Chiral Phase Structure and Thermodynamics

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Overview

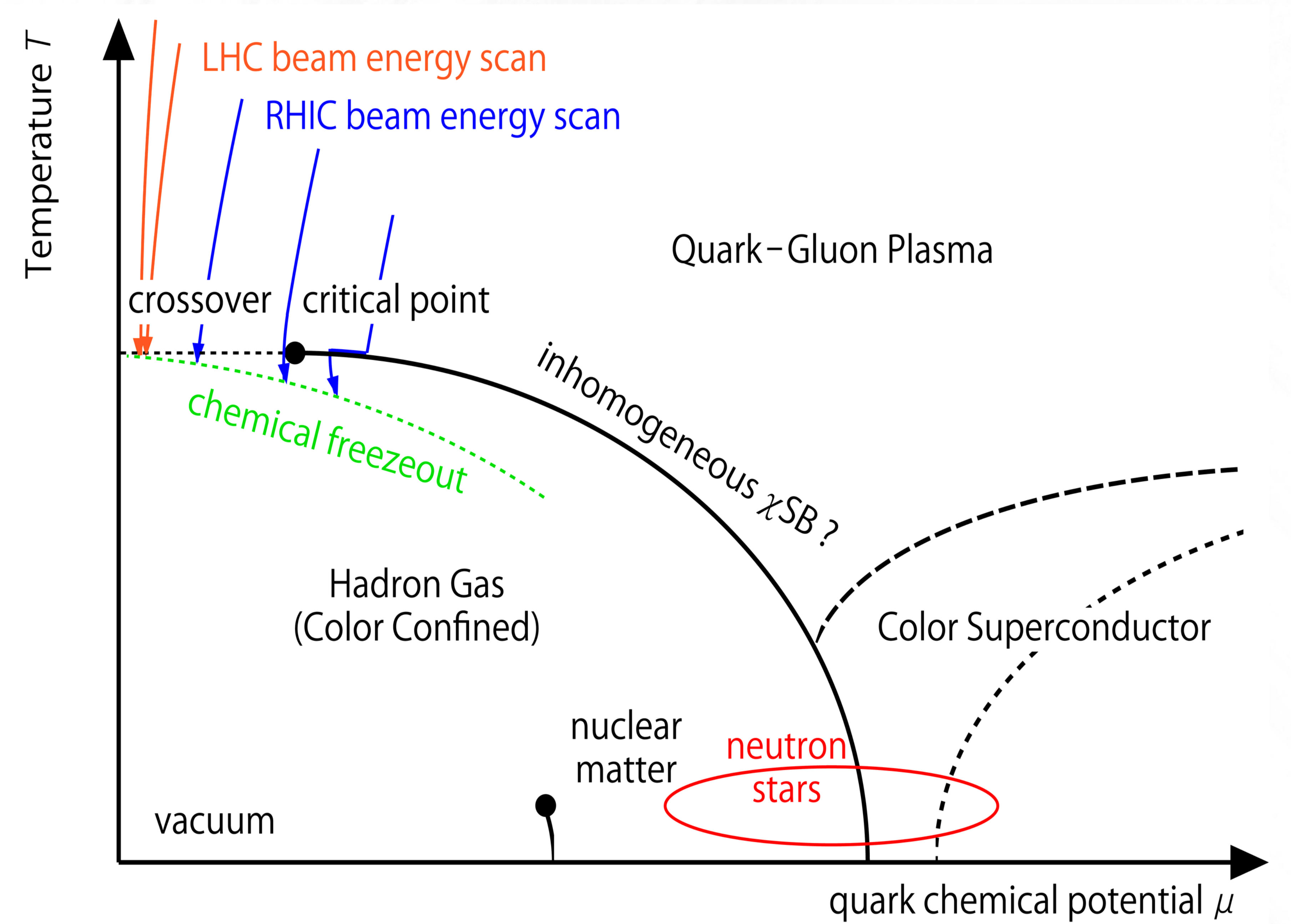
(1) Introduction

(2) Effective Degrees of Freedom

(3) Quark Meson Model

(4) Numerical Results

(1) Introduction



Temperature T
LHC beam energy scan
RHIC beam energy scan
Quark−Gluon Plasma
crossover
critical point
inhomogeneous χSB ?
chemical freezeout
Hadron Gas
(Color Confined)
Color Superconductor
nuclear
matter
neutron
stars
vacuum
quark chemical potential μ

Experimental Efforts

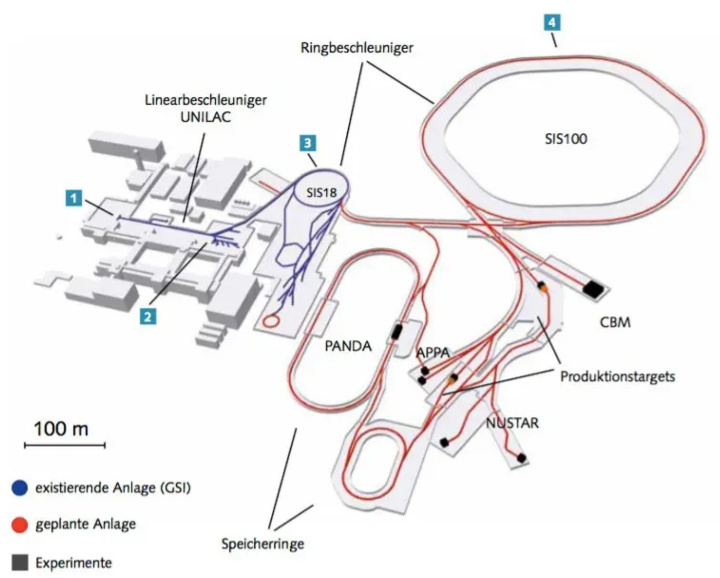
- Indirect determination at heavy ion colliders:

  Freeze-out data

  ➔ LHC, SPS

  ➔ FAIR

  ➔ RHIC



- Data from neutron stars

Theoretical Approach

- Low energy QCD, so perturbation theory is

  not applicable
- Lattice QCD limited to low chemical potential

- Functional Methods: Non-perturbative and not affected by sign-problem (Downside: Truncation needed)

  → Our choice: Functional Renormalization Group

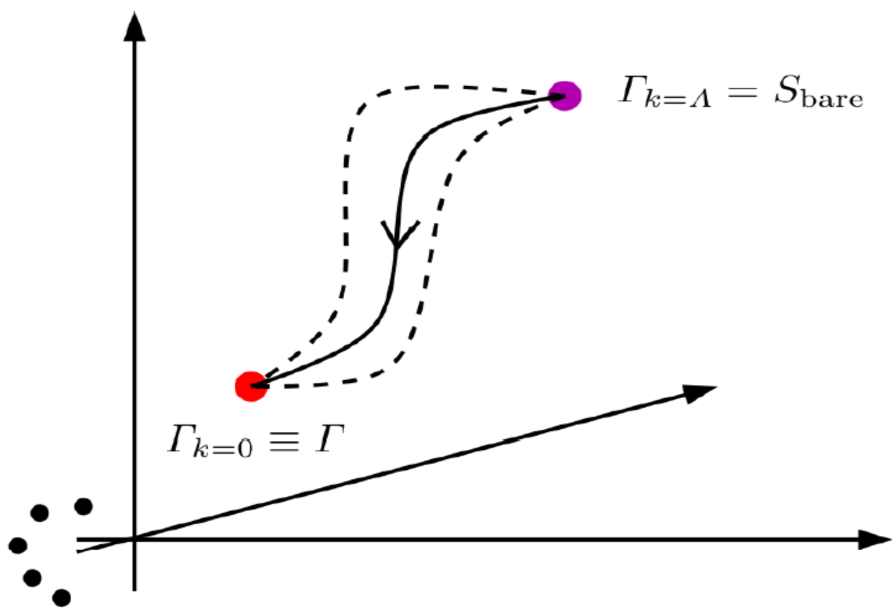
Functional Renormalization Group

- Scale dependent regulated generating functional

$$Z_k[J] = \int_\Lambda \mathfrak{D}\tilde{\Phi} \exp\left\{-\left(S[\tilde{\Phi}] + \Delta S_k[\tilde{\Phi}] + J^T \tilde{\Phi}\right)\right\}$$

leads to an effective action Γ_k

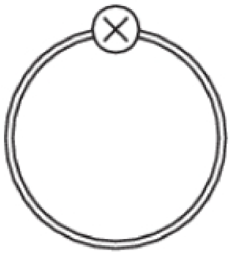
- Idea:



Functional Renormalization Group

- Flow of the effective action described by the Wetterich equation:

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2}\mathrm{STr}\left\{ (\partial_t R_k) \cdot \left(\Gamma_k^{(2)}[\Phi] + R_k \right)^{-1} \right\} = 1/2$$



RG-"time" $t = \ln\left(\frac{k}{\Lambda}\right)$

- Not solvable for full action, truncations/ effective models needed

- Truncation leads to a residual dependence of the effective action on the regulator function R_k

(2) Effective Degrees of Freedom

Four Fermion Interactions

Connection to QCD?

$$\mathcal{L}_{QCD} = \bar{q}\left(i\partial\!\!\!/ - m_0\right) q - \frac{1}{4}\left(F_{\mu\nu}\right)^2 + gA^a_\mu j^\mu_a$$

$$j^\mu_a = \bar{q} T^a \gamma^\mu q$$

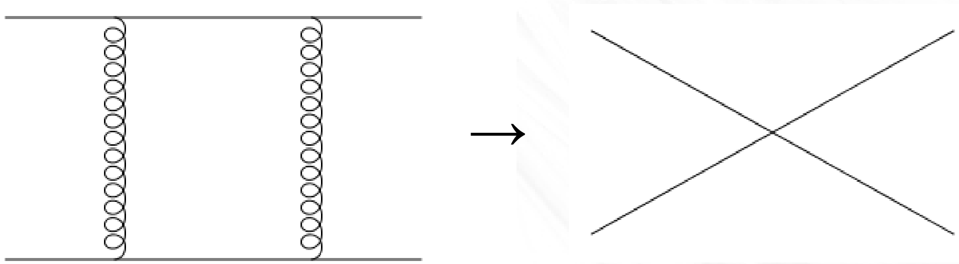
Rewrite the gauge part:

$$G[j] = \ln \int \mathscr{D}A \exp\left\{-\frac{1}{4}\int F^2 + \int gA^a_\mu j^\mu_a\right\}$$

$$\rightarrow \quad Z_{QCD} = \int \mathscr{D}q \mathscr{D}\bar{q} \exp\left\{\int \bar{q}\left(i\partial\!\!\!/\right) q + G[j]\right\}$$

Expansion of G around j=0 and assuming local contact interaction gives leading non-trivial term of the form

$$\sim \int d^4x \frac{g^2}{2} j^\mu_a j^a_\mu$$



→Effective four fermion Interactions

Bosonization vs. Dynamical Hadronization

"Standard": Hubbard-Stratonovich transformation at the UV-scale

Example: $$\Gamma_{\mathrm{NJL},\Lambda}[\psi,\bar{\psi}] = \int d^4x \left\{ Z_\psi \bar{\psi} i \partial\!\!\!/ \psi + \frac{1}{2}\lambda_\Lambda \left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2 \right] \right\}$$

Multiply generating functional with

$$1 = \mathscr{N} \int \mathscr{D}\phi_1 \mathscr{D}\phi_2 \exp\left\{ - \int d^4x \frac{m_\sigma^2}{2} \vec{\phi}^2 \right\}$$

and shift auxiliary fields properly to obtain

$$\Gamma[\psi,\bar{\psi},\phi] = \int d^4x \left\{ Z_\psi \bar{\psi} i \partial\!\!\!/ \psi + \frac{m_\sigma^2}{2} \vec{\phi}^2 + \frac{i h_\sigma}{\sqrt{2}} \bar{\psi} [\phi_1 - i\gamma_5\phi_2] \psi \right\}$$

→All interactions are now mediated by the new bosonic fields!

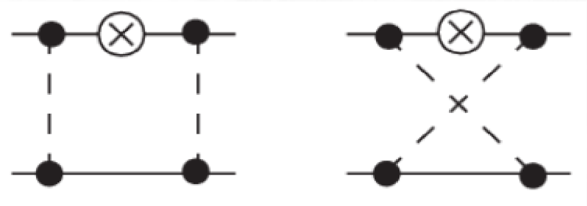
Dynamical Hadronization

Why?

- Fermion interactions which were replaced in the HS-transformation can be re-generated in the flow

- Simply neglecting them at low scales results in an error compared to the original theory

→Solution: Re-bosonize the action in each RG-step



How?

- Allow explicit scale dependence of the mesonic fields

→Additional degrees of freedom can be used to cancel out the flow of the fermionic coupling(s) exactly

Dynamical Hadronization

In our Example: $\Gamma_{\mathrm{NJL,k}}[\psi,\bar{\psi}] = \int d^4x \left\{ Z_\psi \bar{\psi} i\partial\!\!\!/ \psi + \frac{1}{2}\lambda_k \left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2 \right] \right\}$

- Bosonize and assume scale dependence of the form

$$\partial_t \phi_k = \dot{A}_k \begin{pmatrix} \bar{\psi}\psi \\ \bar{\psi}\gamma_5\psi \end{pmatrix}$$

- Modified Wetterich Eq.: $\left(\partial_t + (\partial_t \phi_k) \frac{\delta}{\delta \phi_k} \right) \Gamma_k [\Phi_k] = \frac{1}{2} \mathrm{STr} \left(\frac{\partial_t R_k}{\Gamma_k^{(2)} [\Phi_k] + R_k} \right)$

- Flow for the fermionic coupling: $\partial_t \bar{\lambda}_k \big|_{\bar{\phi}} = (\partial_t^0 + 2\eta_\psi) \bar{\lambda}_k + \xi \dot{\bar{A}}_k$

→ Set the hadronization function to $\dot{\bar{A}} = -(\partial_t^0 \bar{\lambda}_k) \xi^{-1}$

(3) Quark Meson Model

Choice of Truncation

- Scalar and pseudo-scalar channel
- Lowest order derivative expansion: Local Potential Approximation (LPA), where only the effective potential is scale dependent
- Extension: Scale dependent wave fct. renormalizations and Yukawa coupling
- Explicit symmetry breaking (→massive pions)
- Finite temperature and quark chemical potential

$$\Gamma_k[\Phi] = \int_\beta \quad \Big\{ \bar{\psi} \left[Z_{\psi,k} \left(\partial\!\!\!/ - \mu \gamma_0 \right) + g_k \left(\sigma + i \vec{\tau} \vec{\pi} \gamma_5 \right) \right] \psi + \tfrac{1}{2} Z_{\phi,k} \left(\partial_\mu \phi \right)^2 + \Omega_k(\phi^2) - c\sigma \Big\}$$

$$N_f = 2, N_c = 3 \qquad \int_\beta = \int_0^{1/T} dx_0 \int d^3x \qquad \phi = (\sigma, \vec{\pi})$$

- <u>Beyond LPA:</u>
  - ➔ Flow equations depend on external momenta
  - ➔ Quantities (Z's, g) are regarded as field independend, but flows need to be evaluated at some field value

Label	Flows
LPA	Ω_k
LPA'	$\Omega_k, Z_{\psi,k}, Z_{\phi,k}$
LPA+Y	Ω_k, g_k
LPA'+Y	$\Omega_k, Z_{\psi,k}, Z_{\phi,k}, g_k$

(LPA=Local Potential Approx.)

- <u>Choice for fermionic external momenta:</u> $p_{0,ext}^{\psi} = \pi T - i\mu$

- <u>Choice of the evaluation point:</u> "**co-moving"** with scale dependent minimum of the eff. potential vs. always at the vacuum expectation value ("**const.**")

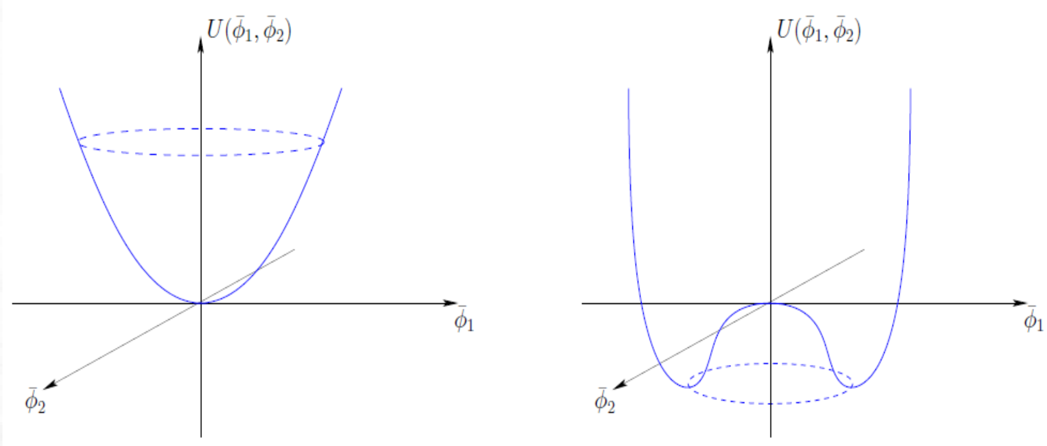
UV-Ansatz

$$\Omega_\Lambda = a_1\phi^2 + \frac{a_2}{2}\phi^4$$

$$\Lambda = 900\,\mathrm{MeV}$$
$$k_0 = 80\,\mathrm{MeV}$$

- Fixing the masses and the minimum of the potential at the
- IR-scale determines the parameters

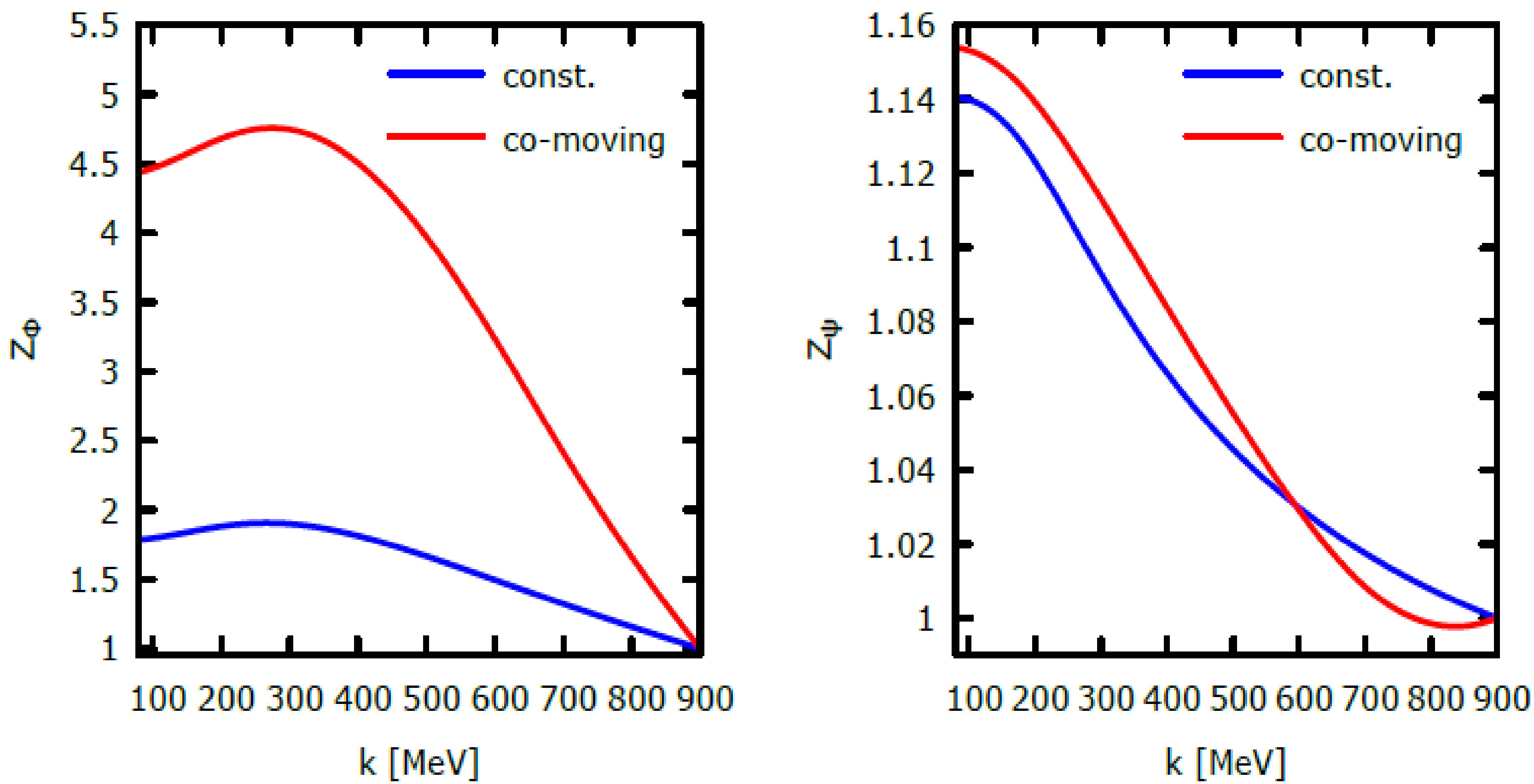
$$\bar{m}_{\psi,vac} = 300\,\mathrm{MeV}$$
$$\bar{m}_{\sigma,vac} = 550\,\mathrm{MeV}$$
$$\bar{m}_{\pi,vac} = 138\,\mathrm{MeV}$$
$$f_{\pi,vac} = 93\,\mathrm{MeV}$$



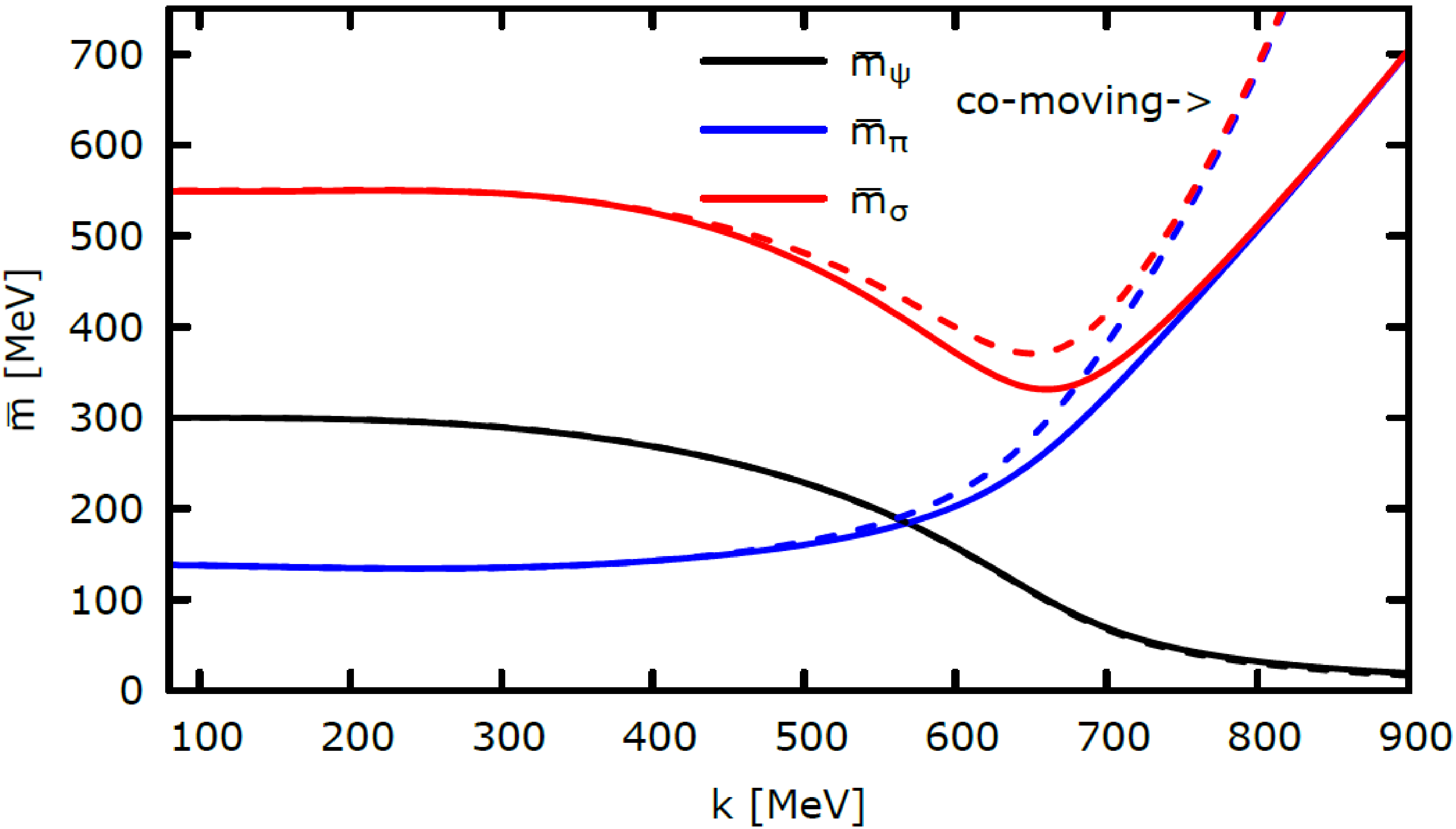
(4) Numerical Results

(LPA'+Y if not stated otherwise)

Wave Function Renormalizations: Vacuum Flow

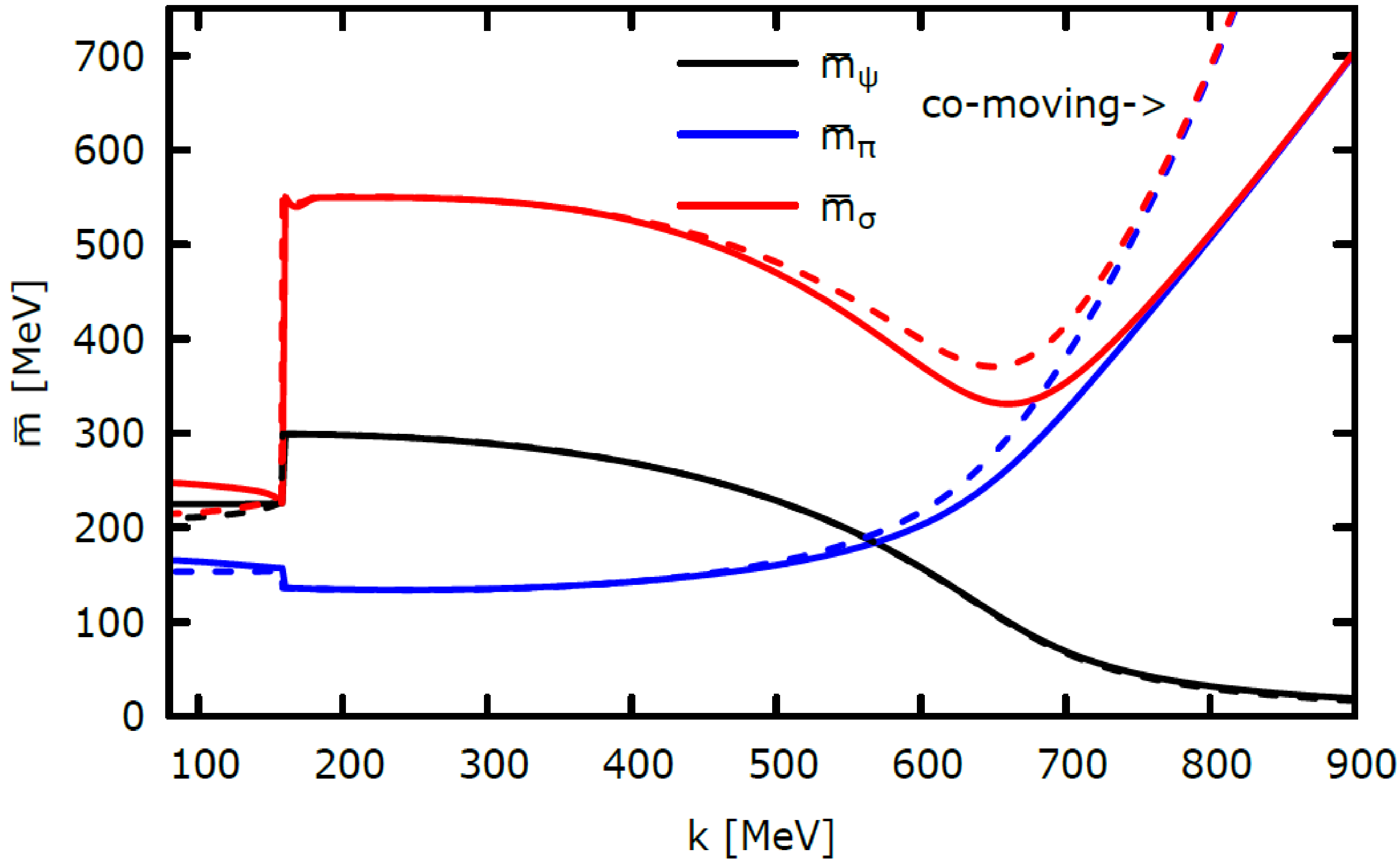


Scale Dependence of the Masses



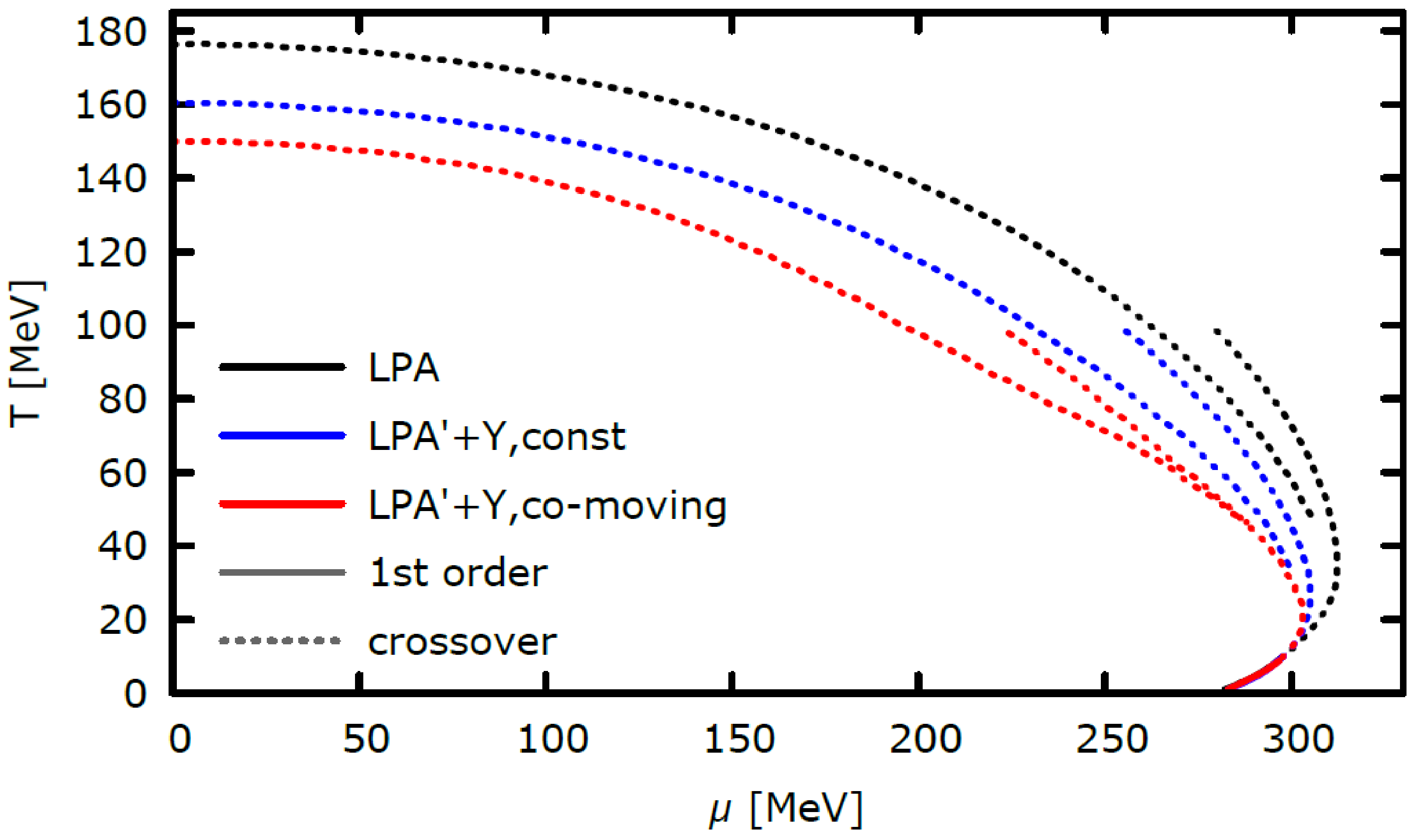
(a) Vacuum flow

Scale Dependence of the Masses



(b) Flow for $T = 5\,\mathrm{MeV}$ and $\mu = 300\,\mathrm{MeV}$

Phase Diagram



Phase Diagram

- <u>Critical temperatures for μ=0:</u>

$$T_c^{\text{const}} = 158.4\,\text{MeV}$$
$$T_c^{\text{co-moving}} = 150.2\,\text{MeV}$$

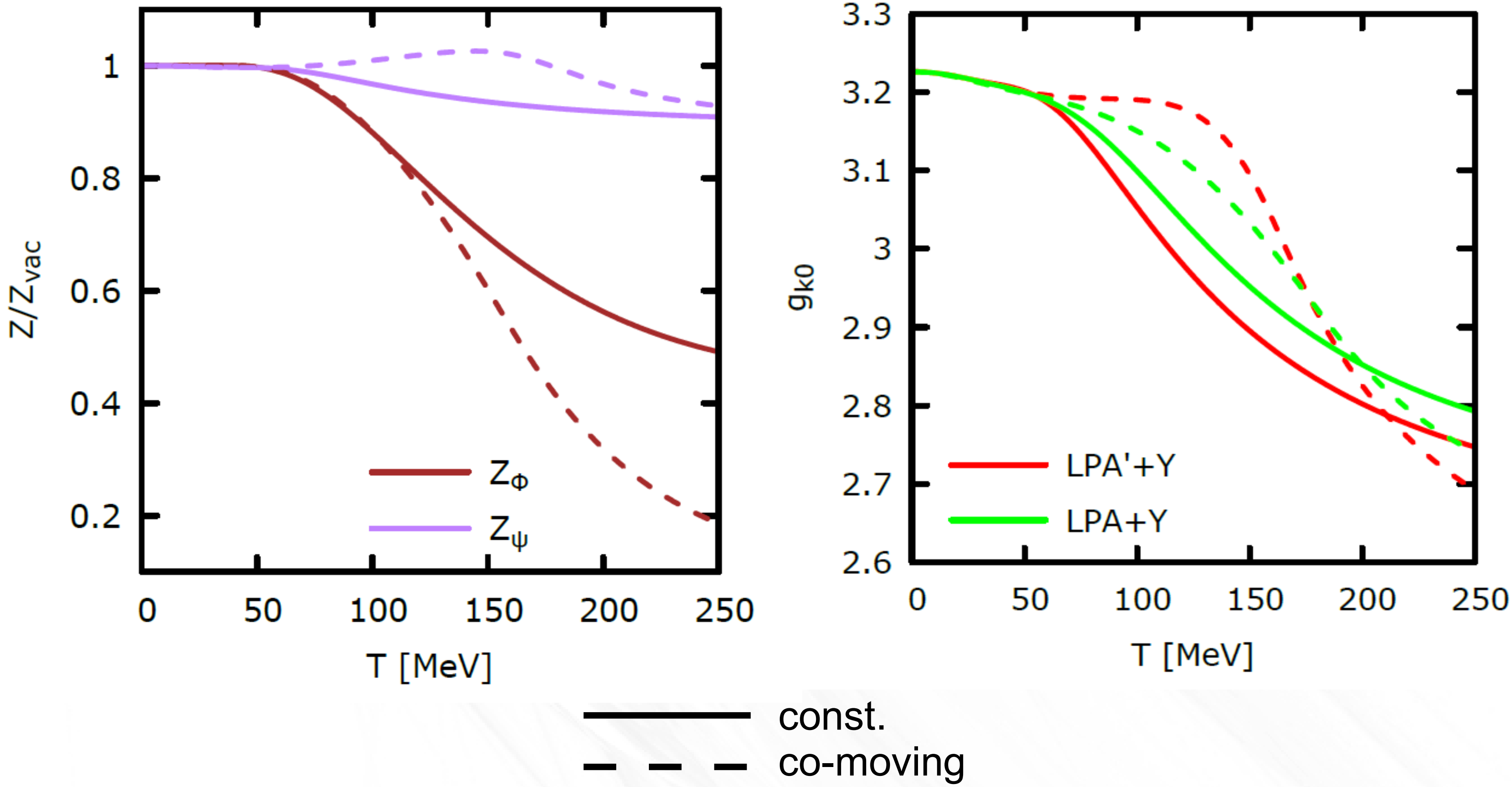
→ good agreement with latice results: $T_c^{\text{lattice}} \approx 157\,\text{MeV}$

R.Bellwied et. al. (2015);HotQCD(2019)

- <u>Critical endpoint:</u> same position in LPA & LPA'+Y, at very low T due to the high sigma mass

$$T_{\text{CEP}} \lesssim 10\,\text{MeV}$$

Temperature Dependence of the Z's and the Yukawa Coupling



Thermodynamics

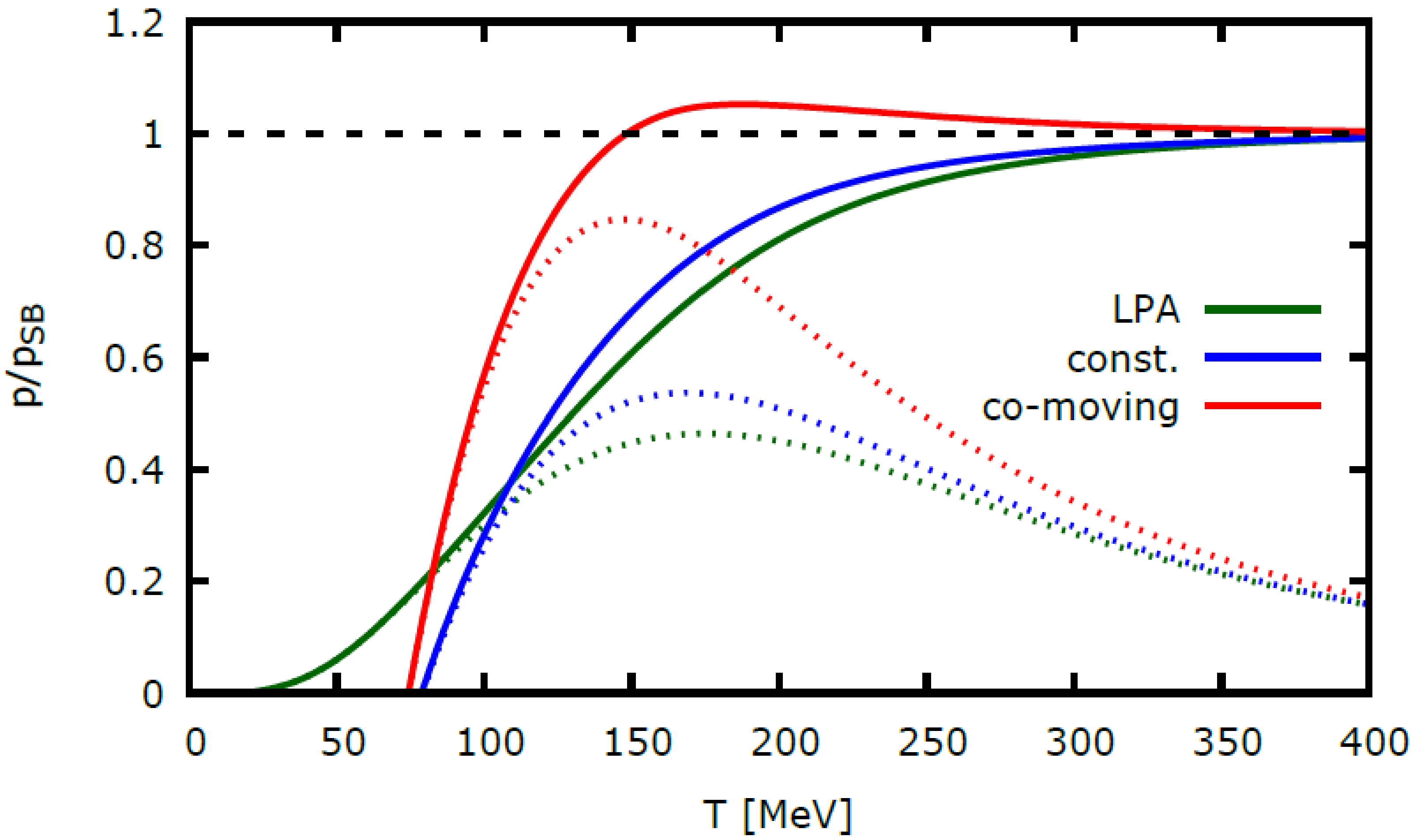
- Pressure given by $$p(T,\mu) = -\left(\bar{\Omega}_{k_0}(T,\mu) - \bar{\Omega}_{k_0}(0,0)\right)\Big|_{\bar{\sigma}=\bar{\sigma}_0}$$

- For high temperatures expect convergence to Boltzmann value $$p_{SB} = 2N_f N_c T^4 \cdot \left[\frac{7\pi^2}{360} + \frac{1}{12}\left(\frac{\mu}{T}\right)^2 + \frac{1}{24\pi^2}\left(\frac{\mu}{T}\right)^4\right]$$

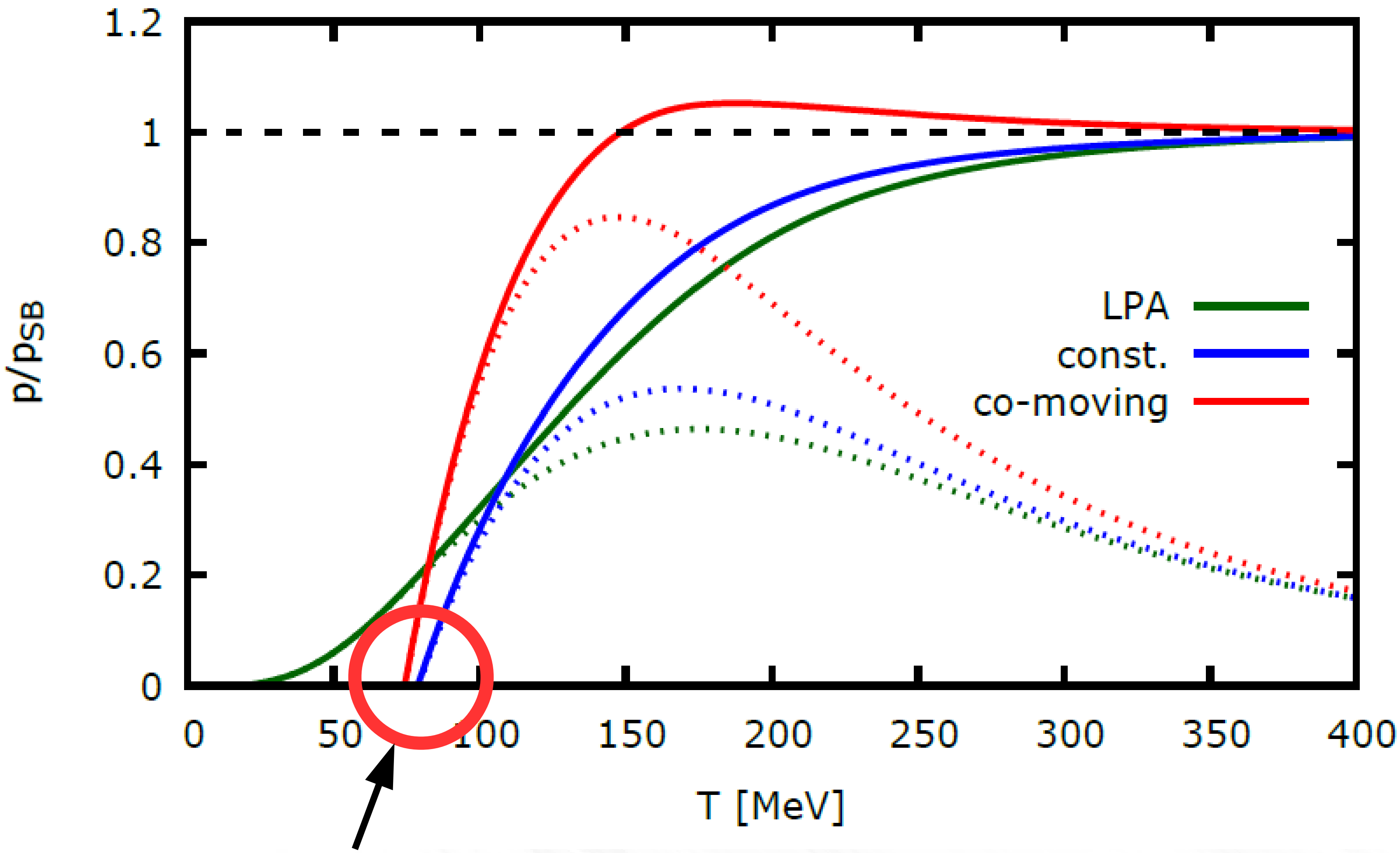
- But: Cutoff effect results in lower values, to take this into account one can add a free fermion gas at higher momenta:

$$\Omega_{UV} = -\frac{4N_f N_c}{12\pi^2}\int_{\Lambda}^{\infty} dk\,k^3 \left\{n_F(k,-\mu) + n_F(k,\mu)\right\}$$

Normalized Pressure at vanishing μ

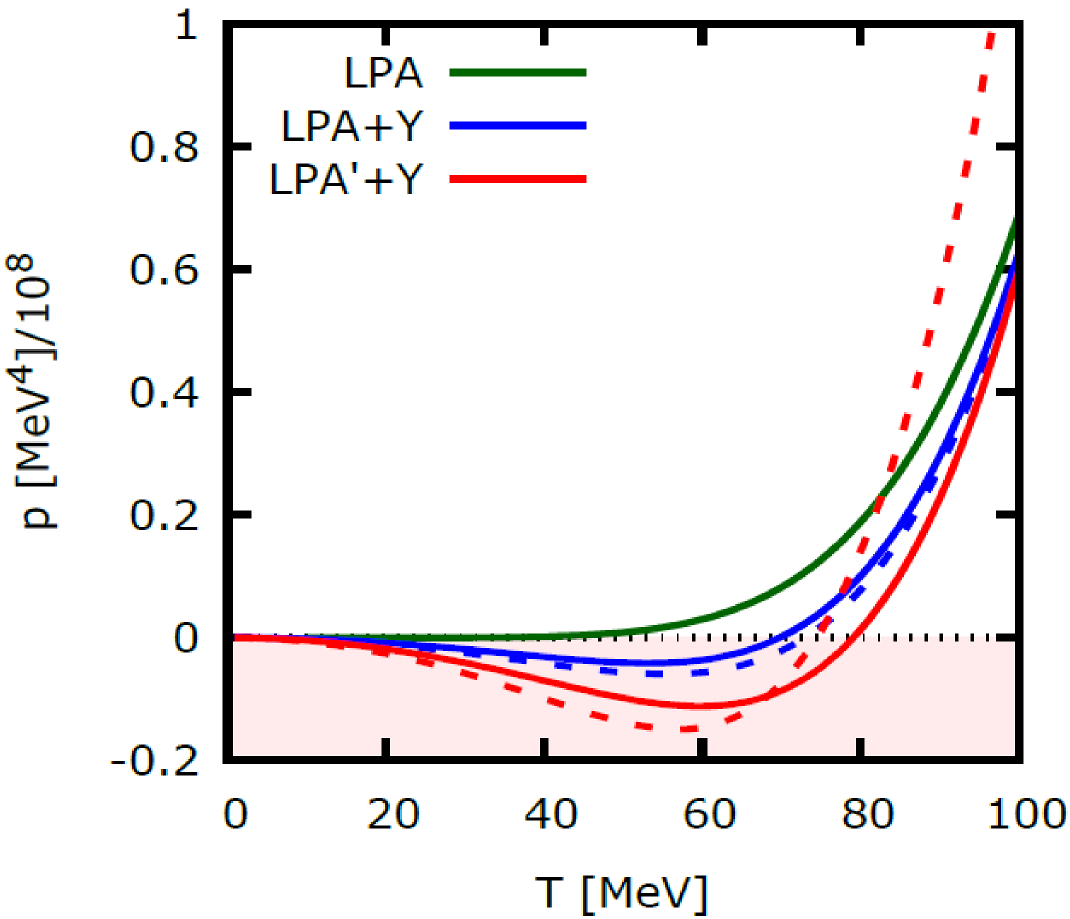


Normalized Pressure at vanishing μ



But what happens at low temperatures?

Absolute values of the pressure:



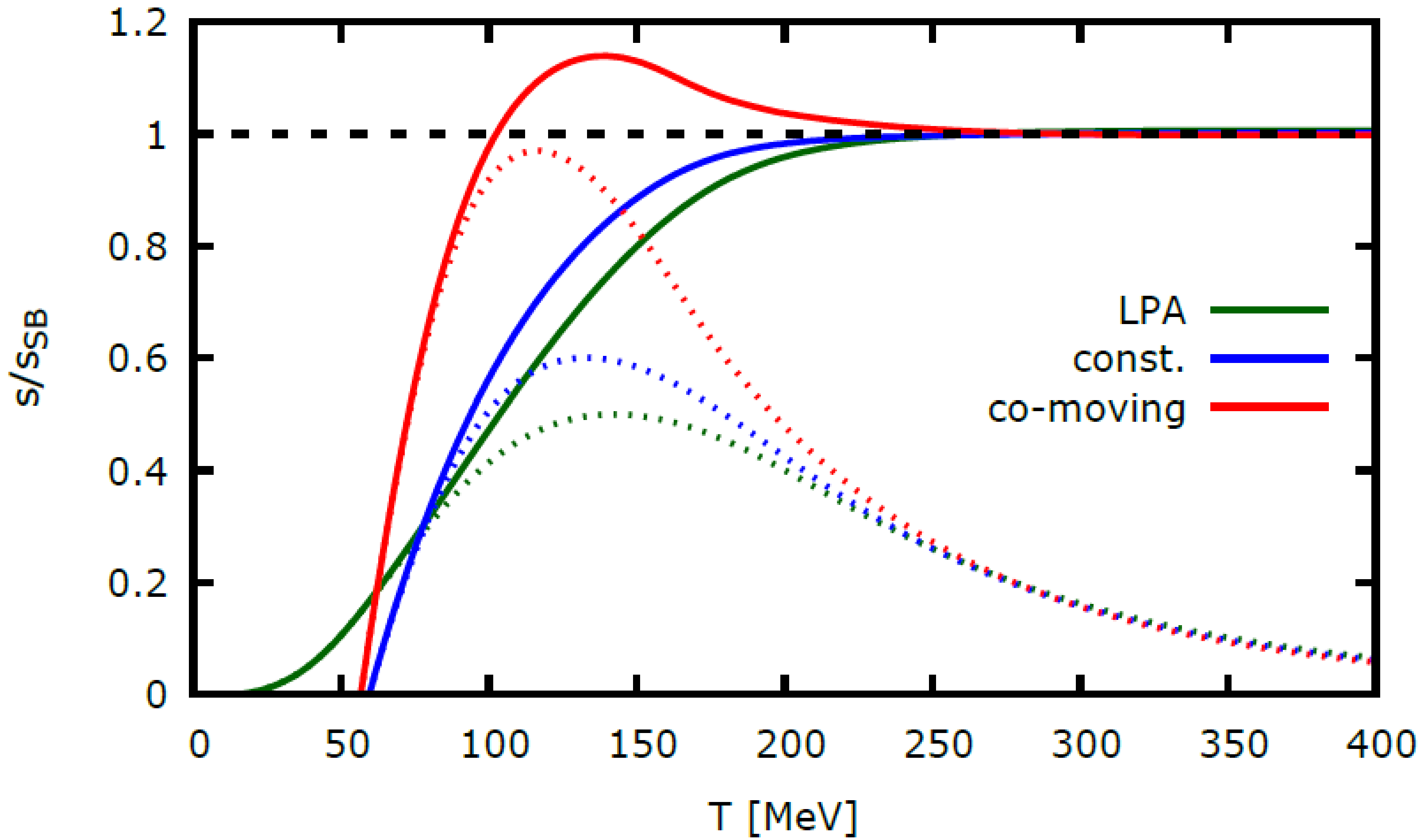
→Can be cured by use of scale dependent external momenta:

Fu, Pawlowski(2015)

$$p_{0,ext}^{\psi} = k\sqrt{1+(\pi\tau)^2\,\Theta_{\varepsilon}(1/\tau)}$$

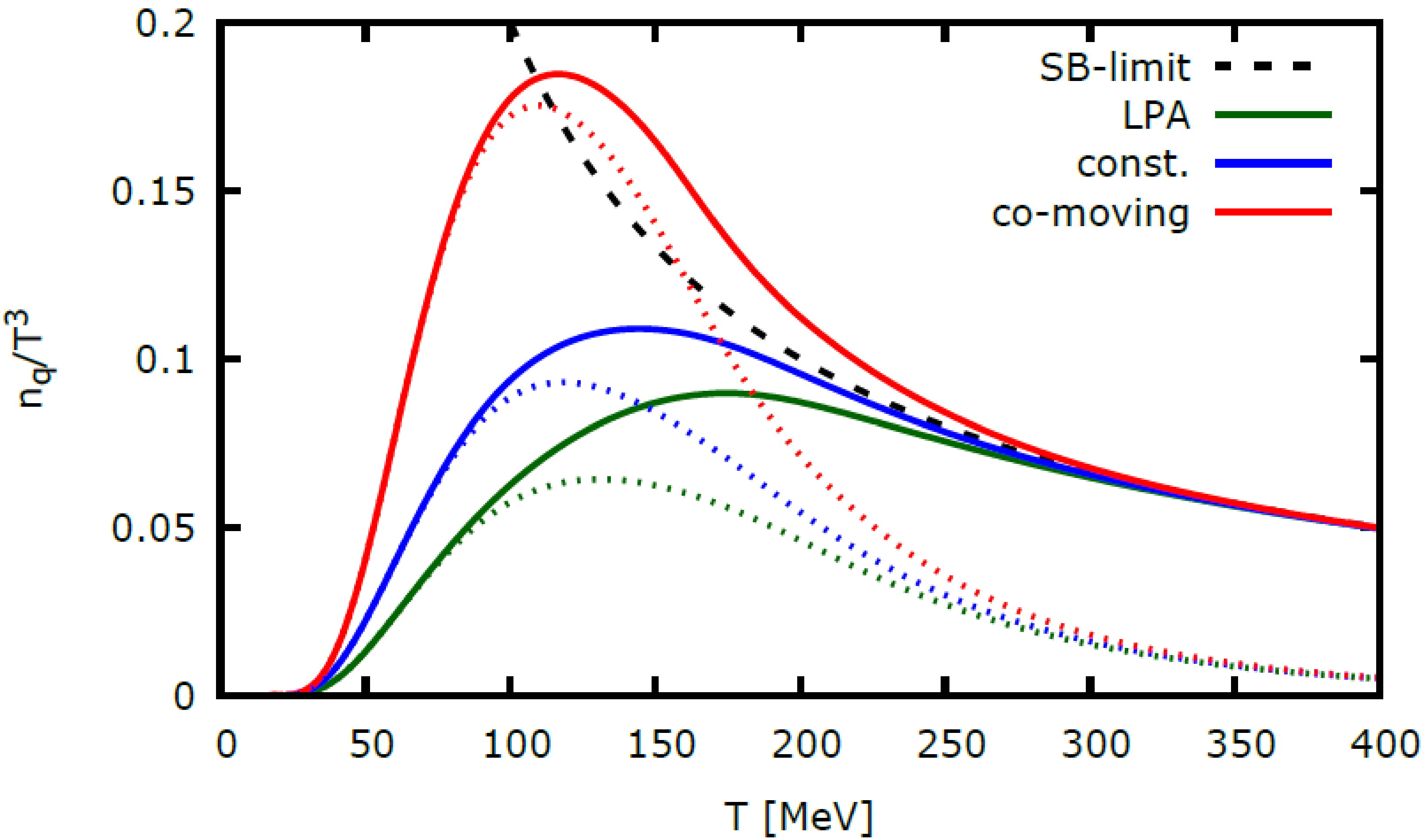
Entropy Density

$$s(T,\mu) := \frac{\partial}{\partial T} p(T,\mu)$$

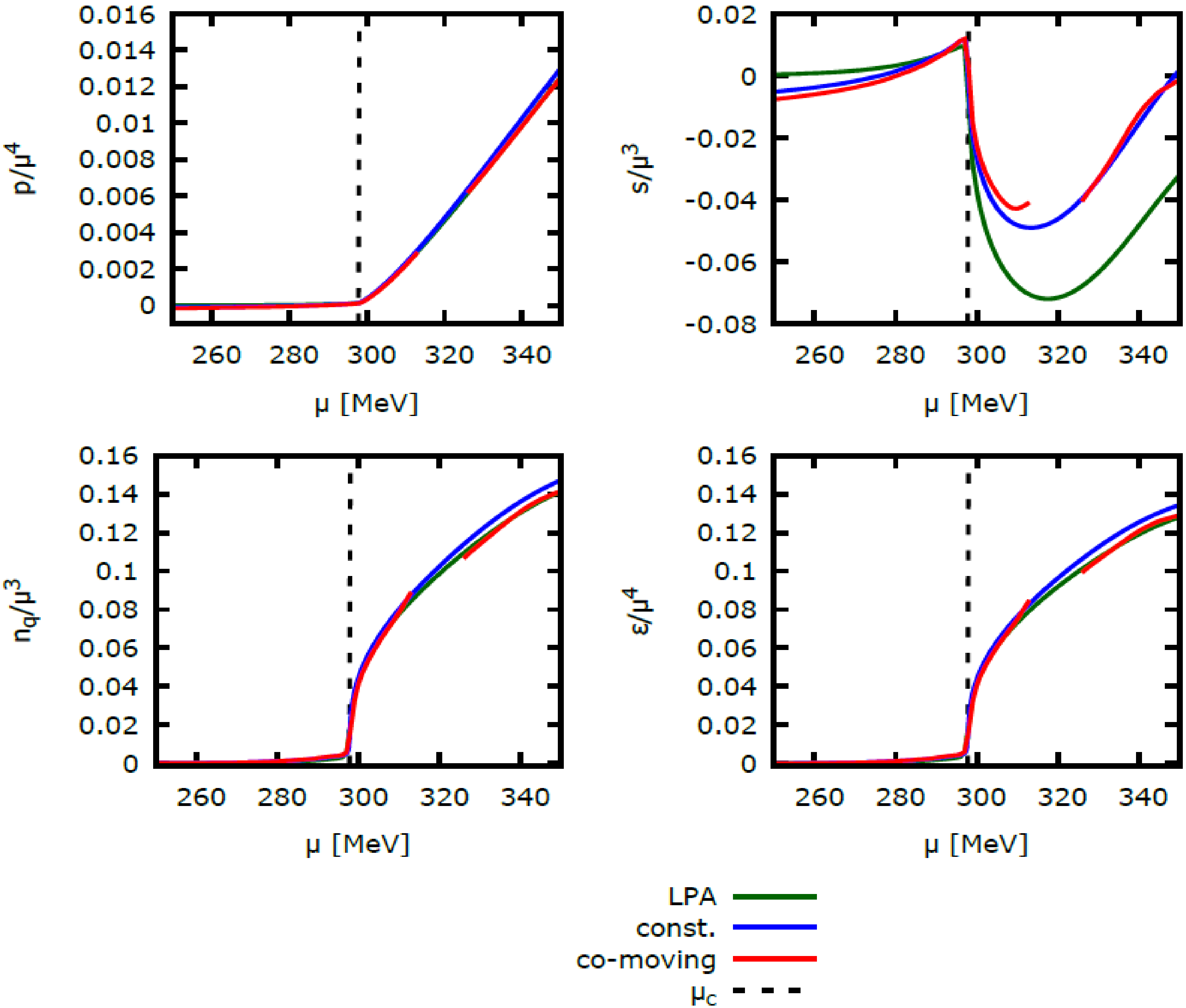


Quark Density (μ=10MeV)

$$n_q(T,\mu) := \frac{\partial}{\partial \mu} p(T,\mu)$$

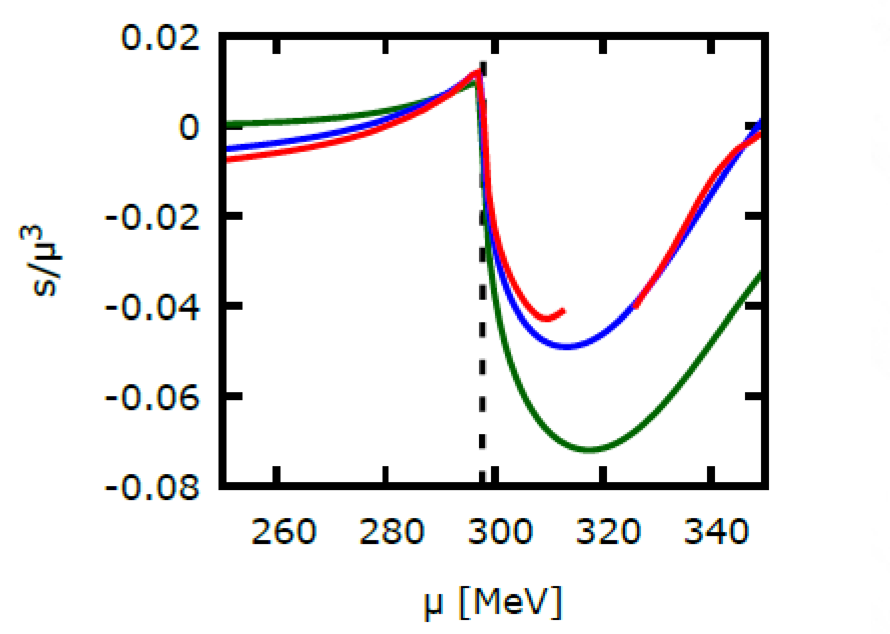


Now at high chemical potential, T=10 MeV:



Negative Entropy Densities

- Were already present in LPA
- A bit improved in LPA'+Y
- Connected to the back-bending in the phase diagram via (Clausius-Clapeyron):



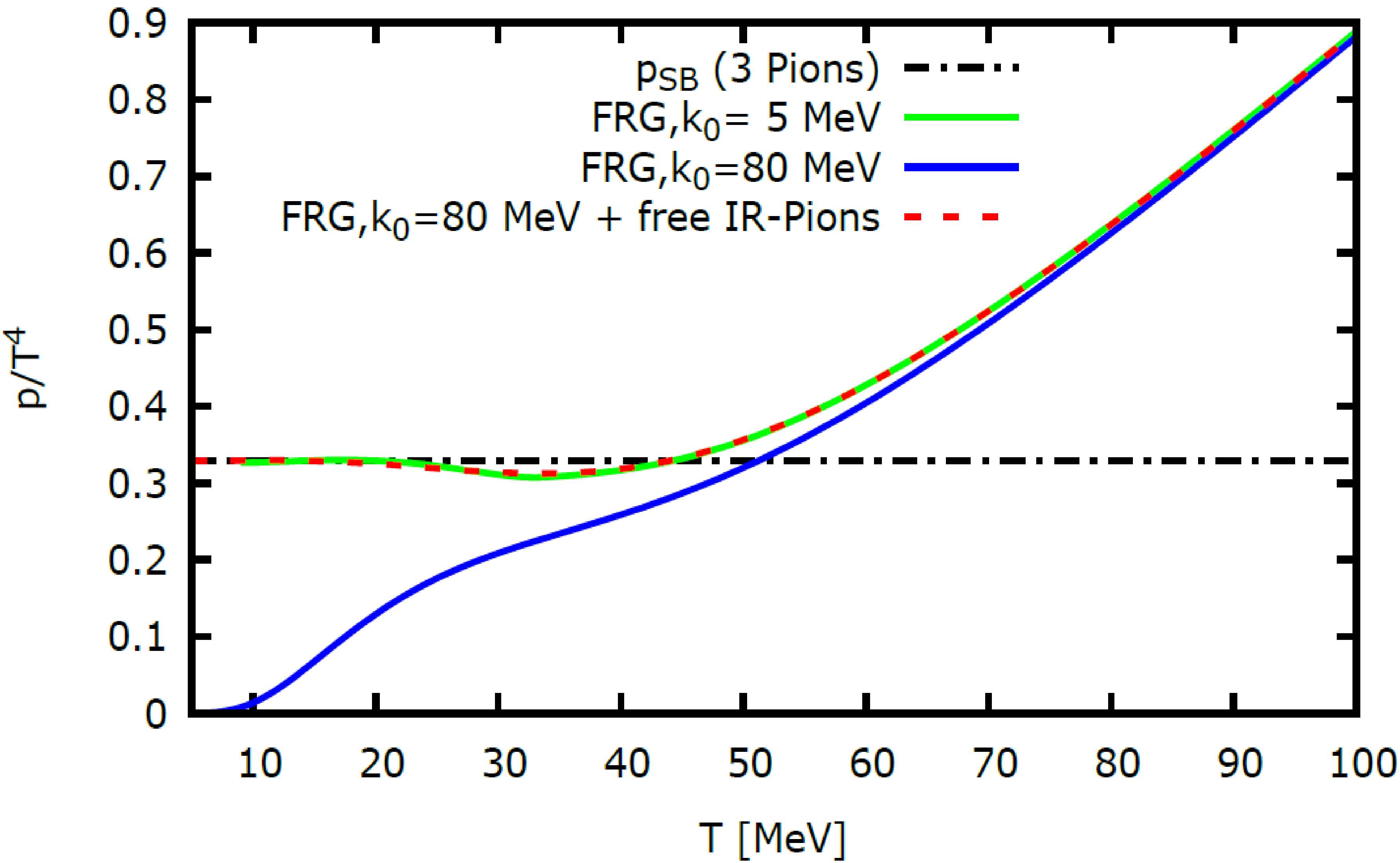
$$\frac{dT_c}{d\mu} = -\frac{\Delta n_q}{\Delta s}$$

→Higher derivatives needed or sign of missing degrees of freedom?

Chiral Limit

- For massless pions the same calculations can be done by simply setting the explicit symmetry breaking term to zero
- But: Fluctuations at low scales become more important, stopping at a scale of 80 MeV is not sufficient anymore
- Problematic: Calculation times "explode" if very small momentum scales are incuded
- Idea: Add gas of free pions at low scales!

$$\partial_k \Omega_k(T) \approx \frac{k^4}{12\pi^2} \left\{ \frac{1}{m_\sigma} + \frac{3}{k} \coth\left(\frac{k}{2T}\right) - \frac{4N_f N_c}{m_q} \right\}$$



0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
p/T^4
p_{SB} (3 Pions)
FRG,k_0= 5 MeV
FRG,k_0=80 MeV
FRG,k_0=80 MeV + free IR-Pions
10
20
30
40
50
60
70
80
90
100
T [MeV]

Summary

- Numerical calculation of the phase diagram

  and thermodynamical quantities pressure, entropy density and

  quark density within the FRG
- Crossover temperature T_c at vanishing μ in good agreement with lattice results
- T_{CEP} very low for the parameters used here
- Back-bending in the phase diagram and corresponding negative entropies not resolved, but slightly reduced

Outlook

- Cross-check numerical methods
- Calculation of additional quantities, e.g. curvature, susceptibilities
- Including more degrees of freedom to get a better description at high μ

  (field dependencies, channels, higher order derivatives(?))
- ...