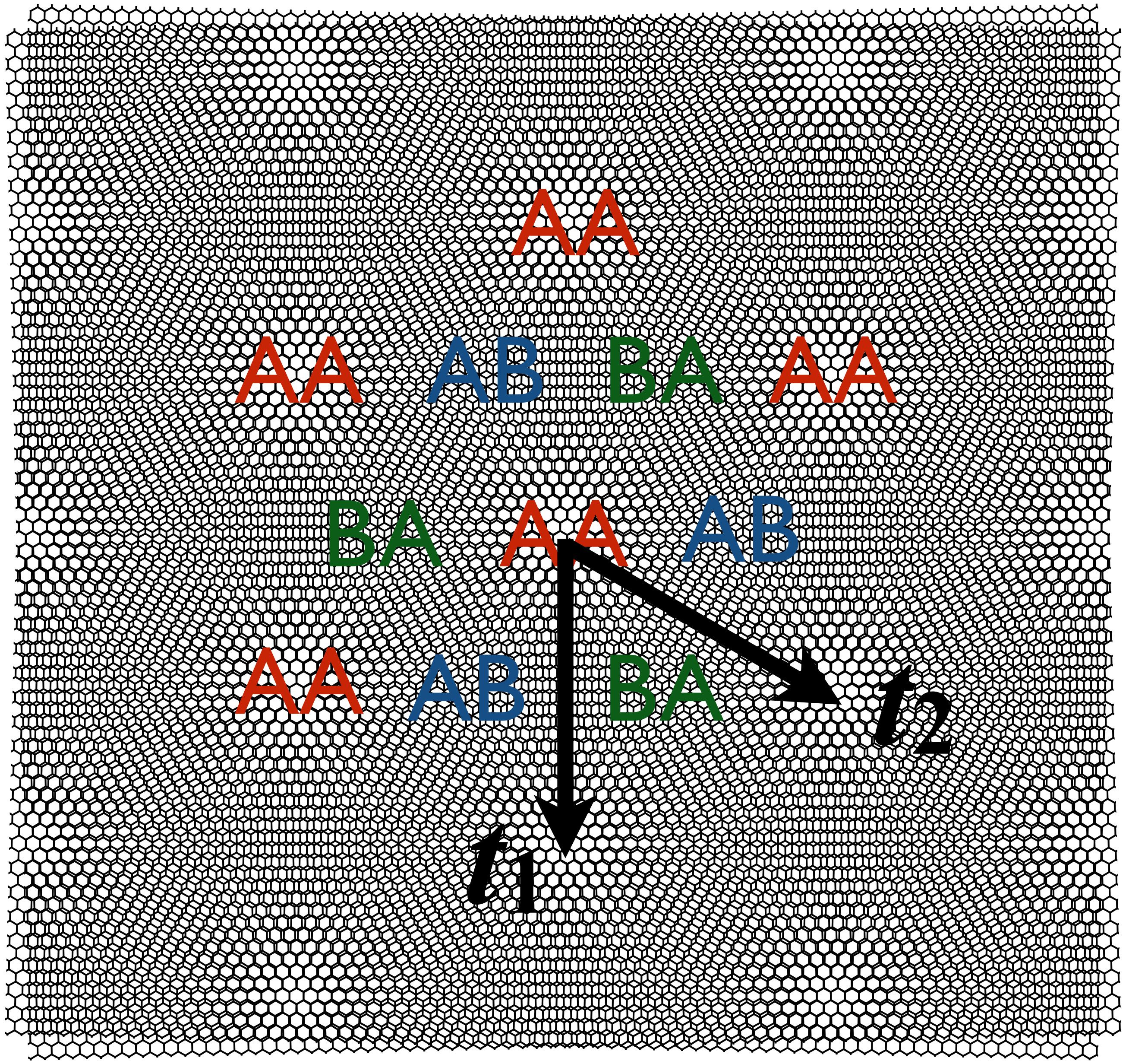


$$\vec{t}_2 = (2i+1)\vec{a}_1 - (i+1)\vec{a}_2$$

- lattice constant: $|\vec{t}_1| = \sqrt{3i^2 + 3i + 1}a_0$
- plot: $i = 1$
- experiment: $i \approx 30$

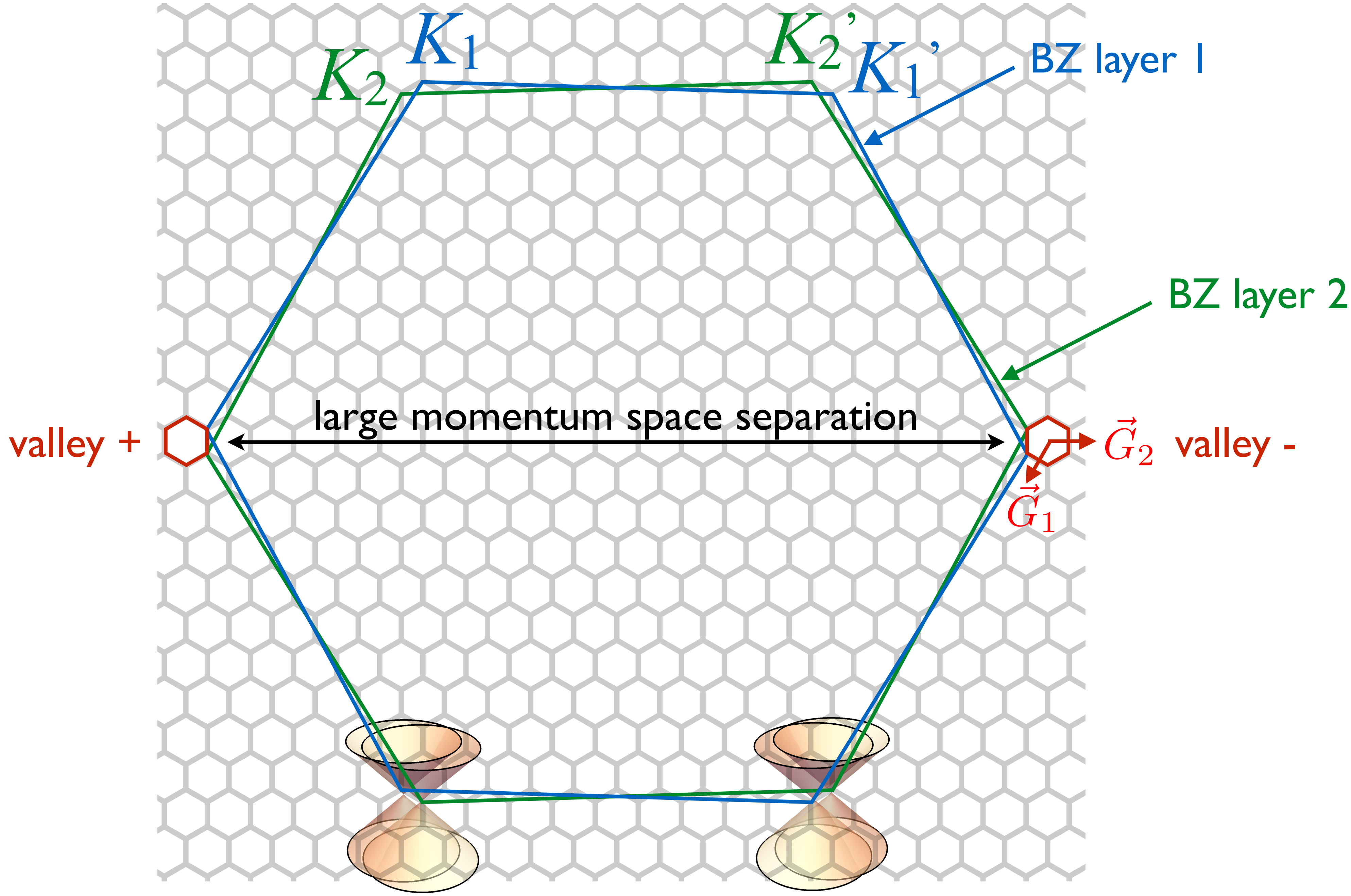
Geometry of **twisted honeycomb bilayers**

- example: $\theta \approx 2.6°$ ($i = 12$)



- local AA, AB and BA stacking regions
- moiré superlattice vectors t_1 and t_2

Mini **Brillouin zone**



- reciprocal lattice vectors:

$$\vec{G}_1 = \frac{4\pi}{3(3i^2+3i+1)}\left[\vec{a}_1 + (3i+1)\vec{a}_2\right]$$

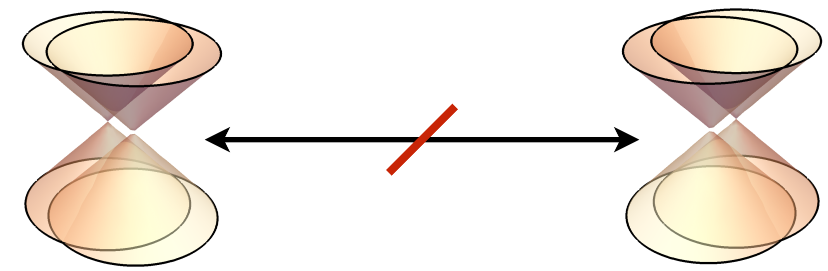
$$\vec{G}_2 = \frac{4\pi}{3(3i^2+3i+1)}\left[(3i+1)\vec{a}_1 - (3i+2)\vec{a}_2\right]$$

- Dirac points of single-layer graphene layers: K_1, K_1', K_2, K_2'

Effective **continuum model**

- moiré period much larger than atomic scale → ***continuum model***

Lopes dos Santos et al., PRL 99, 256802 (2007)
Bistritzer, MacDonald, PNAS 108, 12233 (2011)

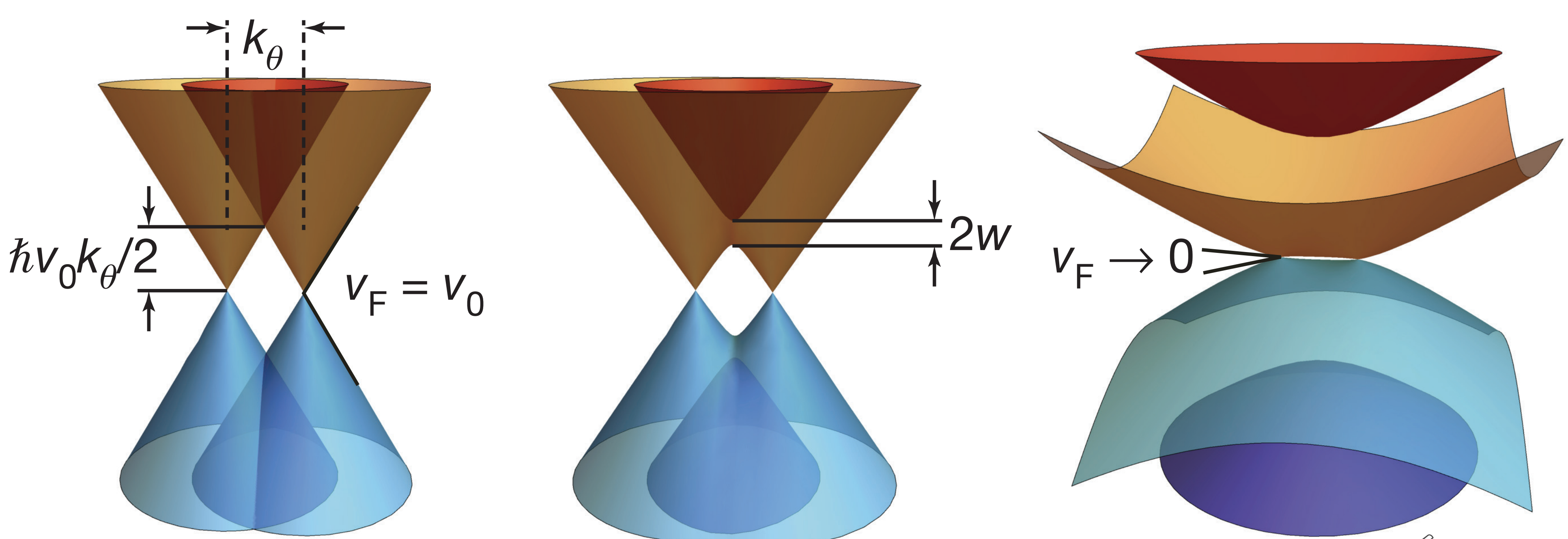


- neglect intervalley mixing (large momentum space separation)
  - independent calculation for each valley $\xi \in \pm$
  - 1 Dirac cone from each layer: interlayer hybridization
  - ***effective 4x4 Hamiltonian***

$$H^{(\xi)} = \begin{pmatrix} H_1 & U^\dagger \\ U & H_2 \end{pmatrix}$$

intralayer Dirac Hamiltonians

effective interlayer coupling

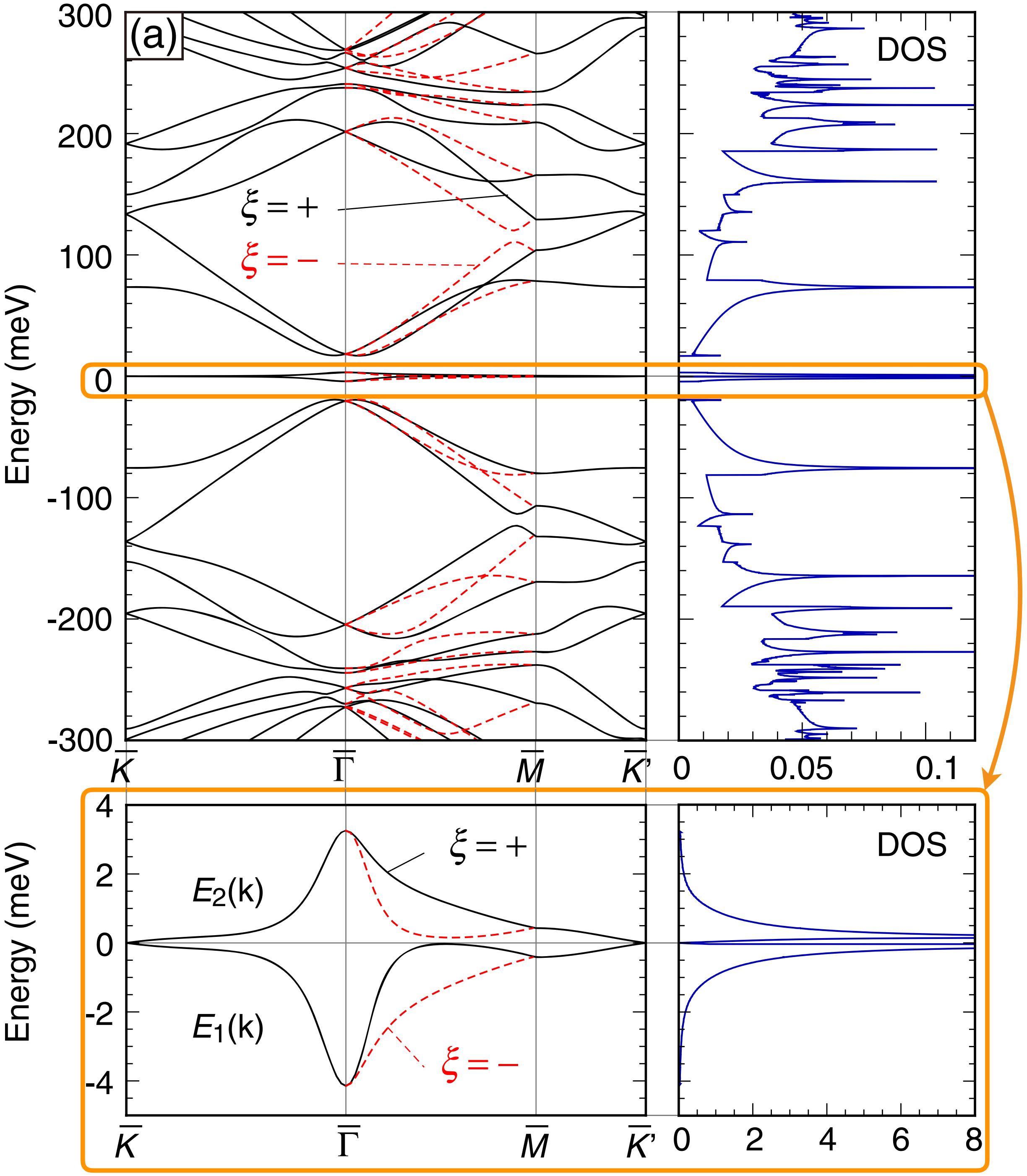


Cao et al, Nature 556, 80 (2018)

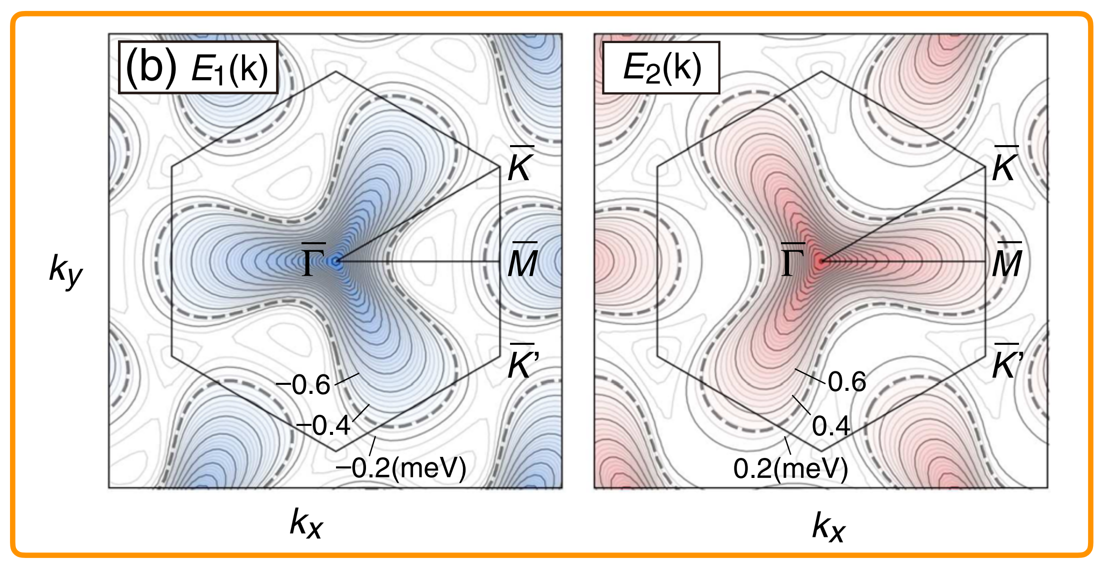
- Dirac cones become nearly flat for ***magic angles***
- dependence on modelling of interlayer coupling (*lattice relaxation, corrugation,…*)

Effective continuum **model**

- full calculation of band dispersion in continuum model



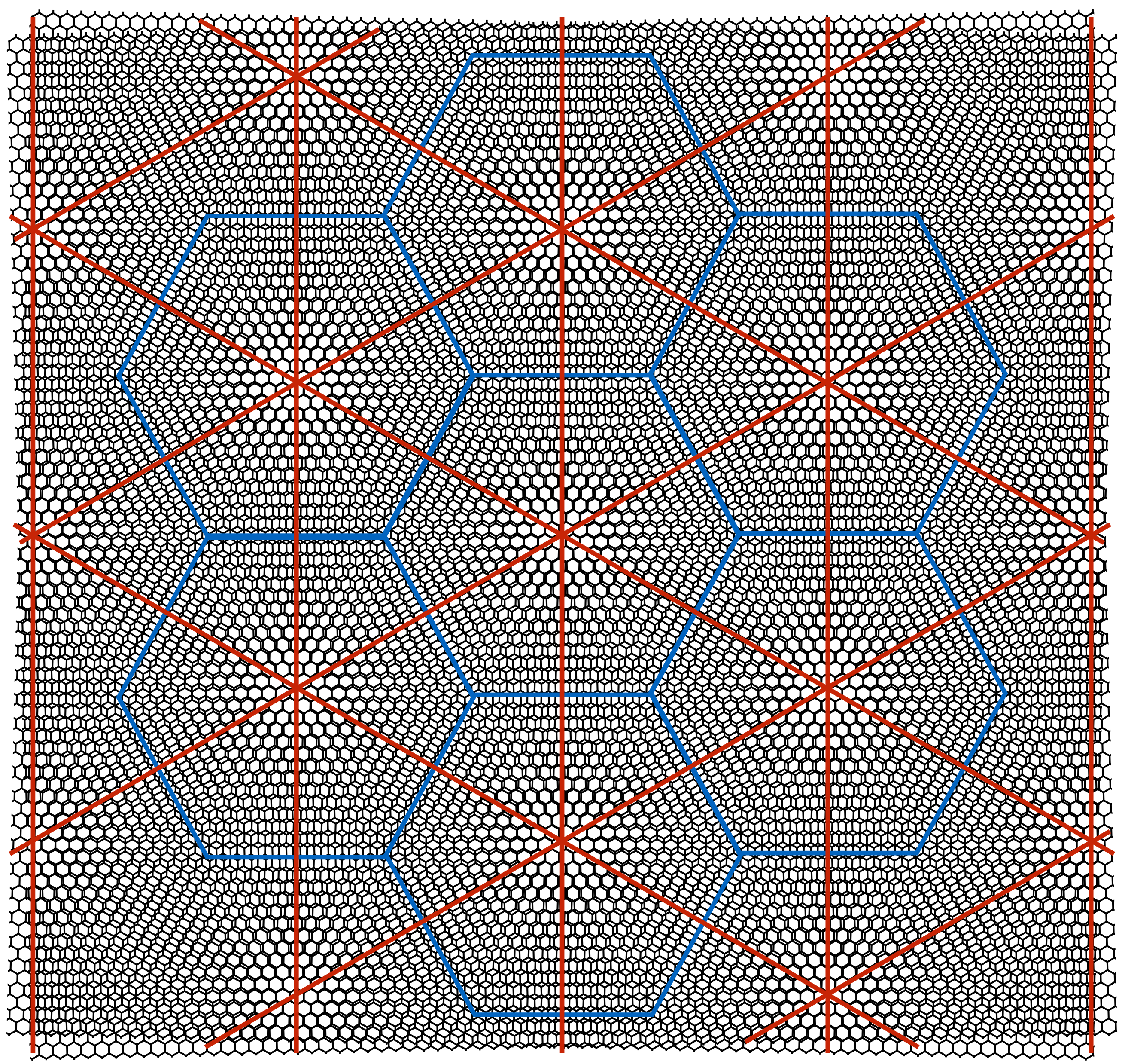
- at *magic angles* θ_M (here: θ_M = 1.05°)
  - emergence of multiple nearly flat bands
  - well-separated from other bands
  - van Hove singularities in flat bands



Koshino et al, PRX 8, 031087 (2018)

Wannier orbitals for flat bands & effective lattice model

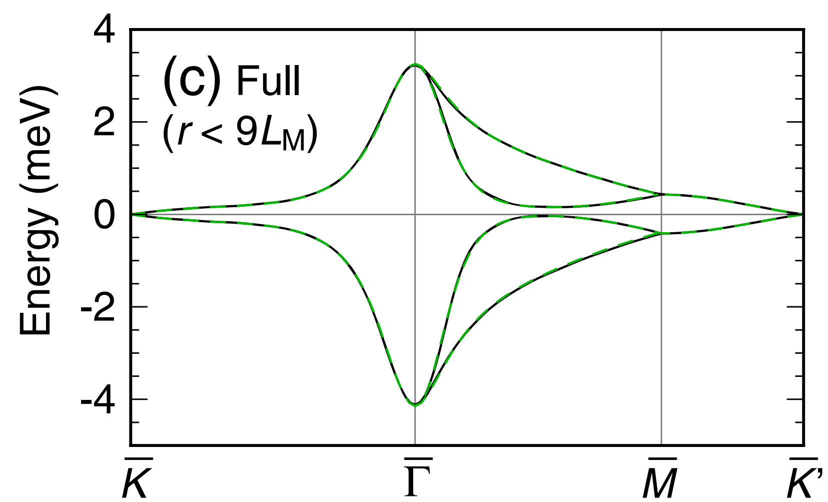
- *symmetry analysis* for tBLG:
  - Wannier orbitals centered at nonequivalent AB and BA spots in moiré pattern
  - formation of emergent honeycomb lattice

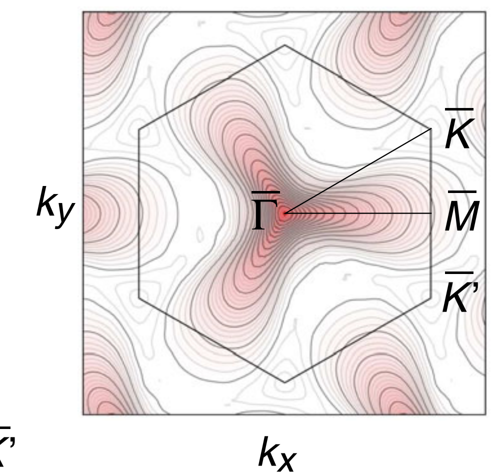


  - emergent *triangular* lattice: ABC trilayer graphene-hBN, twisted double BLG, TMDs

Wannier orbitals for flat bands & effective lattice model

- flat band dispersion from continuum model
  - construct max-localized Wannier orbitals
  - tight-binding + extended Hubbard model

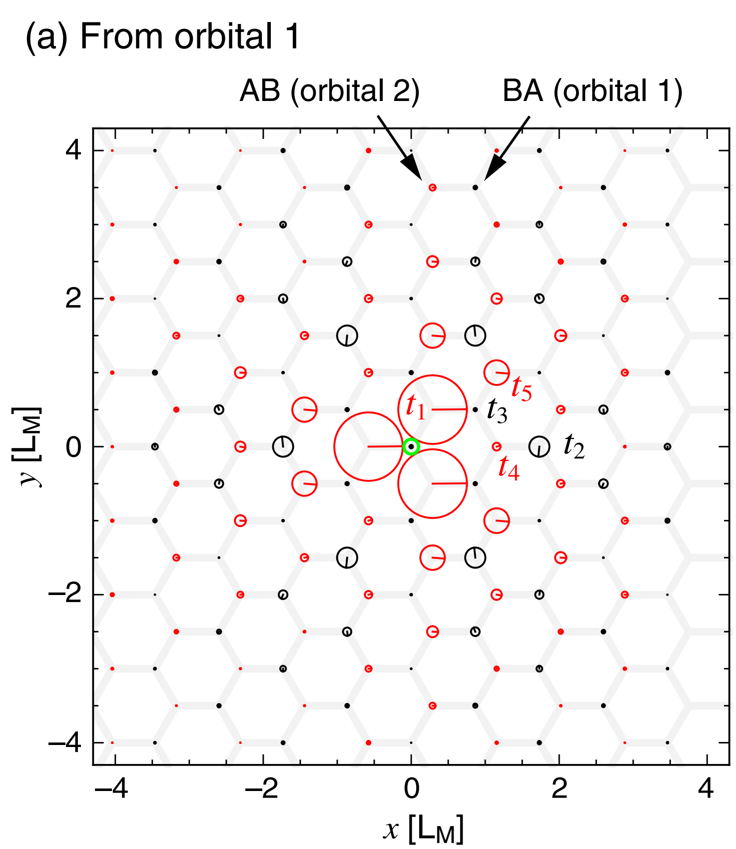


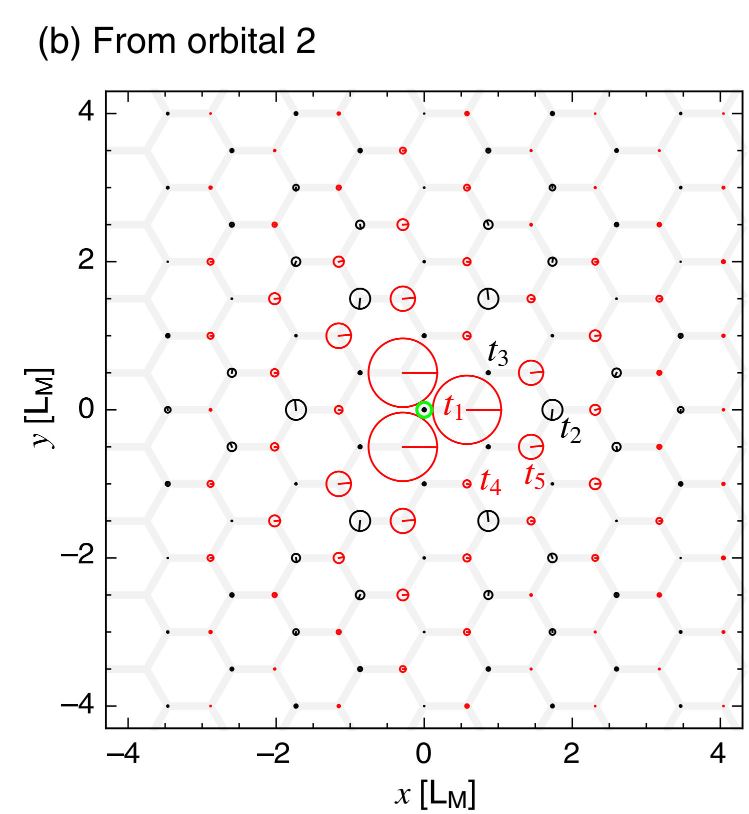


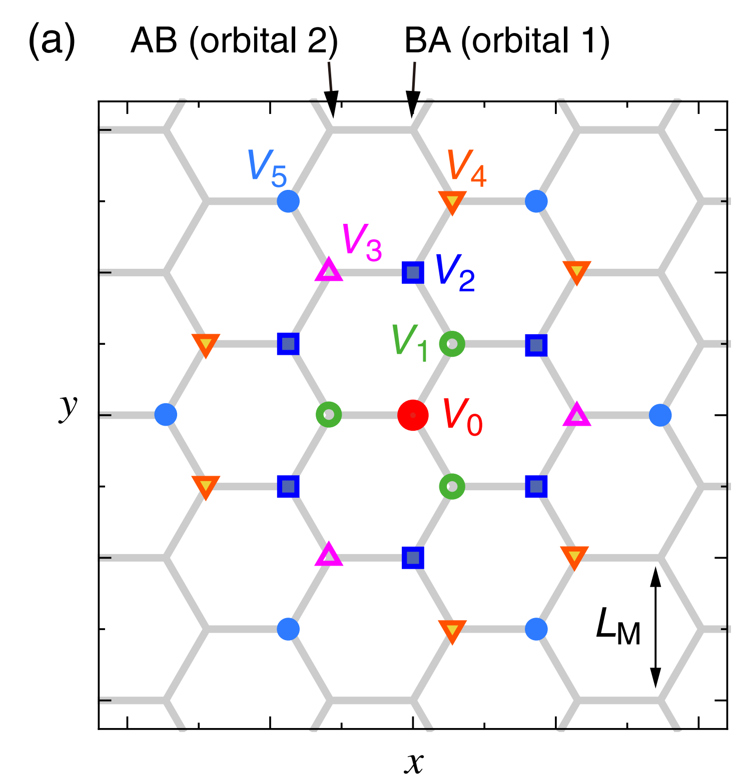
$$H = \sum_{\xi=\pm} \sum_{ij} t(\boldsymbol{r}_{ij}) e^{i\xi\phi(\boldsymbol{r}_{ij})} c^{\dagger}_{i\xi} c_{j\xi} \quad + \quad \textbf{extended Hubbard i.a.}$$

- hopping integrals and electron-electron interaction parameters:

Koshino et al, PRX 8, 031087 (2018)
Kang & Vafek, PRX 8, 031088 (2018)







Correlated moiré heterostructures - **precis**

Precis

- 2D moiré heterostructures:
  - *emergent flat bands,* ***small kinetic energy***
  - ***enhanced interaction effects***

⟹ correlation-driven states!

- construction of effective models on emergent moiré honeycomb/triangular superlattice
  - ***universal features****:*
    - ***multi-orbital*** *structure inherited from two valleys* $\rightarrow$ ***Hund's couplings***
    - *onsite and sizable* ***further-neighbor interactions***

…starting point for application of ***many-body methods…***

- **what is the nature of the correlated insulating and SC states?**
  - *Kekulé valence bond solid, (anti)ferromagnet, interaction-induced top. states,…?*
  - *featureless Mott insulator?*
  - *gapped quantum spin liquid with neutral spin-1/2 excitations?*
  - *topological/chiral d+id superconductor, f-wave superconductor,…?*

Functional **renormalization group approach**

Effective action

- system of interacting fermions: $\mathcal{S}[\psi, \bar{\psi}] = -(\bar{\psi}, G_0^{-1}\psi) + V[\psi, \bar{\psi}]$

general two-particle i.a.

- bare propagator (translation and spin rotation invariance):

single-particle energy

$$G_0(k_0, \mathbf{k}) = \frac{1}{ik_0 - \xi_{\mathbf{k}}}, \quad \xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$$

- generating functional (for connected Green functions):

$$\mathcal{G}[\eta, \bar{\eta}] = -\ln \int \mathcal{D}\psi \mathcal{D}\bar{\psi}\, e^{\mathcal{S}[\psi, \bar{\psi}]} e^{(\bar{\eta}, \psi) + (\bar{\psi}, \eta)}$$

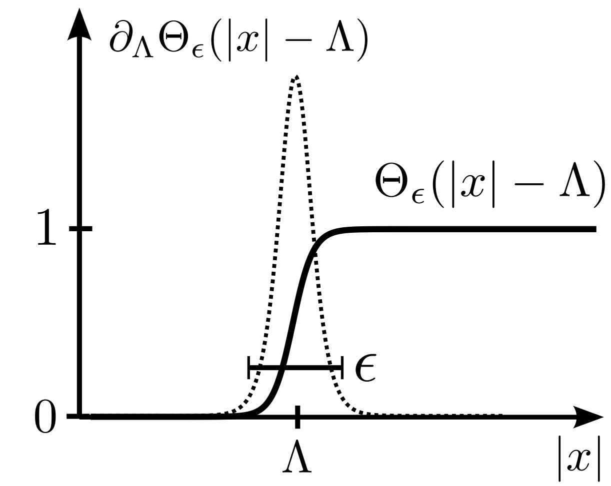
- effective action: $\Gamma[\phi, \bar{\phi}] = (\bar{\eta}, \phi) + (\bar{\phi}, \eta) + \mathcal{G}[\eta, \bar{\eta}], \quad \phi = -\frac{\partial \mathcal{G}}{\partial \bar{\eta}}, \quad \bar{\phi} = \frac{\partial \mathcal{G}}{\partial \eta}$

(generates one-particle irreducible vertex functions)

Functional **flow equations**

- modify bare propagator by introduction of flow parameter

  (IR cutoff, cuts out soft modes $< \Lambda$): $\quad G_0^\Lambda(k_0, \mathbf{k}) = \dfrac{\Theta_\epsilon(|\xi_\mathbf{k}| - \Lambda)}{ik_0 - \xi_\mathbf{k}}$



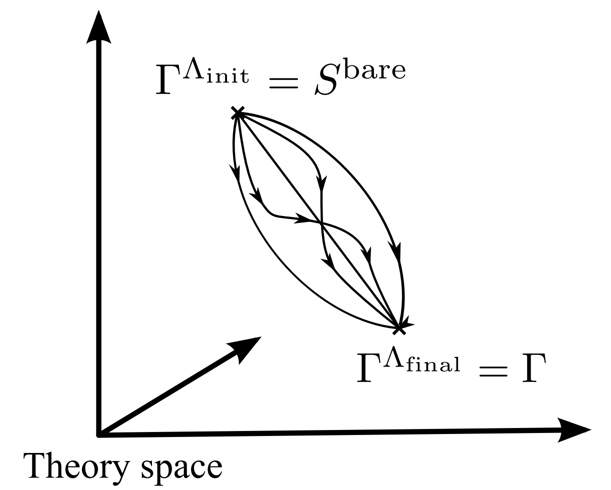
- define all the above quantities with modified bare propagator

  $\longrightarrow$ variation w.r.t to scale provides **exact RG equation**:

$$\boxed{\frac{\partial}{\partial\Lambda}\Gamma^\Lambda[\phi,\bar\phi] = \mathrm{Tr}\left[G_0^\Lambda \frac{\partial (G_0^\Lambda)^{-1}}{\partial\Lambda}\right] - \mathrm{Tr}\left[\left(\frac{\delta^2\Gamma^\Lambda[\phi,\bar\phi]}{\delta\phi\delta\bar\phi} + (G_0^\Lambda)^{-1}\right)^{-1} \frac{\partial (G_0^\Lambda)^{-1}}{\partial\Lambda}\right]}$$

Wetterich (1993)

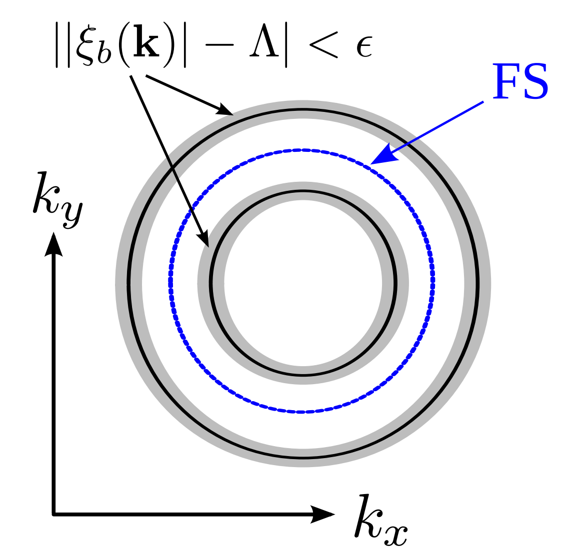
Salmhofer & Honerkamp (2001)



- exact RG equation has one-loop structure

- removing cutoff ($\Lambda \to 0$) yields the full effective action

- lowering cutoff corresponds to momentum-shell integration



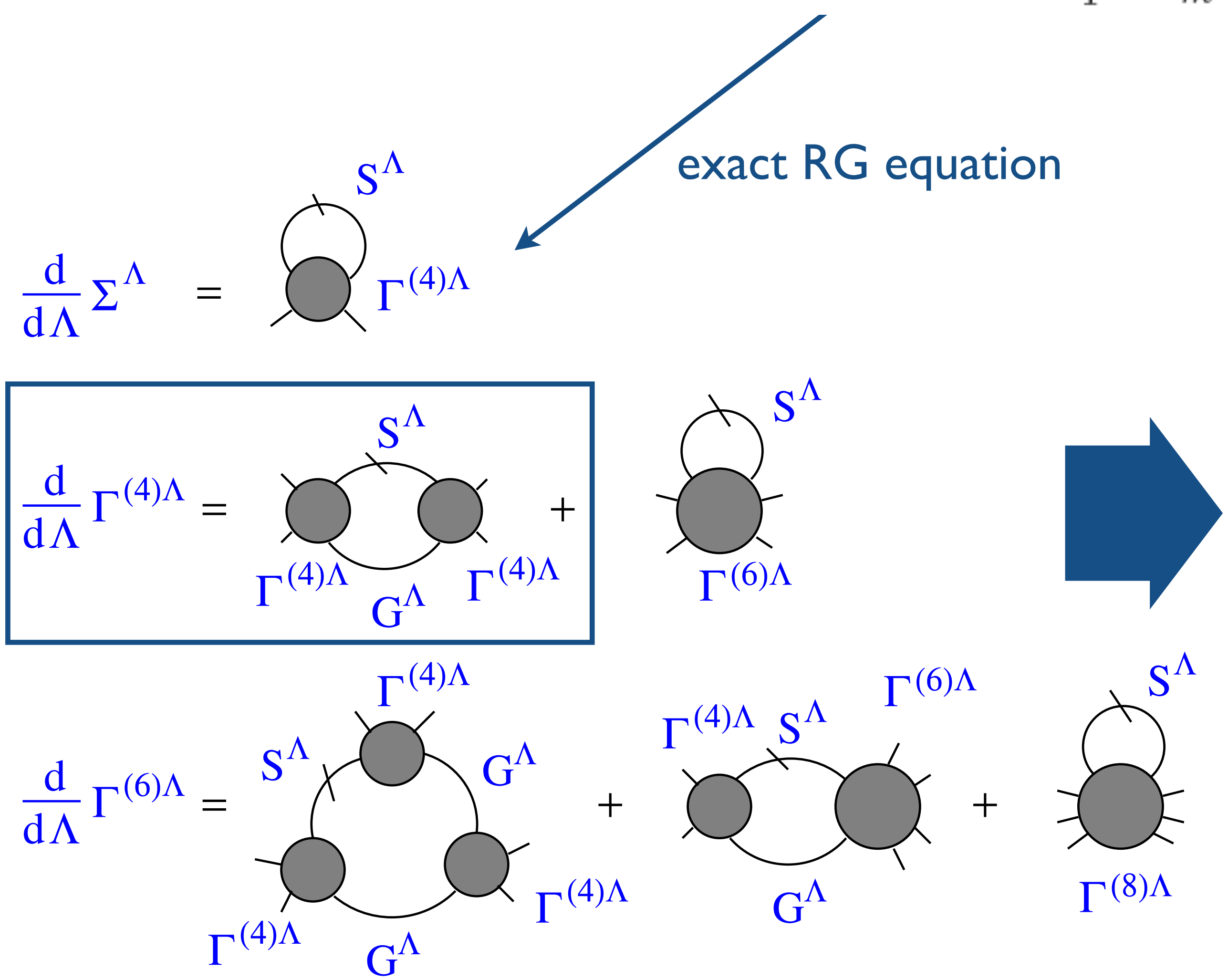
from Platt, Hanke, Thomale (2013)

Truncation and **approximations**

- exact RG equation cannot be solved exactly!

- starting point for systematic approximations (vertex expansion)

$$\Gamma^\Lambda[\phi,\bar\phi] = \sum_{m=0}^{\infty} \frac{1}{(m!)^2} \sum_{K_1 \dots K_m} \sum_{K'_1 \dots K'_m} \gamma_m^\Lambda(K'_1,\dots,K'_m;K_1,\dots,K_m) \prod_{j=1}^{m} \bar\phi_{K'_j}\phi_{K_j}$$



- neglect 6-point and higher vertices
- neglect self-energy feedback

Salmhofer & Honerkamp (2001)

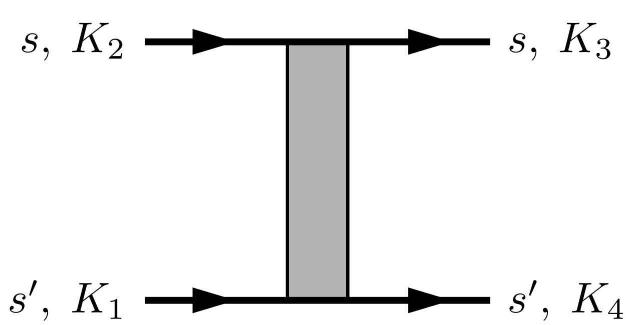
… infinite hierarchy of flow equations!

Symmetries and **approximations**

- system with **spin-rotational invariance**:

  - RG flow of general 4-point function $\Gamma^{(4)\Lambda}$: $\Gamma^{(4)\Lambda}_{\sigma_1,\sigma_2,\sigma_3,\sigma_4} = V^{\Lambda}\delta_{\sigma_1\sigma_3}\delta_{\sigma_2\sigma_4} - V^{\Lambda}\delta_{\sigma_1\sigma_4}\delta_{\sigma_2\sigma_3}$

  ➡️ **interaction vertex *V*^Λ**:



$$V_\Lambda(k_1, k_2, k_3, k_4)$$

- momentum arguments include *frequency*, *wavevector* and *orbital* indices

- ground-state properties: neglect frequency dependence, set external frequencies to zero

Symmetries and **approximations**

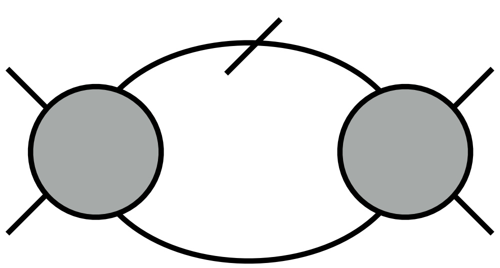
- system with **spin-rotational invariance**:

  - RG flow of general 4-point function $\Gamma^{(4)\Lambda}$: $\Gamma^{(4)\Lambda}_{\sigma_1,\sigma_2,\sigma_3,\sigma_4} = V^{\Lambda}\delta_{\sigma_1\sigma_3}\delta_{\sigma_2\sigma_4} - V^{\Lambda}\delta_{\sigma_1\sigma_4}\delta_{\sigma_2\sigma_3}$

  ➡️ flow of **spin-independent interaction vertex *V*^Λ**:

$$
\begin{aligned}
\frac{d}{d\Lambda}V^{\Lambda}(K_1,K_2;K_3,K_4) = &\int dK\, V^{\Lambda}(K_1,K_2,K)\, L^{\Lambda}(K,-K+K_1+K_2)\, V^{\Lambda}(K,-K+K_1+K_2,K_3)\,, \\
&+ \int dK \Big[-2V^{\Lambda}(K_1,K,K_3)\, L^{\Lambda}(K,K+K_1-K_3)\, V^{\Lambda}(K+K_1-K_3,K_2,K) \\
&\qquad + V^{\Lambda}(K_1,K,K+K_1-K_3)\, L^{\Lambda}(K,K+K_1-K_3)\, V^{\Lambda}(K+K_1-K_3,K_2,K) \\
&\qquad + V^{\Lambda}(K_1,K,K_3)\, L^{\Lambda}(K,K+K_1-K_3)\, V^{\Lambda}(K_2,K+K_1-K_3,K) \Big]\,, \\
&+ \int dK\, V^{\Lambda}(K_1,K+K_2-K_3,K)\, L^{\Lambda}(K,K+K_2-K_3)\, V^{\Lambda}(K,K_2,K_3)\,.
\end{aligned}
$$

  - where $L^{\Lambda}(K,K') = \frac{d}{d\Lambda}[G_0^{\Lambda}(K)G_0^{\Lambda}(K')]$



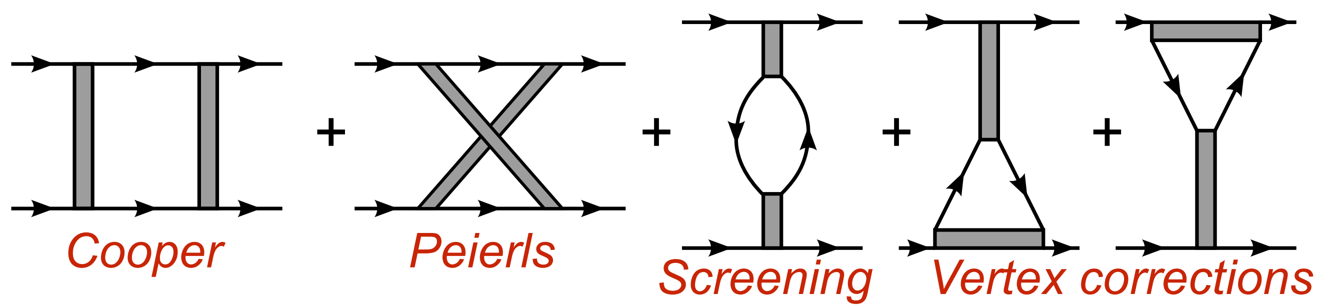
Symmetries and **approximations**

- system with **spin-rotational invariance**:

  - RG flow of general 4-point function $\Gamma^{(4)\Lambda}$: $\Gamma^{(4)\Lambda}_{\sigma_1,\sigma_2,\sigma_3,\sigma_4} = V^{\Lambda}\delta_{\sigma_1\sigma_3}\delta_{\sigma_2\sigma_4} - V^{\Lambda}\delta_{\sigma_1\sigma_4}\delta_{\sigma_2\sigma_3}$

  ➡️ flow of **spin-independent interaction vertex *V*$^{\Lambda}$**:

$$\frac{d}{d\Lambda}V^{\Lambda}(K_1, K_2; K_3, K_4) =$$



- corresponds to *infinite order summation* of one-loop *pp* and *ph* terms
- unbiased investigation of competition between various correlations
- **flow to strong coupling** indicates **ordering transition**: analyze components of V$^{\Lambda}$

Multi-orbital models

- **orbital degrees of freedom**:

  - valley d.o.f. in tBLG, *d*-orbitals of iron pnictides and transition-metal oxides

  - sublattice index of bipartite/multi-layer lattice (e.g., graphene's honeycomb lattice)

  - general non-interacting part of multi-orbital model: $H_0 = \sum_{\vec{k},s} \sum_{o,o'} c^\dagger_{o,s,\vec{k}} K_{o,o'}(\vec{k}) c_{o',s,\vec{k}}$

    orbital indices

    spin projection

    wavevector

  - unitary transformation to **energy band representation**:

$$c_{b,s,\vec{k}} = \sum_o u_{bo,\vec{k}} c_{o,s,\vec{k}}$$

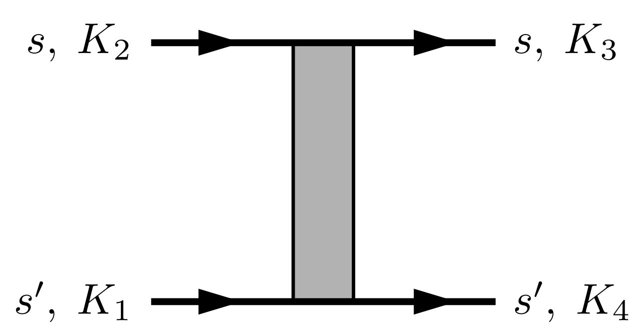
$$c^\dagger_{b,s,\vec{k}} = \sum_o u^*_{bo,\vec{k}} c^\dagger_{o,s,\vec{k}}$$

$$\Longrightarrow \quad H_0 = \sum_{\vec{k},s,b} E_b(\vec{k}) c^\dagger_{b,s,\vec{k}} c_{b,s,\vec{k}}$$

  - interaction part of Hamiltonian has to be transformed accordingly

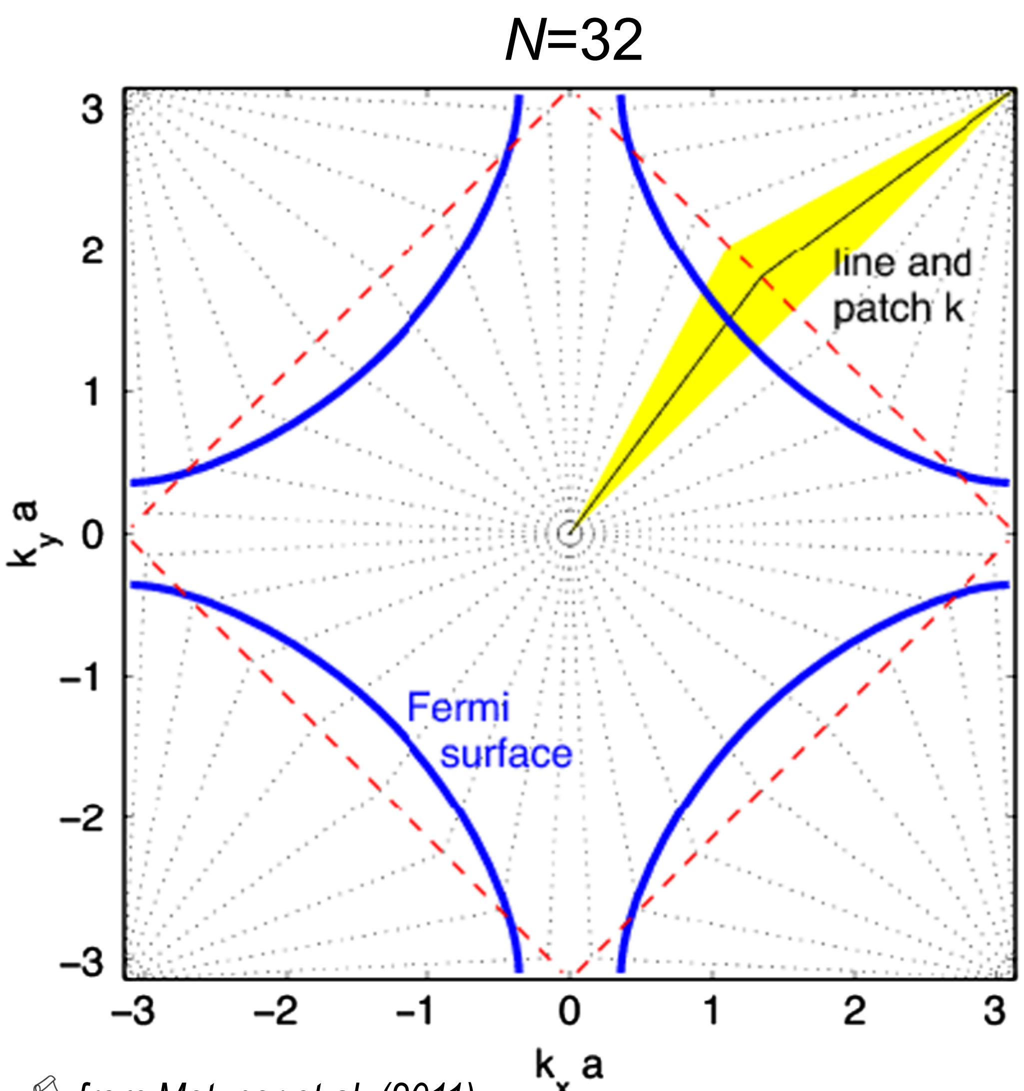
  ➡️ adds momentum dependence to interaction vertex at bare level - **orbital makeup**

Fermi-surface **patching scheme**



$$V_\Lambda(k_1, k_2, k_3, k_4)$$

- wavevector dependence of **Fermi surface** from discretization in ***N* patches:**

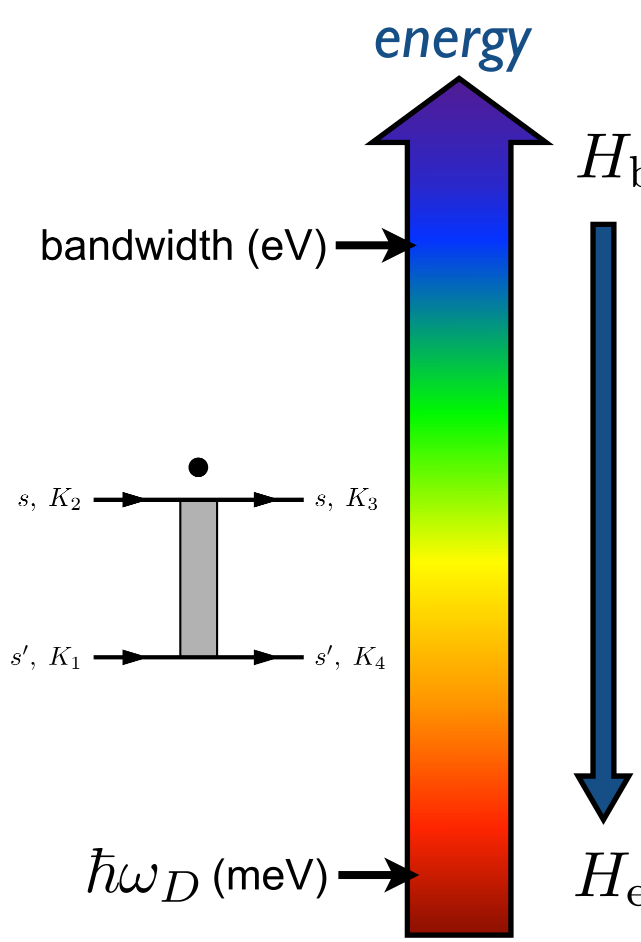


from Metzner et al. (2011)

- **interaction constant within one patch**
- representative momenta lie at Fermi level
- **finite set of coupled flow equations** for components of V^Λ
- facilitates numerical implementation
- **example:**
  - *t*-*t'*-*μ*-Hubbard model on the square lattice: vertex has N^3 components
  - generally: **V^Λ has $N_b^4 N^3$ components**

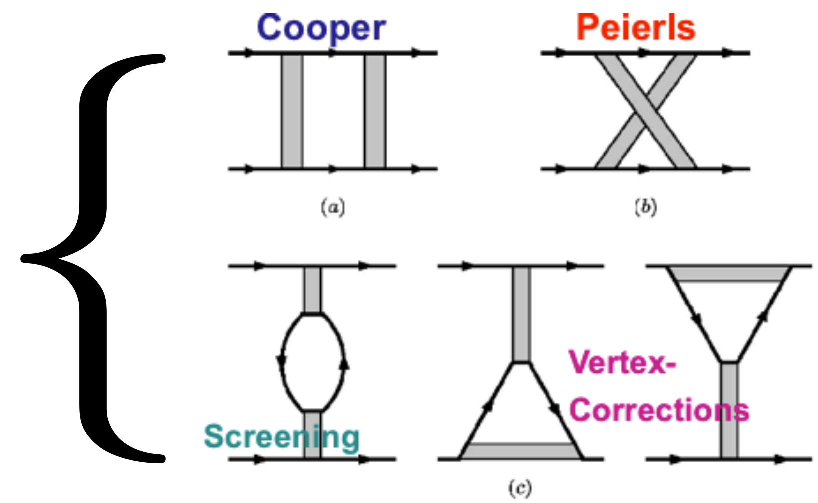
fRG: from **bare** to **effective interaction**

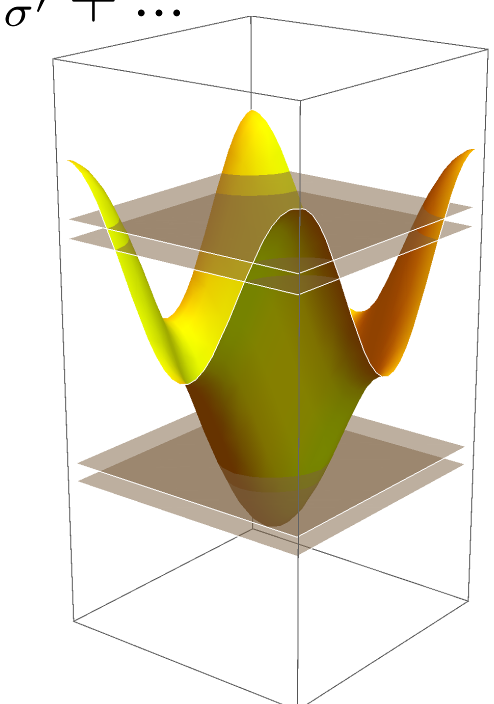
- excitations at intermediate scales generate momentum structure in low-energy interaction



$$H_{\text{bare}} = U\sum_i n_{i,\uparrow}n_{i,\downarrow} + V_1 \sum_{\langle i,j\rangle,\sigma,\sigma'} n_{i,\sigma}n_{j,\sigma'} + \ldots$$

fRG:

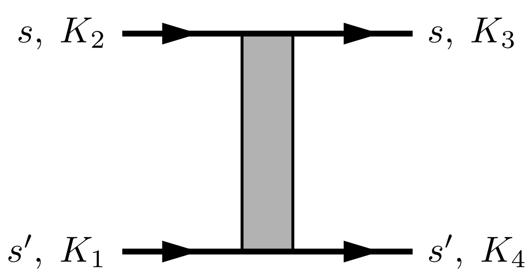




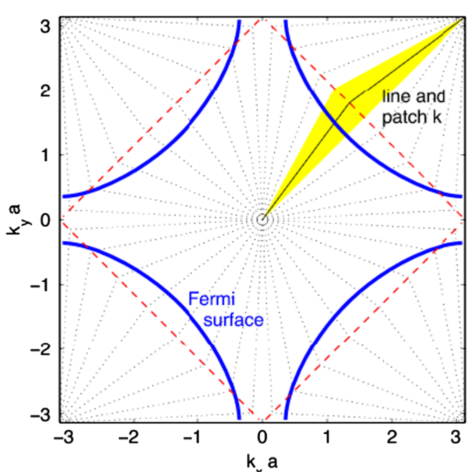
$$H_{\text{eff}} = \frac{1}{2}\sum_{\vec{p},\vec{p}',\vec{q},s,s'} \boxed{V(\vec{p},\vec{p}',\vec{p}+\vec{q})}\, c^\dagger_{\vec{p}+\vec{q},s}c^\dagger_{\vec{p}'-\vec{q},s'}c_{\vec{p}',s'}c_{\vec{p},s}$$

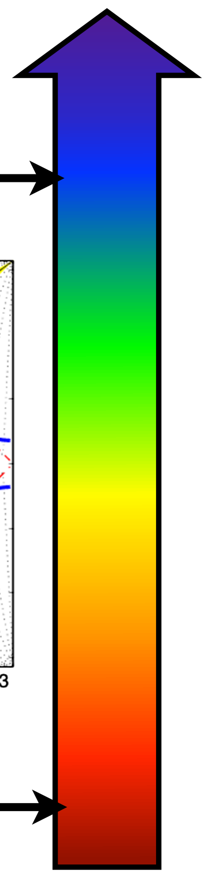
- low-energy effective action & momentum structure
  - ➡️ two-particle interaction vertex $V(\vec{p},\vec{p}',\vec{p}+\vec{q})$
  - ➡️ flow to strong coupling: singularity for $\Lambda \to \Lambda^*$
  - ➡️ read off dominant interactions and e.g. extract form factors of order parameters

fRG: from **bare** to **effective interaction**



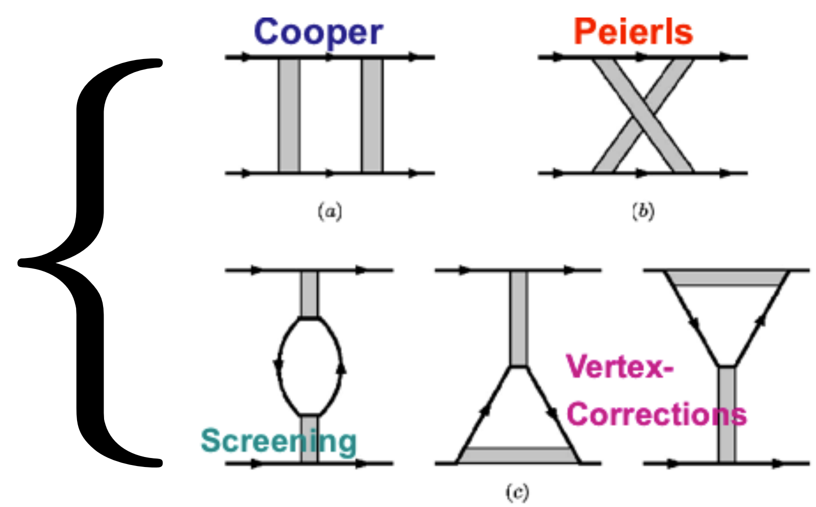
- sharp momentum structures in the interaction vertex emerge
- e.g., onsite interaction (U=3.0t), typical pattern (k_3 fixed at point 1):

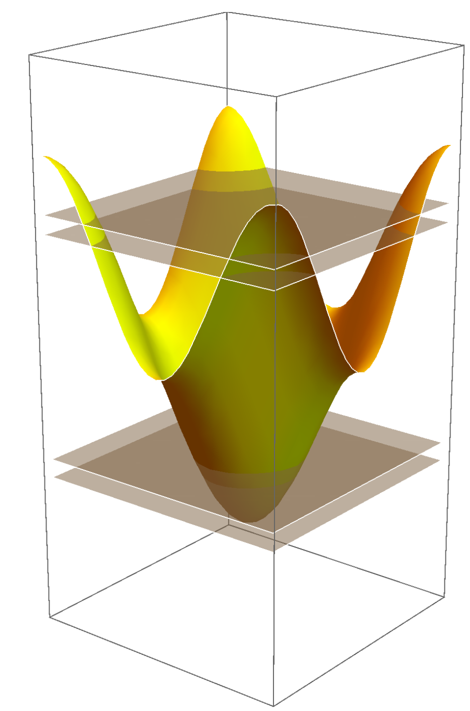




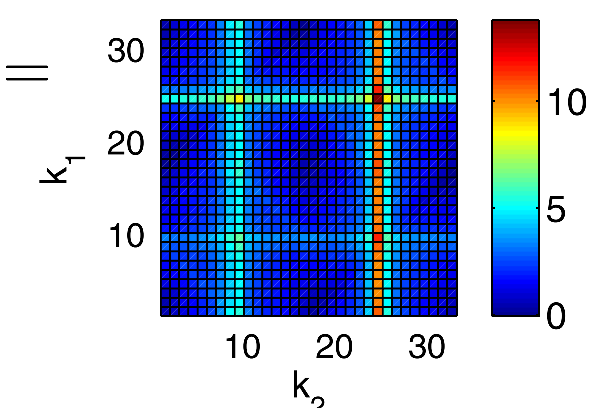
$$H_{\text{bare}} = U \sum_i n_{i,\uparrow} n_{i,\downarrow}$$

fRG:

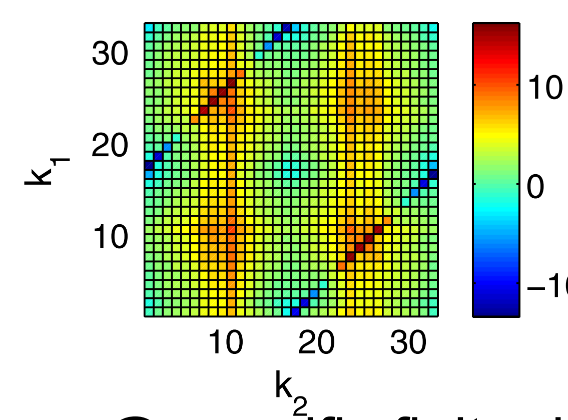




$H_{\text{eff}} =$



or $H_{\text{eff}} =$



@ zero doping

@ specific finite doping

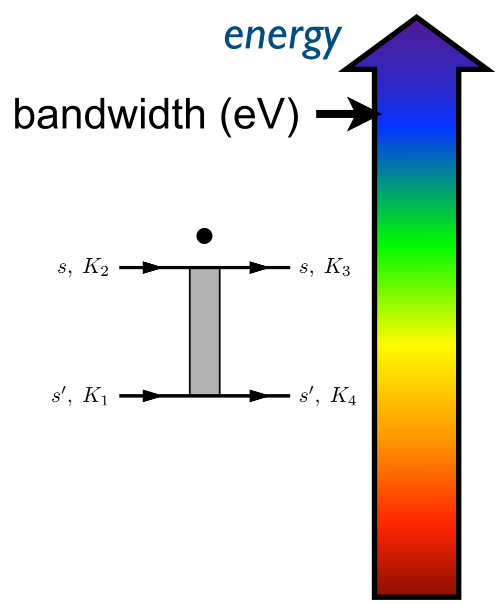
$$J \sum_{\langle i,j \rangle} e^{i\mathbf{Q}\cdot(\mathbf{R}_i - \mathbf{R}_j)} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$H^{\Lambda}_{dSC} = V_{dSC} \sum_{\mathbf{k},\mathbf{k}'} d(\mathbf{k}) d(\mathbf{k}') c^{\dagger}_{\mathbf{k}',\uparrow} c^{\dagger}_{-\mathbf{k}',\downarrow} c_{-\mathbf{k},\downarrow} c_{\mathbf{k},\uparrow}$$

- mean-field decoupling $\longrightarrow$ antiferromagnetic SDW (**AF-SDW**) or ***d*-wave SC**

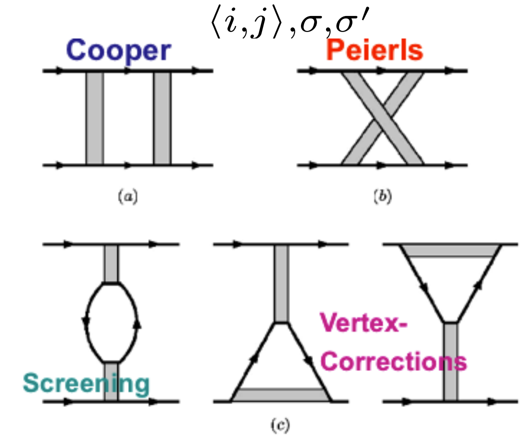
fermion fRG approach — **Precis**

- *fermion fRG:* discovery tool for leading many-body instabilities



$$H_{\mathrm{bare}} = U \sum_i n_{i,\uparrow} n_{i,\downarrow} + V_1 \sum_{\langle i,j\rangle,\sigma,\sigma'} n_{i,\sigma} n_{j,\sigma'} + \ldots$$

fRG:



$$H_{\mathrm{eff}} = \frac{1}{2} \sum_{\vec{p},\vec{p}',\vec{q},s,s'} V(\vec{p},\vec{p}',\vec{p}+\vec{q})\, c^{\dagger}_{\vec{p}+\vec{q},s} c^{\dagger}_{\vec{p}'-\vec{q},s'} c_{\vec{p}',s'} c_{\vec{p},s}$$

Metzner et al., RMP 84 (2012)

- treats all fermionic fluctuation channels on *equal footing*
- *infinite-order resummation* of all fermionic 1-loop diagrams
- can deal with *multi-orbital band structures, non-local i.a. & competing correlations*
- due to truncations/approximations: **qualitative** (not quantitative) tool
- *prediction* of different types of *magnetism / superconductivity / bond order states / …*

Minimal phenomenological model for tDBLG

1. *triangular* lattice near van-Hove filling (twisted **double** bilayer graphene)
2. weak tunnelling between nearest-neighbor unit supercells dominates kinetic energy
3. each cell hosts two degenerate orbitals from original valleys +/-
4. no mixing of orbitals due to large momentum space separation
5. spin-independent hopping of electrons

Balents & Xu, PRL 121, 087001 (2018)

two-orbital tight-binding model $H_{\mathrm{kin}} = -t \sum_{\langle ij \rangle} \sum_{\sigma=\uparrow,\downarrow} \sum_{o=\pm} \left(c^{\dagger}_{i\sigma o} c_{j\sigma o} + \mathrm{h.c.} \right)$

- 4 flavours: $\alpha \in \{(\uparrow,+), (\downarrow,+), (\uparrow,-), (\downarrow,-)\} \rightarrow$ effective SU(4) symmetry
- add SU(4) symmetric Hubbard interaction as minimal interaction

$$H_{\mathrm{int}} = U \sum_i \left(\sum_{\alpha=1}^{4} n_{i\alpha} \right)^2$$

- dominant interaction depends only on total charge of site

Next-to-minimal model

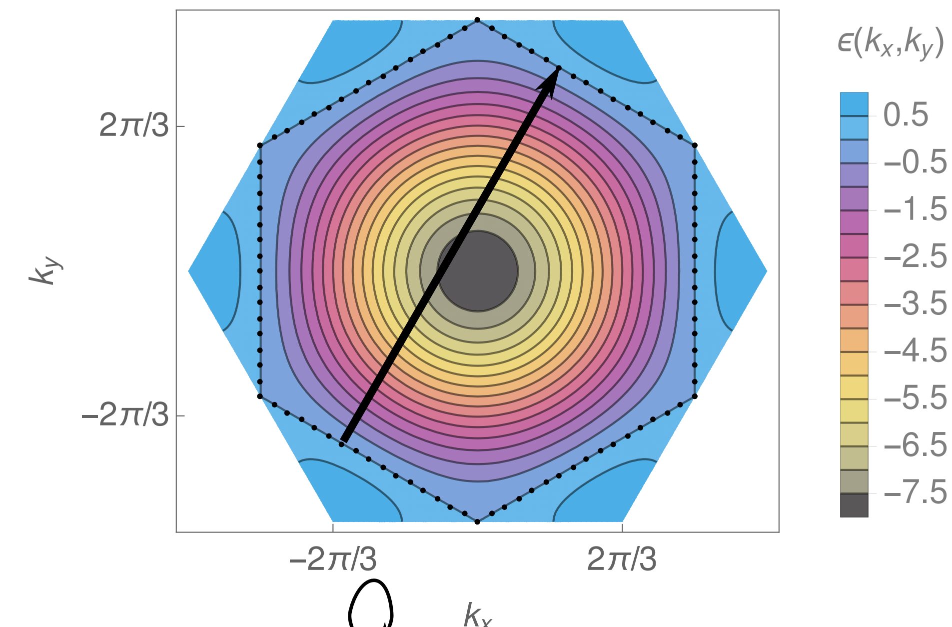
- Include **Hund's couplings**: $H_h = -V_h \sum_i \vec{S}_i \cdot \vec{S}_i$
  - with $\vec{S}_i = \frac{1}{2} c^\dagger_{i\sigma o} \vec{\sigma}_{\sigma\sigma'} c_{i\sigma' o}$
  - breaks SU(4)
  - alternatively "orbital" Hund's coupling (**anti-Hund**):

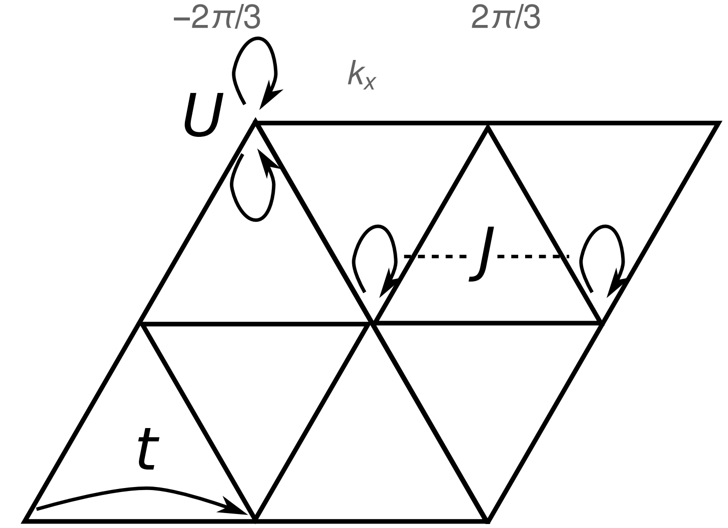
$$H_K = -K \sum_i \vec{L}_i \cdot \vec{L}_i \quad \text{with} \quad \vec{L}_i = \frac{1}{2} c^\dagger_{i\sigma o} \vec{\tau}_{oo'} c_{i\sigma o'}$$

- also add SU(4) **exchange coupling** *J*
- **hierarchy of model parameters**:
  - strong onsite interaction $U > J,\ V_h$,
  - approximate SU(4): $U > V_h, K$
  - tune filling by chemical potential μ

➡️ *fRG phase diagram…*

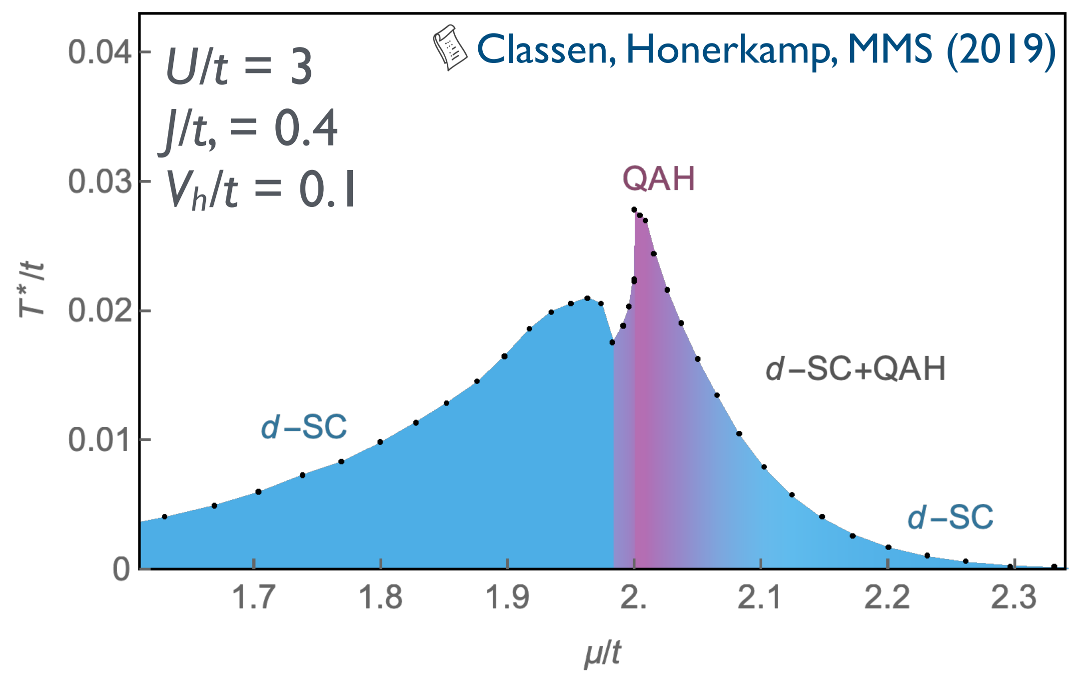
Xu & Balents
Dodaro, Kivelson, Schattner, Sun, Wang
Yuan & Fu
Po, Zou, Vishwanath, Senthil

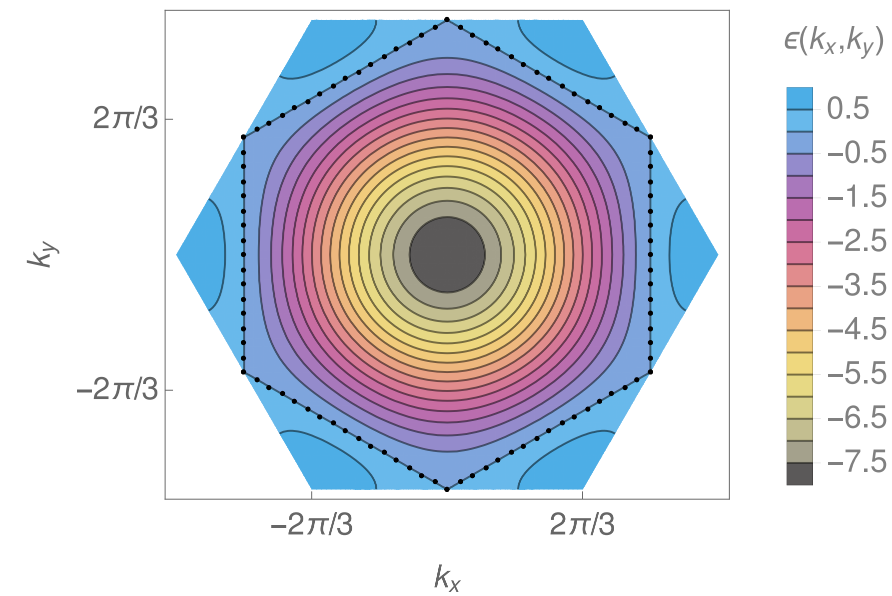




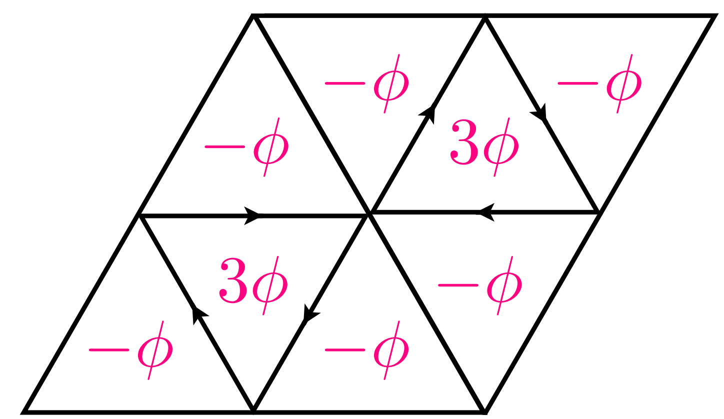
Next-to-minimal models — functional RG approach

- *phase diagram:*

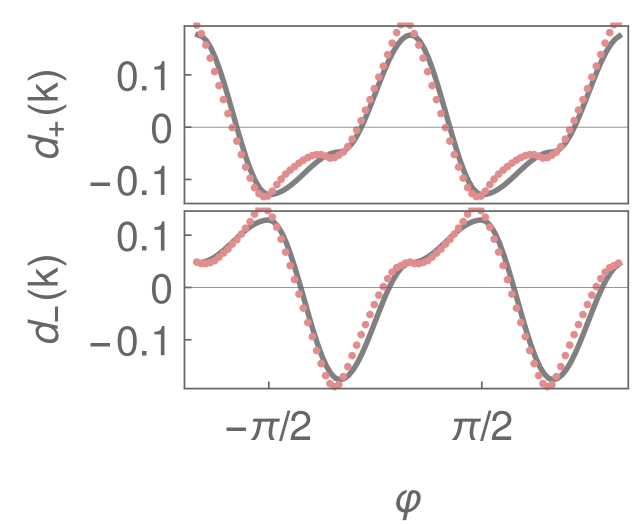




- **interaction-induced quantum anomalous Hall state**:
  - robust near van-Hove filling, fully gapped spectrum
  - breaks time-reversal symmetry
  - Chern insulator: Haldane-like loop current



- ***d±id* superconductivity**:
  - SC form factor superposition of $d_{xy}(k)$ and $d_{x^2-y^2}(k)$
  - expect lowest energy for full gap → $d_{xy} \pm i d_{x^2-y^2}$ superposition
  - for $V_h > K$: (spin-singlet) x (orbital-triblet) pairing function



…next: ***implementation of realistic lattice models…***

Conclusions & Outlook

Conclusions and **Outlook**

- Superlattice modulation of moiré heterostructures
  - flat bands, strongly-correlated physics, "high-T_c" phase diagrams, highly tunable
- **Open questions**
  - appropriate models and characterization of correlated states
  - weak-coupling vs. strong coupling perspective
- Competing orders/instabilities → **fermion fRG** (for tdBLG)
  - next-to-minimal triangular SU(2)xSU(2)
  - main phases:

    ✴️Chern insulating **QAH** state

    ✴️***d±id*** superconductivity