

Solving the transport equations with GiBUU

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Kinetic theory and BUU equation

GiBUU implementation

Hadronization in Cold Nuclear Matter
Coulomb Effects in Heavy Ion Collisions



GiBUU

= The Giessen Boltzmann-Uehling-Uhlenbeck Project

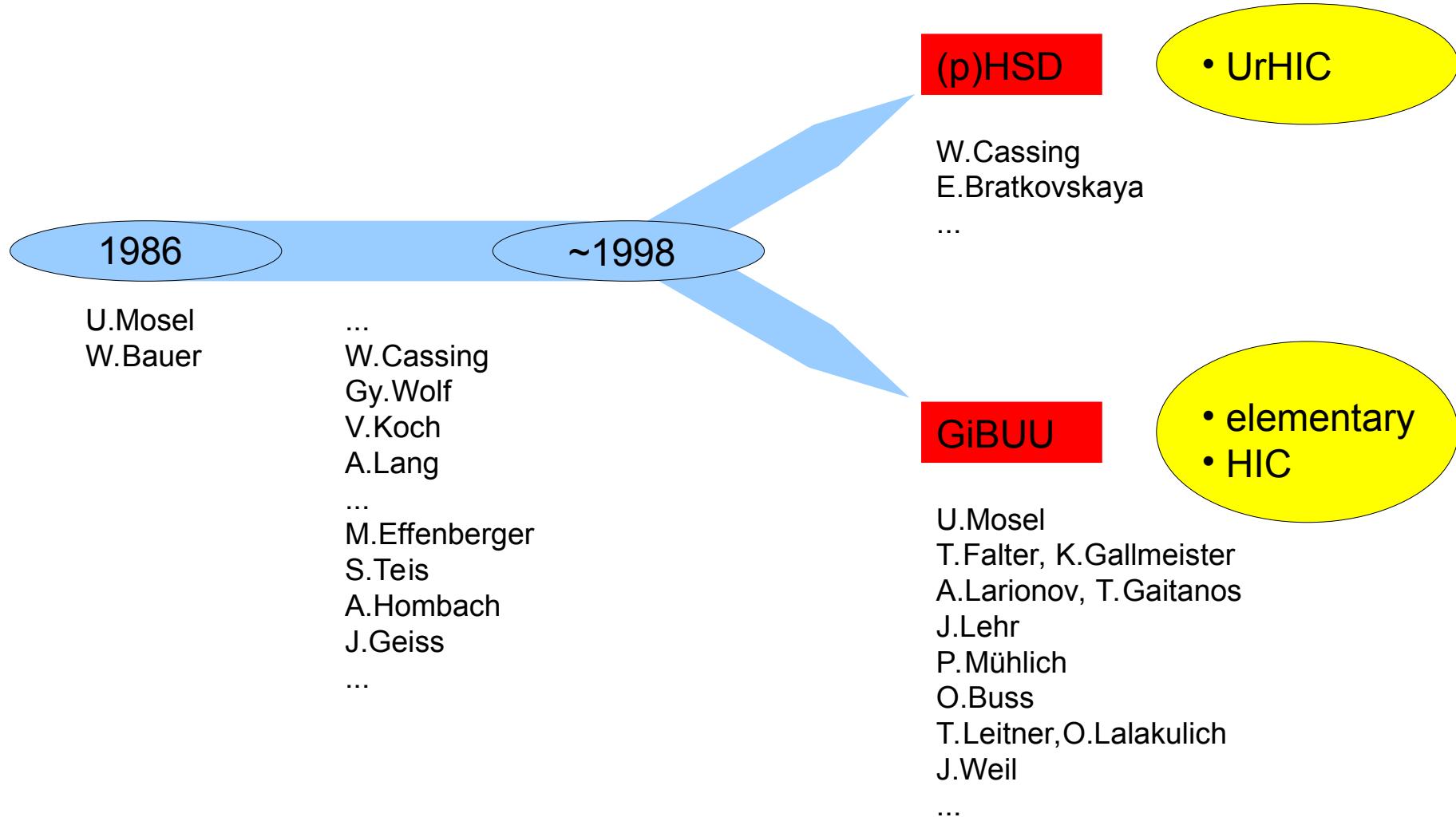
- Gießen: Town in Hesse, Germany
84000 inhabitants (2015)
70 km north of Frankfurt

Institut für Theoretische Physik, Justus-Liebig-Universität

- 'official' pronunciation: ghee – bee – you – you

alternatives: gee – bee – you – you (as "BeeGees")
giii – buuh (as "Hui Buh")

BUU@Gießen and GiBUU



■ Long history

- | | | | |
|------------------------|------------------------------|---|-------------------|
| 1986: first code | (<i>Bauer</i>) | } | lifetime: ~10 yrs |
| ~1996: rewrite of code | (<i>Teis, Effenberger</i>) | | |
| ~2005: rewrite of code | (<i>Buss</i>) | | |

■ Actual version: „GiBUU“

- Modular, Fortran 2003
- Version control (svn + trac)

<https://gibuu.hepforge.org/>

■ Bottlenecks:

- PYTHIA (very slow at low energies)
- Huge code (185 000 lines + Docu + 'Externals')
- „long history“ (old structures)
- ...
- Transparency ratios = ratios of MC calculations: ~1 CPU-year per curve

Some kinetic theory

■ **distribution function** $f(x, p)$ $x = (t, \vec{x})$, $p = (E, \vec{p})$

describes (density) distribution of (single) particles

for each particle species: $f_N, f_\pi, f_\Delta, \dots$

number of particles in a given phase-space volume: $\Delta N = f(x, p) \Delta^3 x \Delta^3 p$

■ **continuity equation** for free, non-interacting particles

$$p^\mu \partial_\mu f(x, p) = 0$$

straight line propagation of particles, no collisions

■ adding external forces (mean field potentials): **Vlasov eq.**

$$[\partial_t + (\nabla_p E) \nabla_r - (\nabla_r E) \nabla_p] f(x, p) = 0$$

propagation in mean field, no collisions

Some kinetic theory: adding collisions

...forget about mean fields, but add collisions:

- continuity eq. + collision term → Boltzmann eq.

$$p^\mu \partial_\mu f(x, p) = C(x, p)$$

straight line propagation of particles, with collisions

collision integral has gain and loss term:

$$C(x, p) = C_{\text{gain}}(x, p) + C_{\text{loss}}(x, p)$$

- mean fields & collision term:

Boltzmann-Uehling-Uhlenbeck equation (BUU)

$$[\partial_t + (\nabla_p H_i) \nabla_r - (\nabla_r H_i) \nabla_p] f_i(\vec{r}, t, \vec{p}) = C [f_i, f_j, \dots]$$

The BUU equation

$$[\partial_t + (\nabla_p H_i) \nabla_r - (\nabla_r H_i) \nabla_p] f_i(\vec{r}, t, \vec{p}) = C [f_i, f_j, \dots]$$

- describes space-time evolution of single particle densities
- index i represents particle species
→ one equation for each species $i = N, \Delta, \pi, \rho, \dots$
- Hamiltonian H_i
 - hadronic mean fields (Skyrme/Welke or RMF)
 - Coulomb
 - „off-shell-potential“
- collision term C
 - decay and scattering processes: 1-, 2- and 3-body
 - (low energy: resonance model, high energy: string model)
 - contains Pauli-blocking
- equations coupled via mean fields and via collision term

Degrees of Freedom

- GiBUU is purely hadronic (no partonic phase)
- 61 baryons, 22 mesons
(strangeness and charm included, no bottom)
properties from Manley analysis (PDG for strange/charm)
- leptons: usually not ‚transported‘, but
 - e+N, nu+N, gamma+N initial events
 - leptonic/photonic decays
- in principle one needs:
 - cross sections for collisions between all of them (all energies)
 - mean-field potentials for all speciesoften not known, thus use hypothesis/models/guesses

Particle species

■ important particles:

particle	mass	width	GiBUU ID	PDG IDs
N	0.983	0	1	p=2212, n=2112
Δ	1.232	0.118	2	2224, 2214, 2114, 1114
N^*			3-18	
Δ^*			19-31	
Λ	1.116	0	32	3122
Σ	1.189	0	33	3222,3212,3112
Λ^*, Σ^*			34-52	
π	0.138	0	101	$\pi^+ = 211, \pi^0 = 111, \pi^- = -211$
η	0.547		102	
ρ	0.775	0.149	103	213,113,-213
σ			104	
ω	0.782	0.004	105	
η'	0.957		106	
K	0.496	0	110	$K^+ = 321, K^0 = 311$
\bar{K}	0.496	0	111	$K^- = -321, \bar{K}^0 = -311$

Mean-field potentials

- two types of mean-field potentials:
 - non-relativistic Skyrme-type potentials
 - relativistic mean fields (RMF)
- potential may enter single-particle energy as
$$H = \sqrt{(m + V)^2 + (\vec{p} + \vec{U})^2} + U_0$$
- RMF is Lorentz vector U^μ
- Skyrme enters as U_0 , bound to specific frame (LRF)
- Scalar Potential V : mass shift

Skyrme/Welke-like potential

$$U_0(x, \vec{p}) = A \frac{\rho}{\rho_0} + B \left(\frac{\rho}{\rho_0} \right)^\gamma$$
$$+ \frac{2C}{\rho_0} \sum_{i=p,n} \int \frac{g d^3 p'}{(2\pi)^3} \frac{f_i(x, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2 / \Lambda^2}$$
$$+ d_{\text{symm}} \frac{\rho_p(x) - \rho_n(x)}{\rho_0} \tau_i$$
$$\rho_0 = 0.168 \text{ fm}^{-3}$$

- defined in local rest frame (LRF, baryon current vanishes)
- six parameters
- fixed to...
 - nuclear binding energy of 16 MeV at $\rho=\rho_0$ (iso-spin symm. matter)
 - nuclear-matter incompressibility $K=200-380$ MeV

RMF potentials

- proper relativistic mean-field description
- based on (nonlinear) Walecka-type Lagrangian

$$\begin{aligned}\mathcal{L} = & \bar{\psi} [\gamma_\mu (i\partial^\mu - g_\omega \omega^\mu - g_\rho \tau \rho^\mu - \frac{e}{2} (1 + \tau^3) A^\mu) - m_N - g_\sigma \sigma] \psi \\ & + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\sigma) - \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^2 \\ & - \frac{1}{4} R_{\mu\nu} R^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho^2 - \frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu}\end{aligned}$$

3 masses, 5 couplings

- difference to nonRMF:
 - theoretically cleaner, computationally more demanding
 - limited range of applicability in energy

Collision term

- contains one-, two-, and three-body collisions

$$C = C_{1 \rightarrow X} + C_{2 \rightarrow X} + C_{3 \rightarrow X}$$

(1) resonance decays

(2) two-body collisions

- elastic and inelastic
- any number of particles in final state
- baryon-meson, baryon-baryon, meson-meson

(3) three-body collisions (only relevant at high densities)

- low energies: cross sections based on resonances

e.g. $\pi N \rightarrow N^*$, $NN \rightarrow NN^*$

- high energies: string fragmentation

Collision term

■ 2-to-2 term ($12 \leftrightarrow 1'2'$)

$$\begin{aligned} C^{(2,2)}(x, p_1) &= C_{\text{gain}}^{(2,2)}(x, p_1) - C_{\text{loss}}^{(2,2)}(x, p_1) \\ &= \frac{\mathcal{S}_{1'2'}}{2p_1^0 g_{1'} g_{2'}} \int \frac{d^4 p_2}{(2\pi)^4 2p_2^0} \int \frac{d^4 p_{1'}}{(2\pi)^4 2p_{1'}^0} \int \frac{d^4 p_{2'}}{(2\pi)^4 2p_{2'}^0} \\ &\quad \times (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_{1'} - p_{2'}) \overline{|\mathcal{M}_{12 \rightarrow 1'2'}|^2} \\ &\quad \times [F_{1'}(x, p_{1'}) F_{2'}(x, p_{2'}) \overline{F}_1(x, p_1) \overline{F}_2(x, p_2) \\ &\quad - F_1(x, p_1) F_2(x, p_2) \overline{F}_{1'}(x, p_{1'}) \overline{F}_{2'}(x, p_{2'})] \end{aligned}$$

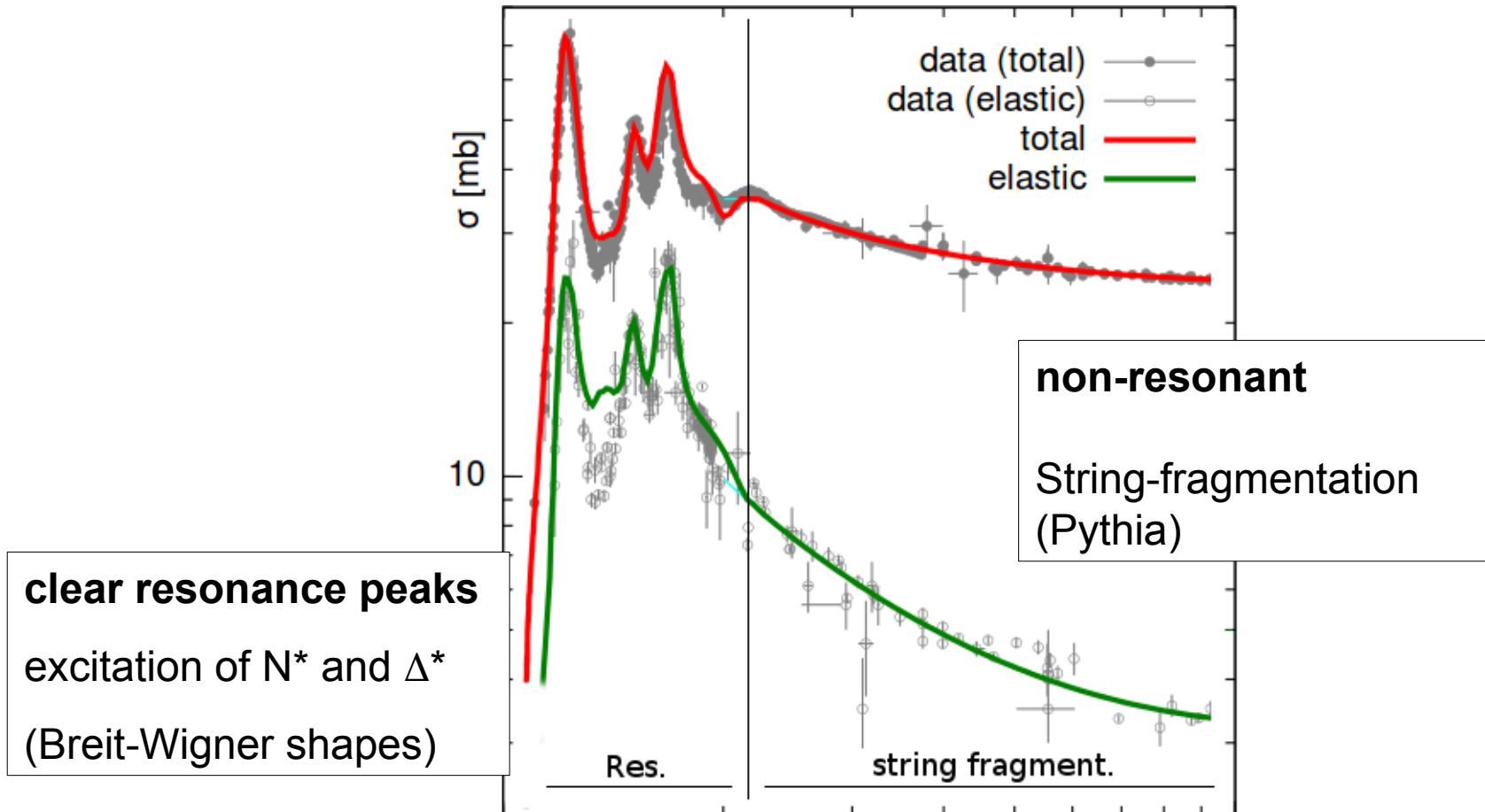
■ $F(x, p) = 2\pi g f(x, p) \mathcal{A}(x, p)$

$$\overline{F}(x, p) = 2\pi g [1 - f(x, p)] \mathcal{A}(x, p)$$

Pauli-blocking

Baryon-Meson collisions

example: πN cross section



$$\mathcal{A}(p) = \frac{1}{\pi} \frac{\sqrt{p^2 \Gamma}}{(p^2 - M_0^2)^2 + p^2 \Gamma^2}$$

Resonance Model

■ resonance parameters, decays modes, widths:

D.Manley, E.Saleski, PRD45 (1992) 4002

= PWA of $\pi N \rightarrow \pi N$ and $\pi N \rightarrow \pi\pi N$ consistency!!!

	rating	M_0 [MeV]	Γ_0 [MeV]	$ \mathcal{M}^2 /16\pi$ [mb GeV 2]		πN	ηN	$\pi\Delta$	branching ratio in %			
				NR	ΔR				ρN	σN	$\pi N^*(1440)$	$\sigma\Delta$
P ₁₁ (1440)	****	1462	391	70	—	69	—	22_P	—	9	—	—
S ₁₁ (1535)	***	1534	151	8	60	51	43	—	$2_S + 1_D$	1	2	—
S ₁₁ (1650)	****	1659	173	4	12	89	3	2_D	3_D	2	1	—
D ₁₃ (1520)	****	1524	124	4	12	59	—	$5_S + 15_D$	21_S	—	—	—
D ₁₅ (1675)	****	1676	159	17	—	47	—	53_D	—	—	—	—
P ₁₃ (1720)	*	1717	383	4	12	13	—	—	87_P	—	—	—
F ₁₅ (1680)	****	1684	139	4	12	70	—	$10_P + 1_F$	$5_P + 2_F$	12	—	—
P ₃₃ (1232)	****	1232	118	OBE	210	100	—	—	—	—	—	—
S ₃₁ (1620)	**	1672	154	7	21	9	—	62_D	$25_S + 4_D$	—	—	—
D ₃₃ (1700)	*	1762	599	7	21	14	—	$74_S + 4_D$	8_S	—	—	—
P ₃₁ (1910)	****	1882	239	14	—	23	—	—	—	—	67	10_P
P ₃₃ (1600)	***	1706	430	14	—	12	—	68_P	—	—	20	—
F ₃₅ (1905)	***	1881	327	7	21	12	—	1_P	87_P	—	—	—
F ₃₇ (1950)	****	1945	300	14	—	38	—	18_F	—	—	—	44_F

$$\Gamma_{R \rightarrow ab}(m) = \Gamma_{R \rightarrow ab}^0 \frac{\rho_{ab}(m)}{\rho_{ab}(M^0)}$$

$$\rho_{ab}(m) = \int p_a^2 p_b^2 \mathcal{A}_a(p_a^2) \mathcal{A}_b(p_b^2) \frac{p_{ab}}{m} B_{L_{ab}}^2(p_{ab} R) \mathcal{F}_{ab}^2(m)$$

(Lund) String-fragmentation (PYTHIA)

idea:

hard $q\bar{q}$ scattering (pQCD)
creates a color flux tube ('string')
which then fragments into hadrons
(via $q\bar{q}$ pair production)

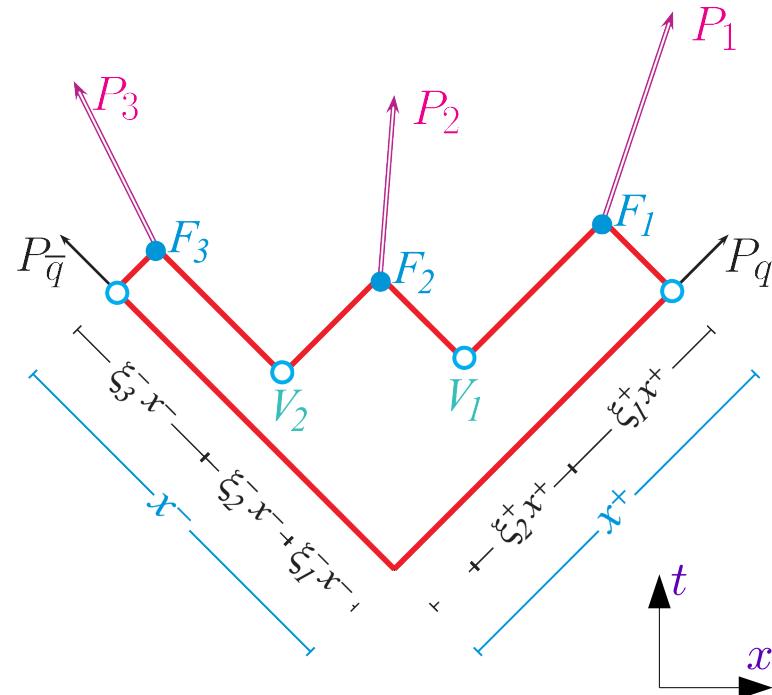
high energy: 10 GeV...

"Lund string model"
implementation: PYTHIA (JETSET)

only low-lying resonances

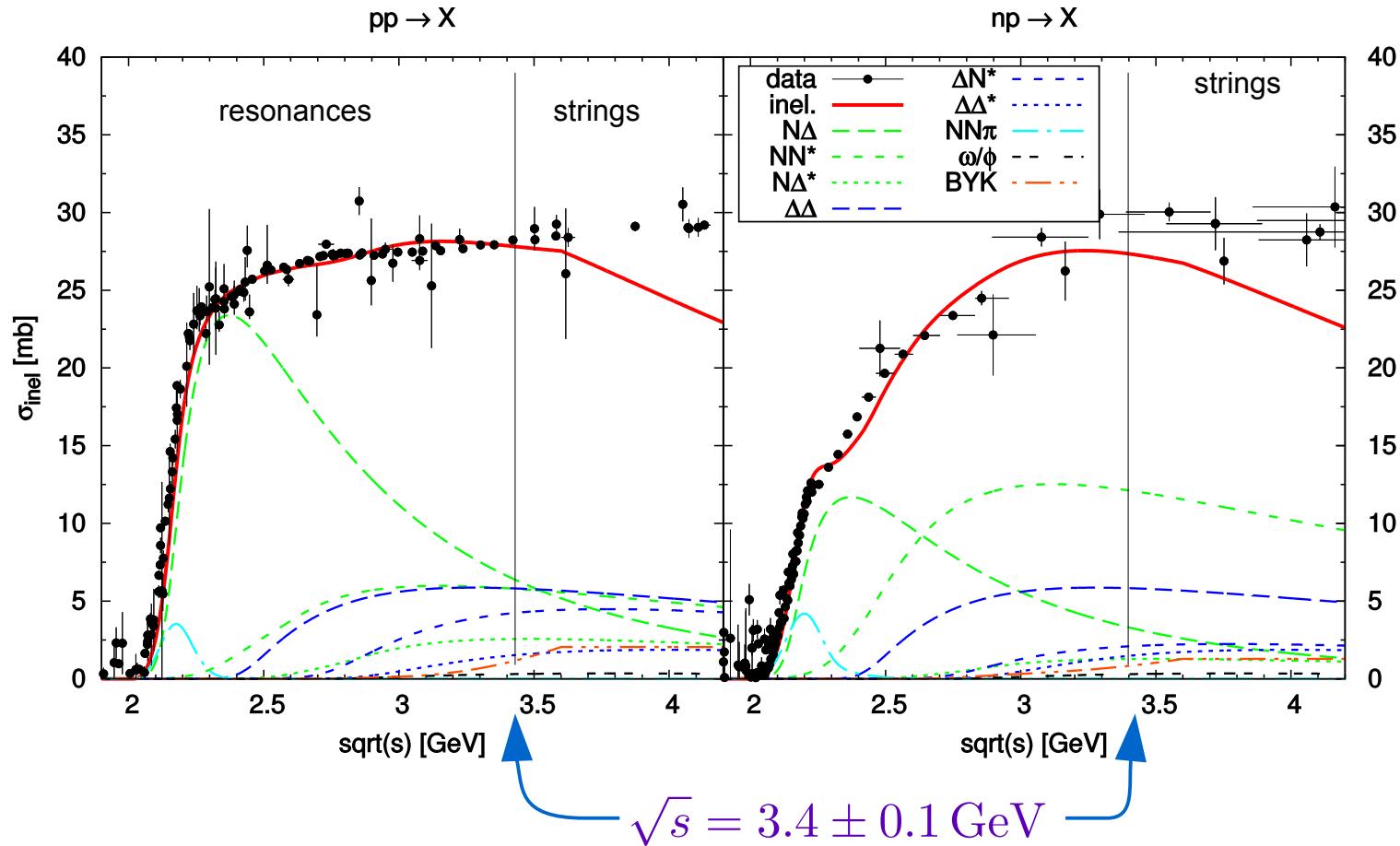
phenomenological fragmentation function
(when and how does a string break?)

parameters fitted to data (different 'tunes' available)



Baryon-Baryon Collisions

- low energy: resonance model, high energy: string model
- no nice peaks due to two-body kinematics
- $NN \rightarrow NR, \Delta R$ ($R = \Delta, N^*, \Delta^*$)



Testparticle ansatz

$$[\partial_t + (\nabla_p H_i) \nabla_r - (\nabla_r H_i) \nabla_p] f_i(\vec{r}, t, \vec{p}) = C [f_i, f_j, \dots]$$

■ *idea:*

approximate phase-space density distribution by sum of delta-functions

$$f(\vec{r}, t, \vec{p}) \sim \sum_{k=1}^{N_{\text{test}}} \delta(\vec{r} - \vec{r}_k(t)) \delta(\vec{p} - \vec{p}_k(t))$$

- each delta-function represents one (test-)particle
with sharp position and momentum
- large number of test particles needed

“parallel ensembles” technique

■ idea:

testparticle index is also ensemble index

■ N_{test} independent runs, densities etc. may be averaged

■ Pros:

- calculational time: collisions scale with N_{test}
- conserved quantum numbers are strictly respected
(‘microcanonical’)

■ Cons:

- non-locality of collisions $\sigma_{ij} \simeq 30 \text{ mb} \rightarrow r = 1 \text{ fm}$

“full ensembles” technique

- each testparticle may interact with every other one
- rescaling of cross section

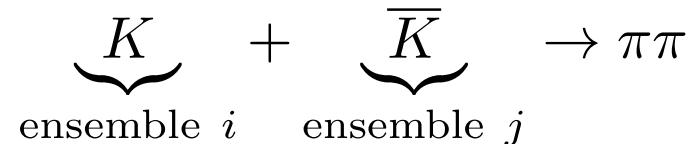
$$\sigma_{ij} \rightarrow \frac{1}{N_{\text{test}}} \sigma_{ij}$$

■ Pros:

- locality of collisions

■ Cons:

- calculational time: collisions scale with $(N_{\text{test}})^2$
- energy not conserved per ensemble, on average only
- conserved quantum numbers are respected on average only
('canonical')



Time evolution

- time axis is discretized
 - collisions only happen at discrete time steps,
 - between collisions: propagation (through mean fields)
- typical time-step size: $\Delta t = 0.1\text{-}0.2 \text{ fm}/c$
- start at $t=0$ and run N timesteps until t_{\max}
- typically:
$$N \Delta t = t_{\max} \approx 20\text{-}50 \text{ fm}/c$$
$$\implies N \approx 100\text{-}1000$$
- density/potentials: if not analytically, recalc at every step

Cross section: Geometric interpretation

particle i and particle j collide, if during timestep Δt

$$r_{ij}(t) = |\vec{r}_i(t) - \vec{r}_j(t)| \stackrel{!}{\leq} \frac{\sqrt{\sigma_{ij}}}{\pi}$$

- problem 1: only for 2-body collisions
- problem 2: not invariant under Lorentz-Trafos
 - different frames may lead to different ordering of collisions
 - specific frame ('calculational frame') needed

Cross section: Stochastic interpretation

- collision rate per unit phase space massless, no $(2\pi)^3$

$$\frac{\Delta N_{\text{coll}}^{2 \rightarrow 2}}{\Delta t \Delta^3 x \Delta^3 p_1} = \frac{\Delta^3 p_2}{2E_1 2E_2} f_1 f_2 \int \frac{d^3 p'_1}{2E'_1} \frac{d^3 p'_2}{2E'_2} |\mathcal{M}| \delta^{(4)}(p_1 + p_2 - p'_1 - p'_2)$$

$$\sigma_{22} = \frac{1}{2s} \int \frac{d^3 p'_1}{2E'_1} \frac{d^3 p'_2}{2E'_2} |\mathcal{M}| \delta^{(4)}(p_1 + p_2 - p'_1 - p'_2)$$

$$f_i = \frac{\Delta N_i}{\Delta^3 x \Delta^3 p}$$

- collision probability in unit box $\Delta^3 x$ and unit time Δt

$$P_{22} = \frac{\Delta N_{\text{coll}}^{2 \rightarrow 2}}{\Delta N_1 \Delta N_2} = v_{\text{rel}} \sigma_{22} \frac{\Delta t}{\Delta^3 x} \quad \left(v_{\text{rel}} = \frac{s}{2E_1 E_2} \right)$$

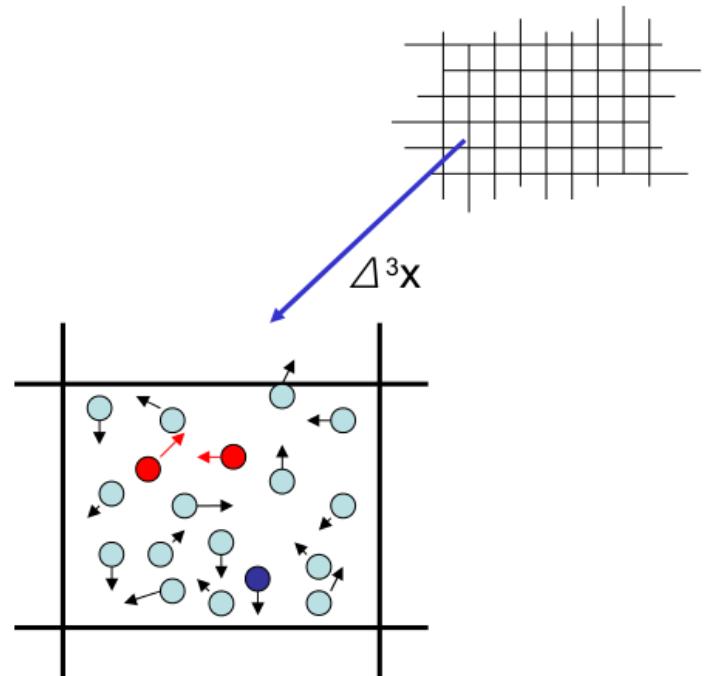
- generalisable to n-body collisions

Cross section: Stochastic interpretation

- discretize time and space

$$P_{2 \rightarrow X} = v_{\text{rel}} \sigma_{2 \rightarrow X} \frac{\Delta t}{\Delta V}$$

$$P_{3 \rightarrow X} = \frac{I_{3 \rightarrow X}}{8E_1 E_2 E_3} \frac{\Delta t}{(\Delta V)^2}$$



- together with ‘full ensemble’
- n particles in cell, randomly select $n/2$ pairs

$$P_2 \rightarrow \frac{n(n-1)/2}{n/2} P_2$$

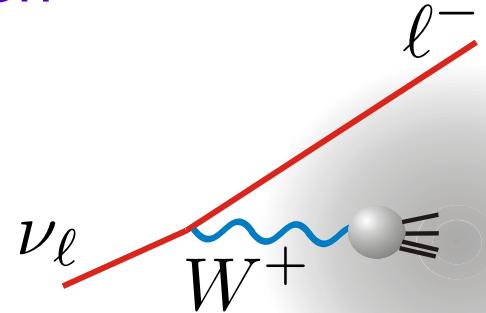
- calculational time: collisions scale approx. with N_{test}
- labeled as “**local ensemble method**”

Nuclear Reactions

■ elementary interaction on nucleon

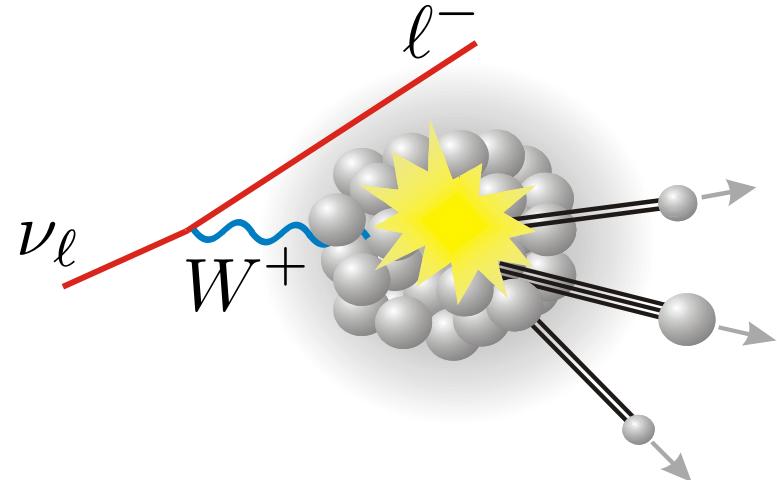
additional:

- binding energies
- Fermi motion
- Pauli blocking
- (coherence length effects)

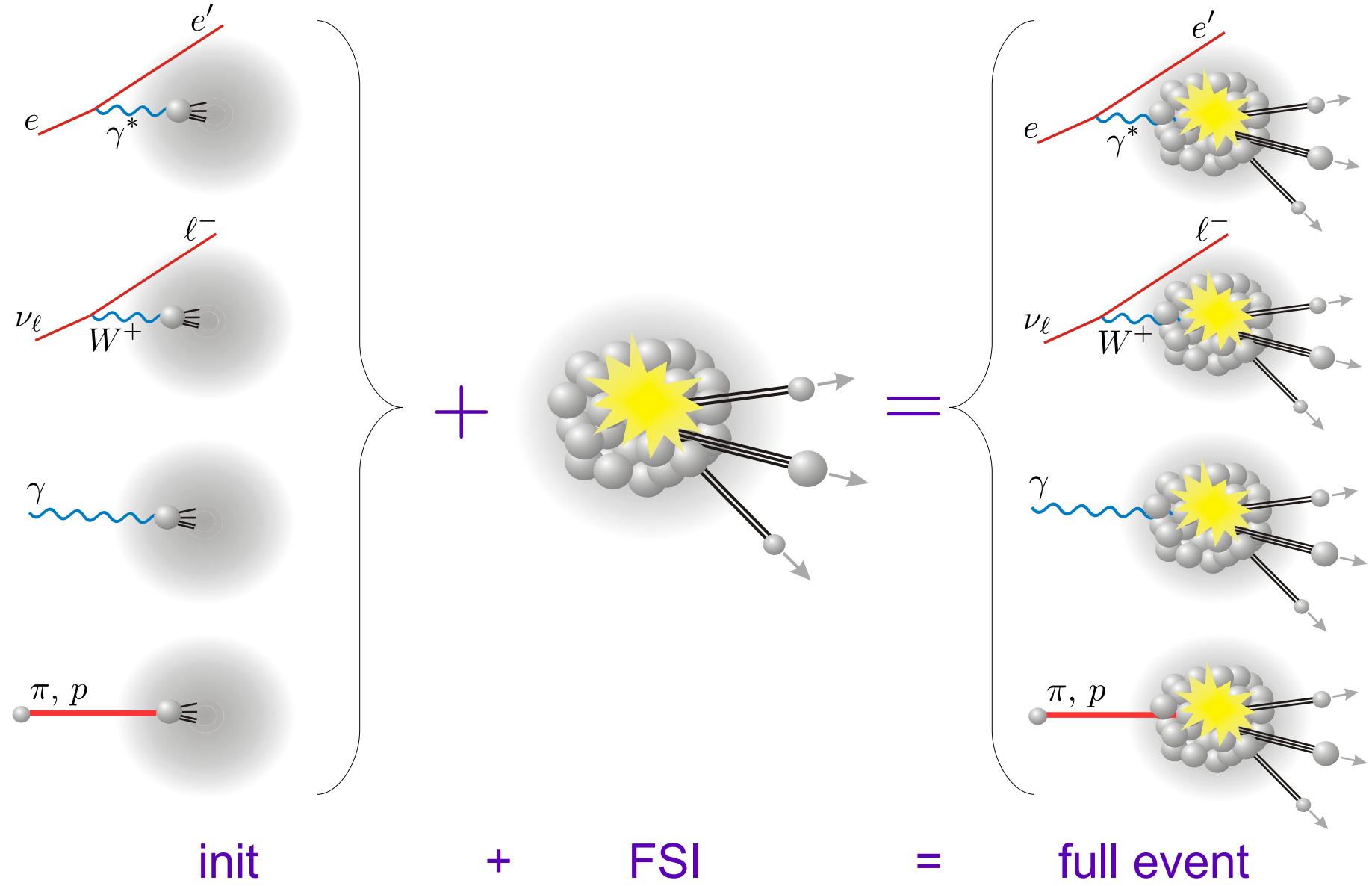


■ propagation of final state

- elastic/inelastic scatterings
- mean fields



GiBUU = plug-in system



Nuclear ground state

- density distribution: Woods-Saxon (or harm. Oscillator)
- particle momenta: ‘Local Thomas-Fermi approximation’

$$f_{(n,p)}(\vec{r}, \vec{p}) = \Theta [p_{F(n,p)}(\vec{r}) - |\vec{p}|]$$

- Fermi-momentum:

$$p_{F(n,p)}(\vec{r}) = (3\pi^2 \rho_{(n,p)}(\vec{r}))^{1/3}$$

- Fermi-energy:

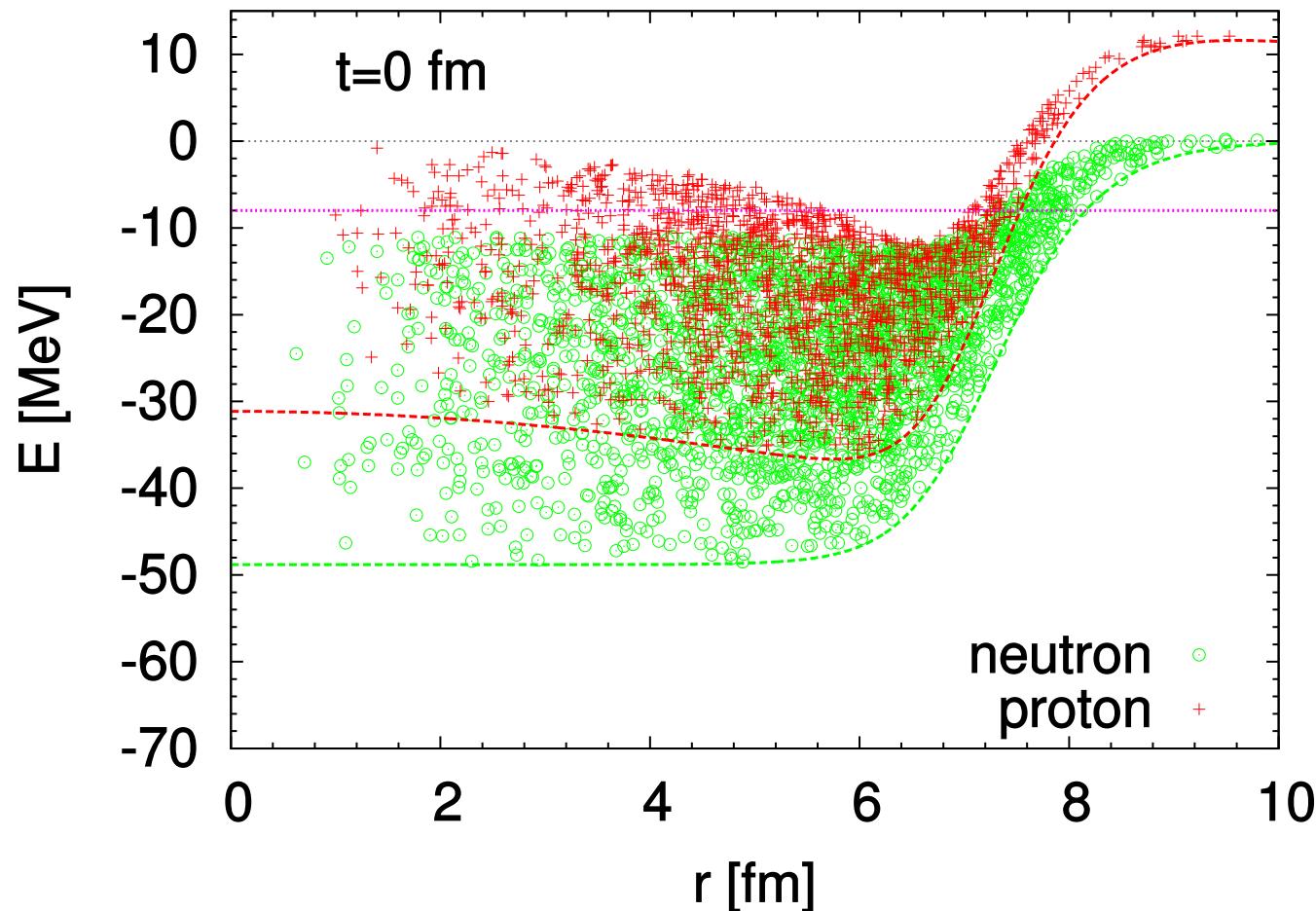
$$E_{F(n,p)} = \sqrt{p_{F(n,p)}^2 + m_N^2} + U_{(n,p)}(\vec{r}, p_F)$$

potential: see above

Nuclear ground state

LTF: time evolution en detail

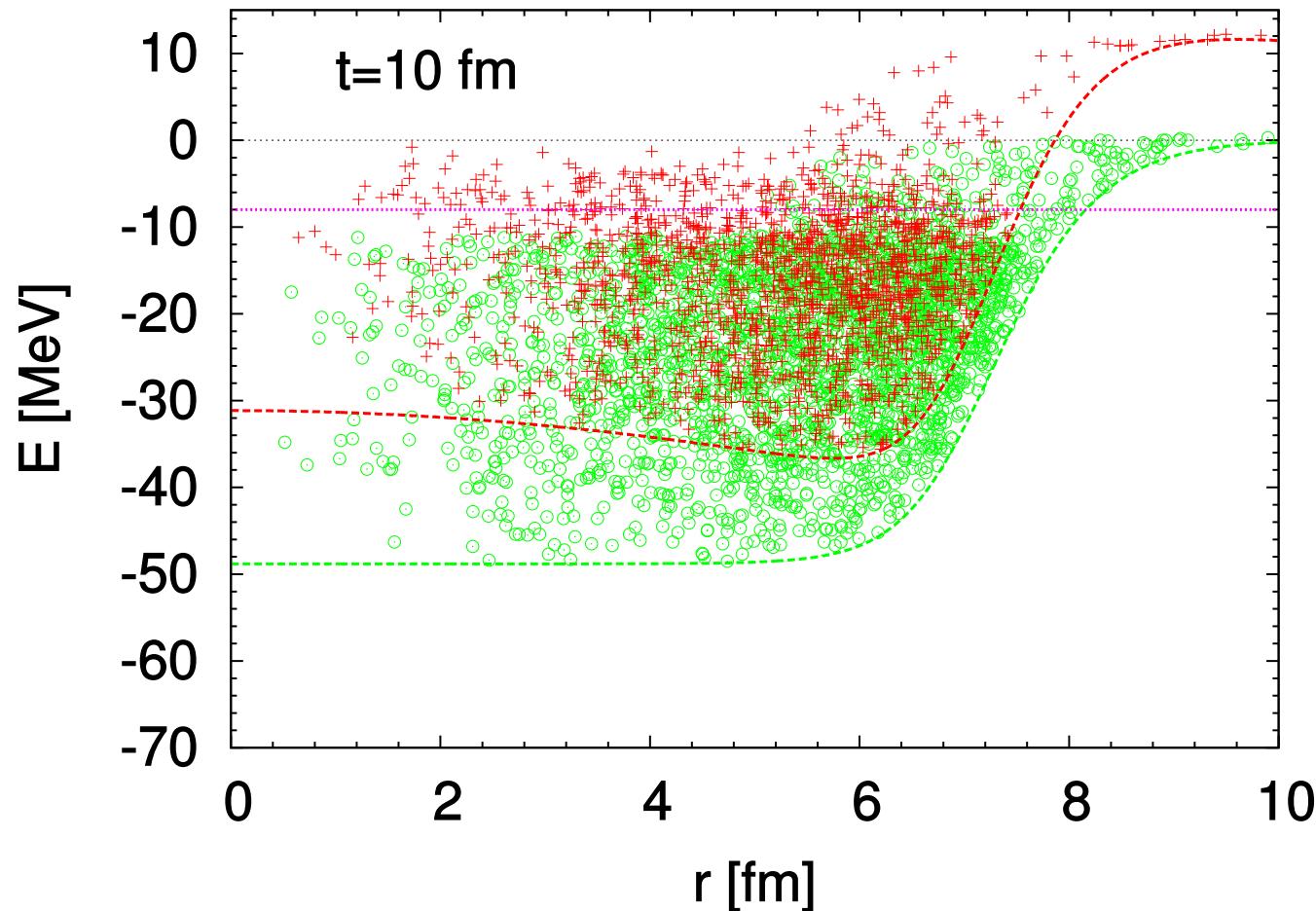
non-mom.dep potential, asymmetry-term, Coulomb



Nuclear ground state

LTF: time evolution en detail

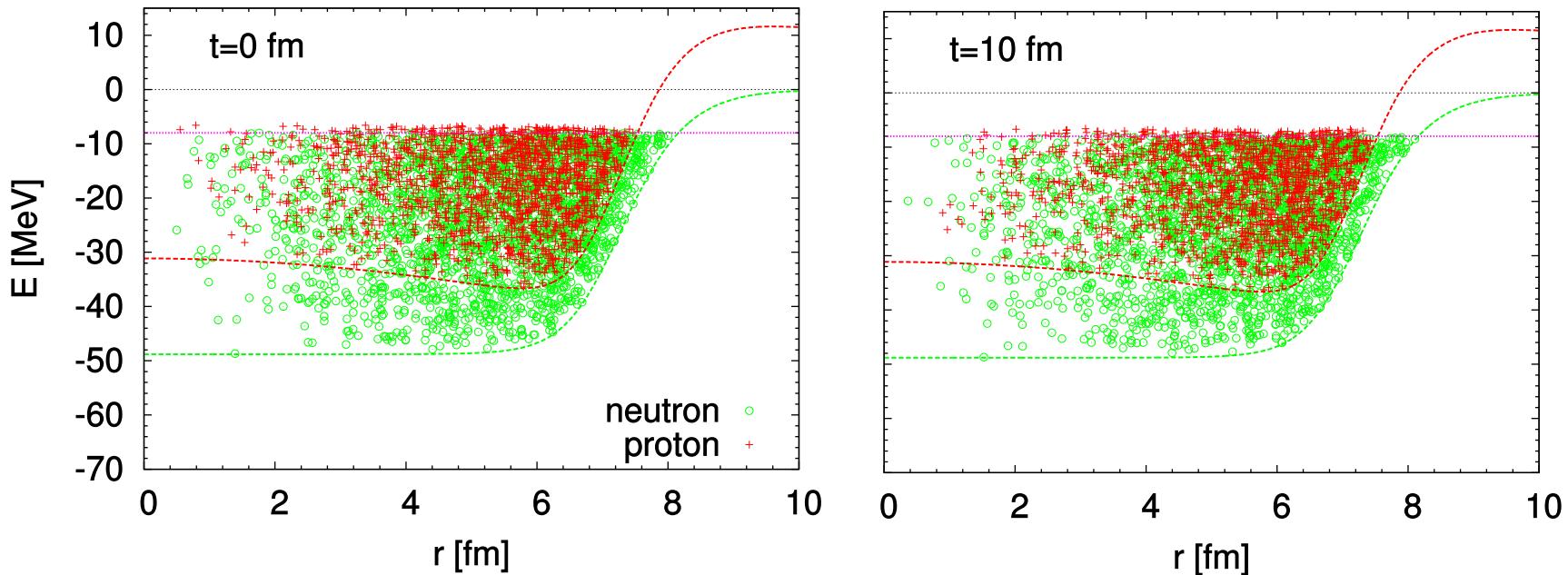
non-mom.dep potential, asymmetry-term, Coulomb



Nuclear ground state

- improvement: ensure constant Fermi-Energy

non-mom.dep potential, asymmetry-term, Coulomb



- needs iteration for mom.dep potential
- important for QE-peak

(Gallmeister, Mosel, Weil, PRC94 (2016) 035502)

■ in principle:

- 1) initialize nucleons
- 2) perform **one** initial elementary event on **one** nucleon
- 3) propagate nucleons and final state particles

■ correct, but ‘waste of time’

■ idea:

final state particles do not really disturb the nucleus

■ 2 particle classes:

- ‘real particles’
- ‘perturbative particles’

Particle classes

■ ‘real particles’

- nucleons
- may interact among each other
- interaction products are again ‘real particles’

■ ‘perturbative particles’

- final state particles of initial event
- may only interact with ‘real particles’
- interaction products are again ‘perturbative particles’

■ ‘real particles’ behave as if other particles are not there

■ total energy, total baryon number, etc. not conserved!

Init with perturbative particles

■ init

- 1) initialize nucleons
- 2) perform **one** initial elementary event on **every** nucleon
- 3) propagate nucleons and final state particles

- final states particles are ‘perturbative particles’
- different final states do not interfere

■ every final state particle gets a ‘perturbative weight’:

- value: cross section of initial event
- is inherited in every FSI
- *for final spectra the ‘perturbative weights’ have to be added, not only the particle numbers*

Init with perturbative particles

simple workaround against oscillating ground states:
("frozen approximation")

■ *idea:*

freeze nucleon testparticles (they are not propagated)

■ since nucleons are real particles, their interactions among each other should not influence final state particles

■ Pros:

- computational time

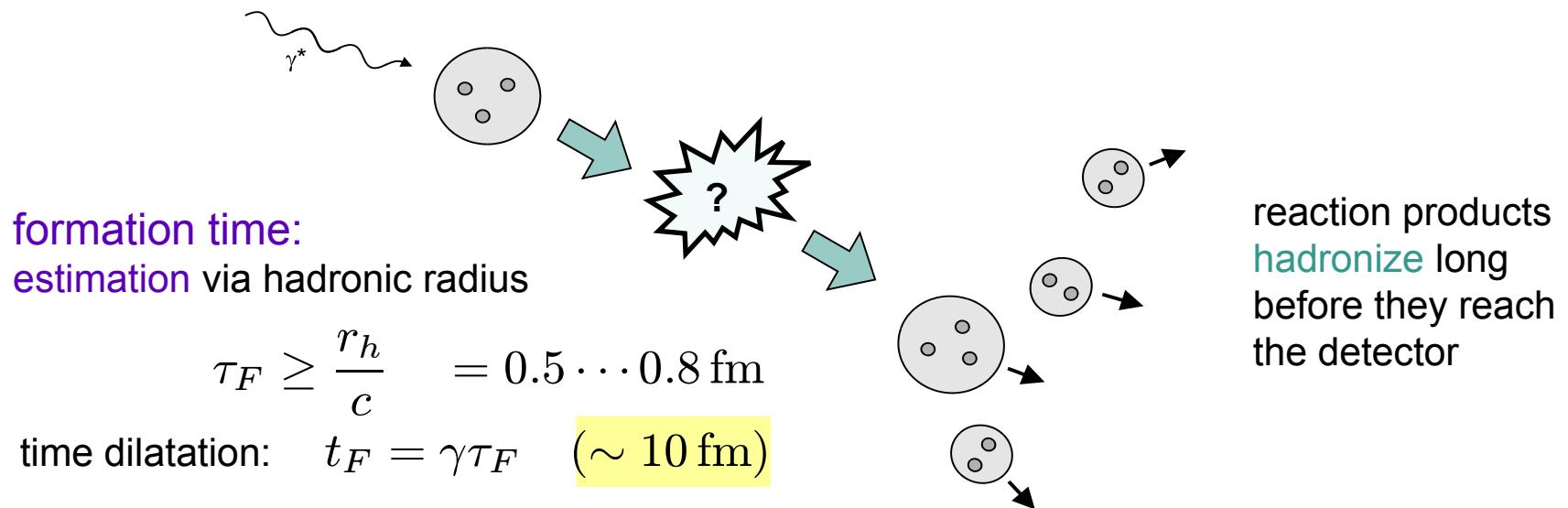
■ Cons:

- ???

Applications

Hadronization in eA

elementary reactions ($eN, \gamma N$) on nucleon:



nuclear reactions ($eA, \gamma A$ @ GeV energies):

interactions with nuclear medium during formation



space-time picture of hadronization

development of wave function

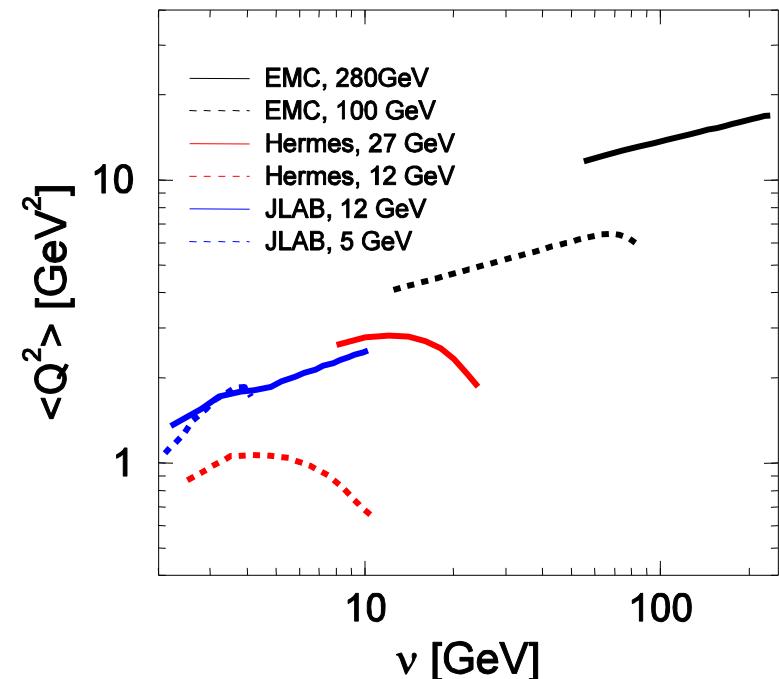
$$\sigma^*/\sigma_H \sim t^{0,1,2,\dots}$$

Observables, Experiments

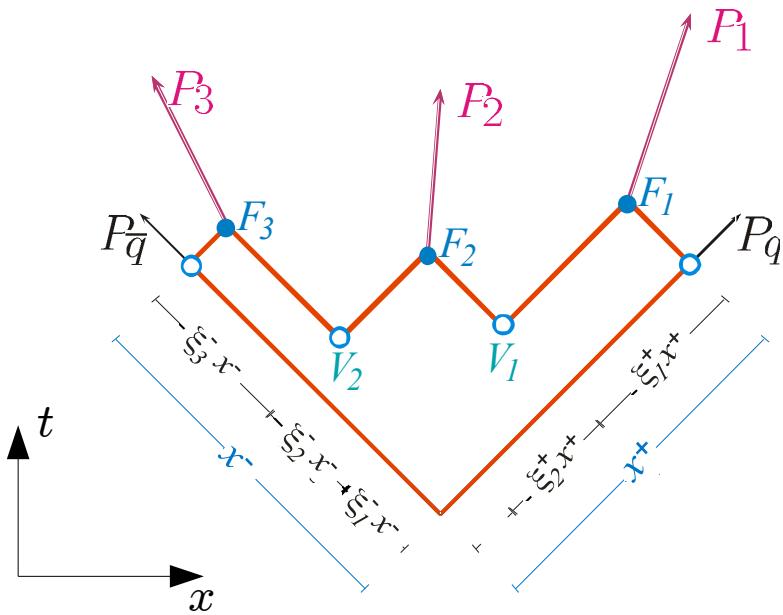
- $R^h(z_h, \dots) = \frac{\frac{N_h(z_h, \dots)}{N_e(\dots)}}{\frac{N_h(z_h, \dots)}{N_e(\dots)}} \Big|_A - \Big|_D$
- hadronic: $z_h = \frac{E_h}{\nu}$, p_T , \dots
- photonic: ν , Q^2 , W , x_B , \dots
- $\Delta p_T^2 = \langle p_T^2 \rangle_A - \langle p_T^2 \rangle_D$

■ experiments:

- $E_{\text{lepton}} =$
- EMC **100...280 GeV**
- Hermes **27 GeV**
 12 GeV
- CLAS **12 GeV**
 5 GeV
- EIC e.g. 3+30 GeV
...multiple combinations of targets



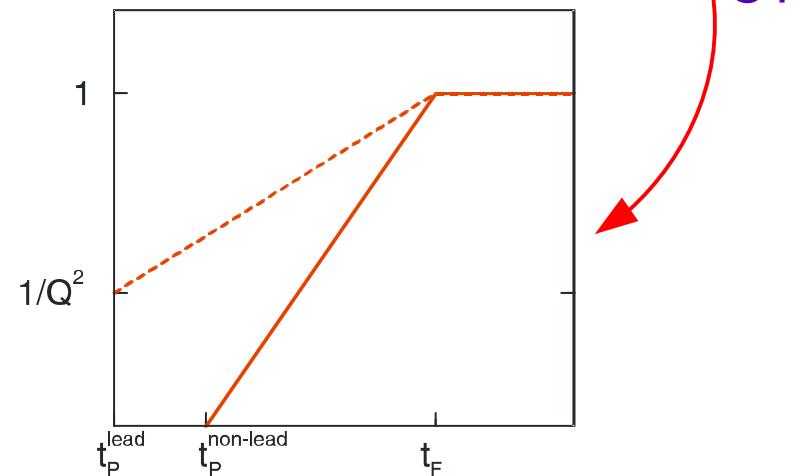
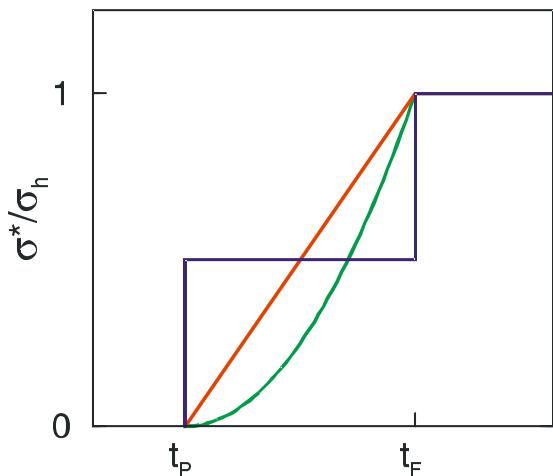
Model: Hadronization in String Model (PYTHIA/JETSET)



- 3 times/points per particle:
 - „production 1“ *string-breaking*
 - „production 2“ *string-breaking*
 - „formation“ *line-meeting*

- leading vs. non-leading
 - connection to interaction vertex

- cross section evolution scenarios: *pre-hadrons*

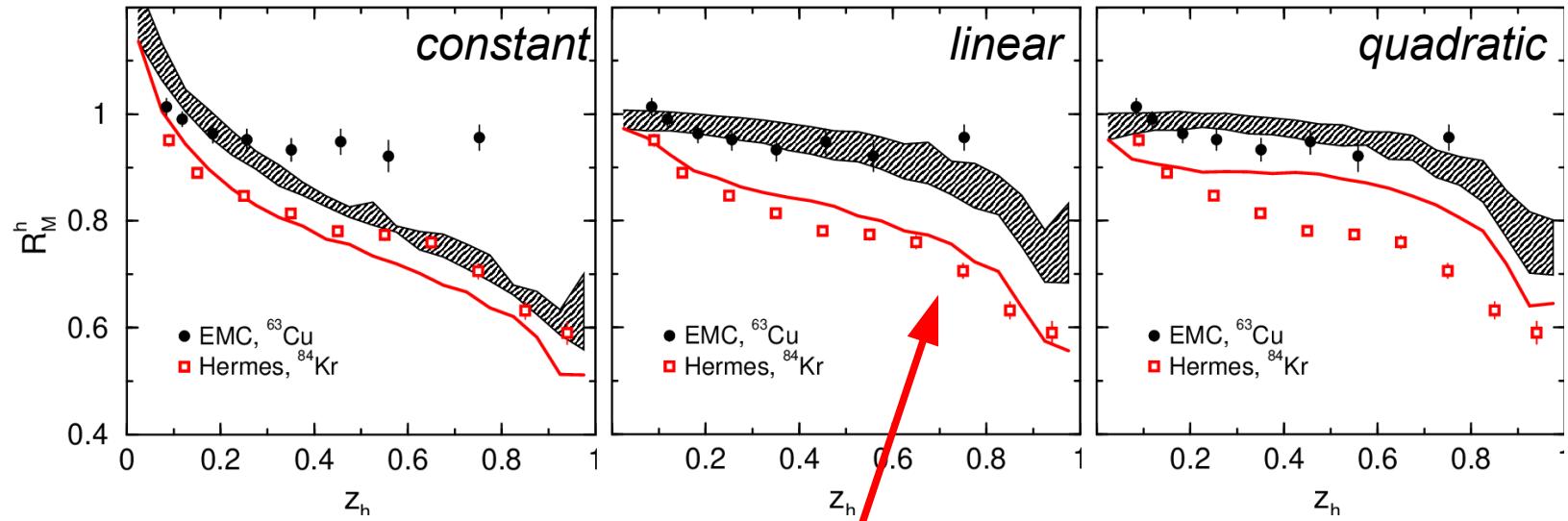


EMC & Hermes

describe simultaneously:

- EMC@100...280 GeV

- Hermes@27 GeV



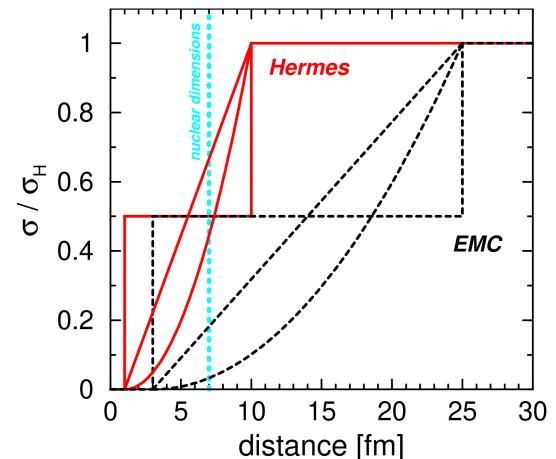
pre-hadronic cross section:

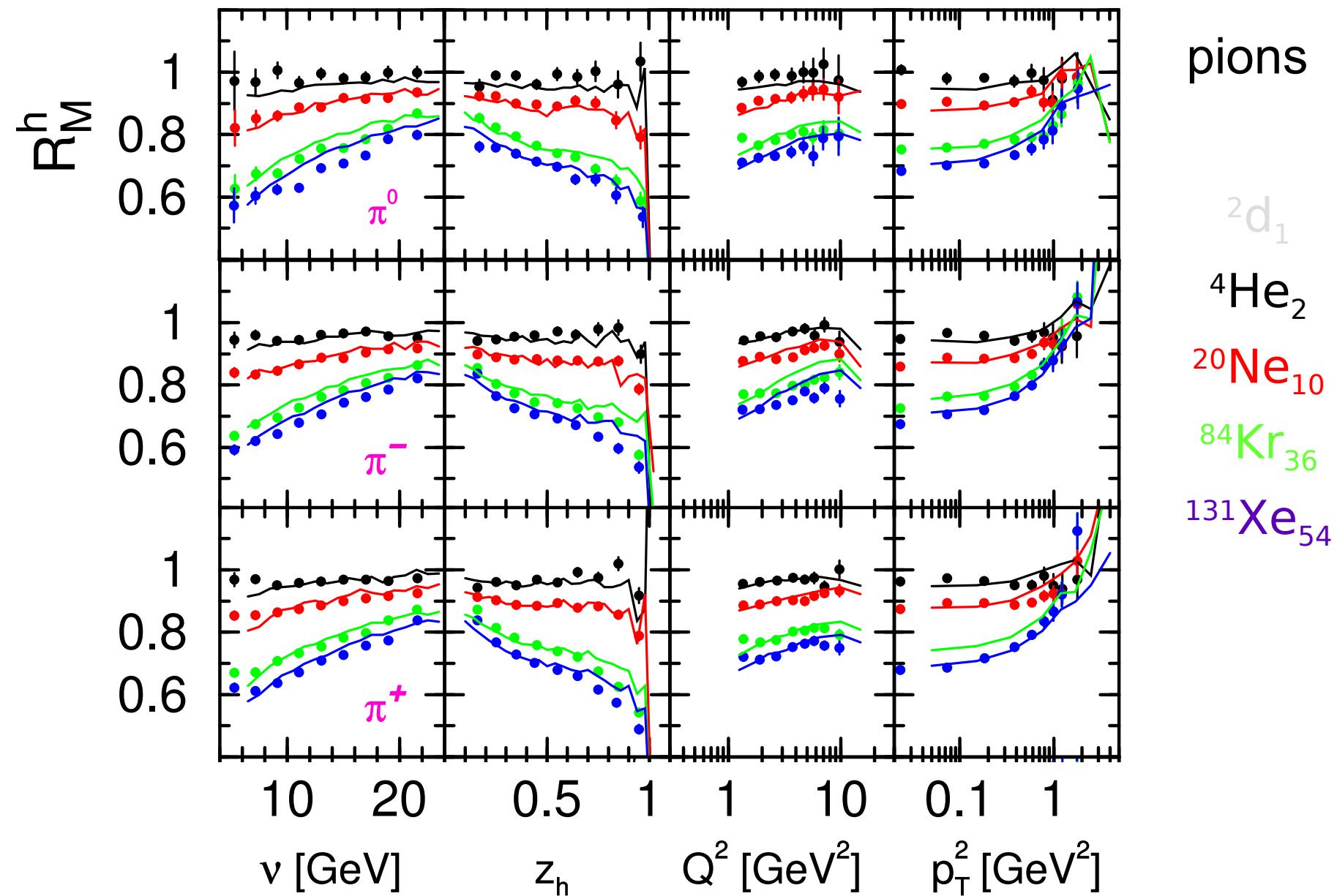
linear increase with time

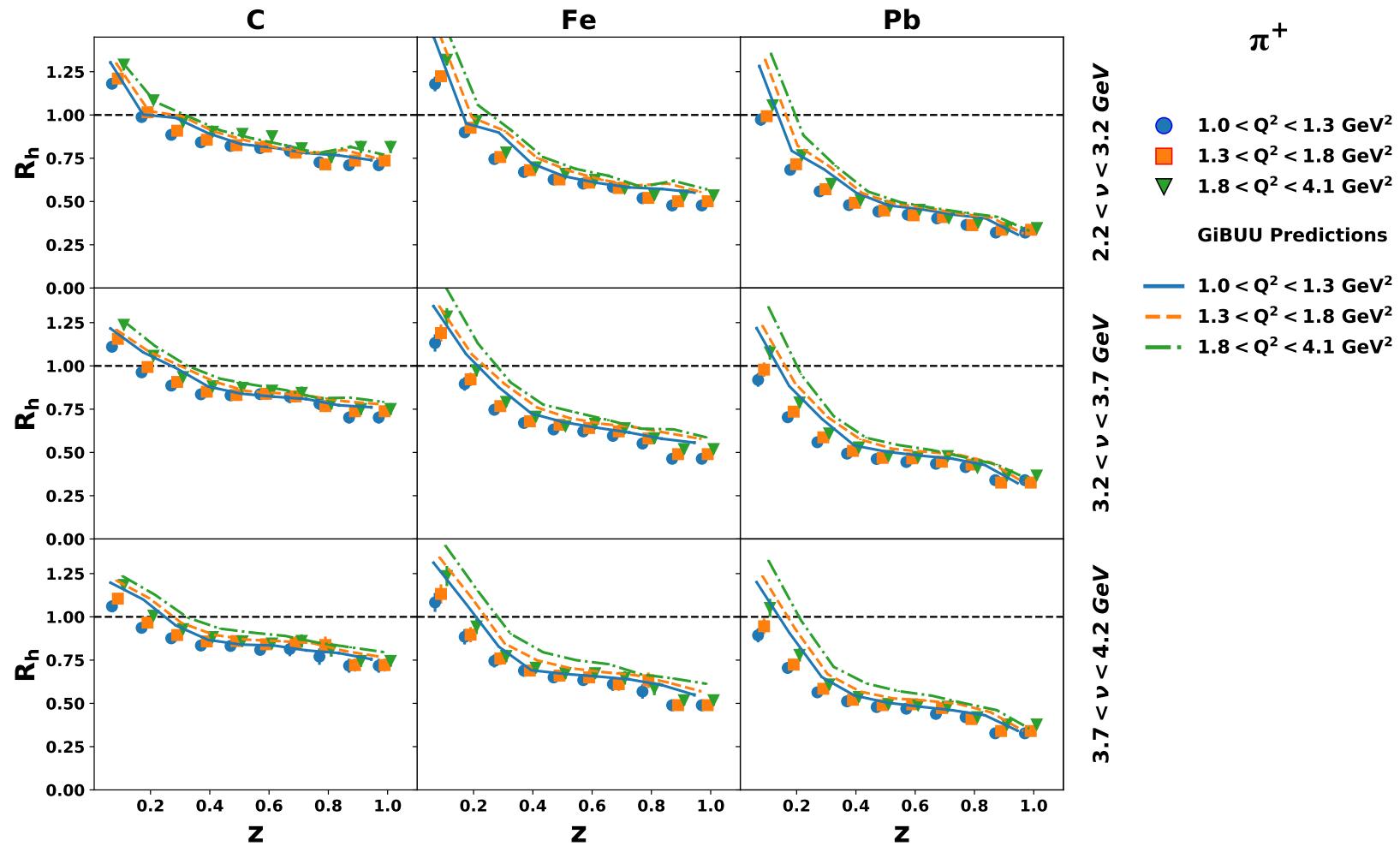
$$\frac{\sigma^*}{\sigma_H} = \frac{r_{\text{lead}}}{Q^2} + \left(1 - \frac{r_{\text{lead}}}{Q^2}\right) \left(\frac{t - t_P}{t_F - t_P}\right)$$

cf. quantum diffusion models,

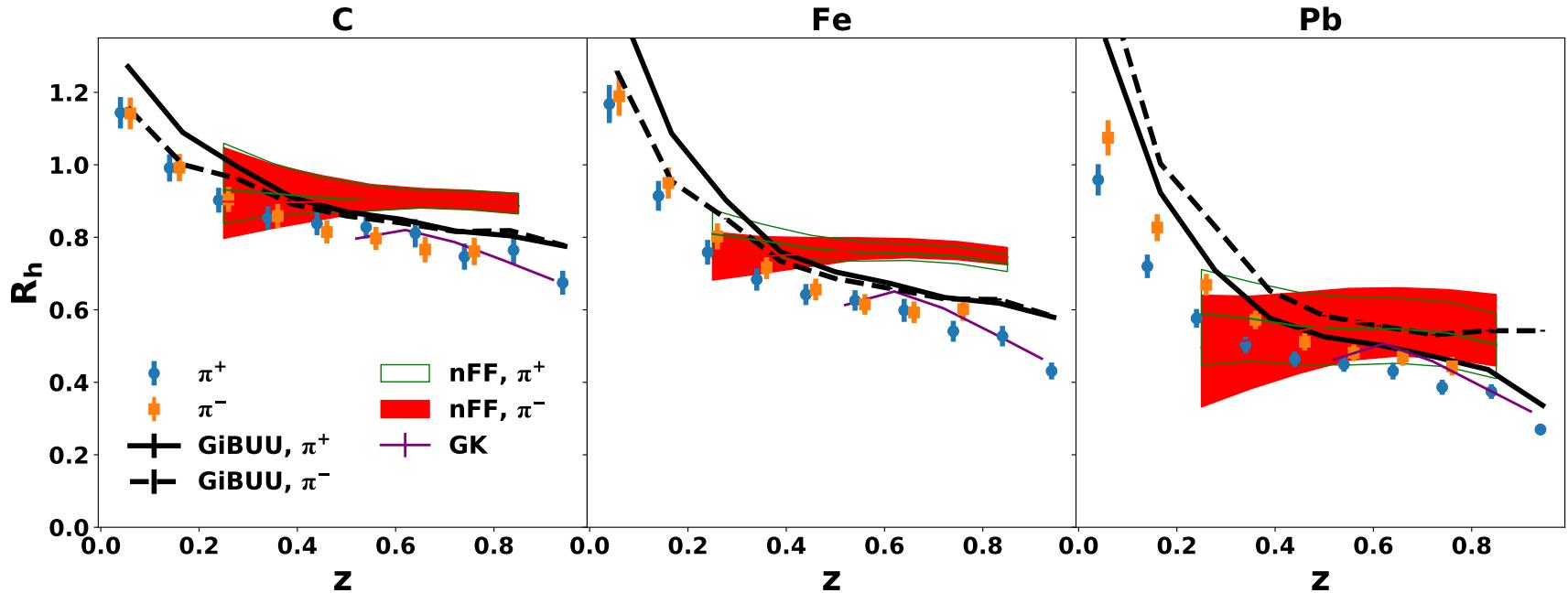
e.g.: Farrar et al., PRL 61(1988) 686
Dokshitzer et al., 1991







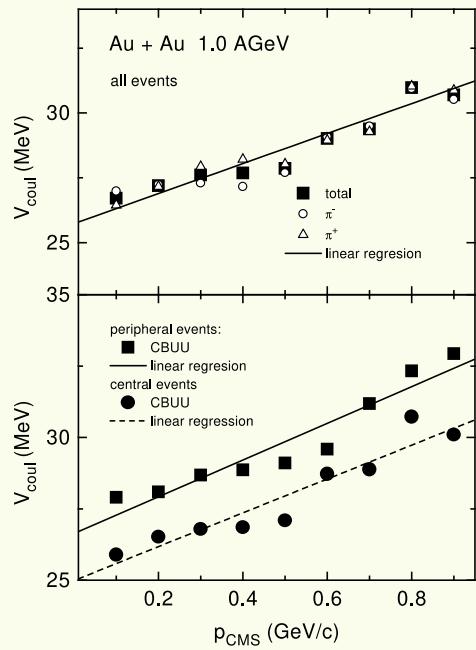
experimental data only published with GiBUU curves!



- GiBUUU: predictions!
- GK: Guiot & Kopeliovich, quark energy loss & pre-hadron absorption
- nFF: nuclear FF, LIKEN21 (fit to Hermes data)

HADES, Au+Au@1.23AGeV: pion Coulomb potential

Teis et al., ZPA 359(1997) 297:



WORK IN PROGRESS

Summary & Conclusions

Kinetic Theory and GiBUU implementation

- BUU equation
- degrees of freedom
- potentials
- collision term
- baryon-meson-, baryon-baryon-collisions
- testparticles, parallel vs. full ensemble
- collision criterion (beyond 2-particle collisions)
- initial state

not in this talk:

- off-shell transport
- photon/dilepton production
(shining method)
- coarse graining / equilibration
- ...

Applications/Results

- Hadronization in cold nuclear matter (color transparency)
- Coulomb effects for pions in heavy ion collisions

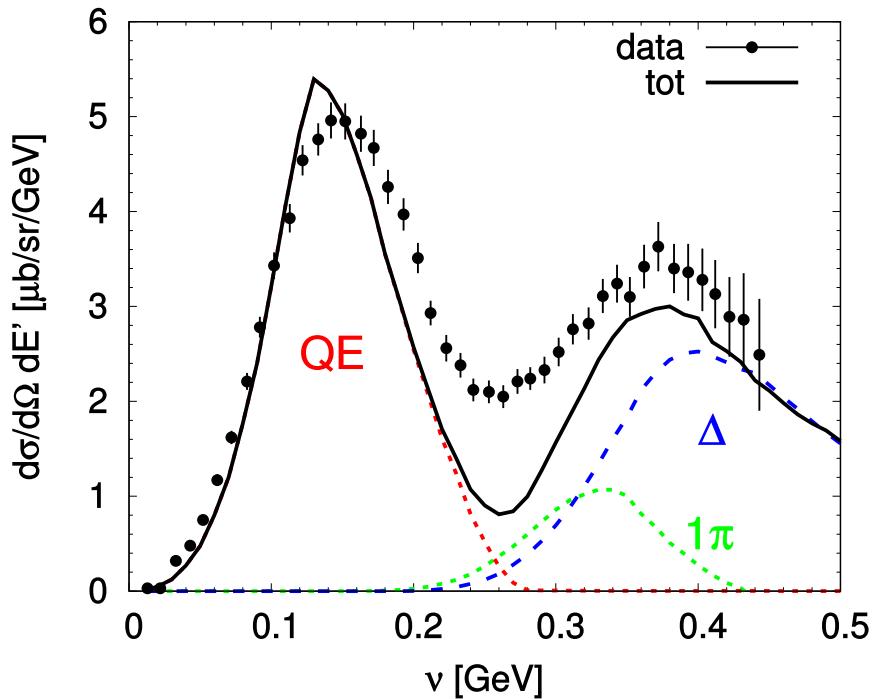
Take home messages

- ...
- GiBUU propagates distributions, not particles
- GiBUU is (partially) **world-leading** (e+A, nu+A, low energy A+A)

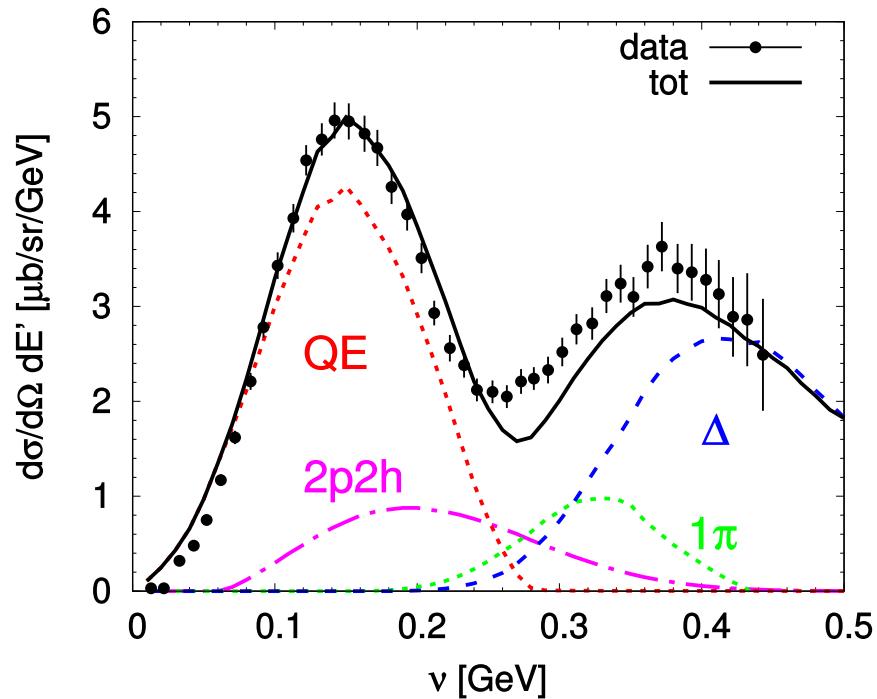
Backup slides

example: eC, $E_e=0.56$ GeV, $\theta=60^\circ$

■ before 2016:



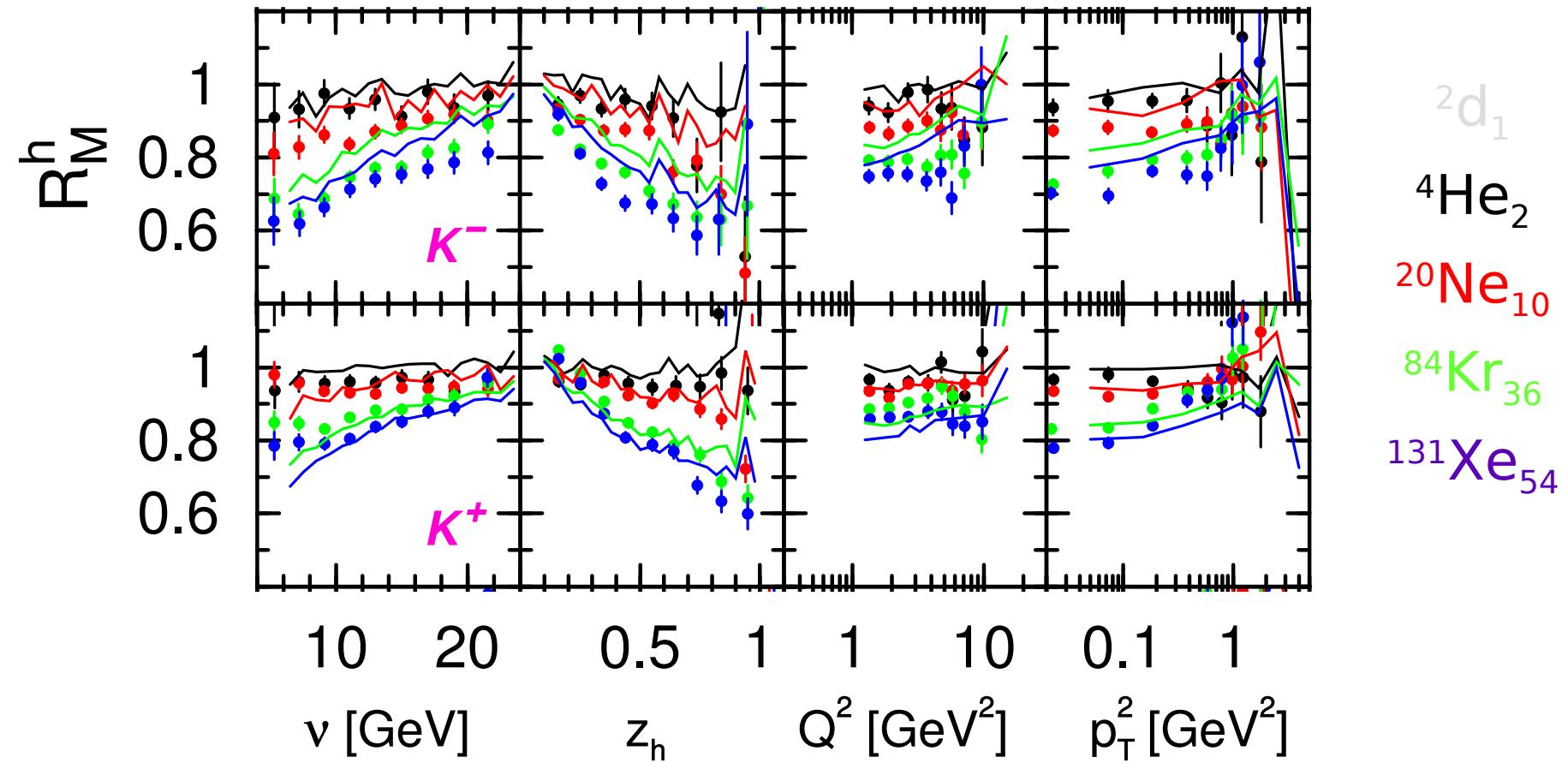
■ since 2016:



- Delta: medium modification à la Oset et al. invalid
- QE: new ground state prescription
- 2p2h: very important contribution

Hermes@27: A.Airapetian et al., NPB780(2007)1

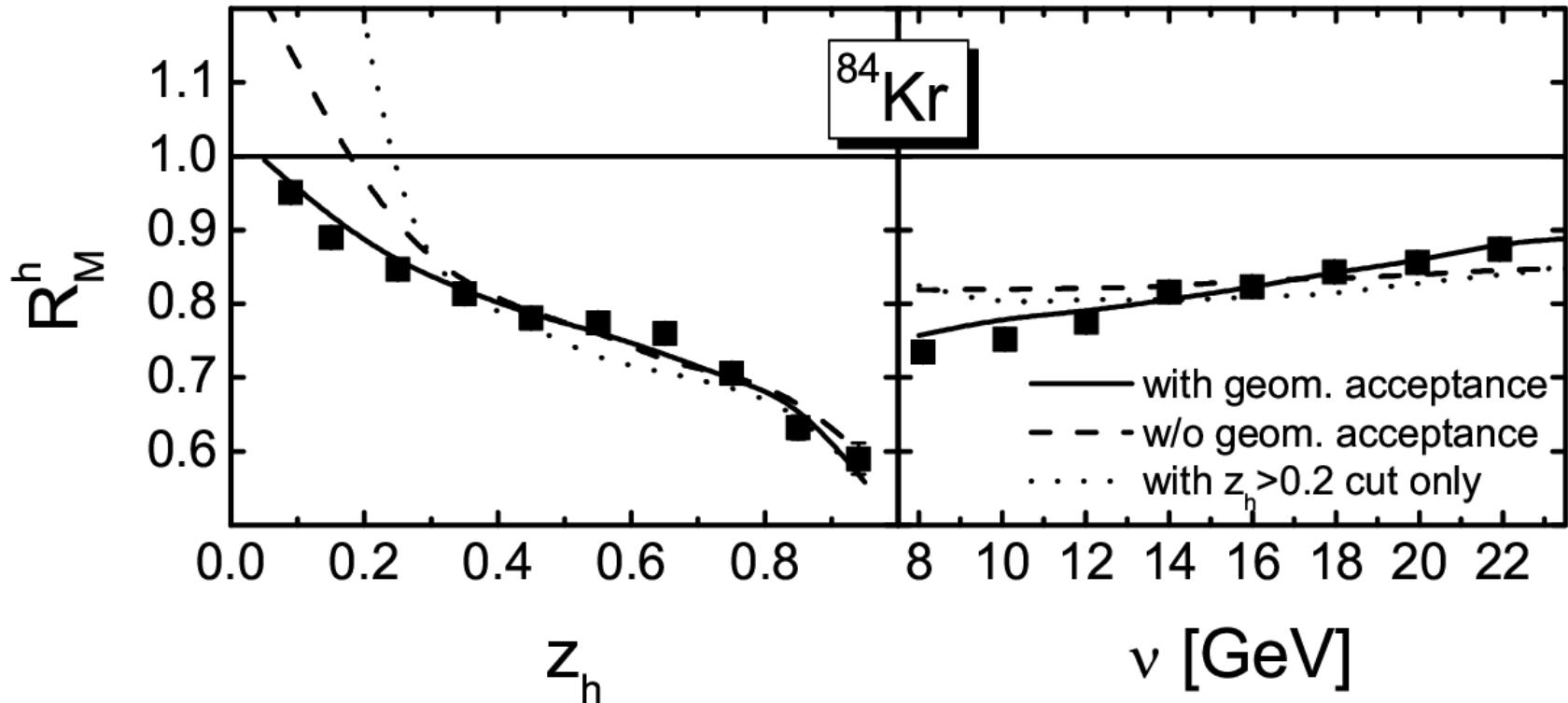
kaons

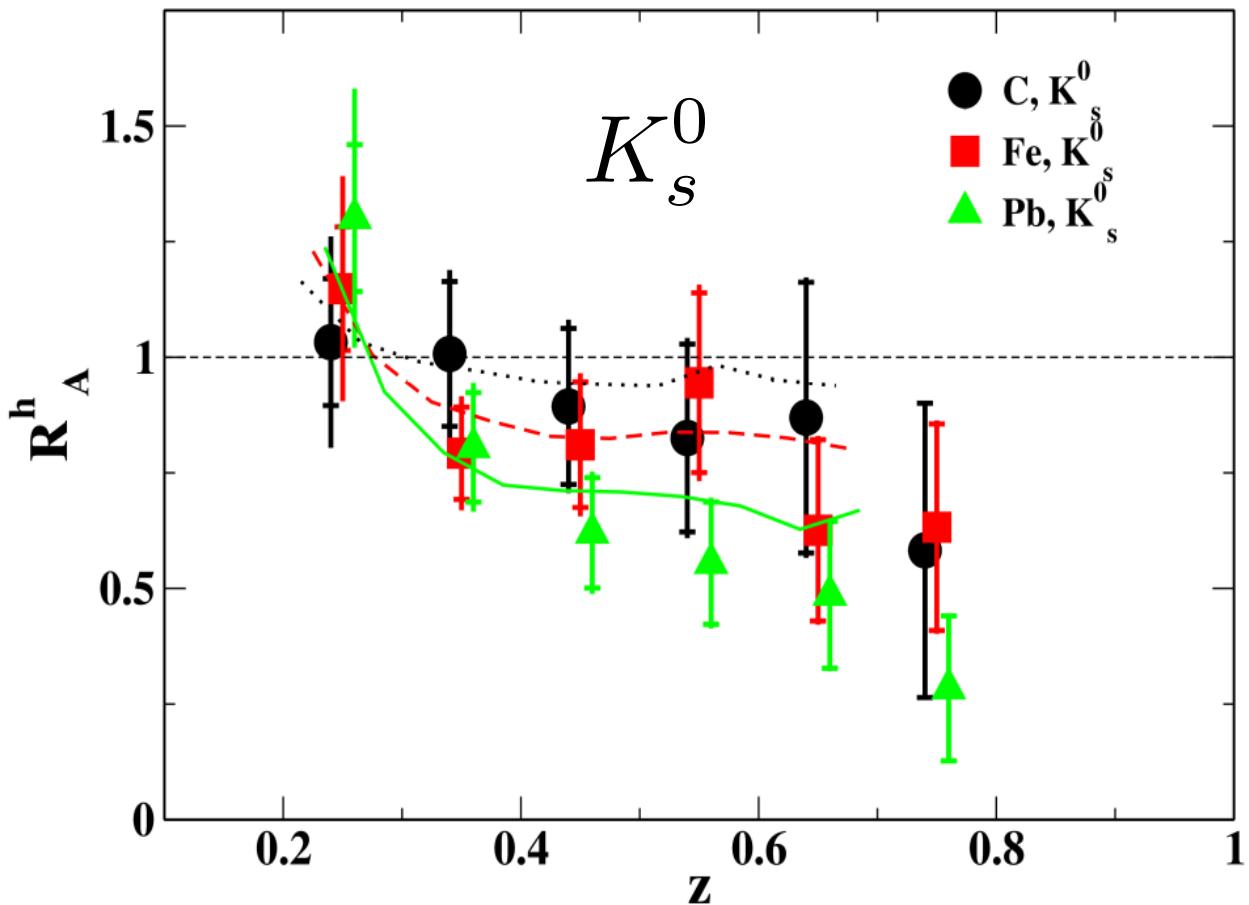


Hermes: Acceptance effects

- Hermes is not a 4pi detector

$$R^h(z_h, \dots) = \frac{\left. \frac{N_h(z_h, \dots)}{N_e(\dots)} \right|_A}{\left. \frac{N_h(z_h, \dots)}{N_e(\dots)} \right|_D}$$





$K^0 : d\bar{s}$

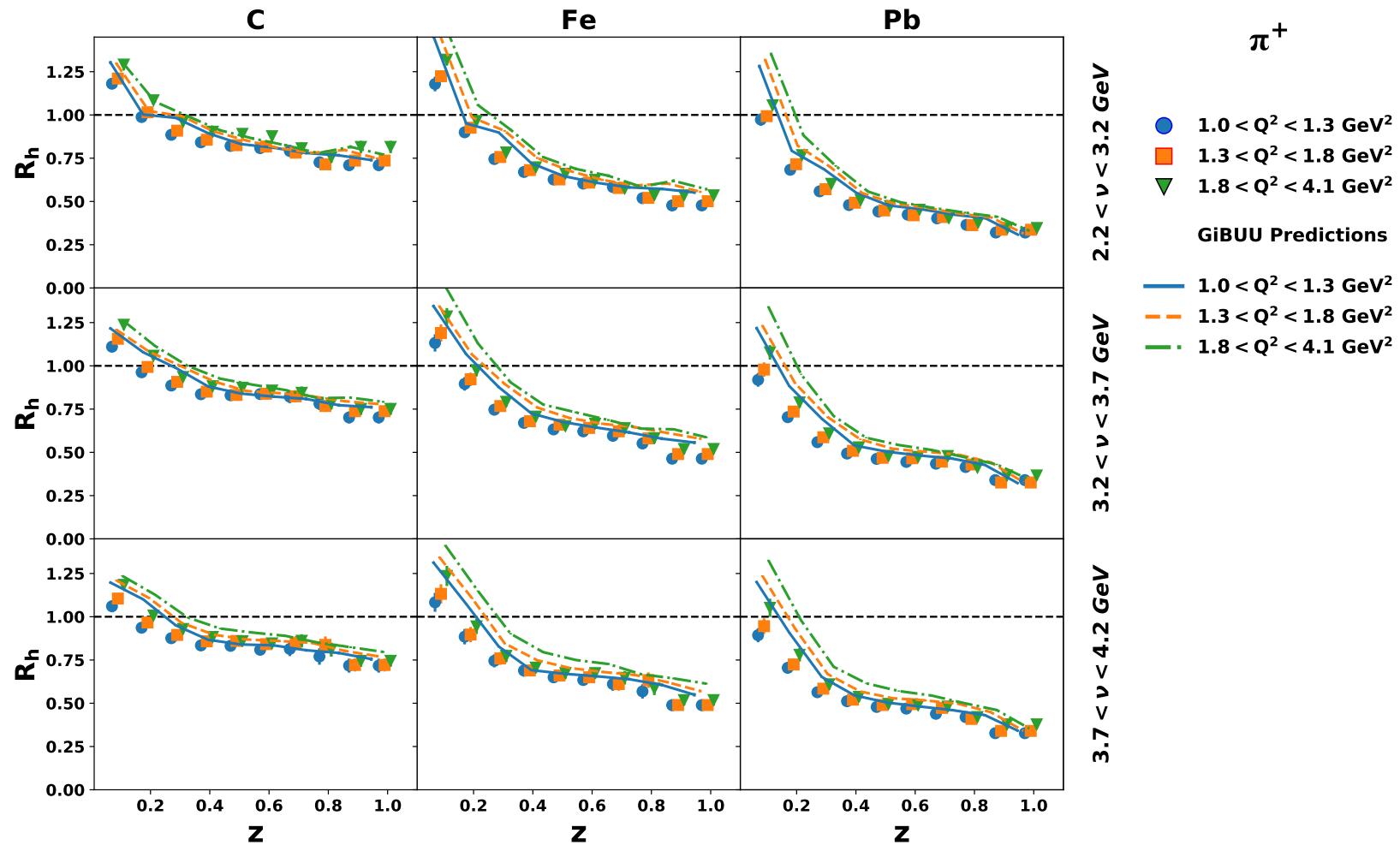
$K^+ : u\bar{s}$

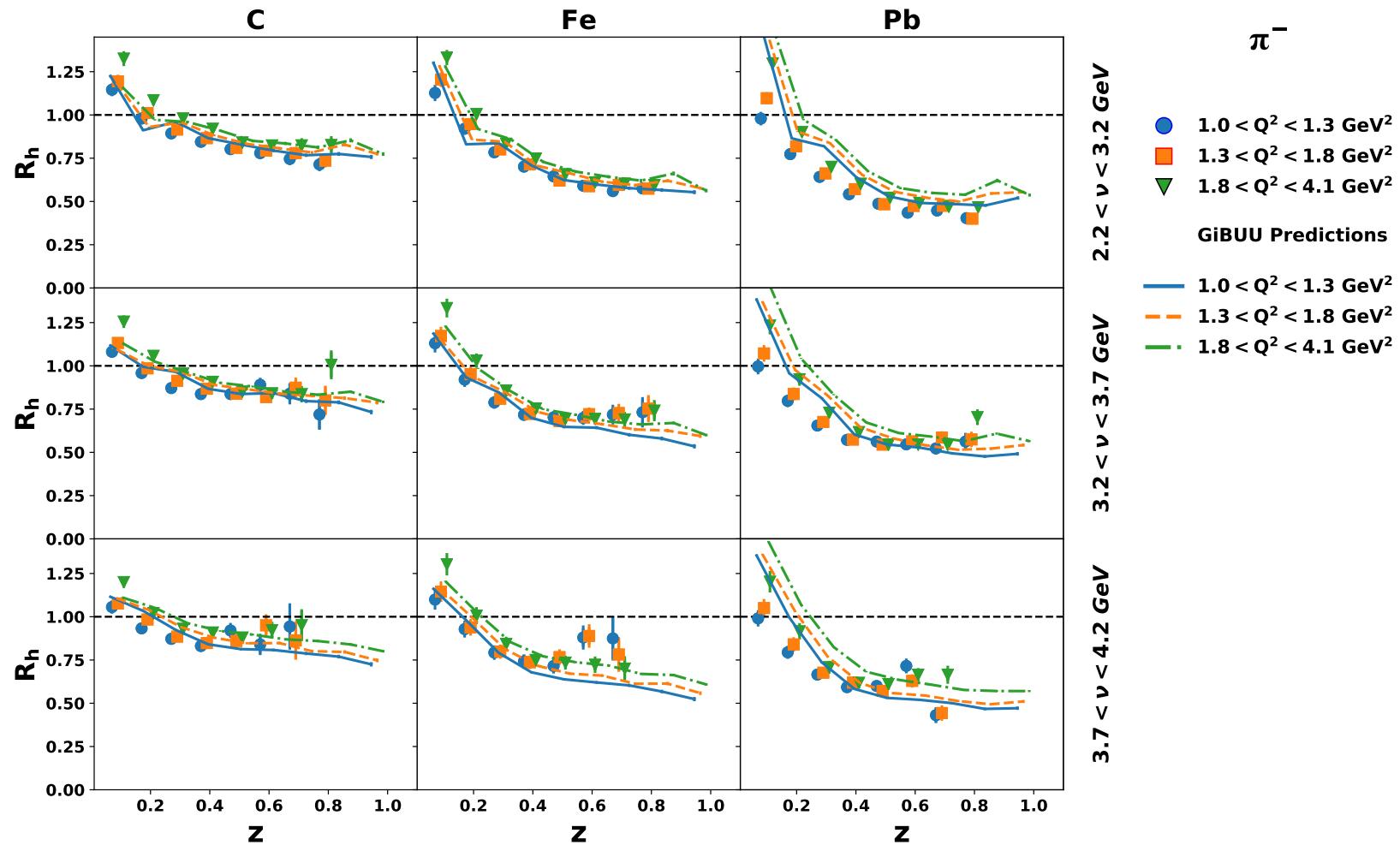
$\bar{K}^0 : \bar{d}s$

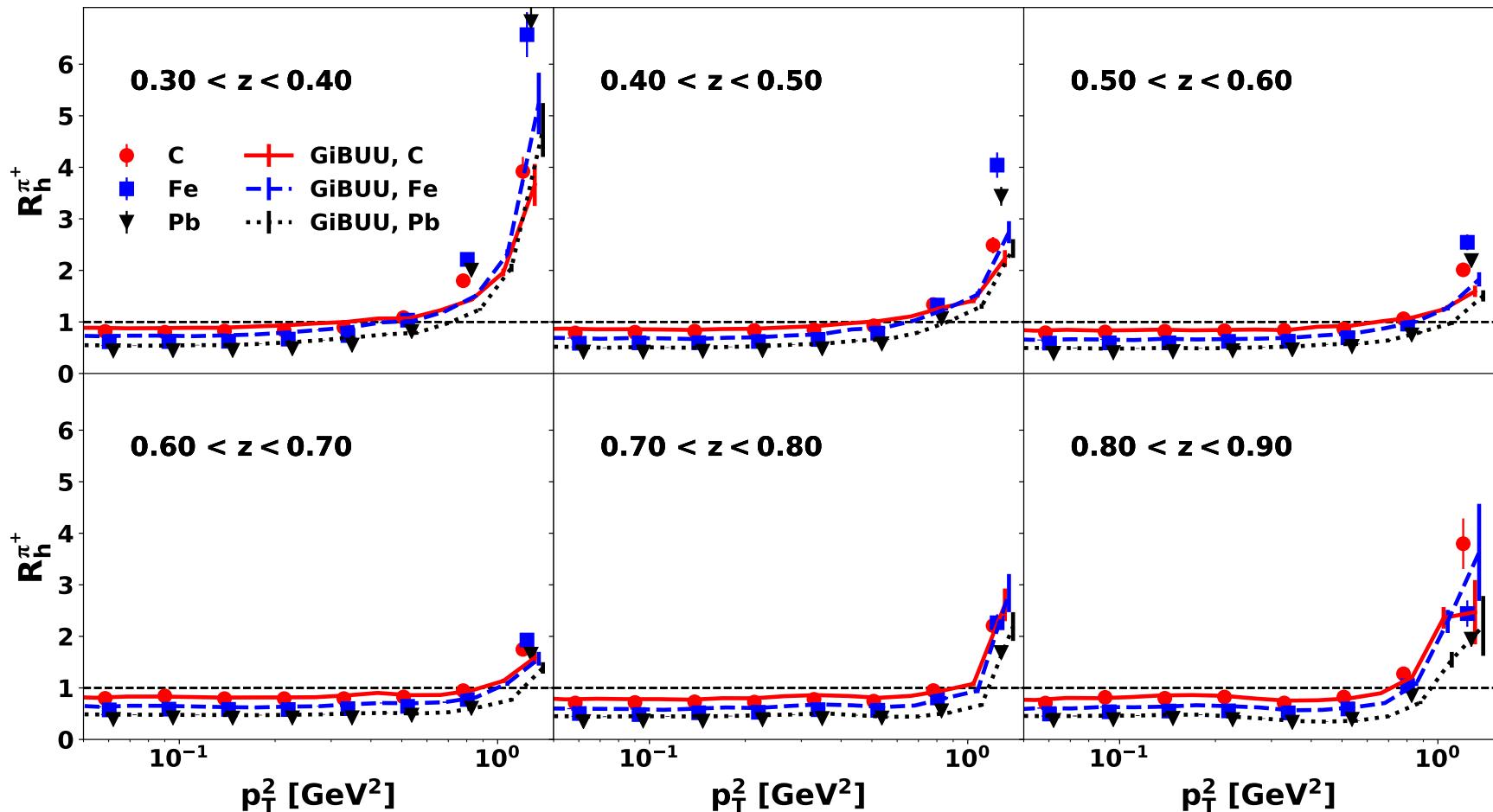
$K^- : \bar{u}s$

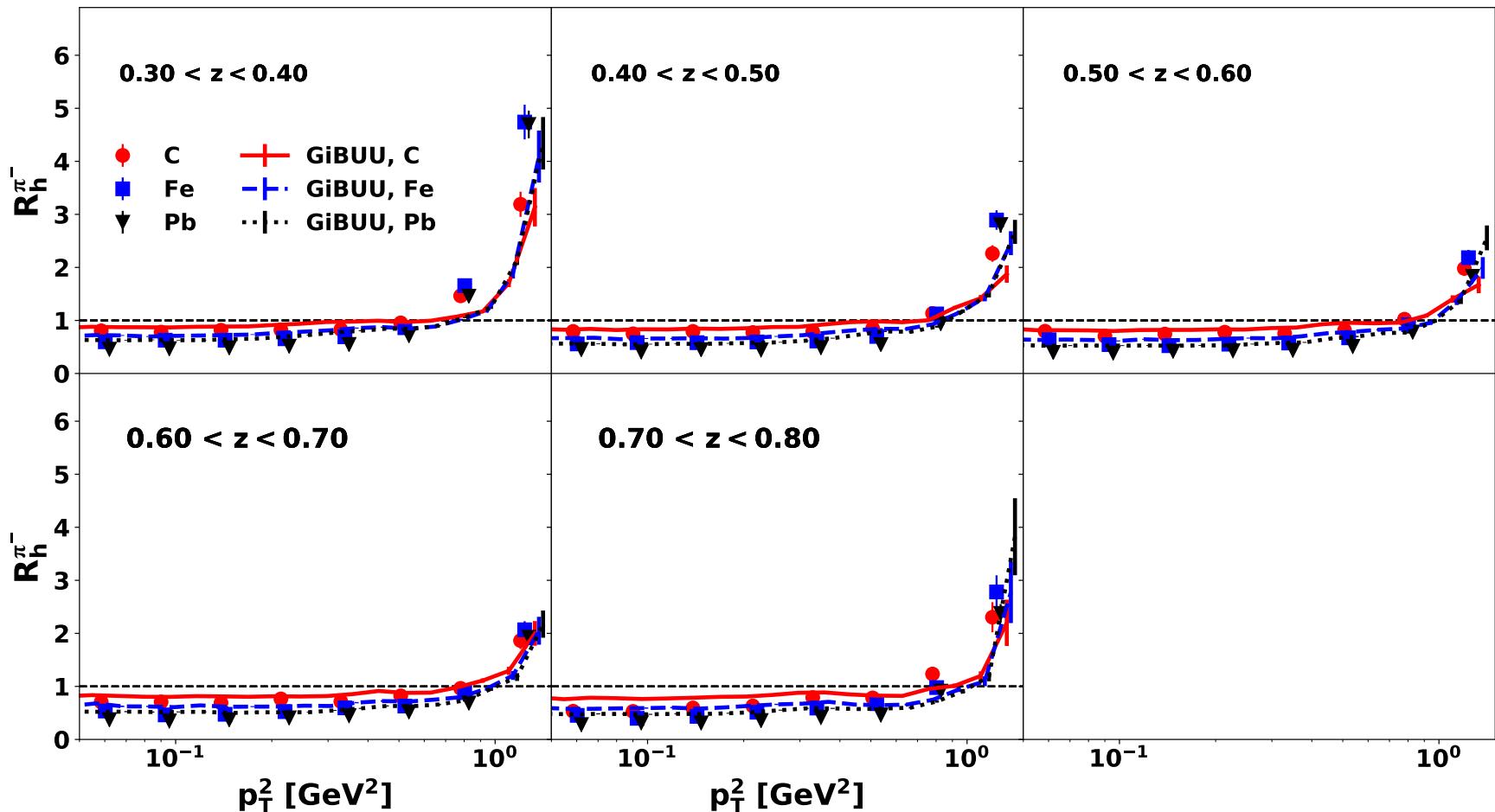
$$K^0 = \begin{cases} 50\% & K_L^0 \\ 50\% & K_S^0 \end{cases}$$

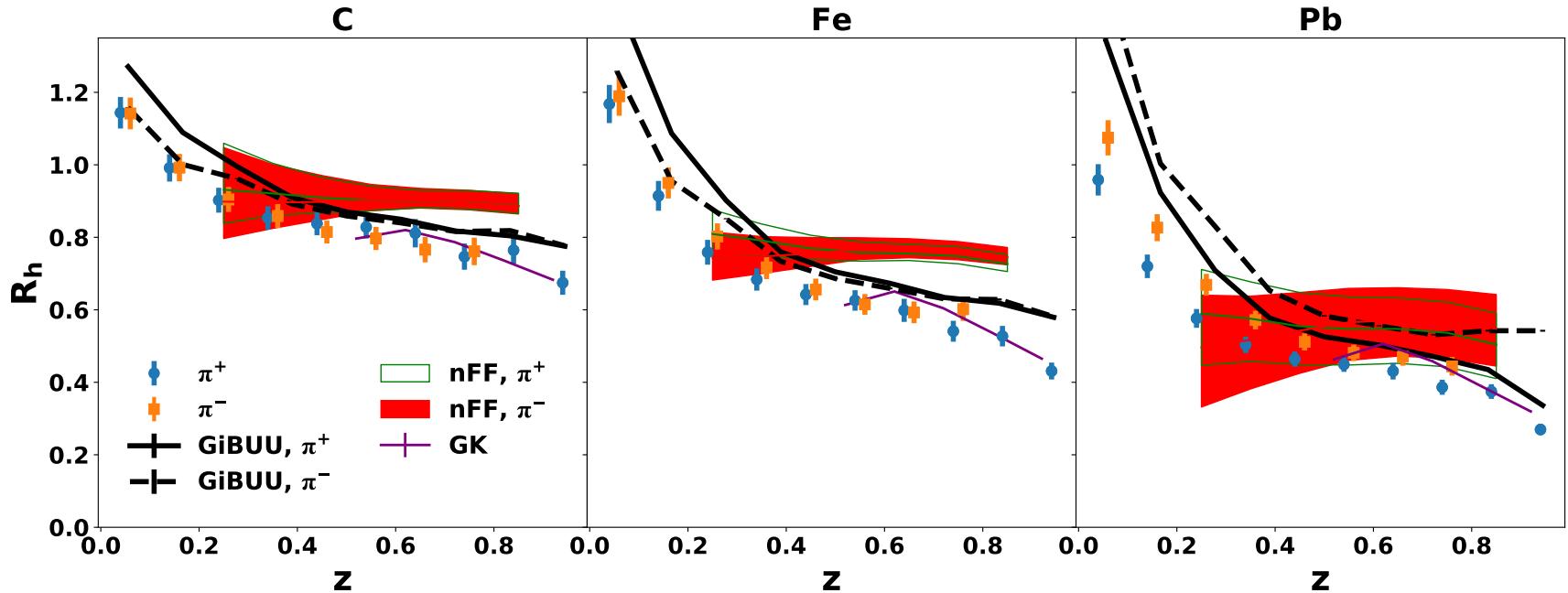
Predictions: K.G., U.Mosel, NPA801(2008)68











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