The QCD phase diagram from Dyson-Schwinger equations

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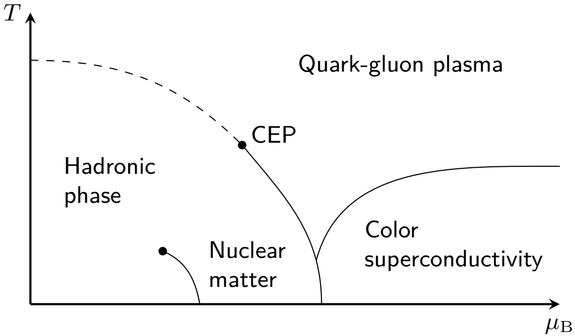
Lunch Club Seminar @ JLU Gießen
January 29, 2020

Phase transitions in QCD

Nonperturbative approaches

- Lattice QCD ... limited to $\mu_{\mathrm{B}}/T \lesssim 3$ due to sign problem
- Effective models ... generalizable?
- Functional methods ... all QCD degrees of freedom & no sign problem (but truncations are necessary)



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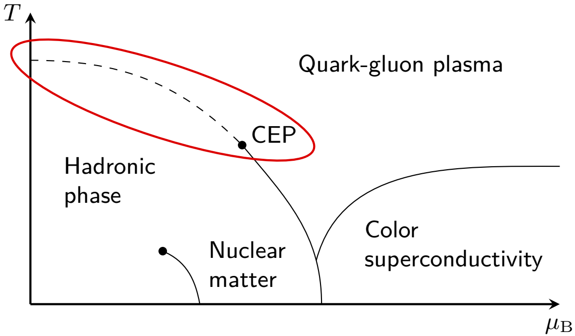

Dyson-Schwinger equations

- Nonperturbative functional approach
- Correlation functions on quark-gluon level
- Bound states as composite objects of quarks and gluons (solve Bethe-Salpeter/Faddeev equations)

Working areas:

Hadron physics

- Meson and baryon spectra
- Scattering amplitudes
- Decays
- Form factors
- Exotics (tetraquarks, glueballs, and hybrids)
- In-medium properties of mesons

Nonzero T and μ

- Phase structure of QCD
- Spectral functions
- Thermodynamics

Additionally

- Muon $g - 2$ (HLbL)
- Higher n-point functions
- ...

Reviews: Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91, 1 (2016)
Fischer, PPNP 105, 1 (2019)

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Setting the scene: Euclidean QCD

Generating functional in imaginary time

$$\mathcal{Z}_{\mathrm{QCD}} = \int \mathcal{D}[\bar{\psi}\psi A\bar{c}c] \exp\Big\{ -\int_0^{1/T} \mathrm{d}x_4 \int \mathrm{d}^3\vec{x}\, \Big(\bar{\psi}\,(-\not{D} + \hat{m} + \gamma_4\,\hat{\mu})\,\psi + \frac{1}{4} F^a_{\nu\sigma} F^a_{\nu\sigma} + \text{gauge fixing} + \text{sources} \Big) \Big\}$$

Propagators in Landau gauge and momentum space, $p = (\omega_p, \vec{p}\,)$:

$$S_f(p) = \Big[\mathrm{i}\,(\omega_p + \mathrm{i}\,\mu_f)\,\gamma_4\, C_f(p) + \mathrm{i}\vec{\not{p}}\, A_f(p) + B_f(p) \Big]^{-1}$$

$$D_{\nu\sigma}(p) = \frac{Z^{\mathrm{T}}(p)}{p^2}\, P^{\mathrm{T}}_{\nu\sigma}(p) + \frac{Z^{\mathrm{L}}(p)}{p^2}\, P^{\mathrm{L}}_{\nu\sigma}(p)$$

Goal: Gauge-independent information from gauge-fixed functional approach

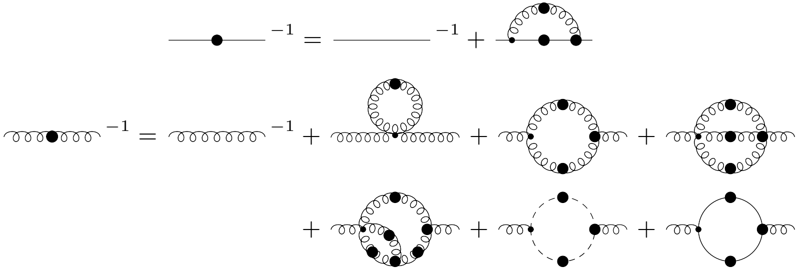
Master equation

$$0 = \int \mathcal{D}[\vec{\varphi}\,]\, \frac{\delta}{\delta \varphi_k} \exp\left(-S_{\mathrm{E}}[\vec{\varphi}\,] + \int \mathrm{d}^4 x\, \vec{J} \cdot \vec{\varphi} \right)$$

$$\Downarrow$$

$$\frac{\delta \Gamma_{1\mathrm{PI}}}{\delta \widetilde{\varphi}_k} = \frac{\delta S_{\mathrm{E}}}{\delta \varphi_k} \left[\varphi_\ell \to \pm \left(\frac{\delta}{\delta J_\ell} + \widetilde{\varphi}_\ell \right) \right] 1$$

DSEs for quark and gluon propagators:



How to truncate?

Quark DSE

Main focus on quark propagator:

- Source for order parameters (chiral symmetry and confinement)
- Starting point for fluctuations

Dressed quark-gluon vertex:

- No lattice results yet
- Explicit solutions at $T = 0$

  Fischer, Williams, PRL 103, 122001 (2009)
  Mitter, Pawlowski, Strodthoff, PRD 91, 054035 (2015)
  Williams, EPJA 51, 53 (2015)
  Williams, Fischer, Heupel, PRD 93, 034026 (2016)
  Sternbeck et al., PoS (LATTICE2016) 349
- $T \neq 0$: Ansatz based on STI and known perturbative behavior

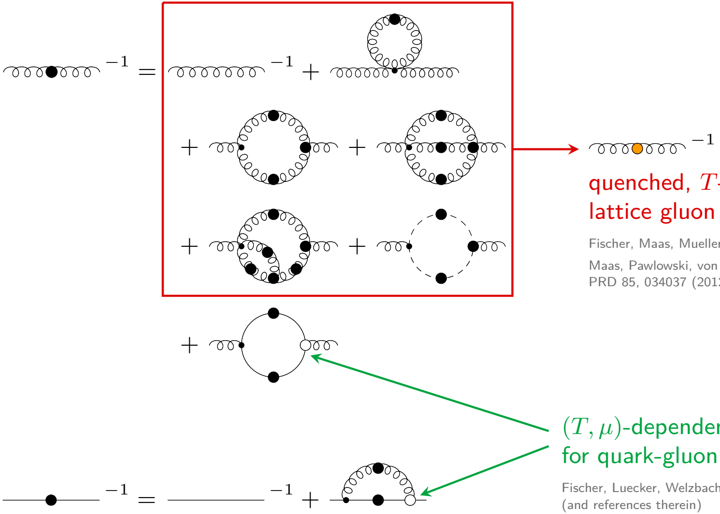
  Preliminary results at $T \neq 0$ (solving vertex DSE):
  Contant, Huber, Fischer, Welzbacher, Williams,
  Acta Phys. Polon. B Proc. Supp. 11, 483 (2018)

Dressed gluon propagator:

- Two strategies:
  - Model gluon propagator

    Qin, Chang, Chen, Liu, Roberts, PRL 106, 172301 (2011)
    Gao, Liu, PRD 94, 076009 (2016)
  - Explicit treatment of gluonic sector
- Here: Use the latter
  - Consistent flavor dependencies
  - Gluon becomes sensitive to chiral dynamics

How to truncate?



quenched, T-dependent lattice gluon propagator

Fischer, Maas, Mueller, EPJC 68, 165 (2010)

Maas, Pawlowski, von Smekal, Spielmann, PRD 85, 034037 (2012)

(T, μ)-dependent ansatz for quark-gluon vertex

Fischer, Luecker, Welzbacher, PRD 90, 034022 (2014) (and references therein)

Ansatz for quark-gluon vertex

$$S_f^{-1}(p) = \mathrm{i}(\omega_p + \mathrm{i}\mu_f)\gamma_4 C_f(p) + \mathrm{i}\vec{p}\!\!/ A_f(p) + B_f(p)$$

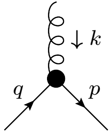
Vertex ansatz:

$$\Gamma_\nu^f(q,p,k) = \widetilde{Z}_3\,\Gamma(k^2)\,\gamma_\nu\left(\delta_{4\nu}\frac{C_f(q)+C_f(p)}{2} + (1-\delta_{4\nu})\frac{A_f(q)+A_f(p)}{2}\right)$$

Phenomenological dressing function:

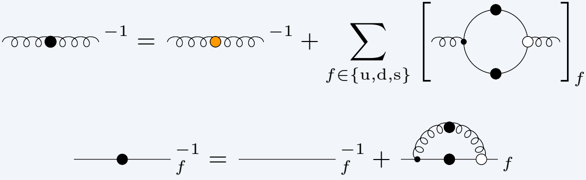
$$\Gamma(k^2) = \frac{d_1}{d_2+k^2} + \frac{1}{1+\Lambda^2/k^2}\left(\frac{\alpha_\mathrm{s}\beta_0}{4\pi}\log\left(1+k^2/\Lambda^2\right)\right)^{2\delta}$$

- Abelian STI (leading term of Ball-Chiu vertex)
  Ball, Chiu, PRD 22, 2542 (1980)
- Perturbative running in the ultraviolet (quantitative)
- Ansatz for IR (qualitative)
  - d_1 fixed via T_c
  - d_2 fixed to match scale of quenched lattice gluon



Three-flavor QCD with Dyson-Schwinger equations

Final set of truncated DSEs

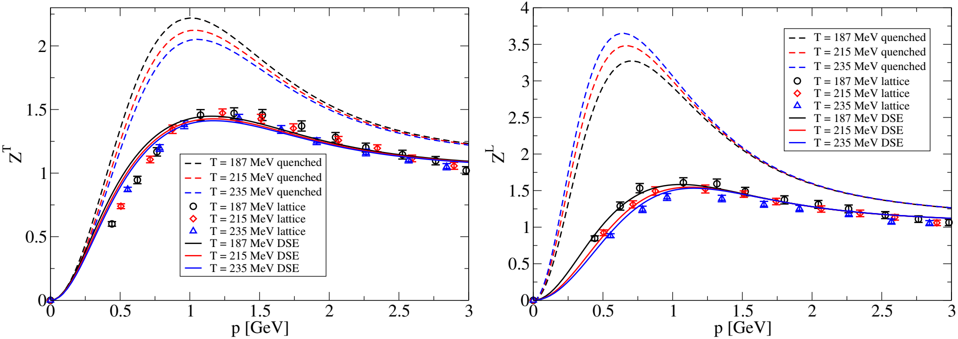


- Quenched lattice gluon propagator as input & unquenching via quark loops
- Nontrivial coupling between different quark flavors
- Vertex ansatz built along STI and perturbation theory

Result: Dressed (i.e., nonperturbative) quark and unquenched gluon propagators

Gluon at nonzero temperature

$$p^2 D_{\nu\sigma}(p) = Z^{\mathrm{T}}(p)\, P^{\mathrm{T}}_{\nu\sigma}(p) + Z^{\mathrm{L}}(p)\, P^{\mathrm{L}}_{\nu\sigma}(p)$$

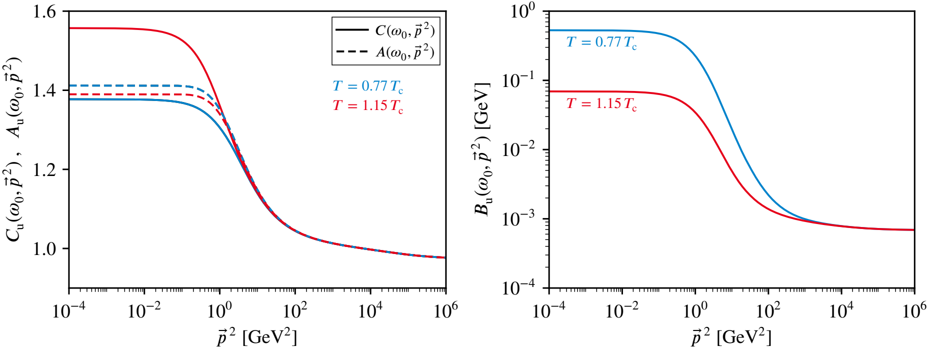


- Very good agreement of DSE and lattice results
- Consistent "melting" of longitudinal dressing function
- Crucial difference between quenched and unquenched gluon

Figures taken from: Fischer, PPNP 105, 1 (2019)
DSE: Fischer, Luecker, PLB 718, 1036 (2013)
Lattice: Aouane, Burger, Ilgenfritz, Müller-Preussker, Sternbeck, PRD 87, 114502 (2013)

Quark at nonzero temperature

$$S_f^{-1}(p) = \mathrm{i}(\omega_p + \mathrm{i}\mu_f)\gamma_4 C_f + \mathrm{i}\vec{p\!\!/} A_f(p) + B_f(p)$$



- Nonperturbative effects in the IR; perturbative/bare in the UV
- Mass generation due to DCSB
- Chiral symmetry (partially) restored at large temperatures

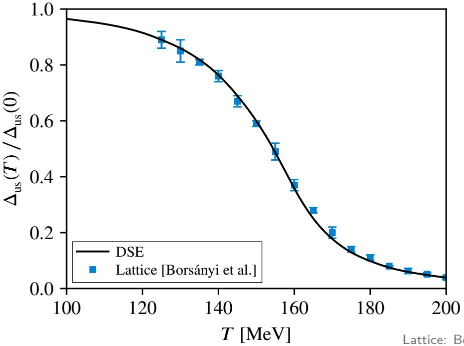
Chiral order parameter:

Quark condensate

$$\langle \bar{\psi}\psi \rangle_f = -Z_2^f Z_m^f \,\mathrm{Tr}[S_f]$$

Subtracted condensate

$$\Delta_{ff'} = \langle \bar{\psi}\psi \rangle_f - \frac{m_f}{m_{f'}} \langle \bar{\psi}\psi \rangle_{f'}$$



Lattice: Borsányi et al., JHEP09(2010)073

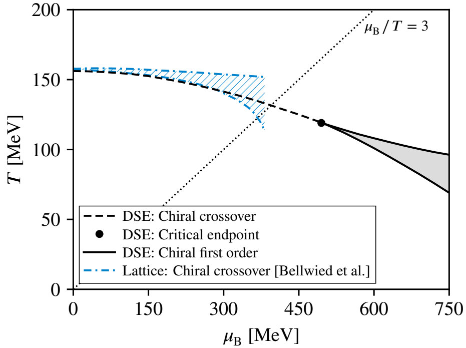
Phase structure: $N_\mathrm{f} = 2 + 1$ at physical point

PI, Buballa, Fischer, Gunkel, PRD 100, 074011 (2019)

Main result

Second-order CEP at large chemical potential:

$$\mu_\mathrm{B}^\mathrm{CEP} = 495\,\mathrm{MeV}\,, \quad T^\mathrm{CEP} = 119\,\mathrm{MeV}$$



Lattice:
Bellwied et al., PLB 751, 559 (2015)

- Ratio: $\mu_\mathrm{B}^\mathrm{CEP} / T^\mathrm{CEP} \approx 4.2$
- Crossover temperature: $T_\mathrm{c}^{(\mu=0)} = 156\,\mathrm{MeV}$

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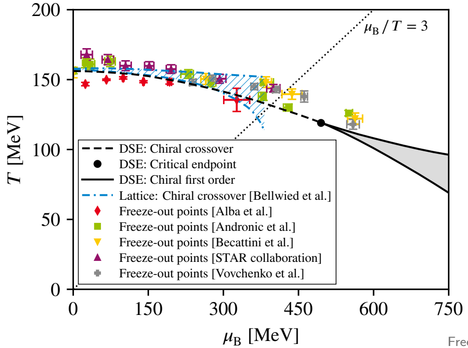


Lattice:
Bellwied et al., PLB 751, 559 (2015)

Freeze-out:
Alba et al., PLB 738, 305 (2014)
Andronic et al., JPCS 779, 012012 (2017)
Becattini et al., PLB 764, 241 (2017)
STAR collab., PRC 96, 044904 (2017)
Vovchenko et al., PRC 93, 064906 (2016)

- Ratio: $\mu_B^{\mathrm{CEP}} / T^{\mathrm{CEP}} \approx 4.2$
- Crossover temperature: $T_c^{(\mu=0)} = 156\,\mathrm{MeV}$

Contact to experiment: Fluctuations

Fluctuations from QCD's grand-canonical potential

$$\chi_{ijk}^{\mathrm{uds}} = -\frac{1}{T^{4-(i+j+k)}} \frac{\partial^{i+j+k}\,\Omega}{\partial\mu_{\mathrm{u}}^i\,\partial\mu_{\mathrm{d}}^j\,\partial\mu_{\mathrm{s}}^k}$$

Relation to conserved charges:
("quark basis $\leftrightarrow$ phenomenological basis")

$$\mu_{\mathrm{u}} = \mu_{\mathrm{B}}/3 + 2\mu_{\mathrm{Q}}/3$$
$$\mu_{\mathrm{d}} = \mu_{\mathrm{B}}/3 - \mu_{\mathrm{Q}}/3$$
$$\mu_{\mathrm{s}} = \mu_{\mathrm{B}}/3 - \mu_{\mathrm{Q}}/3 - \mu_{\mathrm{S}}$$

Ratios related to experimental quantities, e.g.:

$$\frac{\chi_2^{\mathrm{B}}}{\chi_1^{\mathrm{B}}} = \frac{\sigma_{\mathrm{B}}^2}{M_{\mathrm{B}}}\,, \qquad \frac{\chi_4^{\mathrm{B}}}{\chi_2^{\mathrm{B}}} = K_{\mathrm{B}}\,\sigma_{\mathrm{B}}^2$$

Sensitive to phase structure: $\chi_2^{\mathrm{B}} \sim \xi^{\kappa}$ (with $\kappa > 0$) and $\xi \to \infty$ at CEP

Reviews: Luo, Xu, Nucl. Sci. Tech. 28, 112 (2017)
Bzdak, Esumi, Koch, Liao, Stephanov, Xu, arXiv:1906.00936

PI, Buballa, Fischer, Gunkel, PRD 100, 074011 (2019)

Grand-canonical potential from 2PI formalism:

Cornwall, Jackiw, Tomboulis, PRD 10, 2428 (1979)

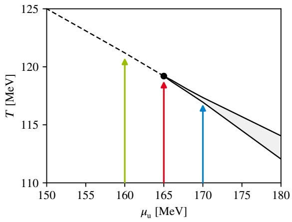
$$\Omega = -\frac{T}{V}\left(\mathrm{Tr}\log\frac{S^{-1}}{T} - \mathrm{Tr}\left[\mathbb{1} - S_0^{-1}S\right] + \Phi_{\mathrm{int}}[S]\right) + \Omega_{\mathrm{YM}}$$

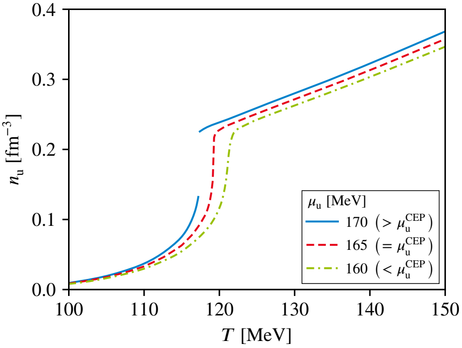
Quark number density:

$$n_f = T^3\chi_1^f = -\partial\Omega/\partial\mu_f = -Z_2^f\,\mathrm{Tr}[\gamma_4 S_f]$$

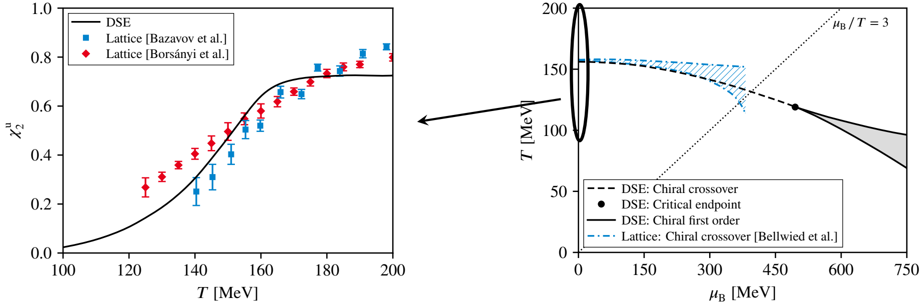
Behavior around CEP:
Consistent with effective models

e.g. Buballa, Phys. Rep. 407, 205 (2005)
Schaefer, Wambach, PRD 75, 085015 (2007)





PI, Buballa, Fischer, Gunkel, PRD 100, 074011 (2019)



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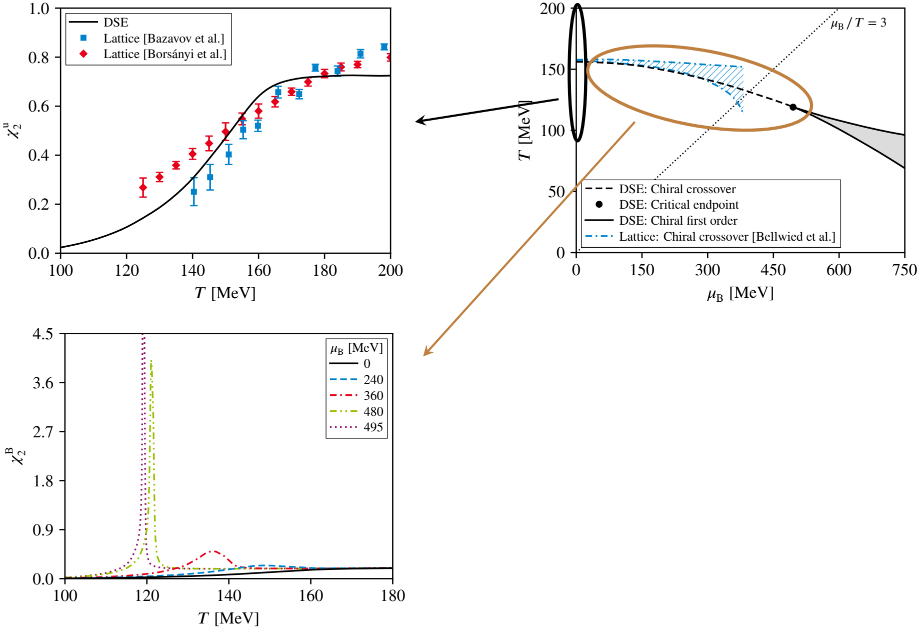
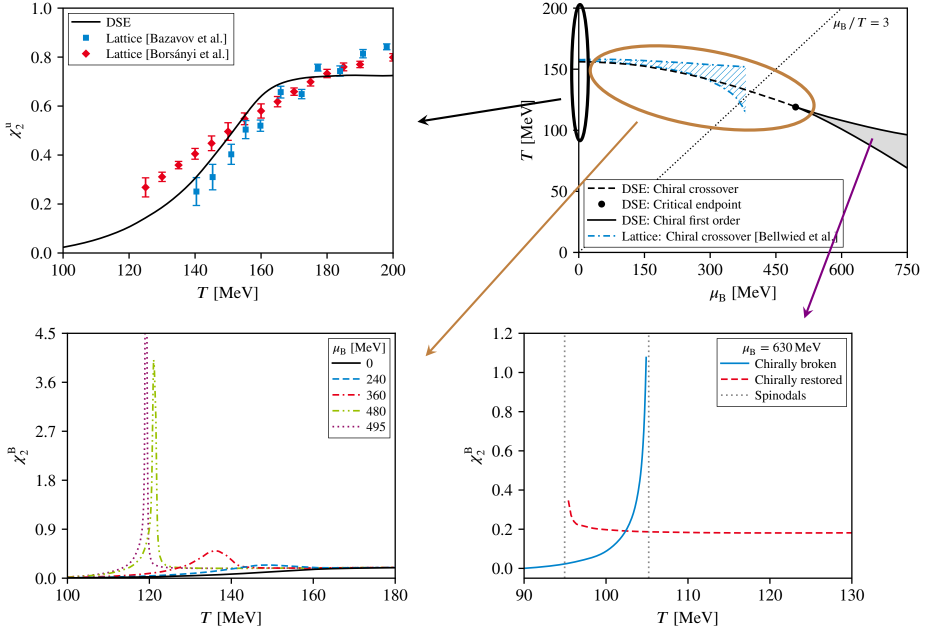


Fluctuations from DSEs

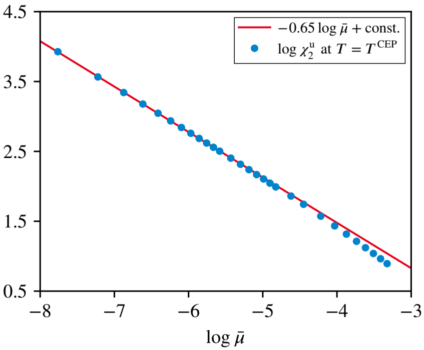
PI, Buballa, Fischer, Gunkel, PRD 100, 074011 (2019)



Detour: Scaling at CEP

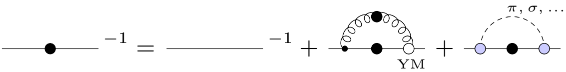
- Sufficiently close to CEP: $\chi_2^{\mathrm{u}} \sim |\bar{\mu}|^{-\varepsilon}$ along $T = T^{\mathrm{CEP}}$
- $\bar{\mu} = 1 - \mu_{\mathrm{u}}/\mu_{\mathrm{u}}^{\mathrm{CEP}} > 0$ (approach from crossover side)

- Fit: $\log \chi_2^{\mathrm{u}} = -\varepsilon \log \bar{\mu} + \mathrm{const.}$
  $\Rightarrow \varepsilon = 0.65 \pm 0.02$
  $\Rightarrow$ mean-field scaling



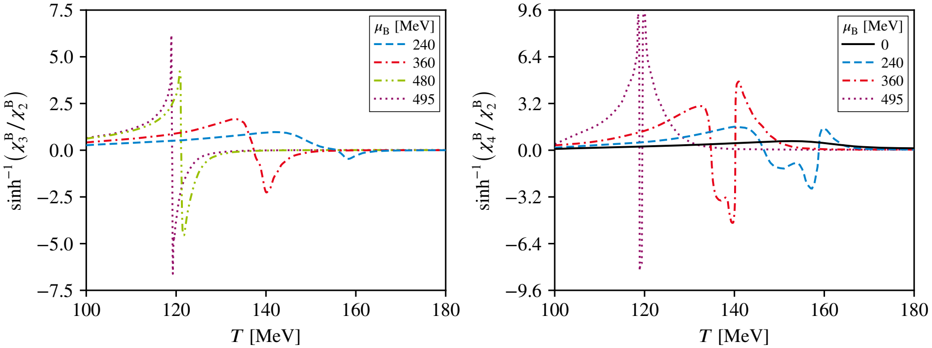
Scaling beyond mean-field: Mesonic contributions are important!

Fischer, Mueller, PRD 84, 054013 (2011)



Need mesons in medium ... work in progress (→ Lunch Club talks by Pascal Gunkel)

First step: Gunkel, Fischer, PI, EPJA 55, 169 (2019)

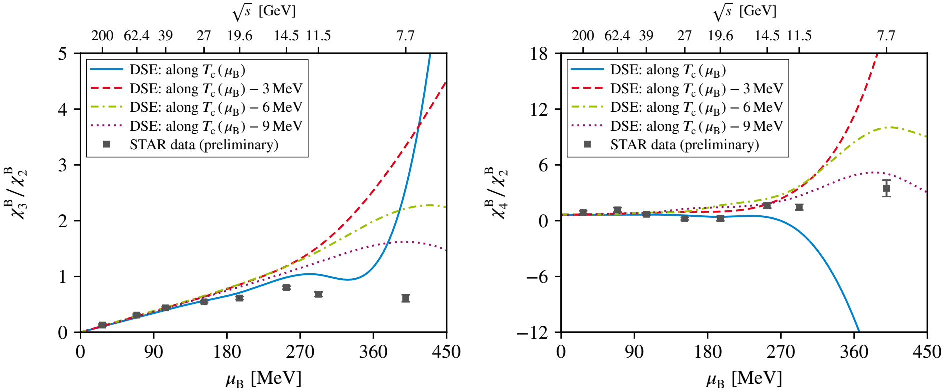


Very sensitive to phase structure & clear signals for CEP

Caveats when comparing with experiment:

- No off-diagonal fluctuations yet
- Only "naive" strangeness neutrality
- Critical region may be too large
- Ordering of freeze-out points

Skewness and kurtosis ratios



- $\sqrt{s} > 14.5\,\mathrm{GeV}$: Good agreement; variation of $T_c(\mu_B)$ has only mild impact
- $\sqrt{s} = 14.5\,\mathrm{GeV}$: Trend ok; freeze-out close to crossover favored
- $\sqrt{s} \leq 11.5\,\mathrm{GeV}$: freeze-out line $\neq$ crossover line ?!

Summary

Phase diagram with DSEs:

- Backcoupling of quarks onto gluons important
- $N_\mathrm{f} = 2+1$: CEP at large chemical potential; $\mu_\mathrm{B}^\mathrm{CEP}/T^\mathrm{CEP} \approx 4.2$
- Combined result from DSE and FRG: No CEP at $\mu_\mathrm{B}/T < 3$
  FRG study: Fu, Pawlowski, Rennecke, arXiv:1909.02991

Fluctuations with DSEs:

- First calculation in truncation with reasonable CEP location
- Results suggest/support that freeze-out line bends below CEP

Outlook and work in progress:

- Systematic control over error budget
- Off-diagonal fluctuations and "proper" strangeness neutrality
- Finite-volume effects