Four- vs. Two-Quark States from Bethe-Salpeter Equations

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Lunch Club, Uni Giessen, 09.12.2020

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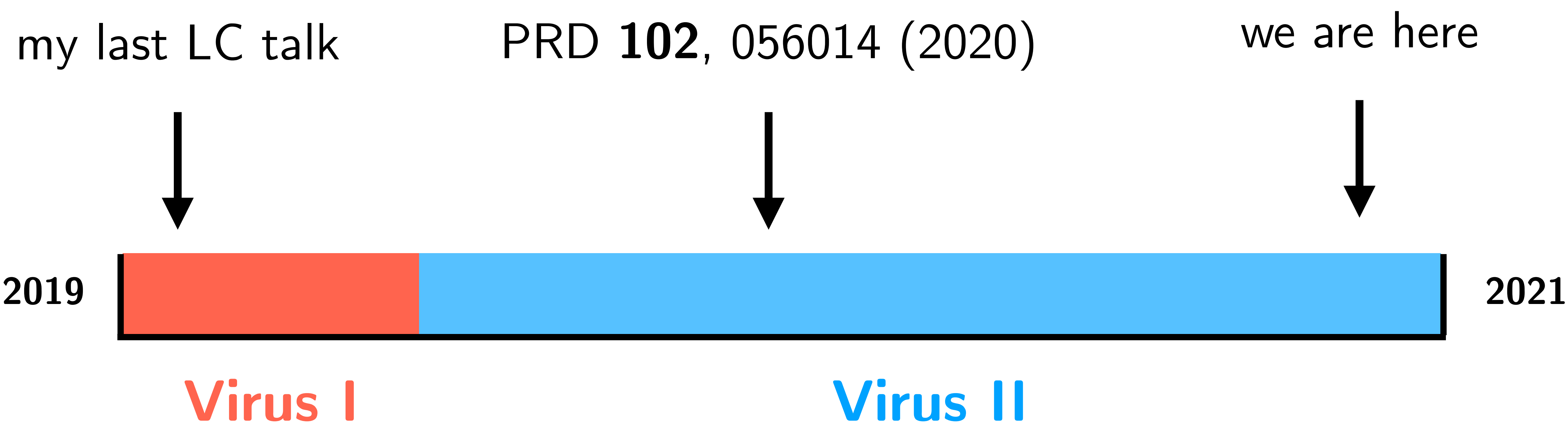
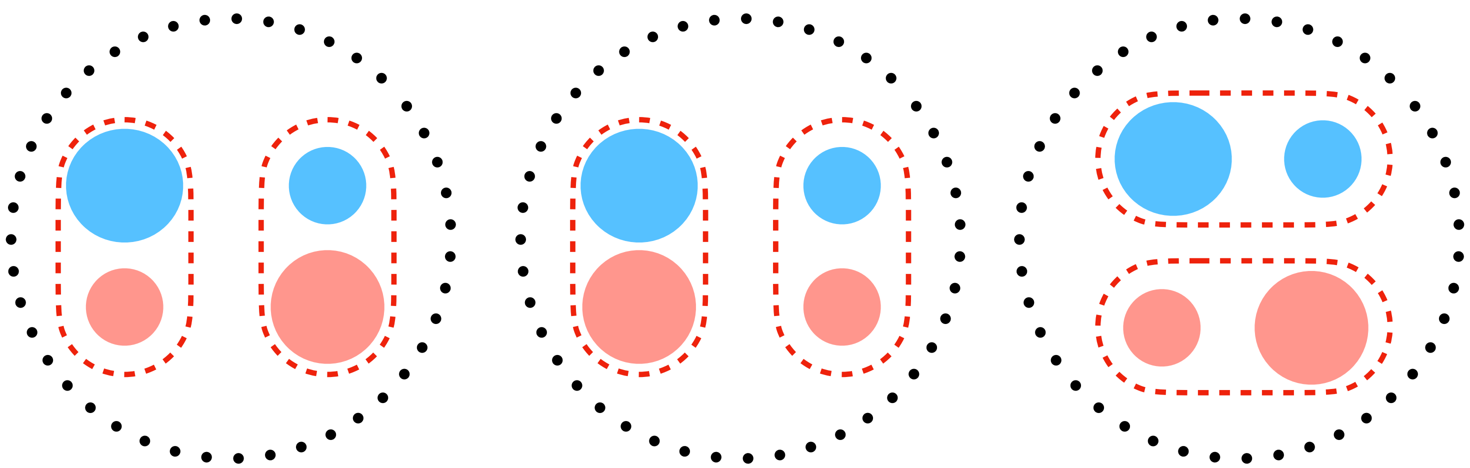
2020

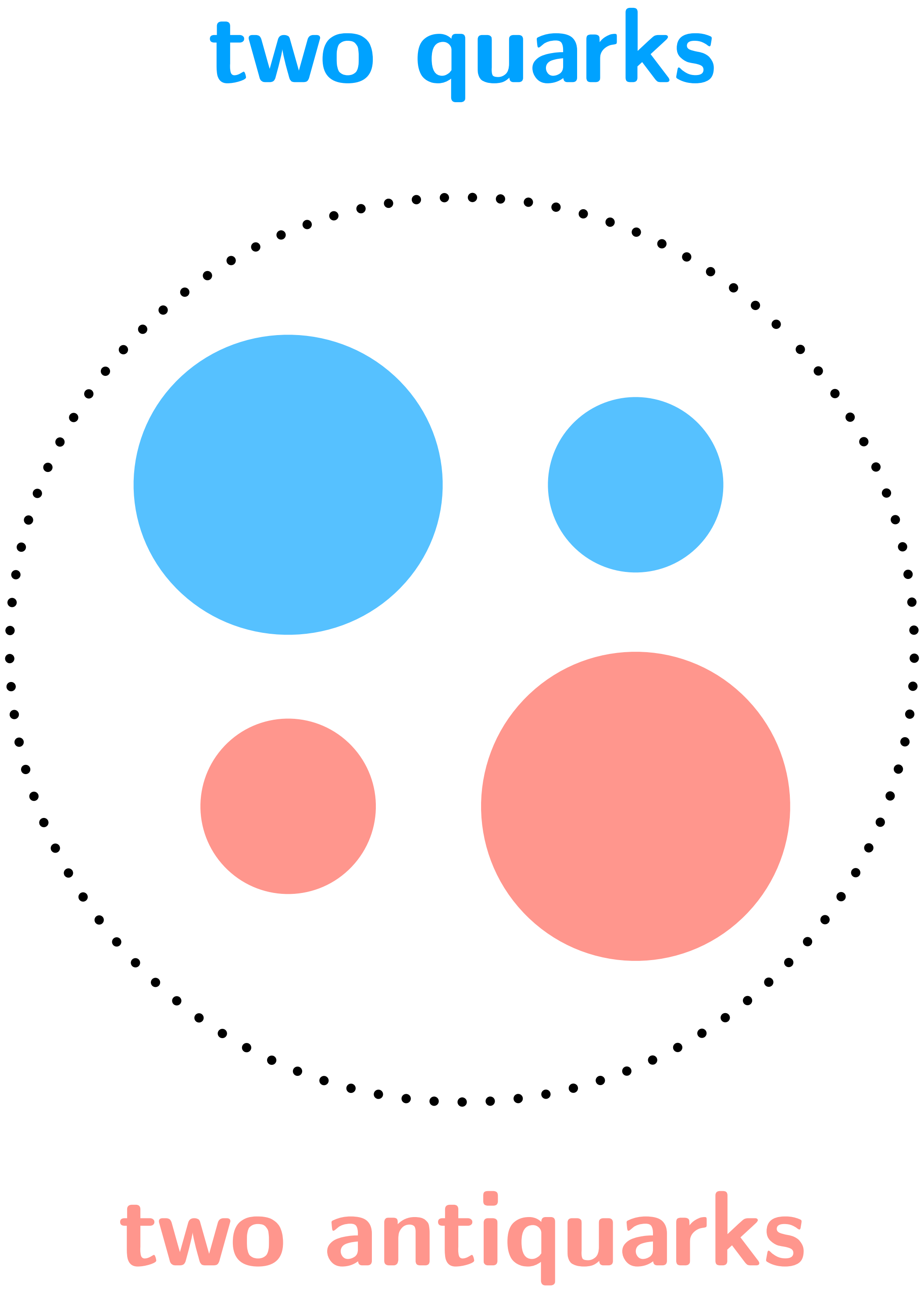

my last LC talk
PRD **102**, 056014 (2020)
we are here
2019
2021
Virus I
Virus II

My Last LC Talk

- talk about tetraquark candidates in the charmonium spectrum
  ➡ heavy-light ($c\bar{c}q\bar{q}$)
- identification of <u>inner structure</u> of XYZ states



HL meson molecule | **hadro-charmonium** | **diquark-antidiquark**



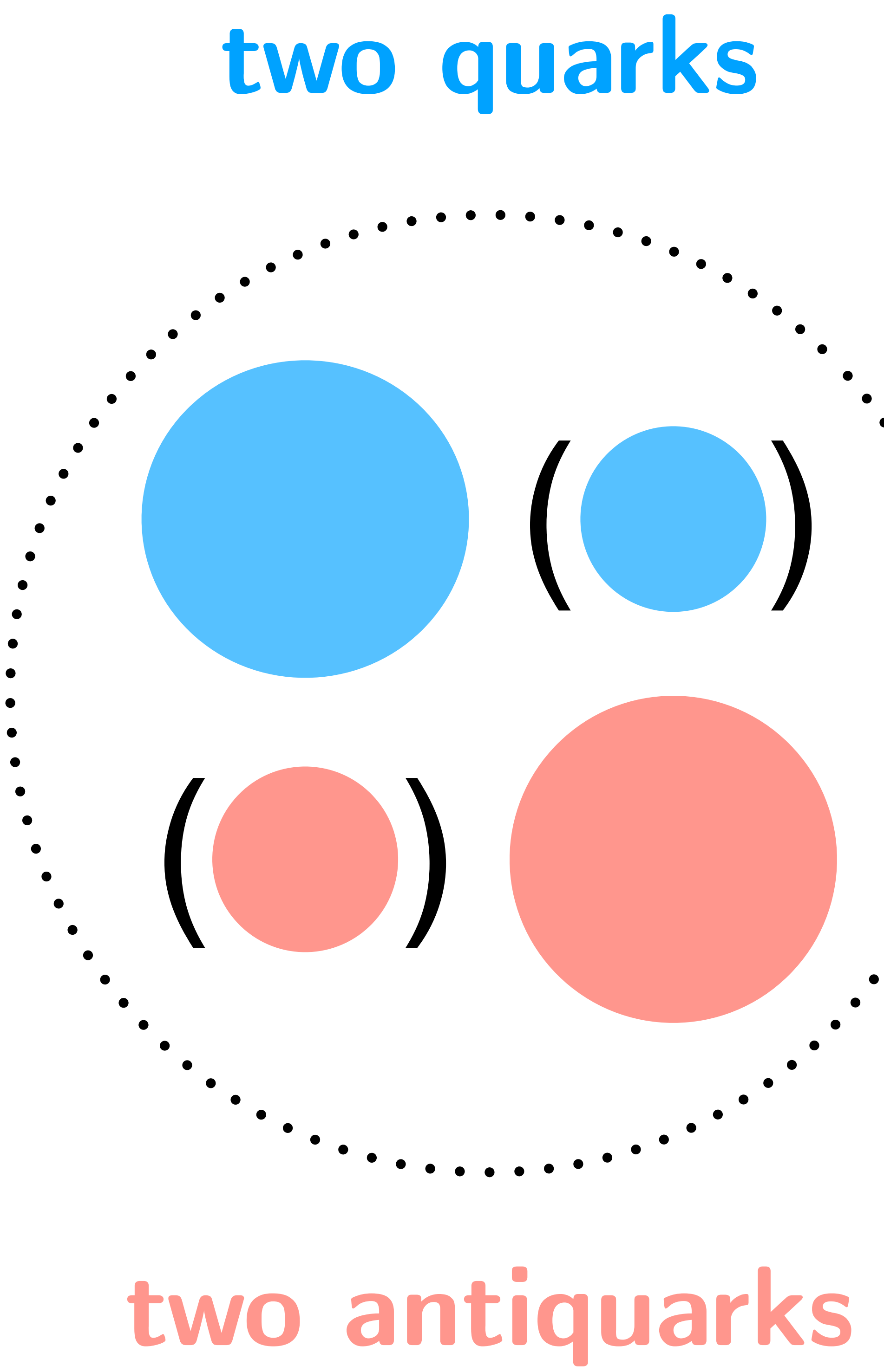
My Last LC Talk

- issue:

  Isoscalars ($I = 0$) may mix with ordinary quarkonia

  ➡ for investigation of the inner structure, it is necessary to take mixing diagrams into account!

  ($\neq$ last LC talk)

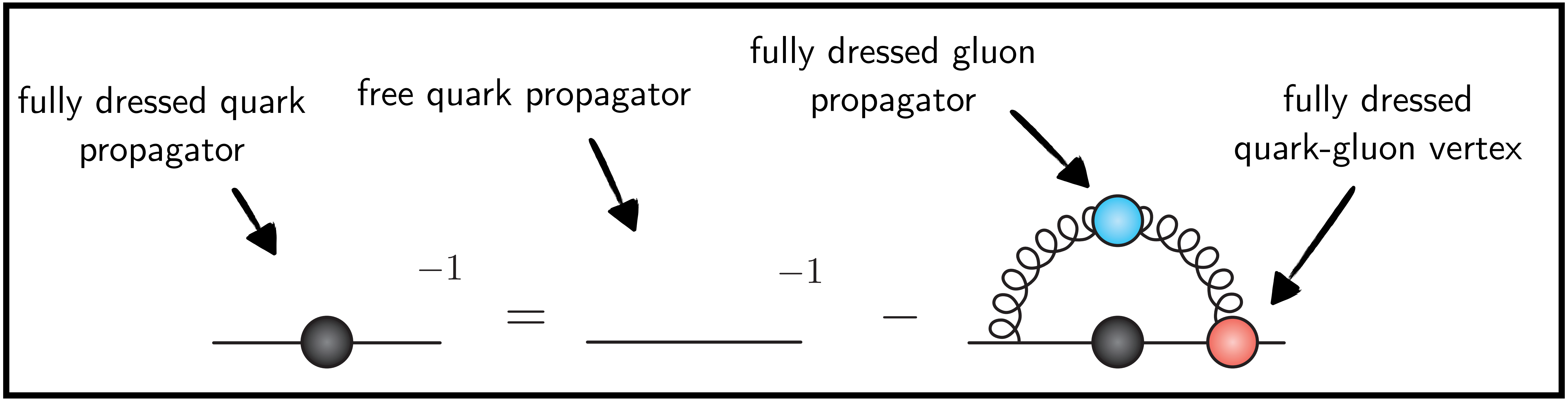


Dyson-Schwinger Equations
Bethe-Salpeter Equations

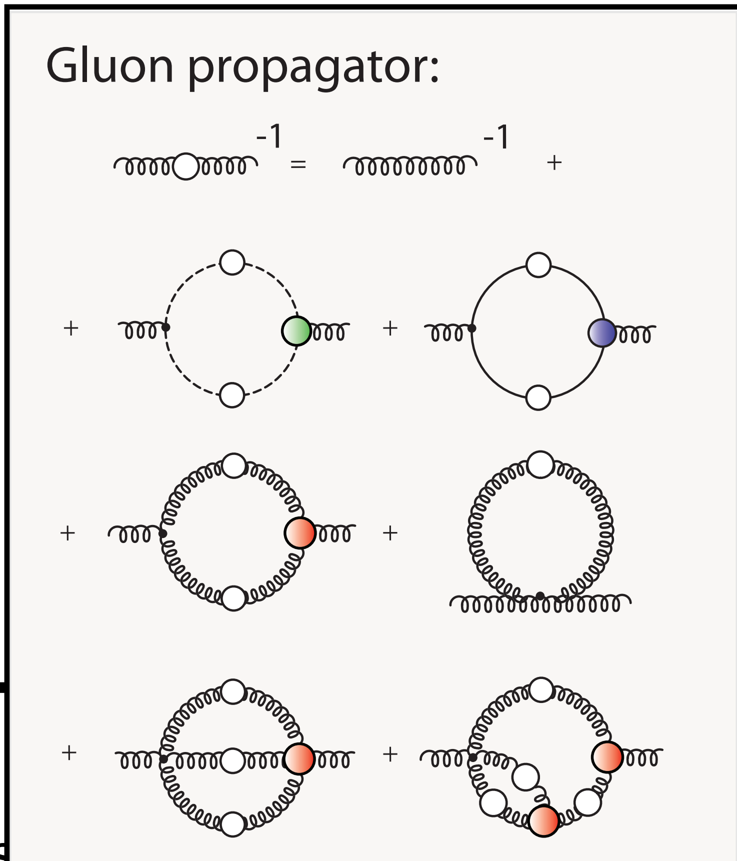
Dyson-Schwinger equations (DSEs)

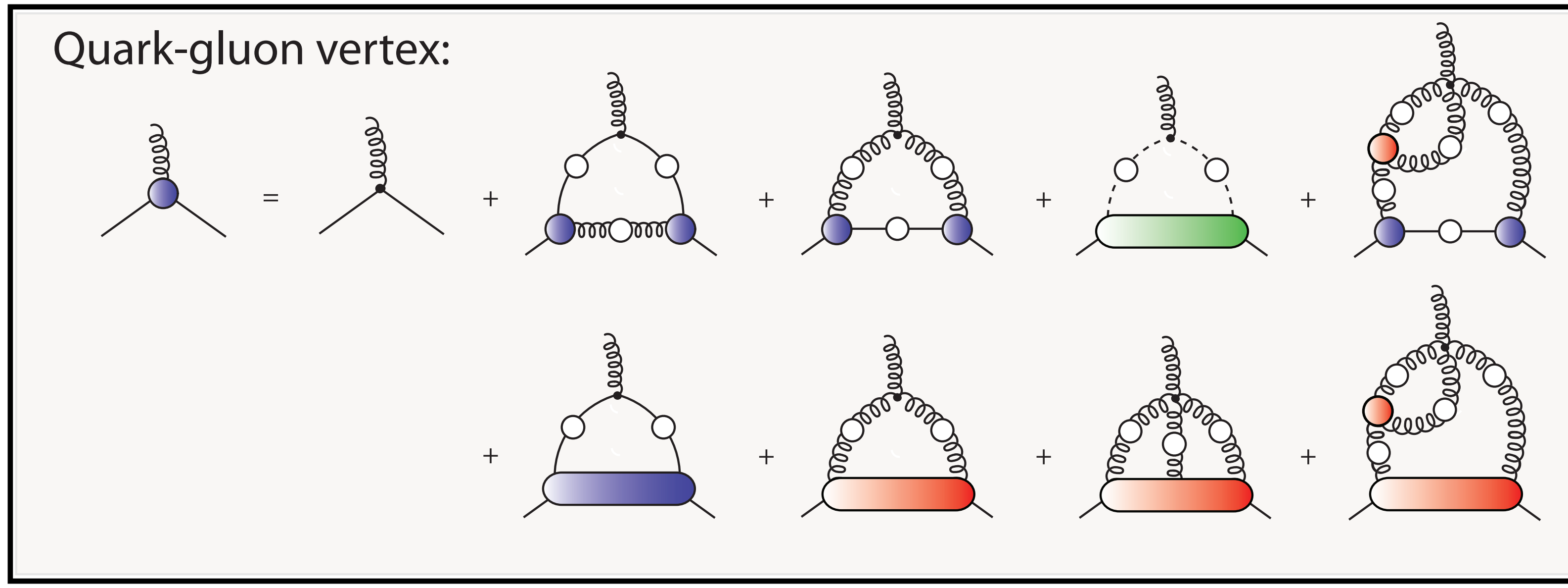
(exact) equations of motion of the Green's functions of a quantum field theory

In QCD: **quark propagator**, gluon propagator, quark-gluon vertex, ...



$$S^{-1}(p) = S_0^{-1}(p) - Z_{1f}\,\frac{4g^2}{3}\int \mathrm{d}^4q\,\gamma^{\mu} D_{\mu\nu}(q-p)\Gamma^{\nu}_{\mathrm{qg}}(q,p)S(q)$$





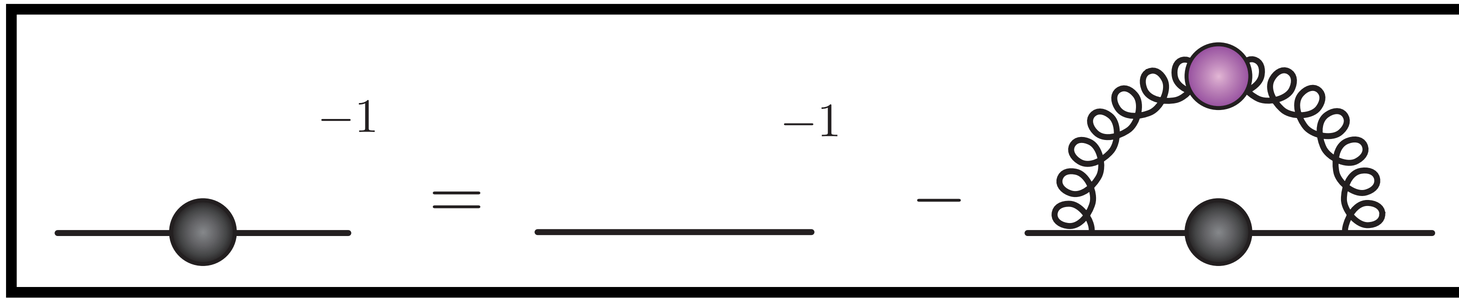
For a full solution, we need to know the solutions of other DSEs!

figures above taken from G. Eichmann, arXiv:0909.0703 [hep-ph]

Dyson-Schwinger equations (DSEs)

Model: "effective gluon"

P. Maris, P. Tandy, Phys. Rev. C **60**, 055214 (1999)



Replacement of the gluon propagator and the quark-gluon vertex with an effective gluon and a bare vertex

$$Z_{1f} g^2 D_{\mu\nu}(k) \Gamma^\nu_{\mathrm{qg}} \to Z_2^2 \, \frac{\mathcal{G}(k^2)}{k^2} \, \mathrm{T}_{\mu\nu}(k) \gamma^\nu$$

with a model function $\mathcal{G}(k^2)$. (parameters fixed on π, K physics)

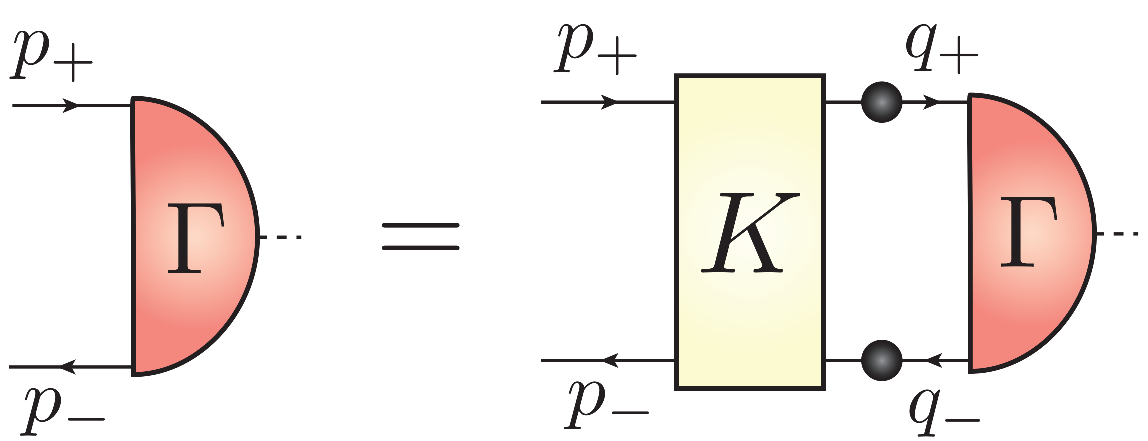
$$\frac{\mathcal{G}(k^2)}{k^2} = \underbrace{\frac{8\pi^2 \gamma_m \left[1 - \exp\left(-\frac{k^2}{4m_t^2}\right)\right]}{\ln\left[\tau + (1 + \frac{k^2}{\Lambda^2_{\mathrm{QCD}}})^2\right]}}_{\text{UV part}} + \underbrace{\frac{4\pi^2}{\omega^6} \cdot D k^4 \, \exp\left(-\frac{k^2}{\omega^2}\right)}_{\text{IR part}}$$

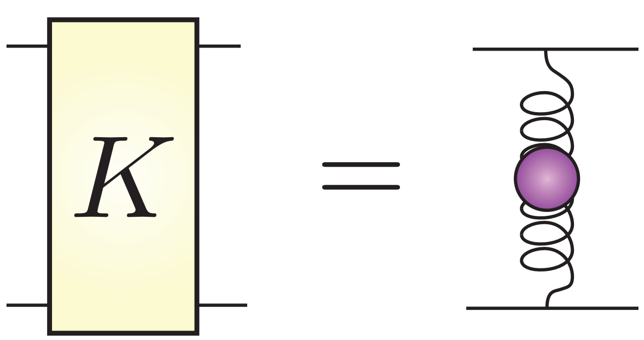
Bethe-Salpeter equations (BSEs)

In the Bethe-Salpeter formalism, a bound state is determined by its amplitude Γ, the solution of the Bethe-Salpeter equation.

meson case:

$$\Gamma(P,p) = \int_q K(P,p,q)S(q_+)\Gamma(P,q)S(q_-)$$





for on-shell solutions: $P^2 = -M^2$

same kernel as in the quark self energy integral (➡ "effective gluon")

Bethe-Salpeter equations (BSEs)

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meson case:

$$\Gamma(P,p) = \int_q K(P,p,q) S(q_+) \Gamma(P,q) S(q_-)$$

structure: $\Gamma = \sum_i \tau_i(P,p) \cdot \; f_i(P,p) \otimes \Gamma_{\text{color}} \otimes \Gamma_{\text{flavour}}$

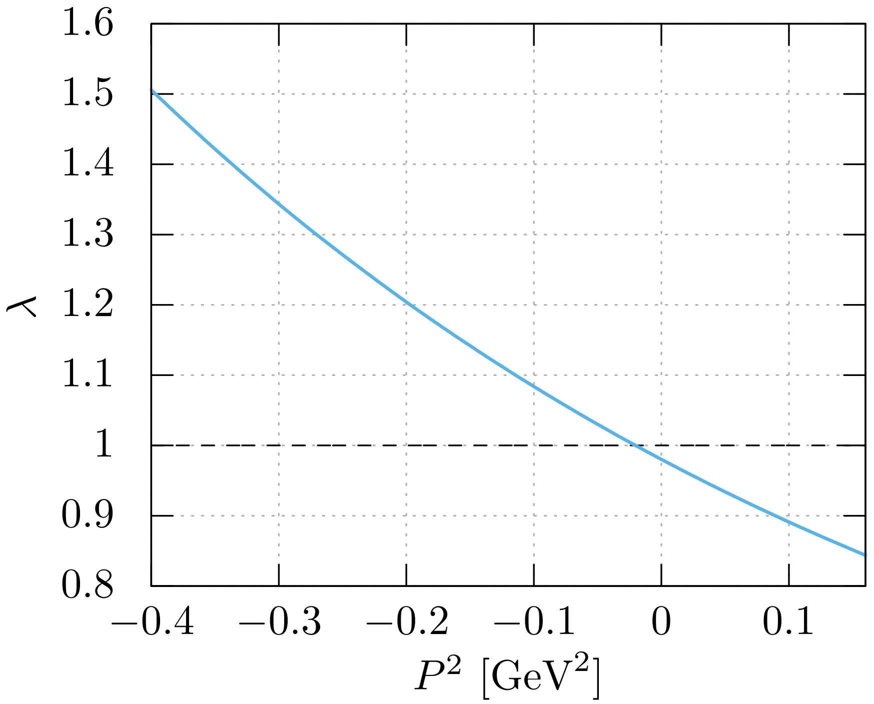
τ_i: Dirac basis elements — f_i: scalar dressing functions

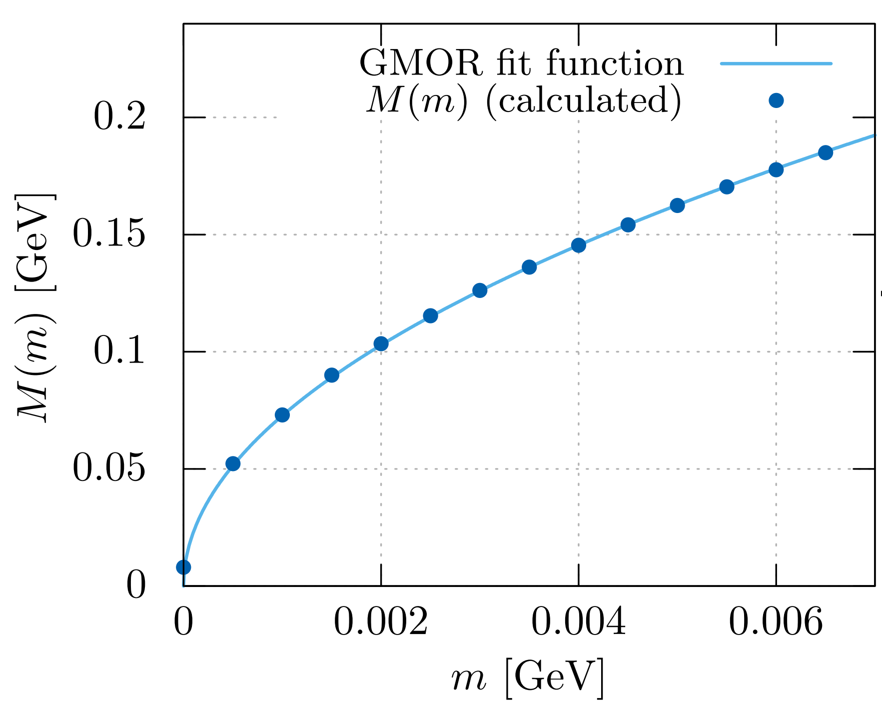
Bethe-Salpeter equations (BSEs)

structure: $\Gamma = \sum_i \tau_i(P,p) \cdot \; f_i(P,p) \otimes \Gamma_{\text{color}} \otimes \Gamma_{\text{flavour}}$

simplest setup: pseudoscalar meson (e.g. pion)

$$\tau_i \in \{\gamma_5, \gamma_5 \not{P}, \gamma_5 \not{p}, \gamma_5 [\not{P}, \not{p}]\}$$

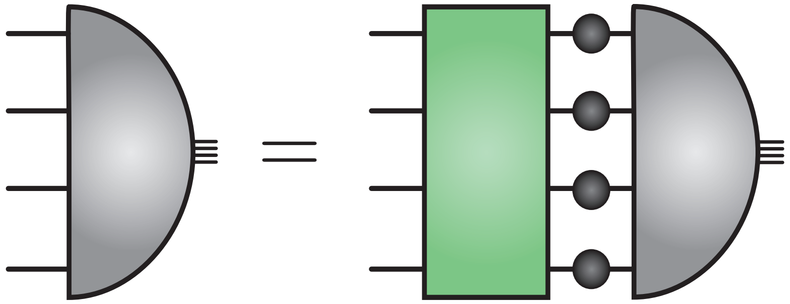




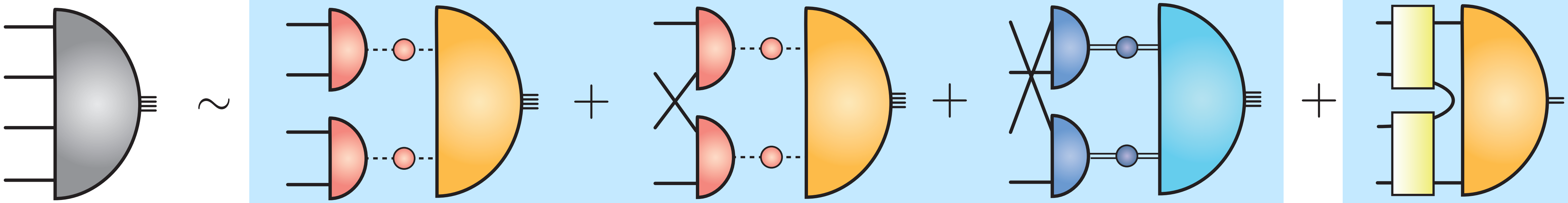
GMOR-relation
$M_\pi \propto \sqrt{m_q}$
✅

Results for "physical pion": $m_\pi = 139.0\,\text{MeV}, \quad f_\pi = 92.5\,\text{MeV}$

The Tetraquark Bethe-Salpeter equation

Full four-body tetraquark BSE: 

Two-body structure:

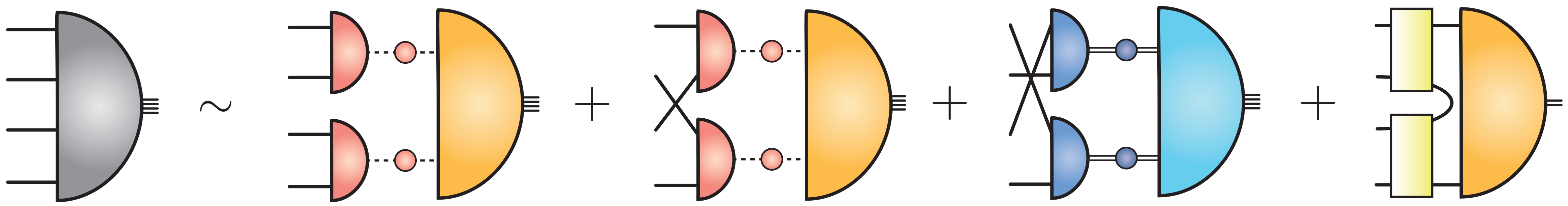


three tetraquark components

one meson comp.

phenomenological

The Tetraquark Bethe-Salpeter equation

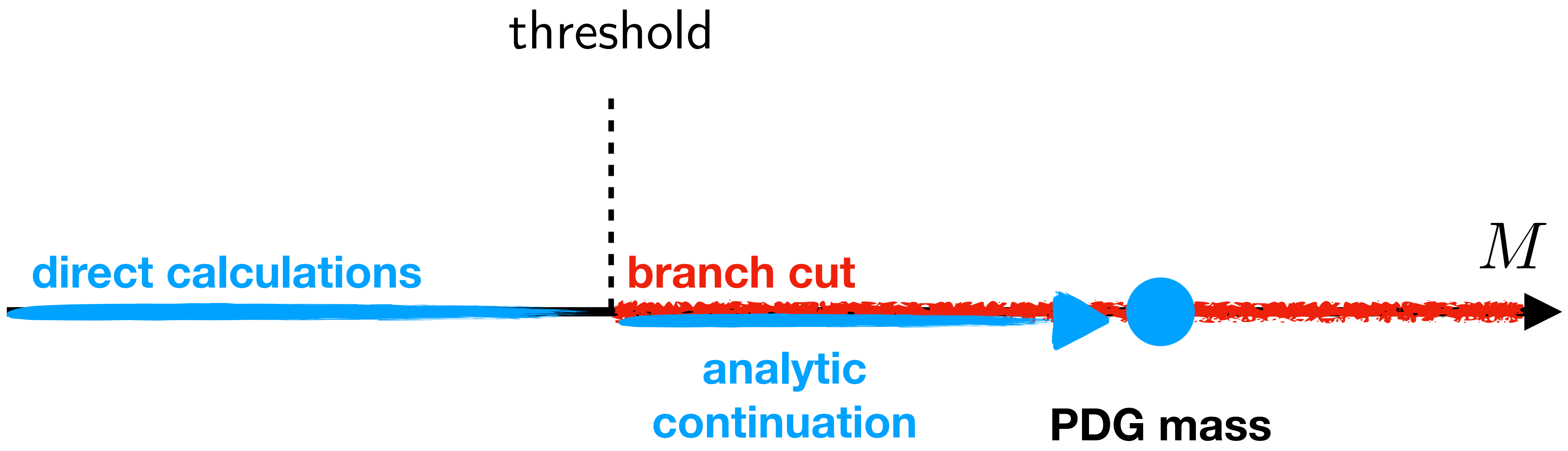


Amplitude exclusions:

- Exclusion of one, two or three components is possible, e.g.
  - ➡ pure tetraquark (exclude [4])
  - ➡ pure mesonic tetraquark (exclude [3], [4])
  - ➡ pure quark-antiquark state (exclude [1], [2], [3])

Eigenvalue curves

σ meson: $f_0(500)$ BREIT-WIGNER MASS (400 - 550) MeV

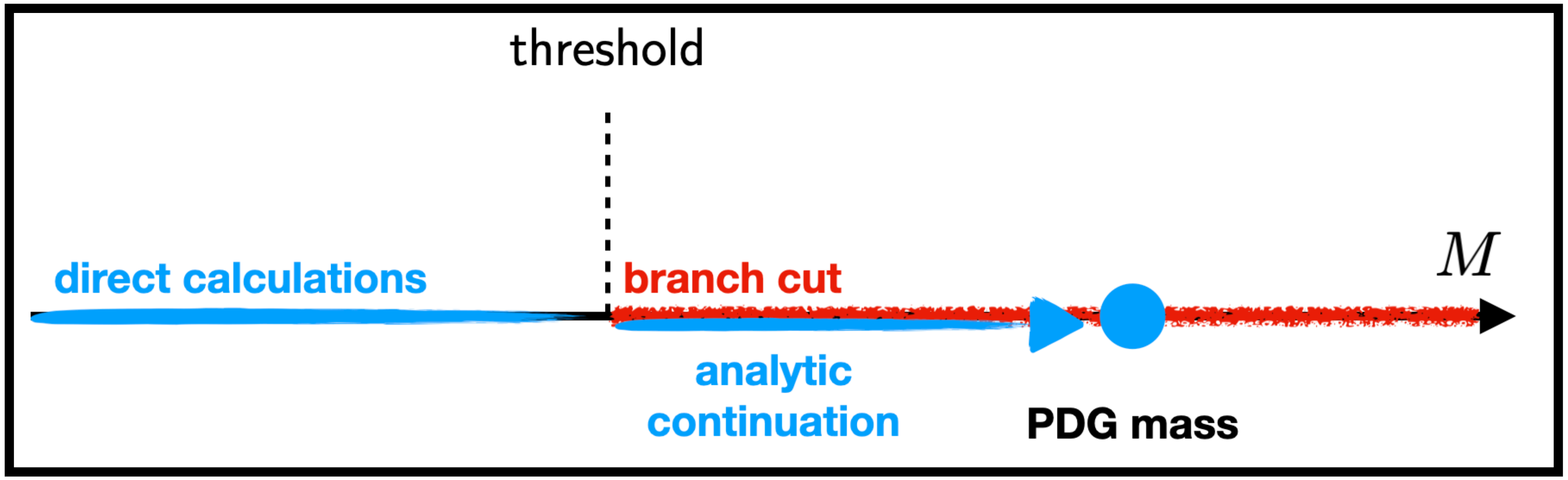


Results

- for now: treat the sigma meson (scalar meson) as a mixed state:

  $$|\sigma\rangle = |q\bar{q}q\bar{q}\rangle + |q\bar{q}\rangle$$

- calculation on the real axis



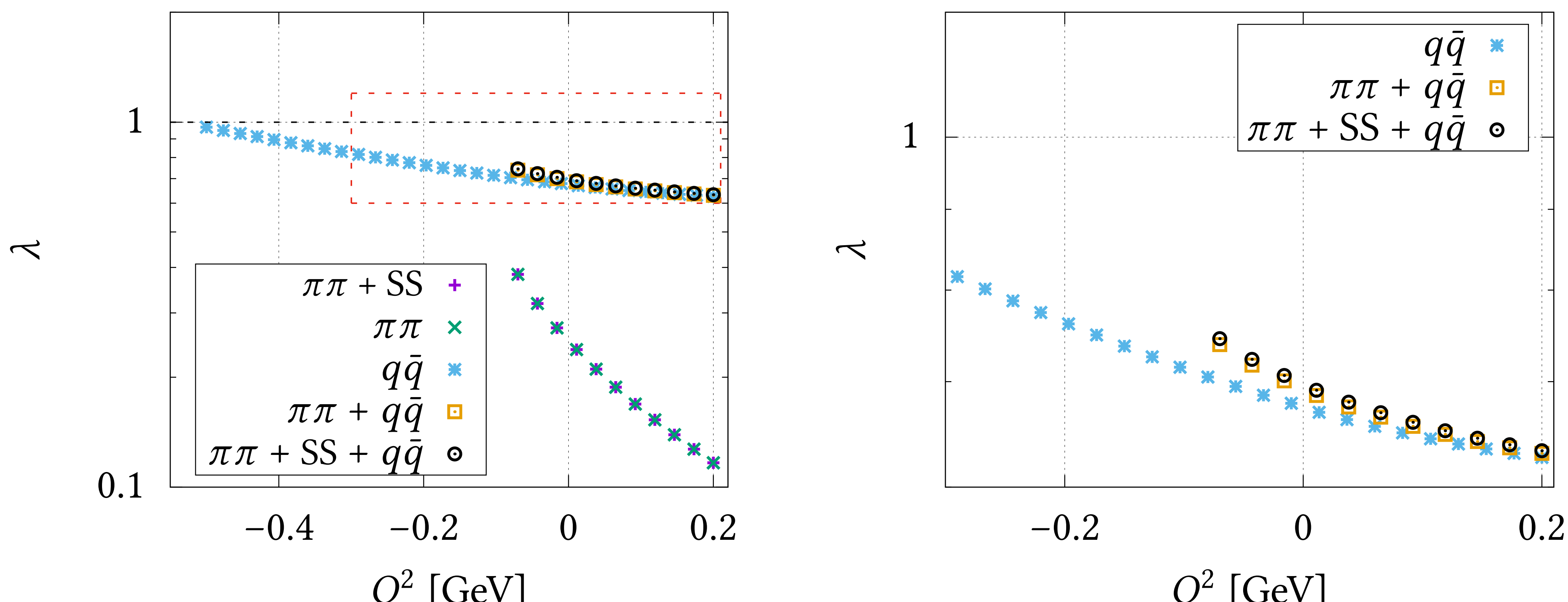
Results: Mixed σ on the real axis

- for now: treat the sigma meson (scalar meson) as a mixed state:

$$|\sigma\rangle = |q\bar{q}q\bar{q}\rangle + |q\bar{q}\rangle$$

- calculation on the real axis

NS et. al. PRD **102**, 056014 (2020)
arXiv:2007:06495



$$Q^2 = -M^2$$

Results: Mixed σ on the real axis

- for now: treat the … meson) as a mixed state:

$M_{qq\bar{q}\bar{q}} \approx 416(26)$ MeV

$M_{q\bar{q}} \approx 667(2)$ MeV

$M_{qq\bar{q}\bar{q}+q\bar{q}} \approx 456(24)$ MeV

- calculation on the real axis

NS et. al. PRD **102**, 056014 (2020)
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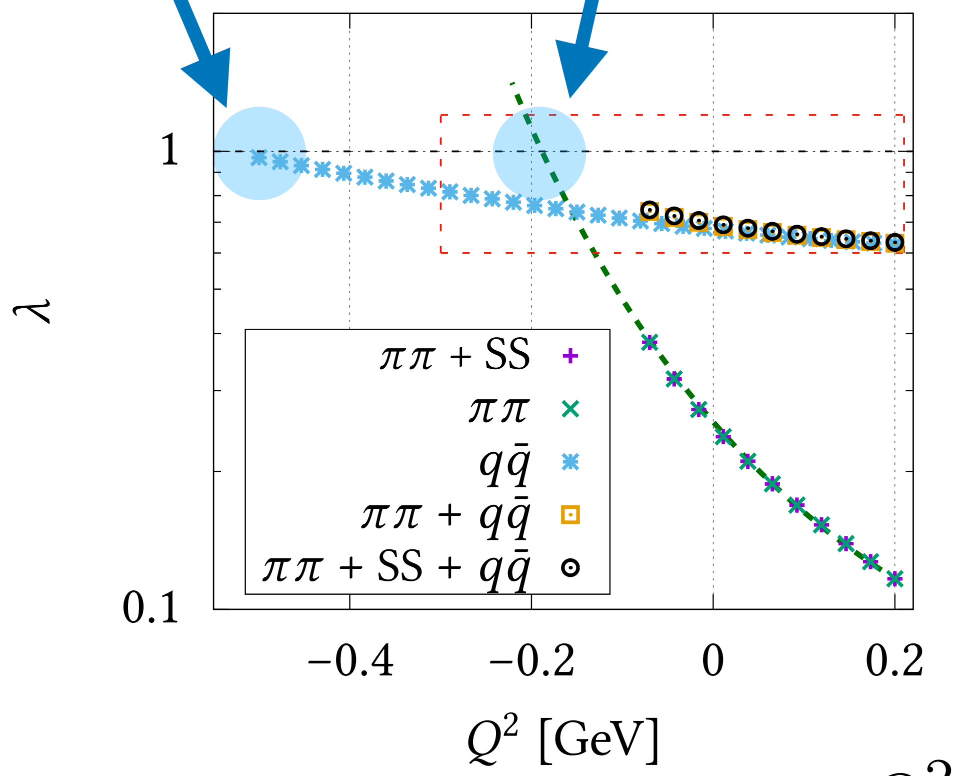
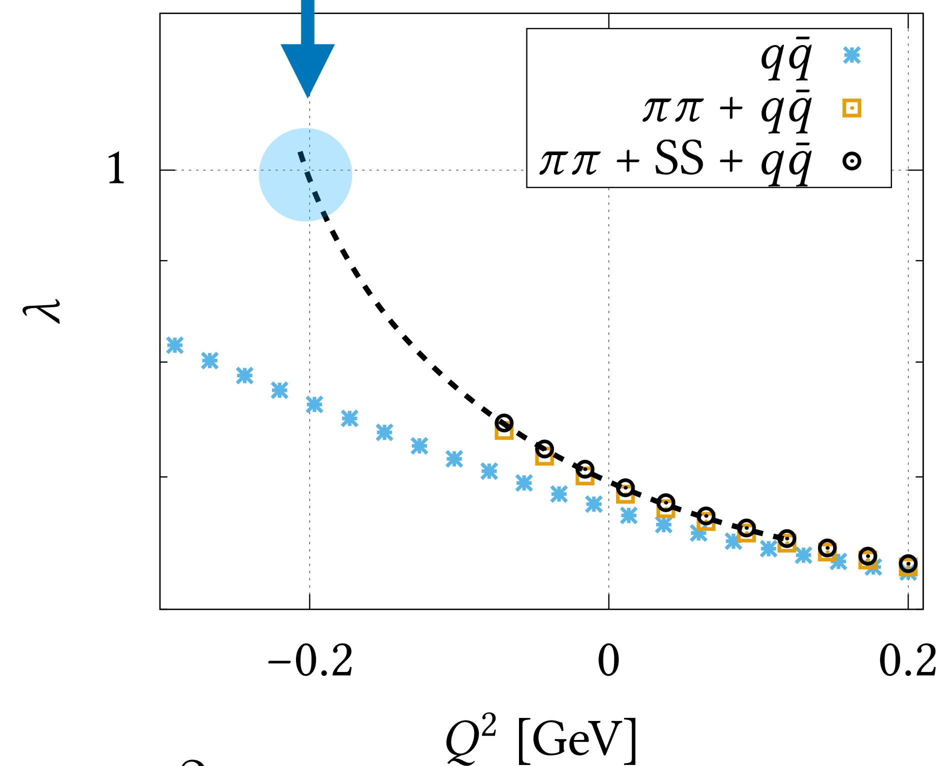
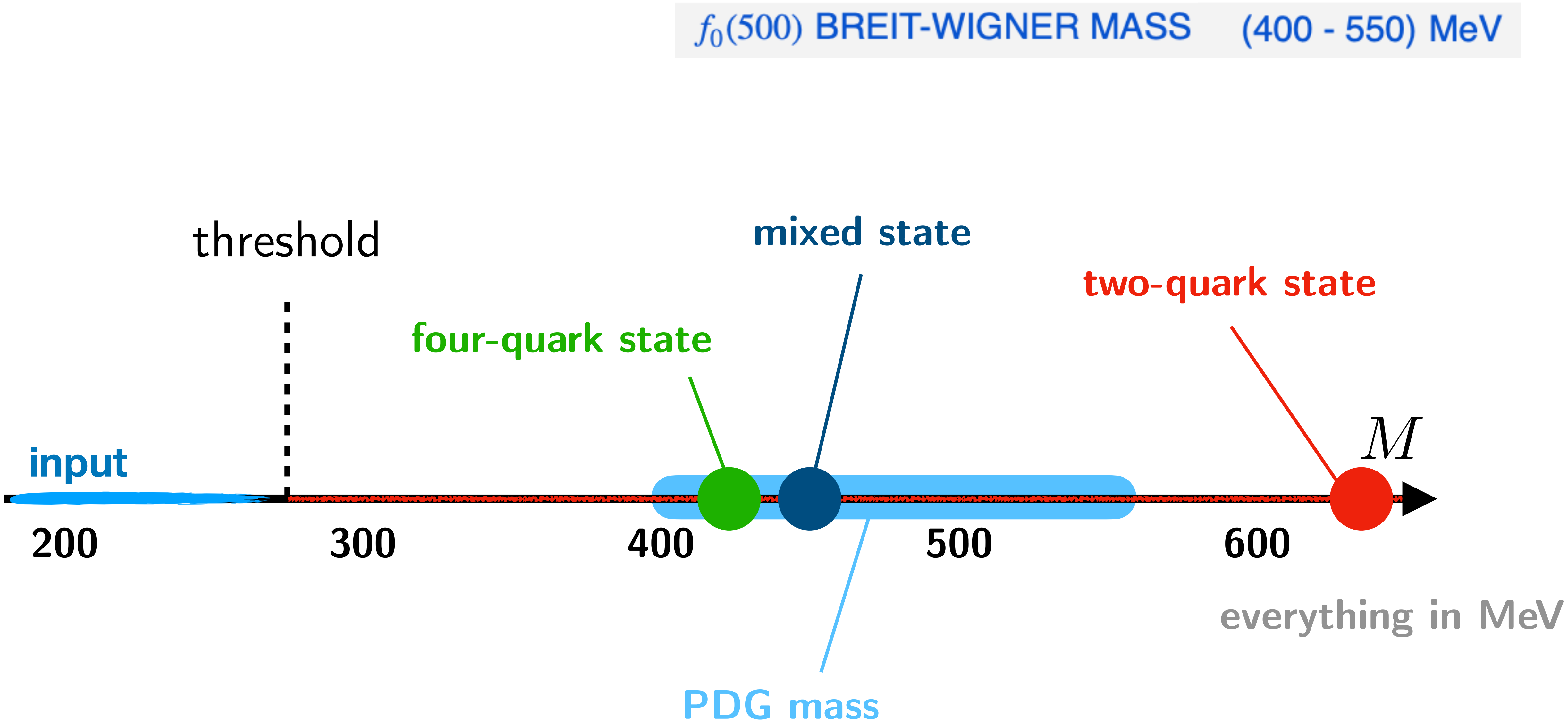




$$Q^2 = -M^2$$

Results: σ on the real axis



We identify the σ meson as a four-quark state

NS et. al. PRD **102**, 056014 (2020)
arXiv:2007:06495

But there is width...

σ meson: $f_0(500)$ BREIT-WIGNER MASS (400 - 550) MeV

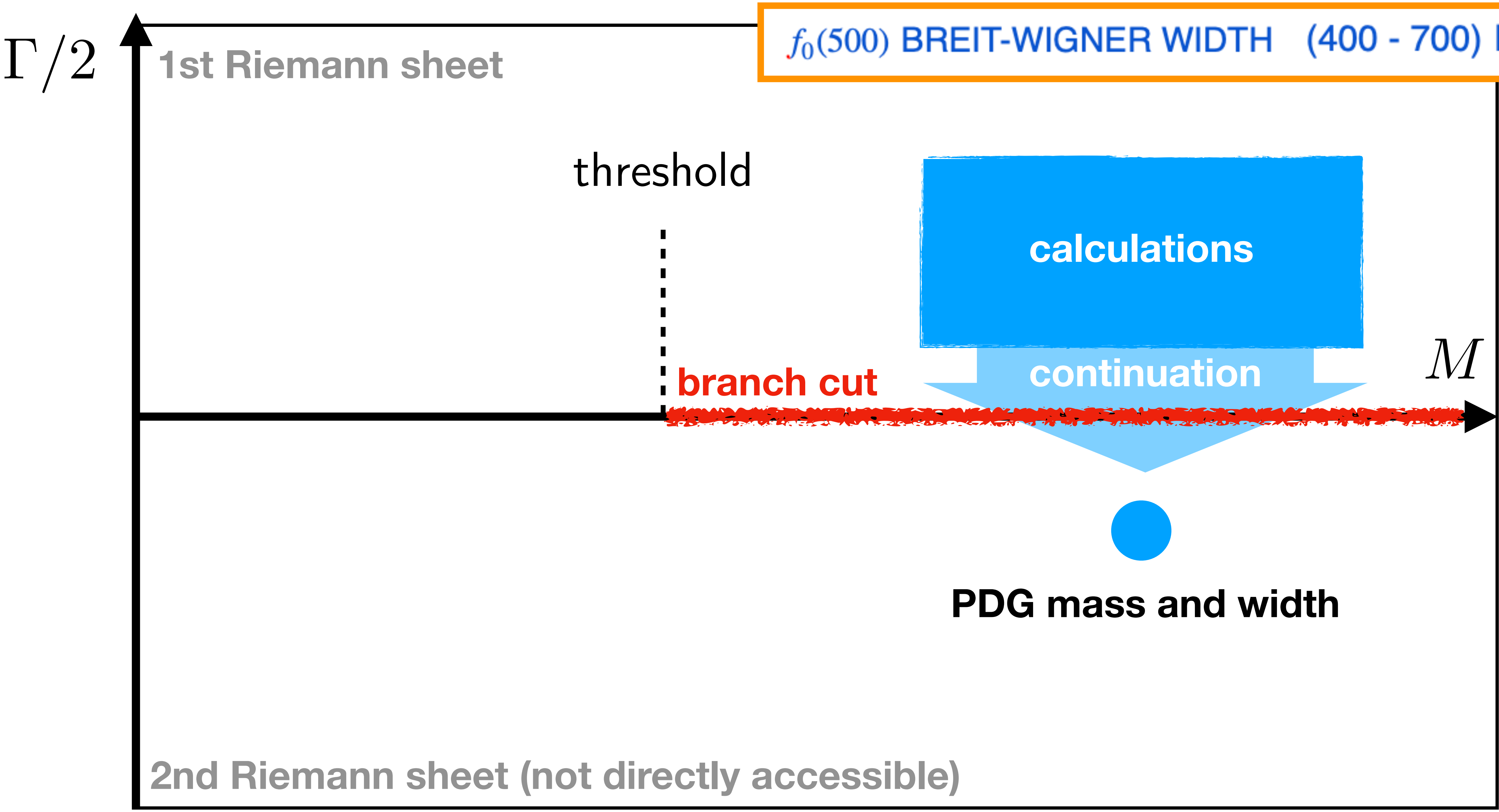
$f_0(500)$ BREIT-WIGNER WIDTH (400 - 700) MeV

but there is width...

But there is width...

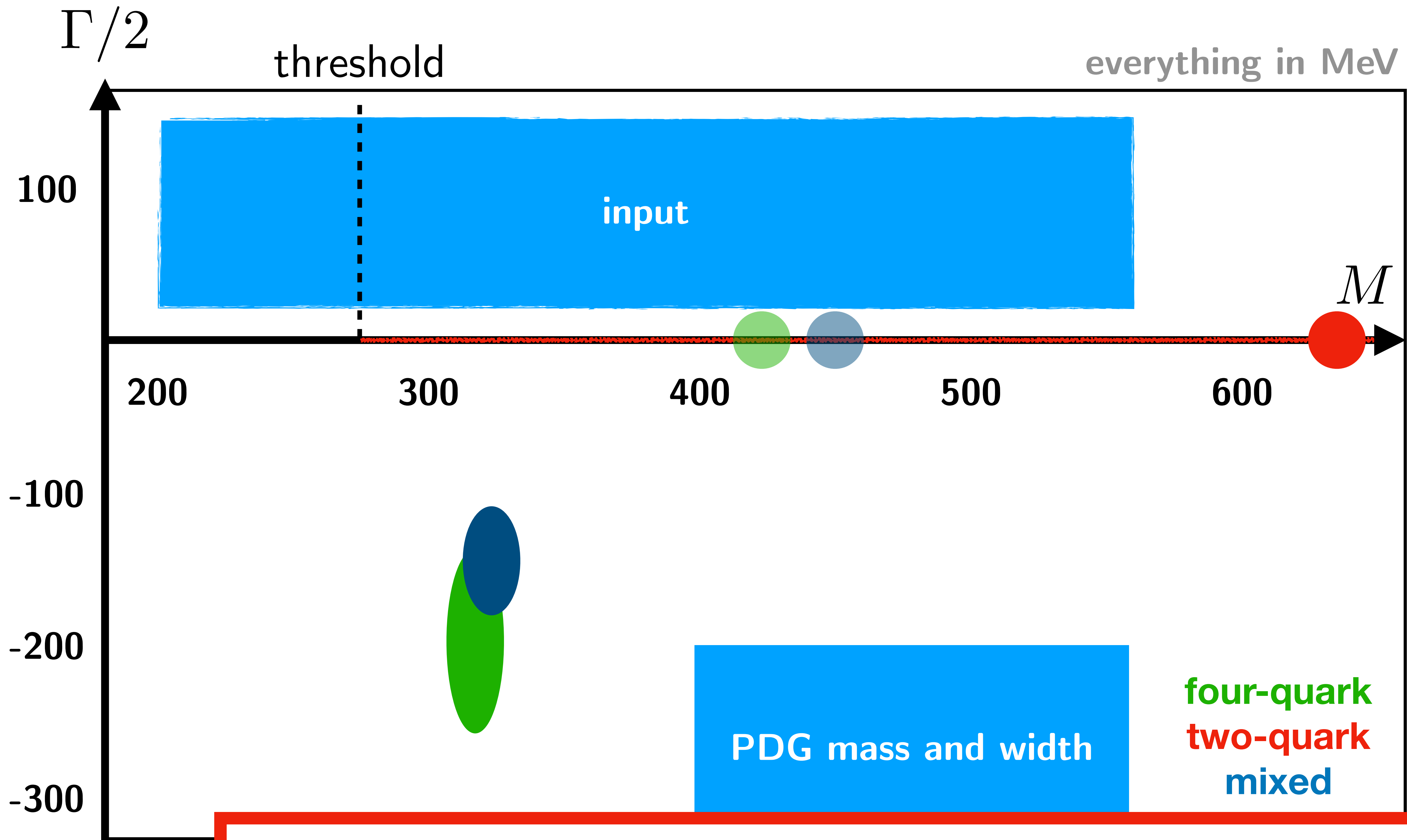
σ meson:

$f_0(500)$ BREIT-WIGNER MASS	(400 - 550) MeV
$f_0(500)$ BREIT-WIGNER WIDTH	(400 - 700) MeV



$$\sqrt{s} = \mathrm{i}\,(M - \mathrm{i}\,\Gamma/2)$$

Results: σ in the complex plane



still four-quark dominance, but with reduced mass

Potential

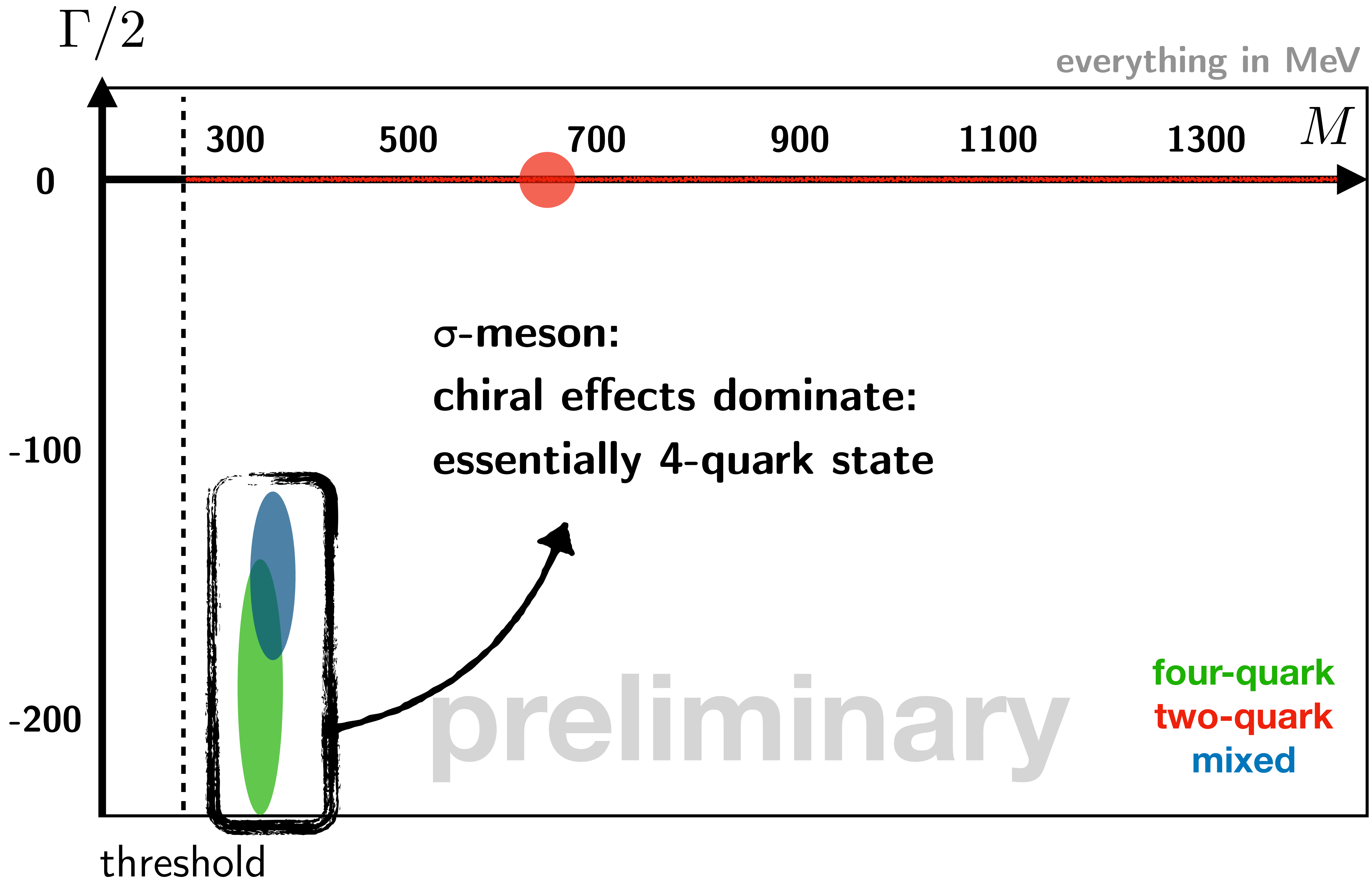
- Proof of principle done with sigma meson: $q\bar{q} + q\bar{q}q\bar{q}$
- How about other scalar states ...
  - $s\bar{s} + s\bar{s}s\bar{s}$

$$q\bar{q}q\bar{q} \xrightarrow{\text{increase all quark masses simultaneously}} s\bar{s}s\bar{s}$$

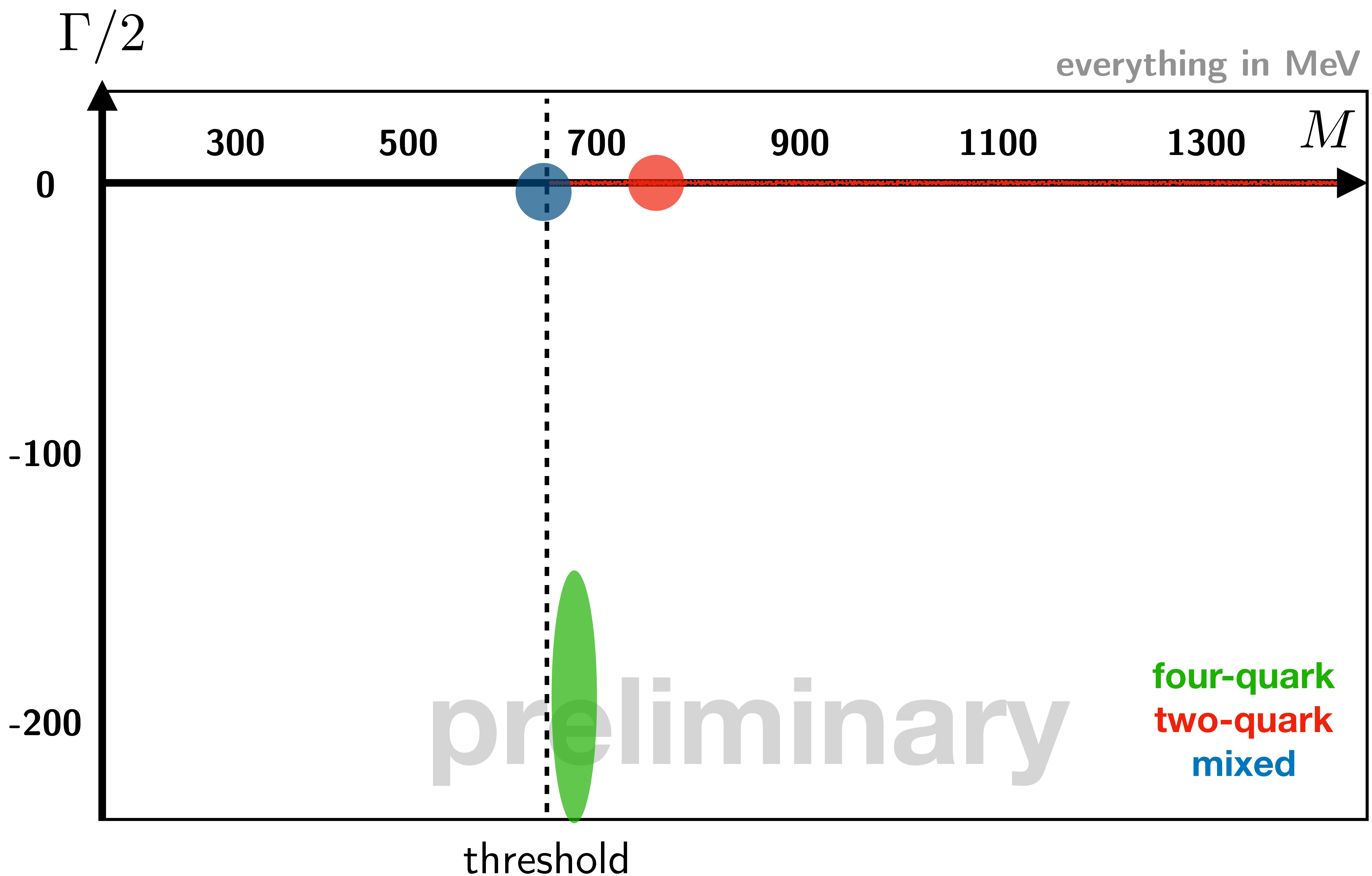
  - $s\bar{s} + s\bar{s}q\bar{q}$ ($a_0(980)$ candidate!)

$$q\bar{q}q\bar{q} \xrightarrow{\text{increase two quark masses simultaneously}} s\bar{s}q\bar{q}$$

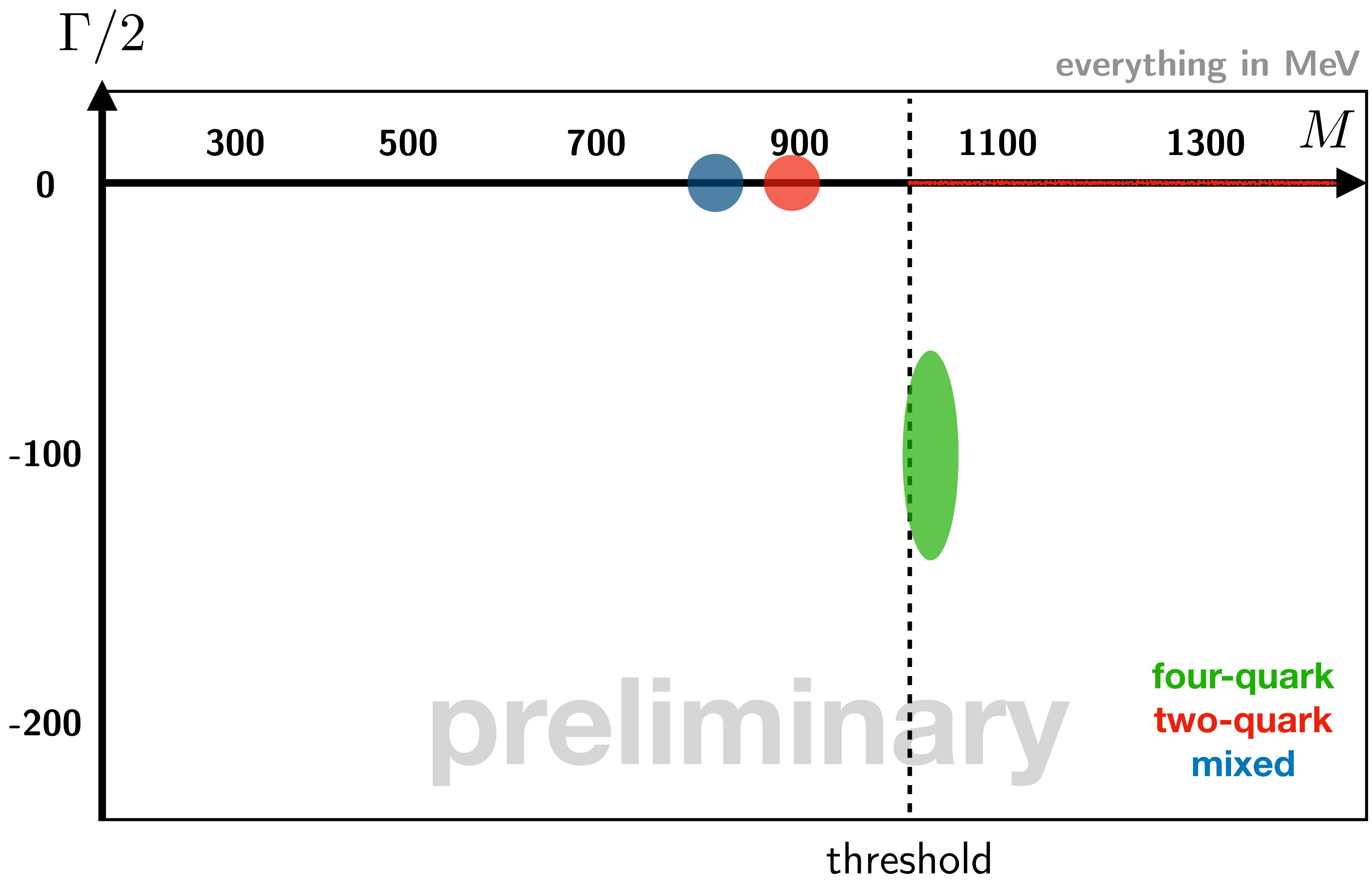
$qq[qq]$ state ($m_q = 3.8$ MeV — u/d)



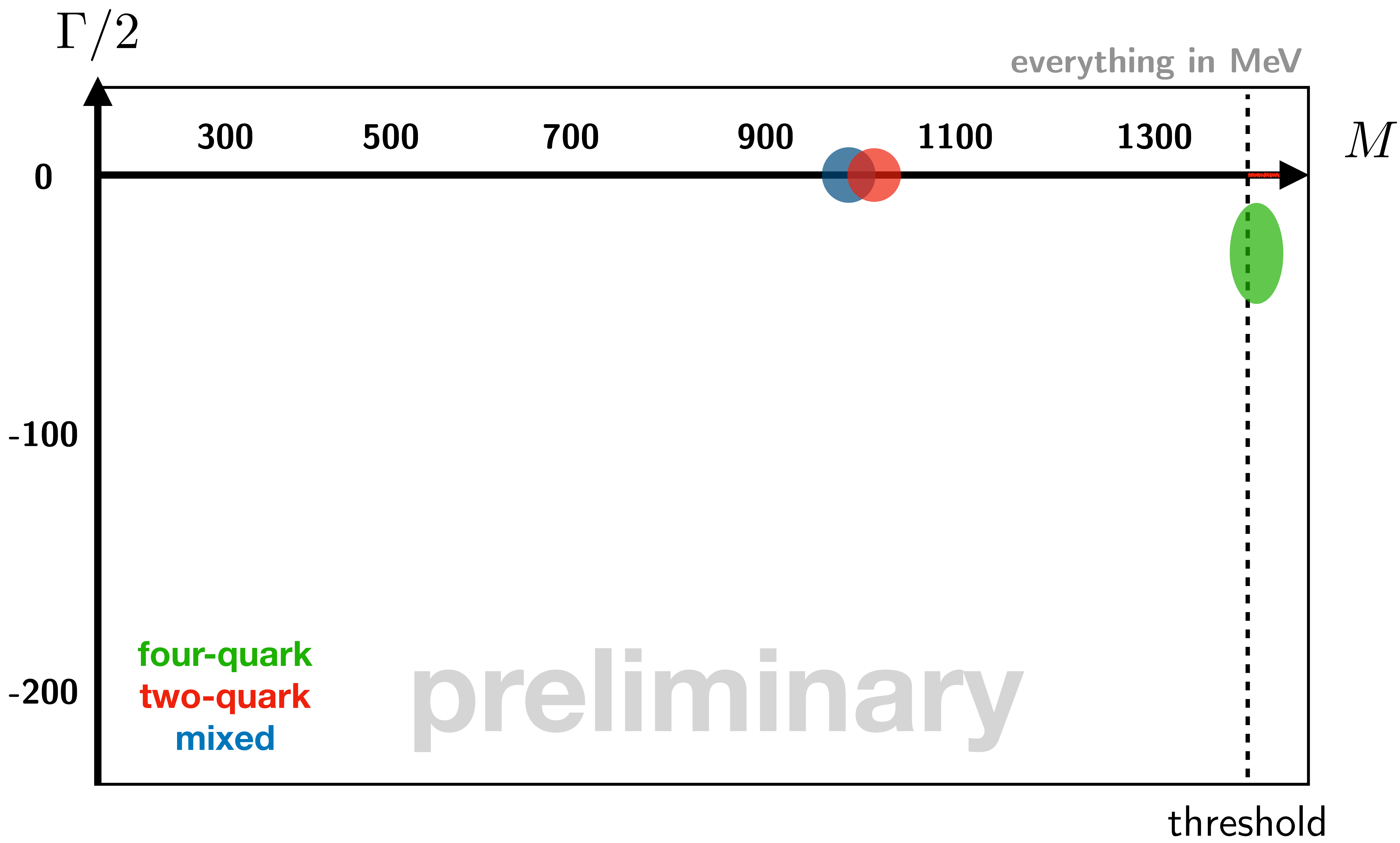
$qq[qq]$ state ($m_q = 20$ MeV)



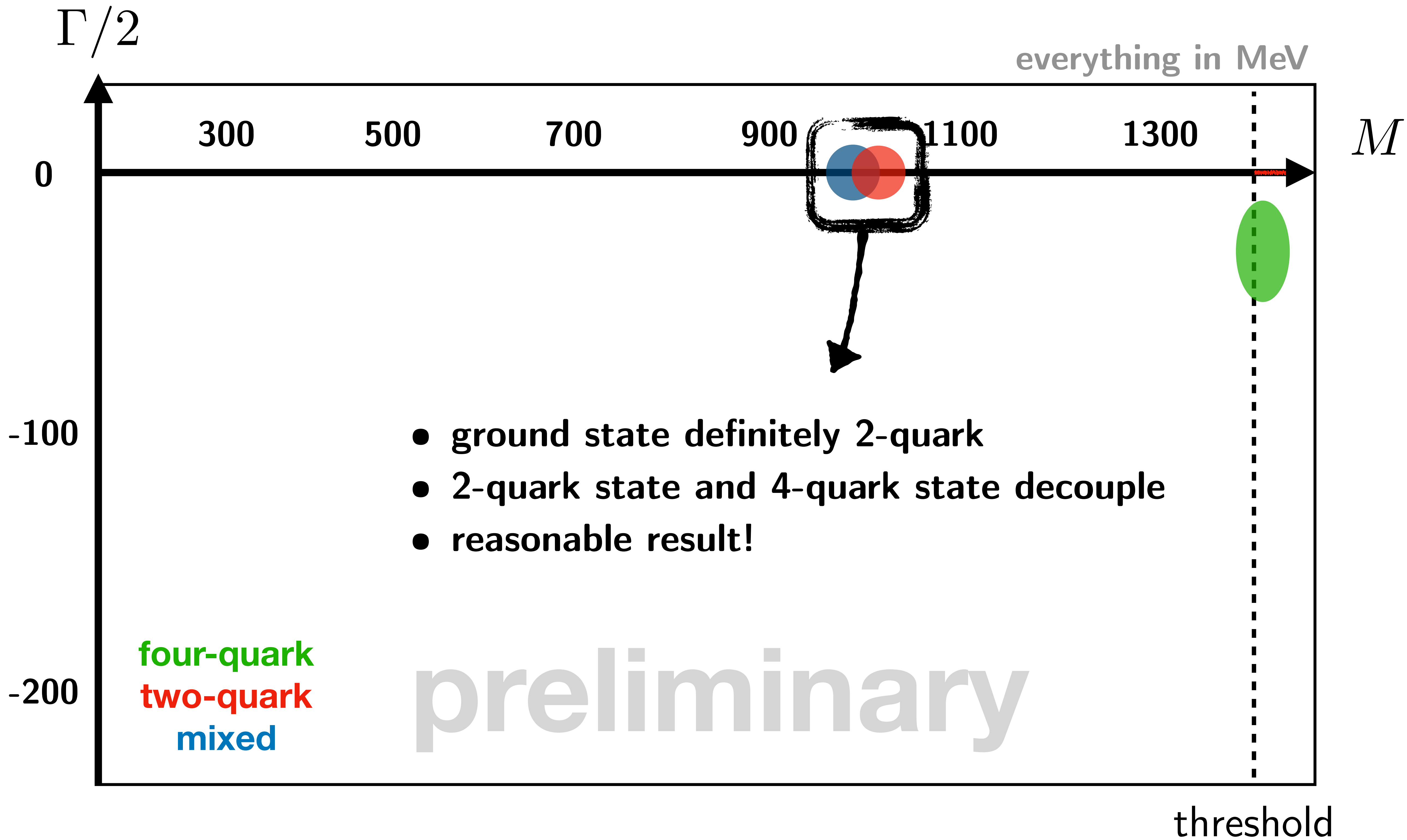
$qq[qq]$ state ($m_q = 50$ MeV)



$qq[qq]$ state ($m_q = 85.5$ MeV — s)



$qq[qq]$ state ($m_q = 85.5$ MeV — s)



- ground state definitely 2-quark
- 2-quark state and 4-quark state decouple
- reasonable result!

Potential

- Proof of principle done with sigma meson: $q\bar{q} + q\bar{q}q\bar{q}$

- How about other scalar states ...

  - $s\bar{s} + s\bar{s}s\bar{s}$

$$q\bar{q}q\bar{q} \xrightarrow{\text{increase all quark masses simultaneously}} s\bar{s}s\bar{s}$$

  - $s\bar{s} + s\bar{s}q\bar{q}$ $(a_0(980)$ candidate!)

Ongoing work!

$$q\bar{q}q\bar{q} \xrightarrow{\text{increase two quark masses simultaneously}} s\bar{s}q\bar{q}$$

Summary and Outlook

- Huge technical improvements within the last 10 months
  - ✓ quarkonium mixing *(never before!)*
  - ✓ complex plane and decay width *(only for 2-quark states yet!)*
- σ meson is a 4-quark state (ππ)
  - ➡ chiral effect: gets lost by increasing quark mass!
- Masses are lower than experimental masses in our equations
  - ➡ possibly model + truncation issue

- Huge potential to investigate nature of exotica
- Long term goal: access to XYZ states in the complex plane

Thank you for your attention!