MOAT REGIMES & THEIR SIGNATURES IN HEAVY-ION COLLISIONS

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[Pisarski, FR, PRL 127 (2021)]

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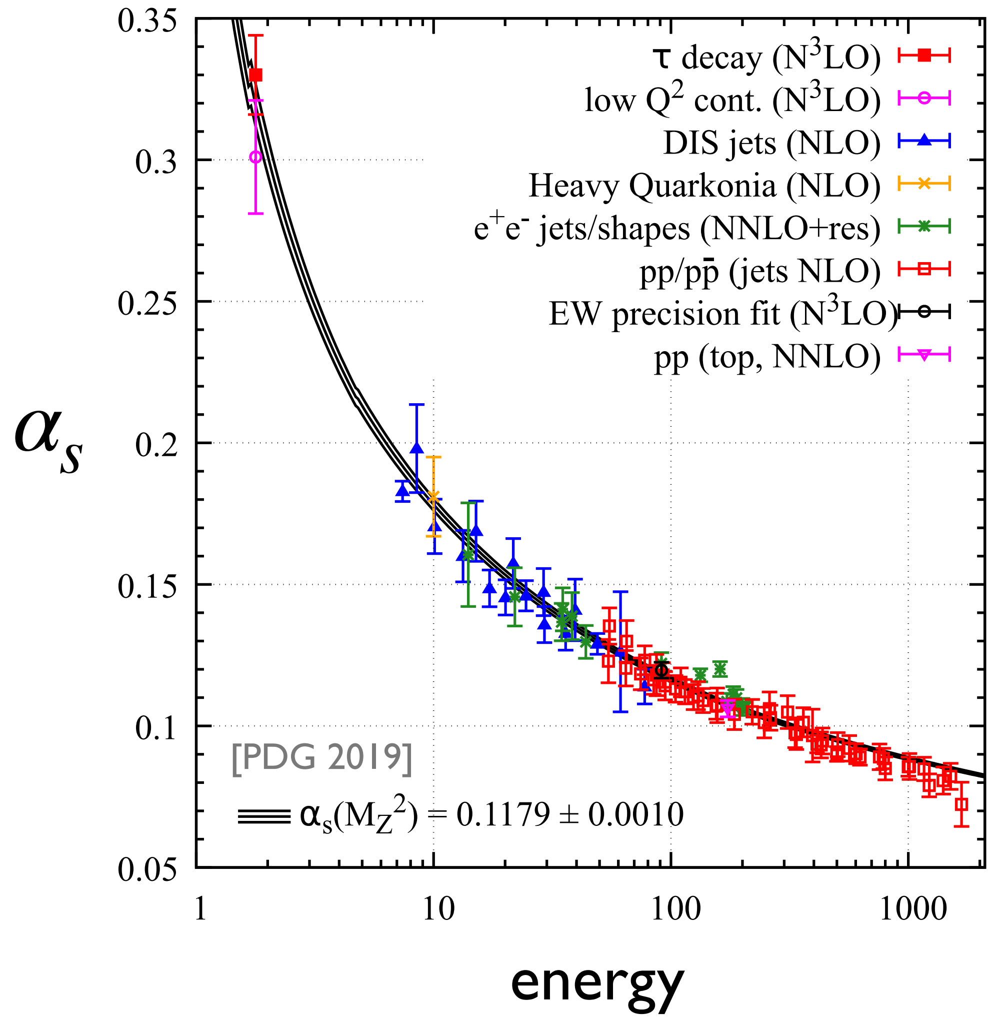
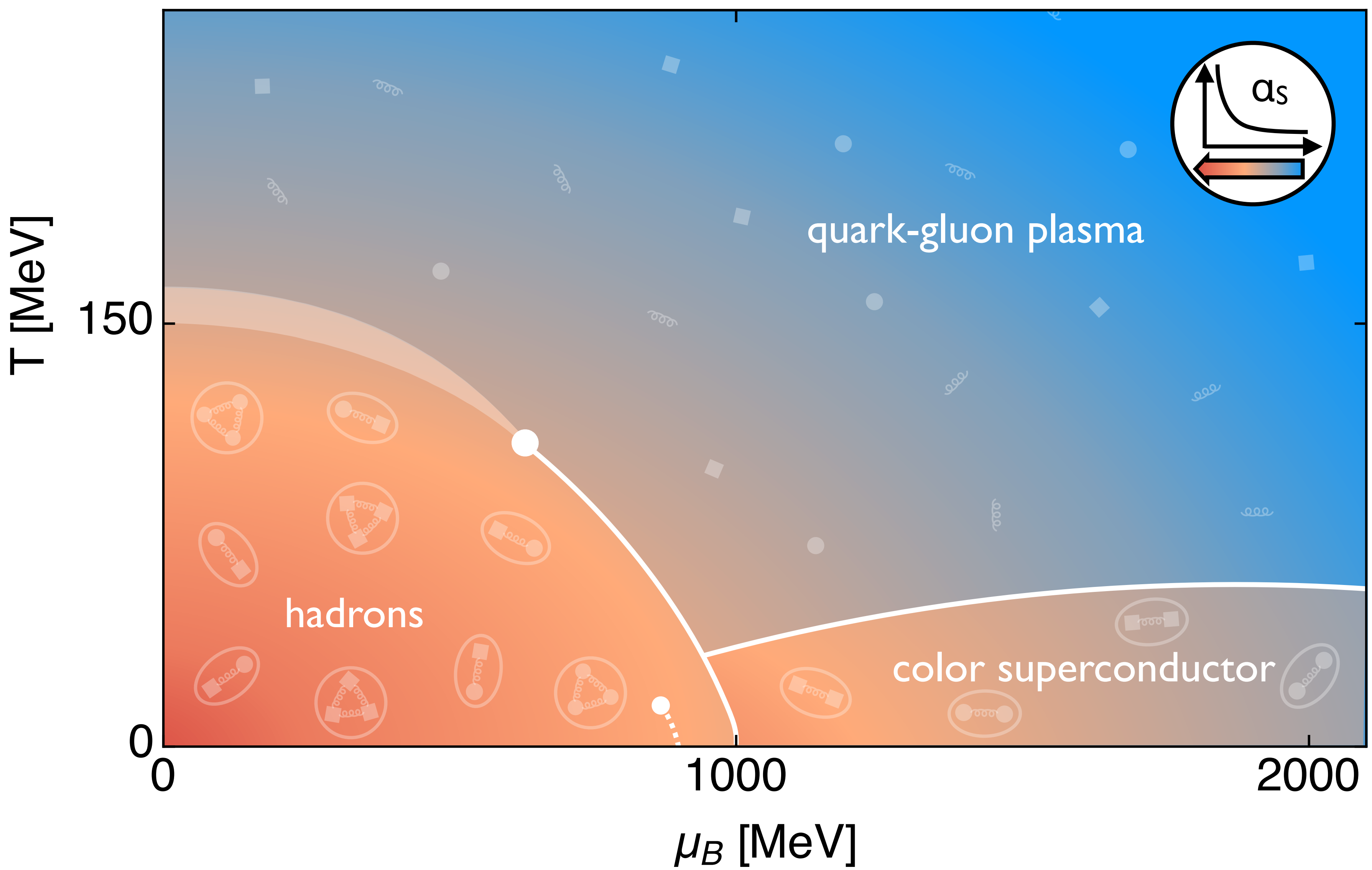
QUANTUM CHROMODYNAMICS

- describes the strong interaction between quarks and gluons
- strong coupling α_s grows with decreasing energy scale



QUANTUM CHROMODYNAMICS

"external" parameters control scale: **phase diagram**



QUANTUM CHROMODYNAMICS

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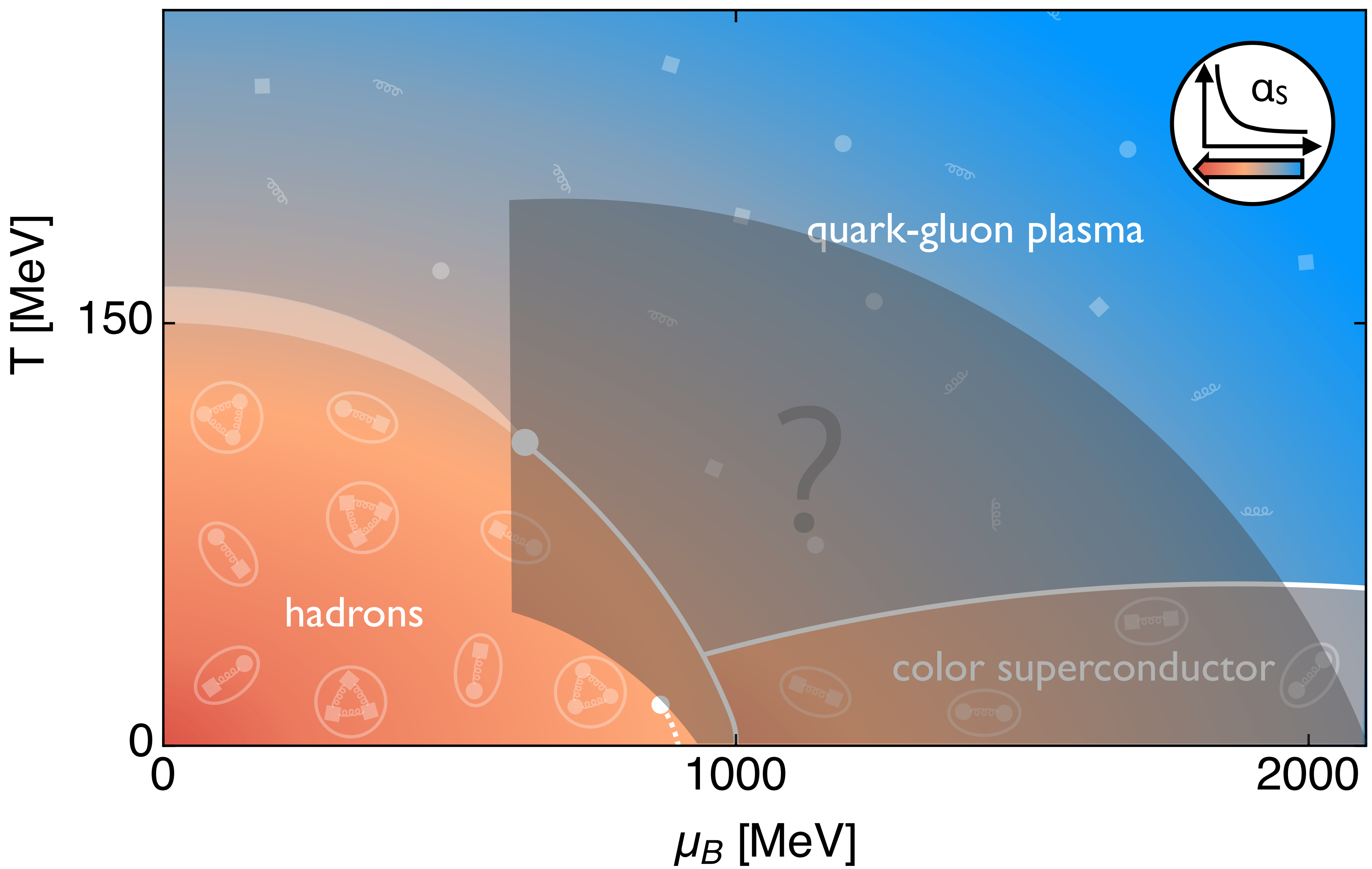
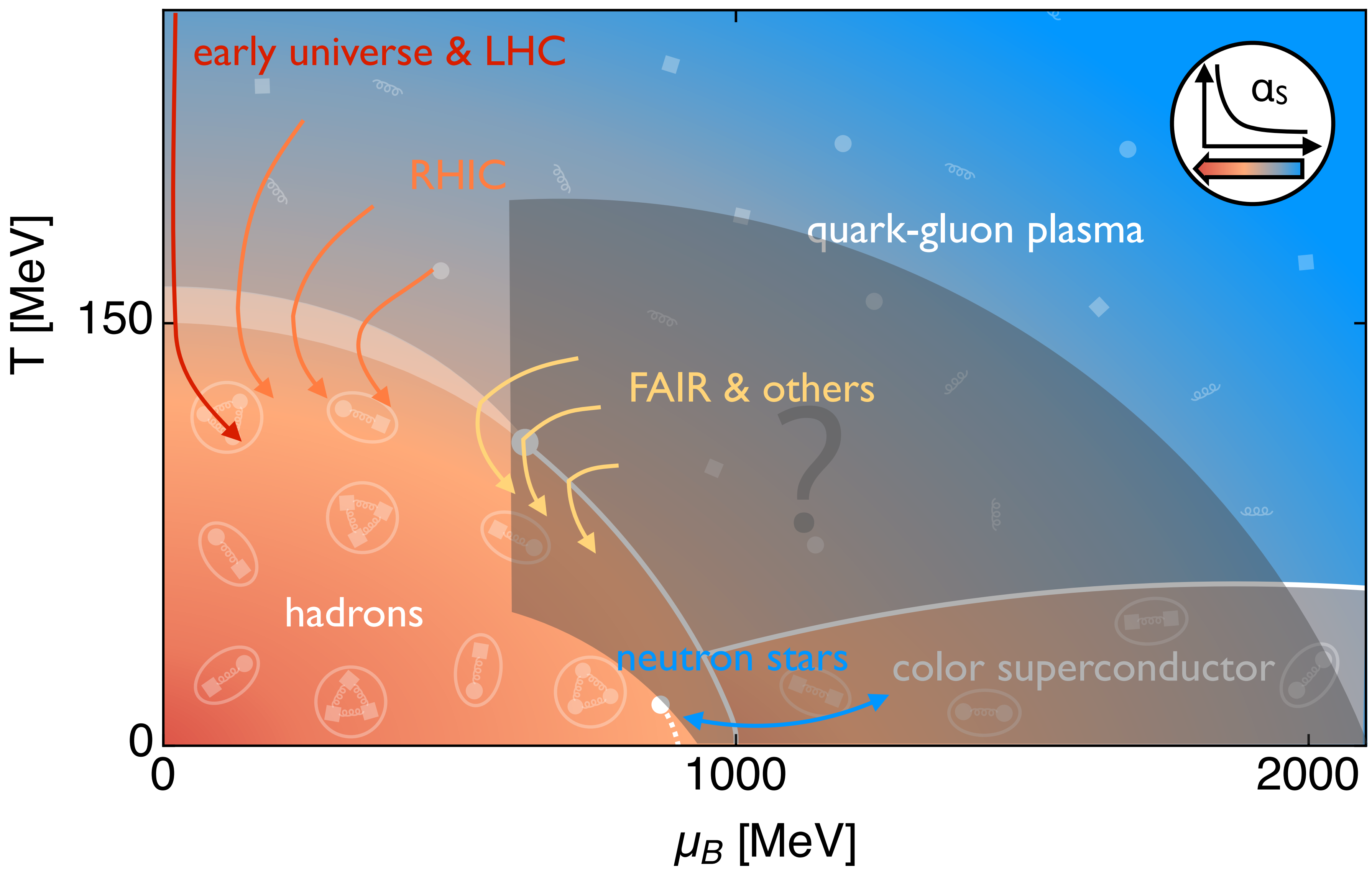


possible phases:

- inhomogeneous phases
- no CEP, but Lifshitz point
- various CSC phases
- quarkyonic matter
- quantum pion liquid

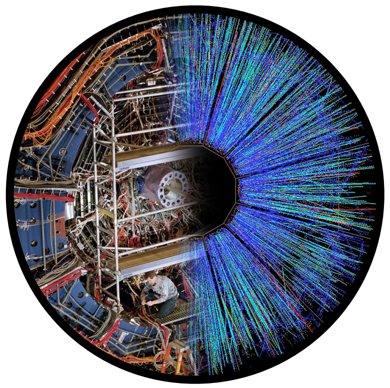
QUANTUM CHROMODYNAMICS

"external" parameters control scale: **phase diagram**

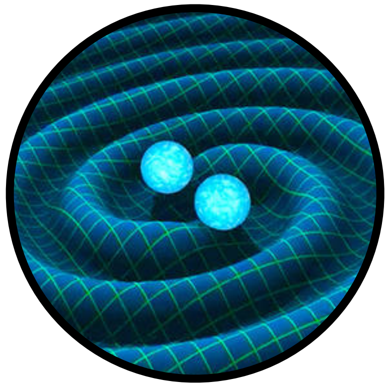


Experiments:

heavy-ion collisions



e.g. gravitational waves

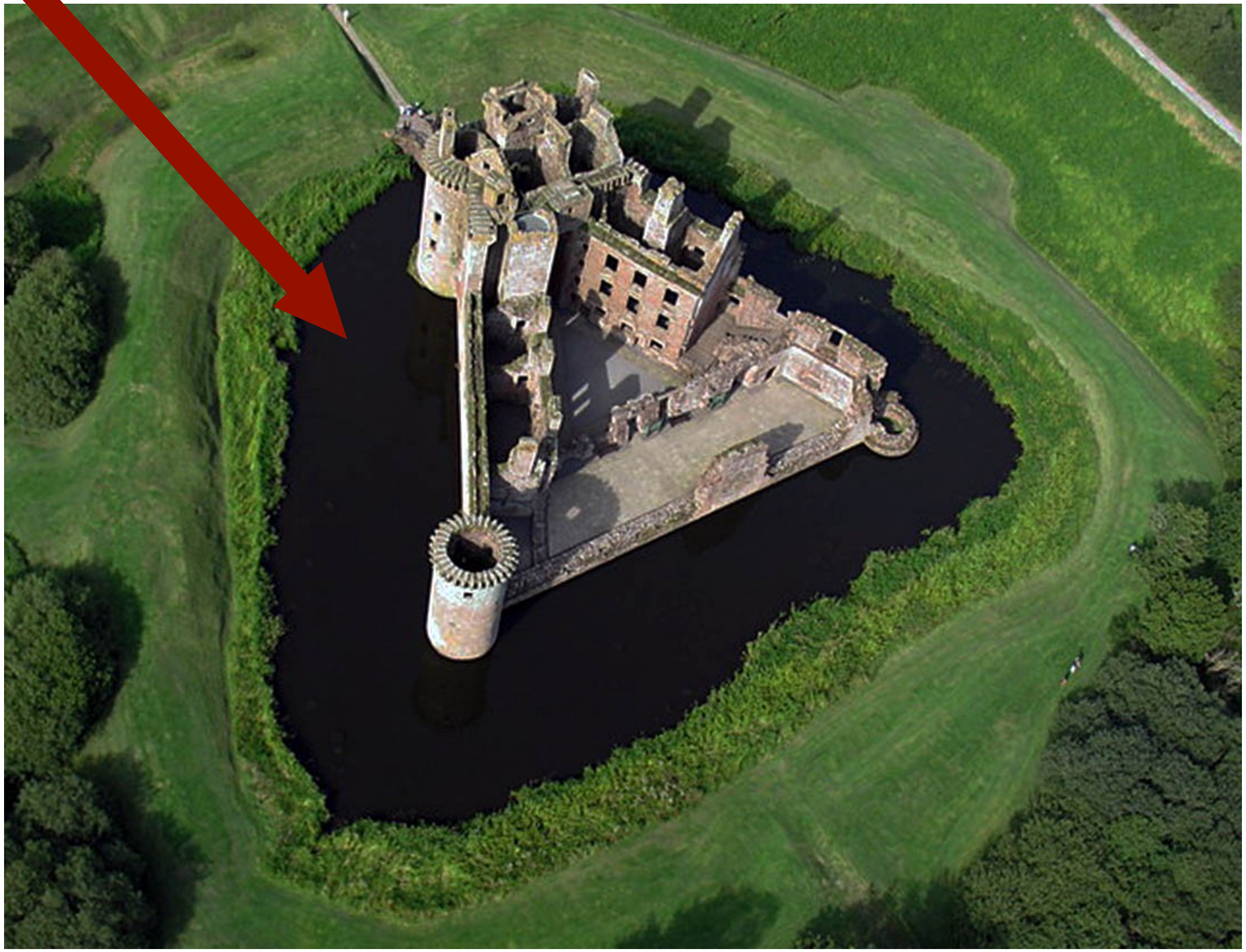


OUTLINE

- moat regimes
- how to find them

MOAT REGIMES

A MOAT

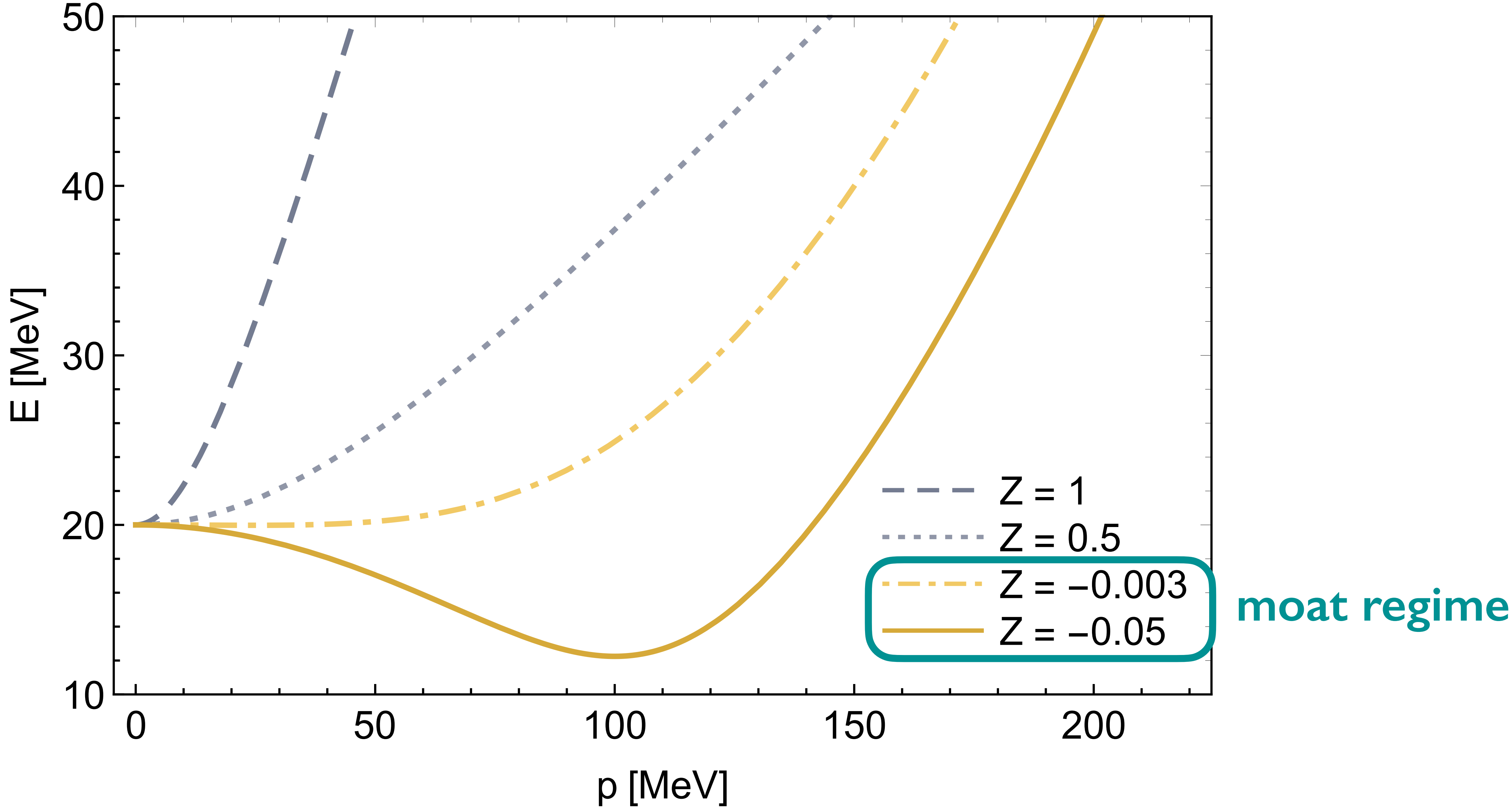


[Caerlaverock Castle, Scotland (source: Wikipedia)]

A MOAT

energy dispersion of particle ϕ:

$$E_\phi(\mathbf{p}^2) = \sqrt{Z\,\mathbf{p}^2 + W\left(\mathbf{p}^2\right)^2 + m_{\text{eff}}^2}$$



particles can be favored to have a nonzero momentum

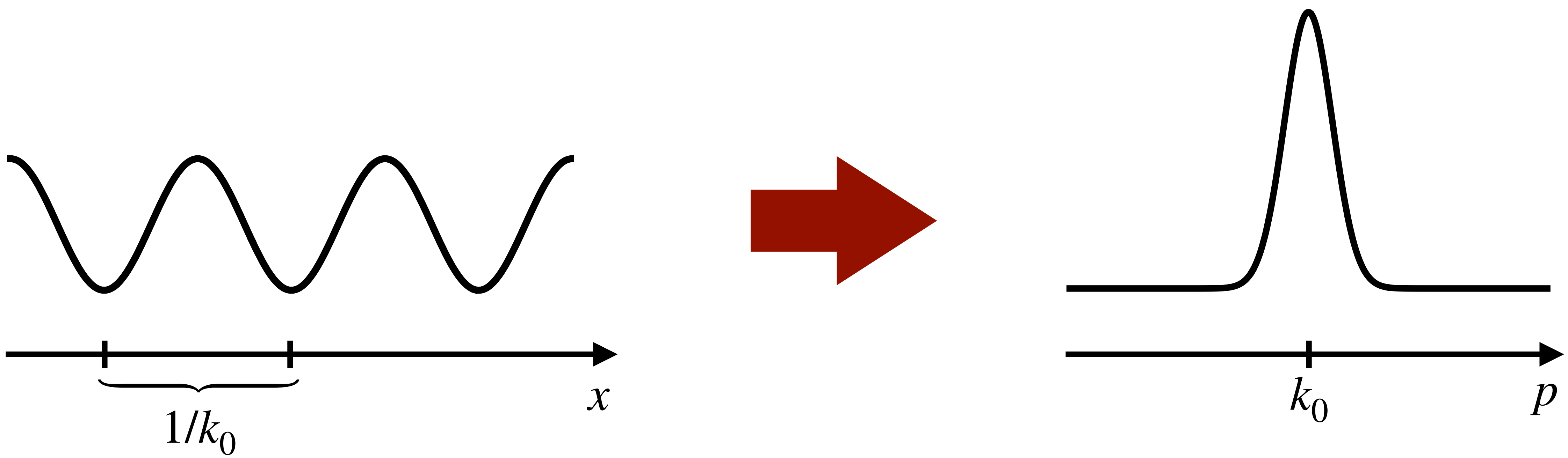
"gain energy by going faster"

WHERE DOES THE MOAT COME FROM?

heuristic picture:

spatial oscillation
$\cos(2\pi k_0 x)$

momentum space peak
$\delta(p - k_0)$



- particles subject to a spatial modulation are favored to have finite momentum k_0

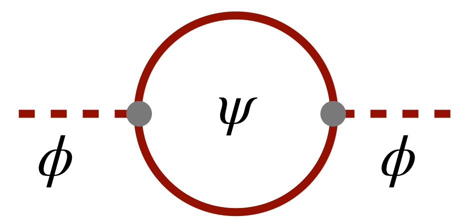
$\longrightarrow$ moat energy dispersion
(minimal energy at k_0)

- typical for inhomogeneous/crystalline phases or a quantum pion liquid (QπL)

WHERE CAN MOAT REGIMES APPEAR?

simplistic consideration:

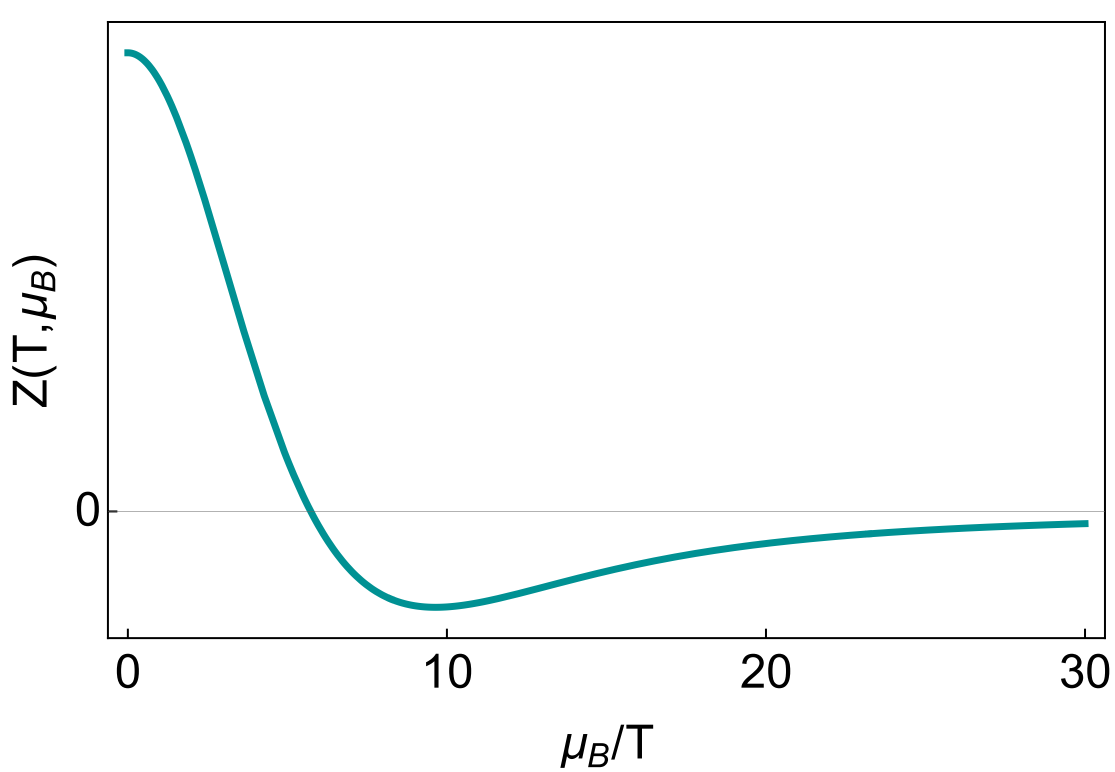
- consider "meson" coupled to a massless quark in 1+1 dimensions
- 1-loop meson self-energy

$\Gamma^{(2)} =$ 

- Z in dispersion relation is the coefficient of p^2 in $\Gamma^{(2)}$

polygamma function (not a quark)

$$Z(T,\mu_B) = \frac{1}{2}\frac{\partial^2}{\partial p^2}\Gamma^{(2)}\bigg|_{p=0} \propto -\operatorname{Re}\psi^{(2)}\left(\frac{1}{2} + \frac{i}{2\pi}\frac{\mu_B}{3T}\right)$$

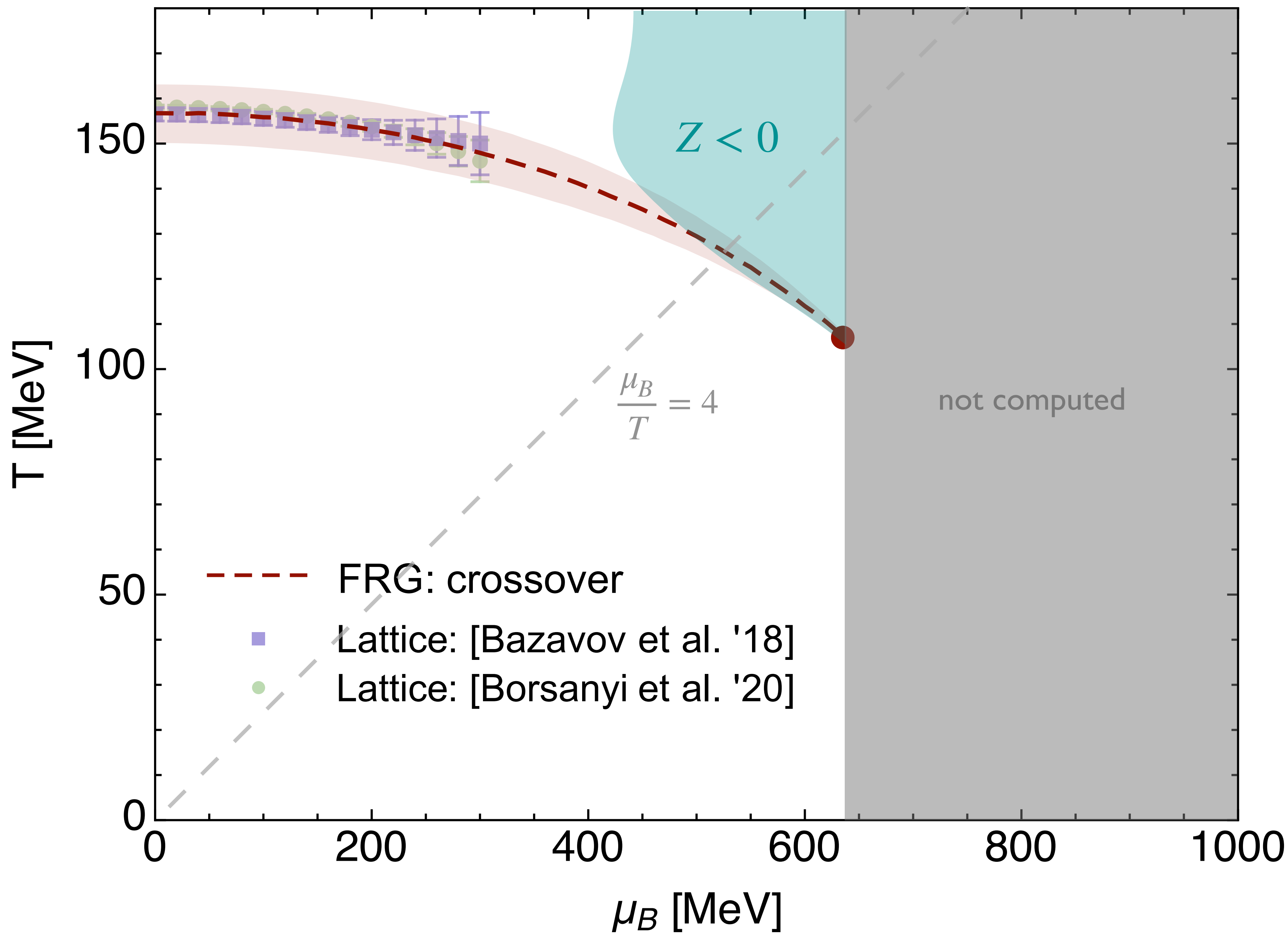


moat regime ($Z < 0$)
at large density

WHERE CAN MOAT REGIMES APPEAR?

At large μ_B in the QCD phase diagram:

[Fu, Pawlowski, FR (2019)]

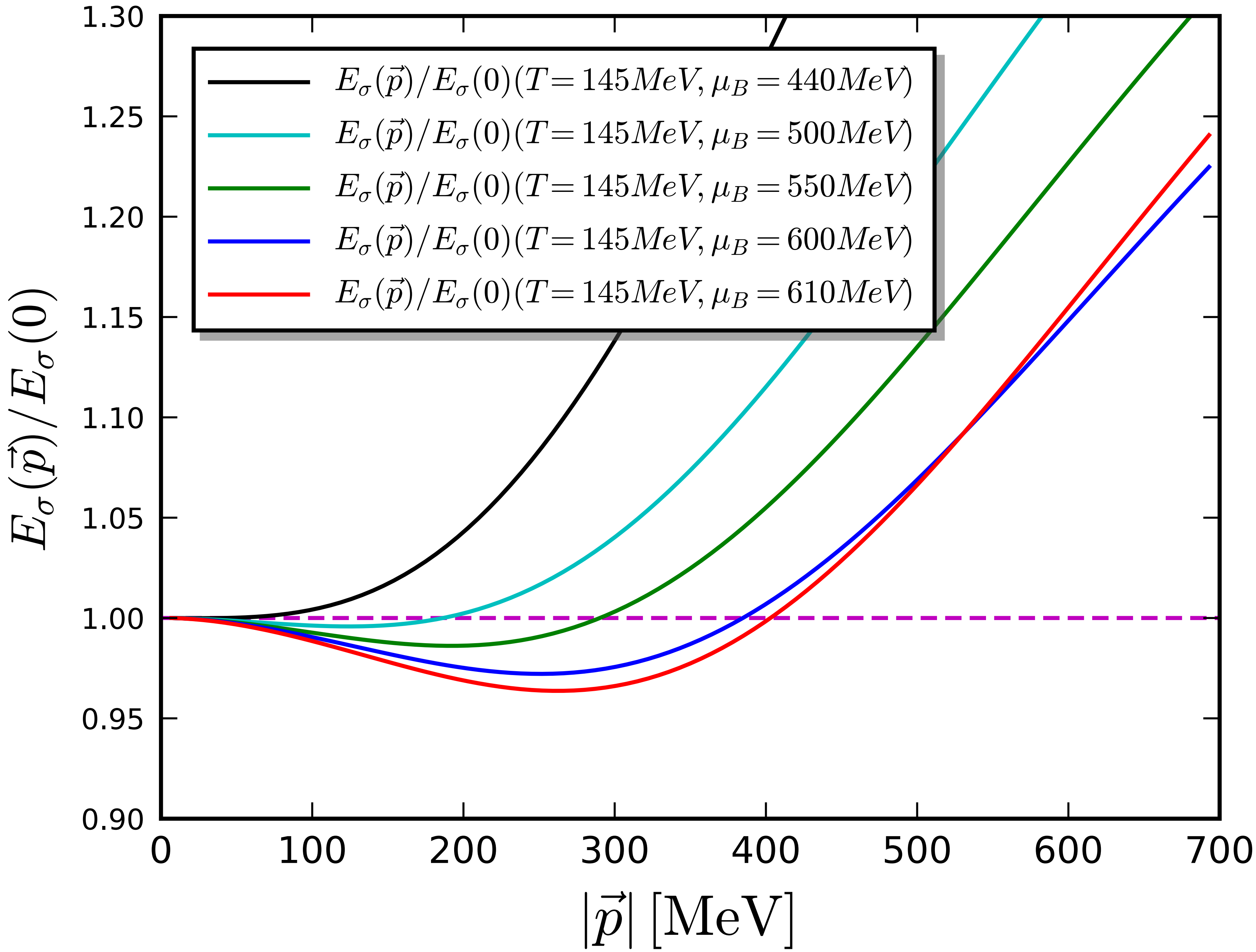


→ indication for extended region with $Z < 0$ in QCD: moat regime

MOAT REGIME IN THE QCD PHASE DIAGRAM

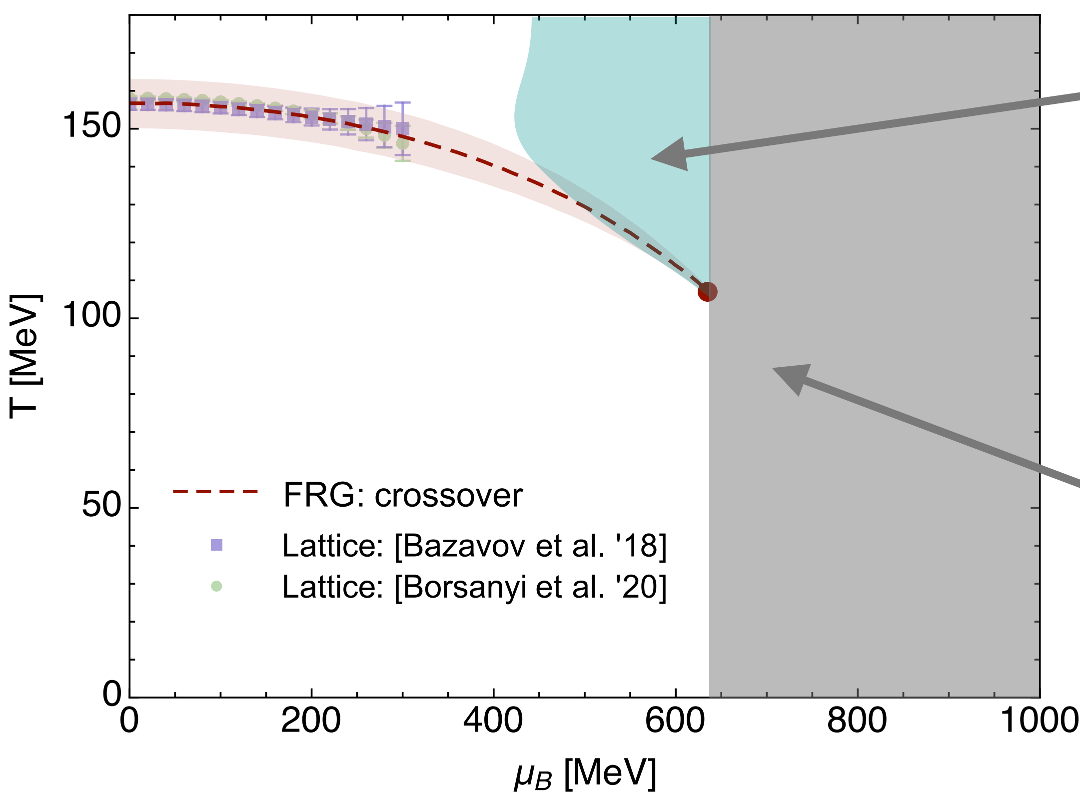
- **preliminary** result for the energy dispersion of pions/sigmas

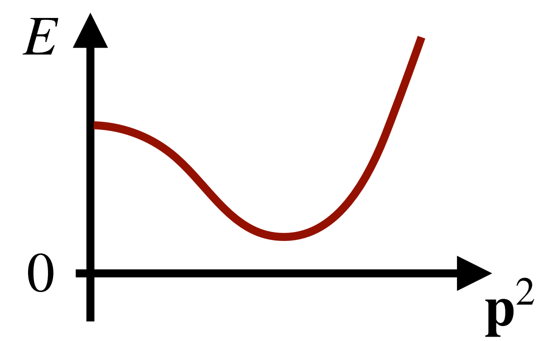
$$E_\sigma(\mathbf{p}^2) = \sqrt{Z_\sigma(\mathbf{p}^2)\,\mathbf{p}^2 + m_\sigma^2}$$



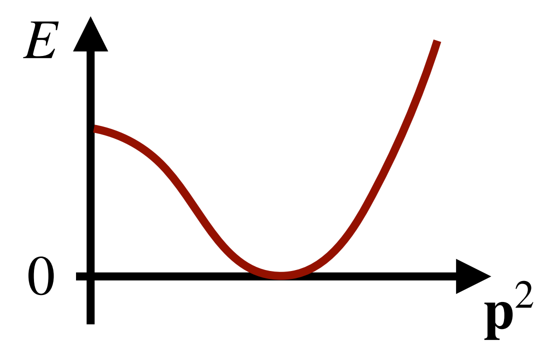
IMPLICATIONS OF THE MOAT

The energy gap might close at lower T and larger μ_B :





$E > 0$ for all $\mathbf{p}^2$



$E = 0$ at $\mathbf{p}^2 > 0$:

→ instability towards formation of an **inhomogeneous condensate**

INHOMOGENEOUS PHASE

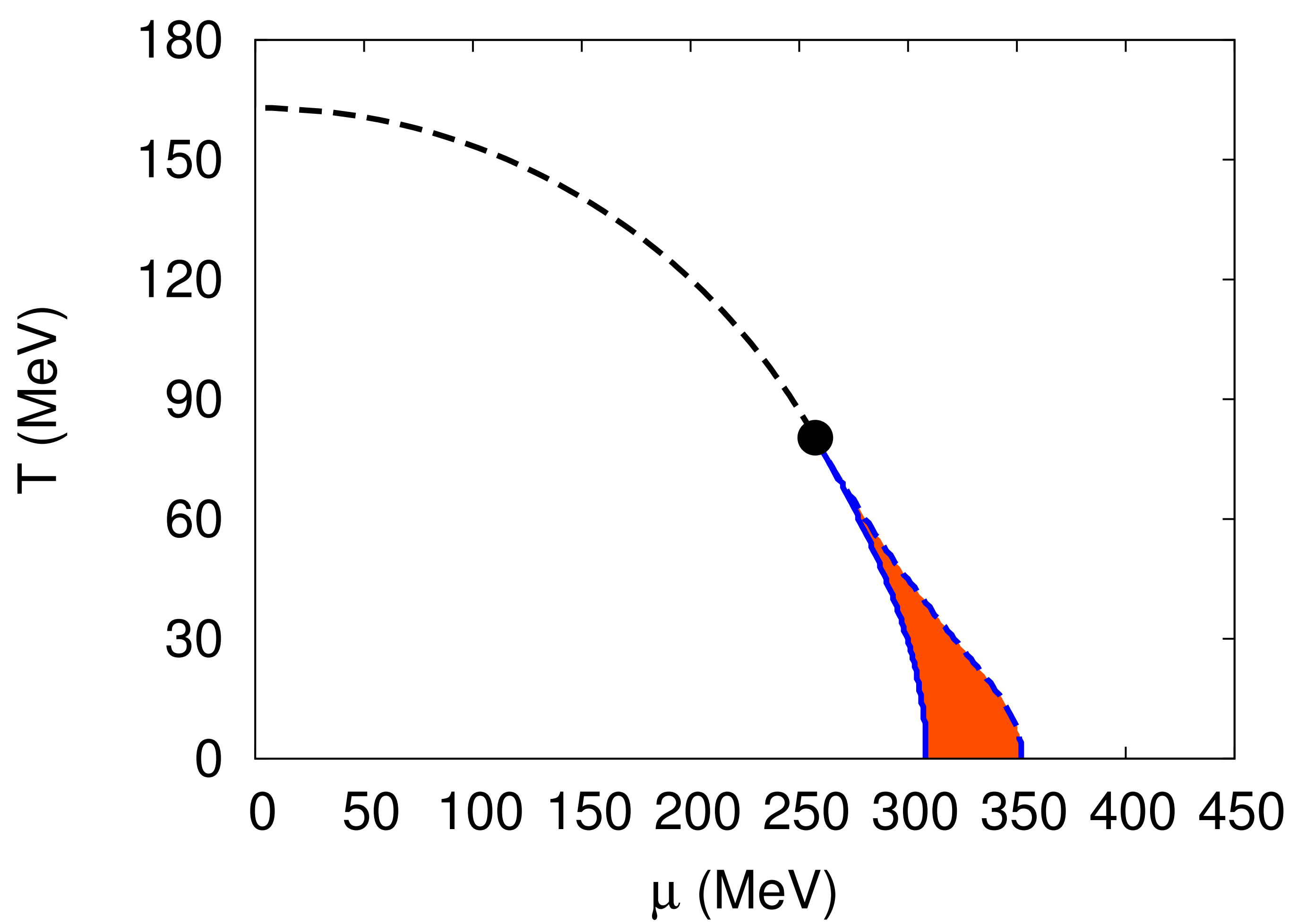
emerges if energy gap closes

- $E_\phi(k_0^2) = 0$: particles with momentum k_0 condense
- basic example: $O(N_f)$ chiral density wave

field   condensate/VEV

$$\phi = \begin{pmatrix} \sigma \\ \pi_{N-1} \\ \pi_{N-2} \\ \vdots \\ \pi_1 \end{pmatrix}, \quad \phi_0 = \Delta \begin{pmatrix} \cos(k_0 z) \\ \sin(k_0 z) \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

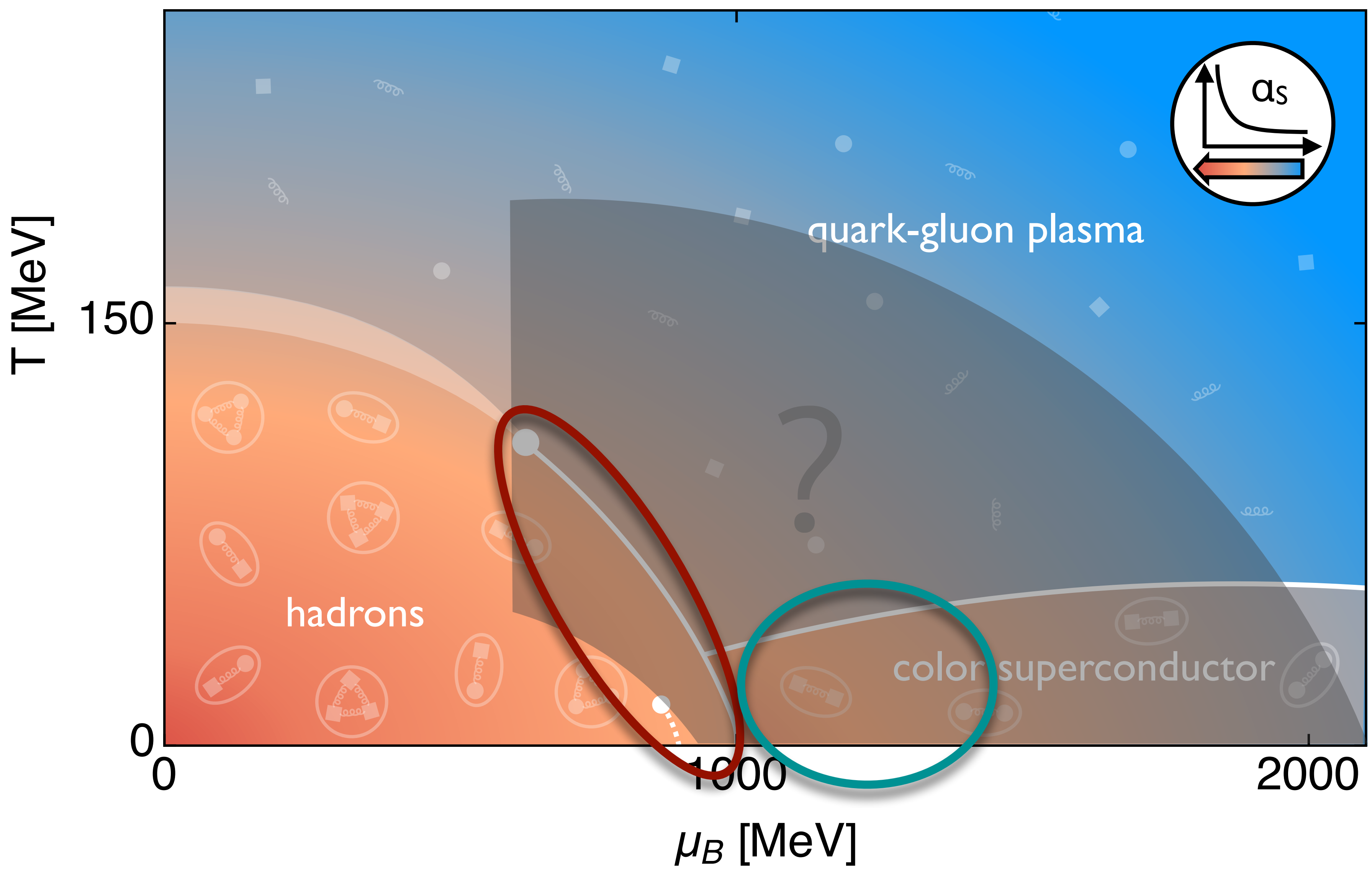
mean-field phase diagram



[Carignano, Buballa, Schaefer '14]

IMPLICATIONS OF THE MOAT

option 1: moat is a precursor for an inhomogeneous phase



possibilities: inhomogeneous chiral condensate or crystalline CSC

INHOM. PHASES & FLUCTUATIONS I

Inhomogeneous phases are mostly studied in mean-field.
But associated spontaneous symmetry breaking gives rise to massless modes (Goldstones).
Their fluctuations must be relevant!

Two types of symmetry breaking for inhomogeneous phases:

- continuous spatial symmetries (rotations, translations) broken down to discrete ones
- global flavor symmetries are broken (e.g. $O(N_f) \to O(N_f - 2)$ for chiral density wave)

It has been argued that 1d modulations are favored against higher-dimensional ones

[Abuki, Ishibashi, Suzuki '12]

Goldstone bosons from spatial symmetry breaking (e.g. phonons) lead to Landau-Peierls instability of 1d inhomogeneous condensates (e.g. chiral density wave)

- Goldstones lead to **logarithmic divergences**

  $\longrightarrow$ 1d condensate is destroyed; the system is disordered

- algebraically instead of exponentially decaying correlations still possible

  $\longrightarrow$ quasi-long-range order (e.g. liquid crystal)

[Landau, Lifshitz, Stat. Phys. I, §137]
[Lee, Nakano, Tsue, Tatsumi, Friman '15]

Option 2: moat is a precursor for a liquid-crystal-like phase

INHOM. PHASES & FLUCTUATIONS 2

even "worse" for fluctuations of Goldstones from broken flavor symmetry

- basic example: fluctuations around $O(N)$ chiral density wave

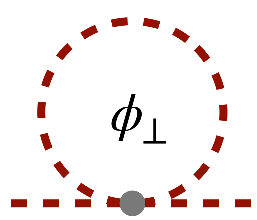
$$\phi = \Delta \begin{pmatrix} \cos(k_0\, z) \\ \sin(k_0\, z) \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \begin{pmatrix} \delta\phi_{\parallel} \\ \delta\phi_{\perp} \end{pmatrix}$$

static (large T) propagator of transverse (Goldstone) modes

$$G_{\phi_\perp} = \frac{1}{W\,(\mathbf{p}^2 - k_0^2)^2} \longrightarrow$$

double pole at nonzero momentum

- tadpole corrections in any dimension lead to linear IR divergences at finite T:



$$\sim T \int \frac{d^d\mathbf{p}}{(2\pi)^d}\, G_{\phi_\perp} \sim \frac{T}{W}\, k_0^{d-3} \int_{|\mathbf{p}|\sim k_0} \frac{d\,|\mathbf{p}|}{(\,|\mathbf{p}| - k_0)^2}$$

transverse fluctuations $\delta\phi_\perp$ disorder the system:

$\longrightarrow$ **no inhomogeneous phase for $N > 2$**

not even quasi-long-range order

(rigorous for $O(N)$ chiral density wave at $N \to \infty$)

[Pisarski, Tsvelik, Valgushev '20, Pisarski '21]

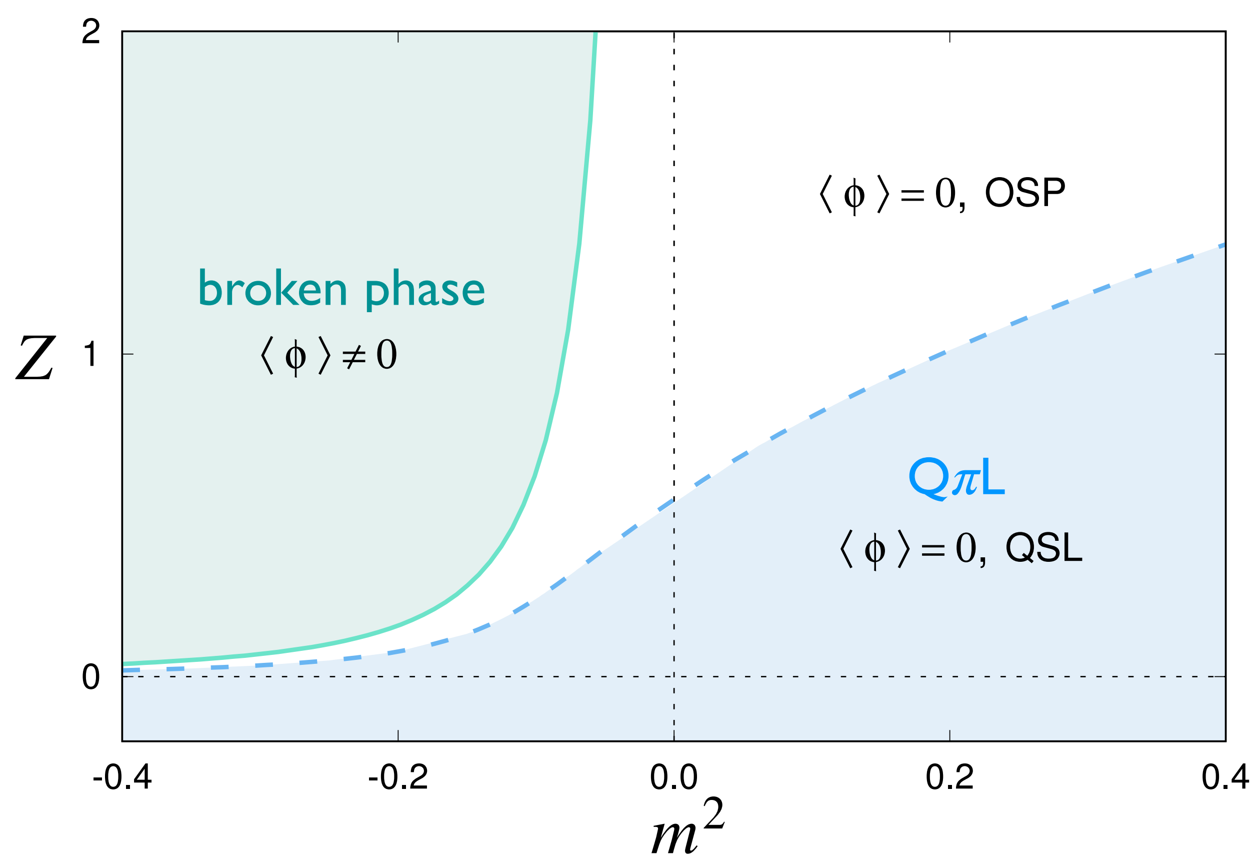
QUANTUM PION LIQUID

in this case we are left with another unusual disordered phase:

- disordered phase with a moat spectrum ($E > 0$ for all $\mathbf{p}^2$)
- instead of double pole, $G_{\phi_\perp}$ has complex poles $|\mathbf{p}| = m_r + im_i$
- lead to spatial modulations: $\langle \phi(x)\phi(0) \rangle \sim e^{-m_r x} \cos(m_i x)$ for large x

→ **quantum pion liquid**
(in analogy to quantum spin liquids)

[Pisarski, Tsvelik, Valgushev '20]
[Pisarski '21]



Option 3: moat signals a quantum pion liquid

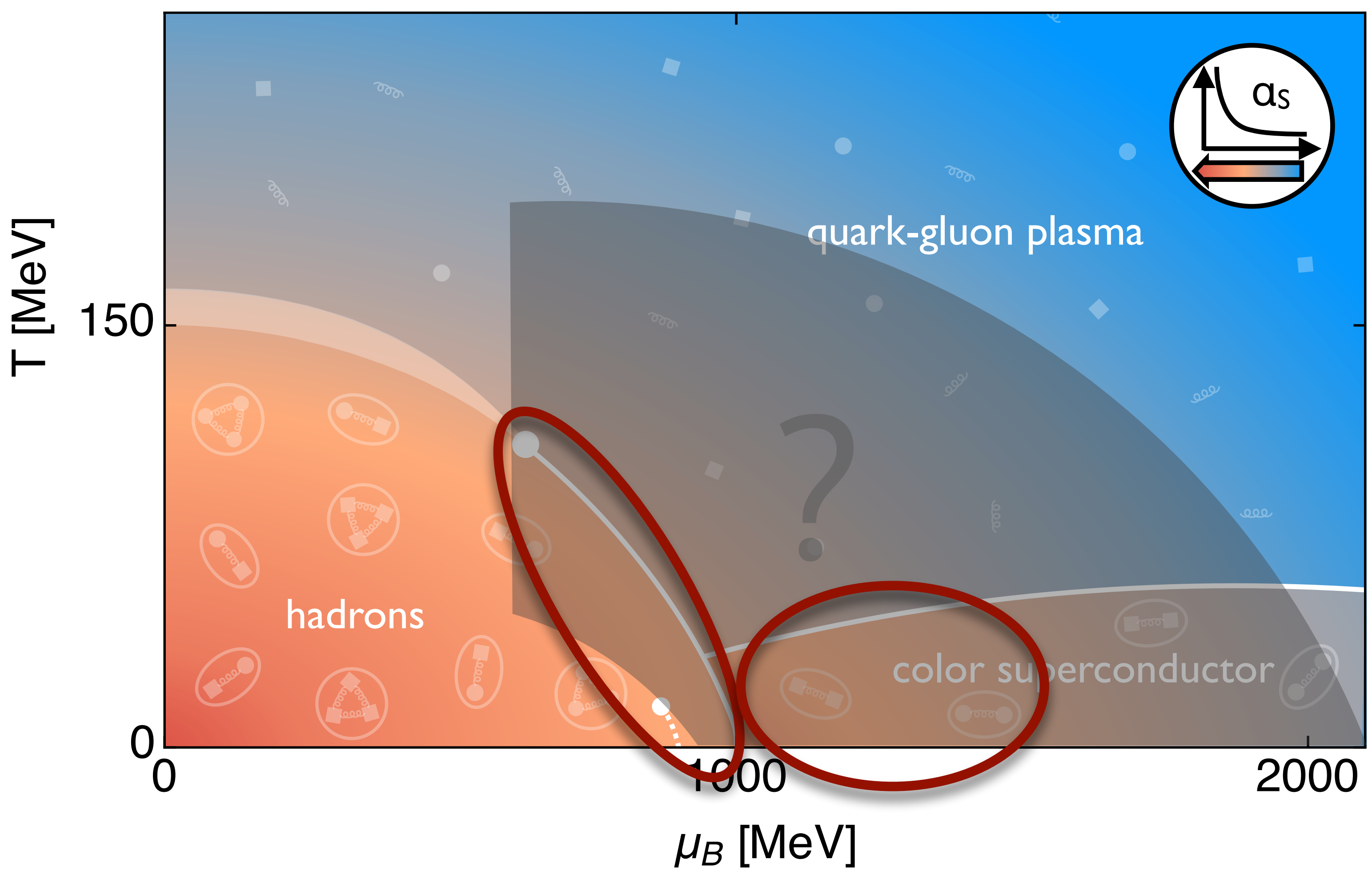
IMPLICATIONS OF THE MOAT

the moat regime could be an indication that dense QCD has:

option 1:
inhomogeneous phase
- only if there are no Goldstone bosons

option 2:
liquid-crystal-like
- only if there are only Goldstones from spatial symmetry breaking

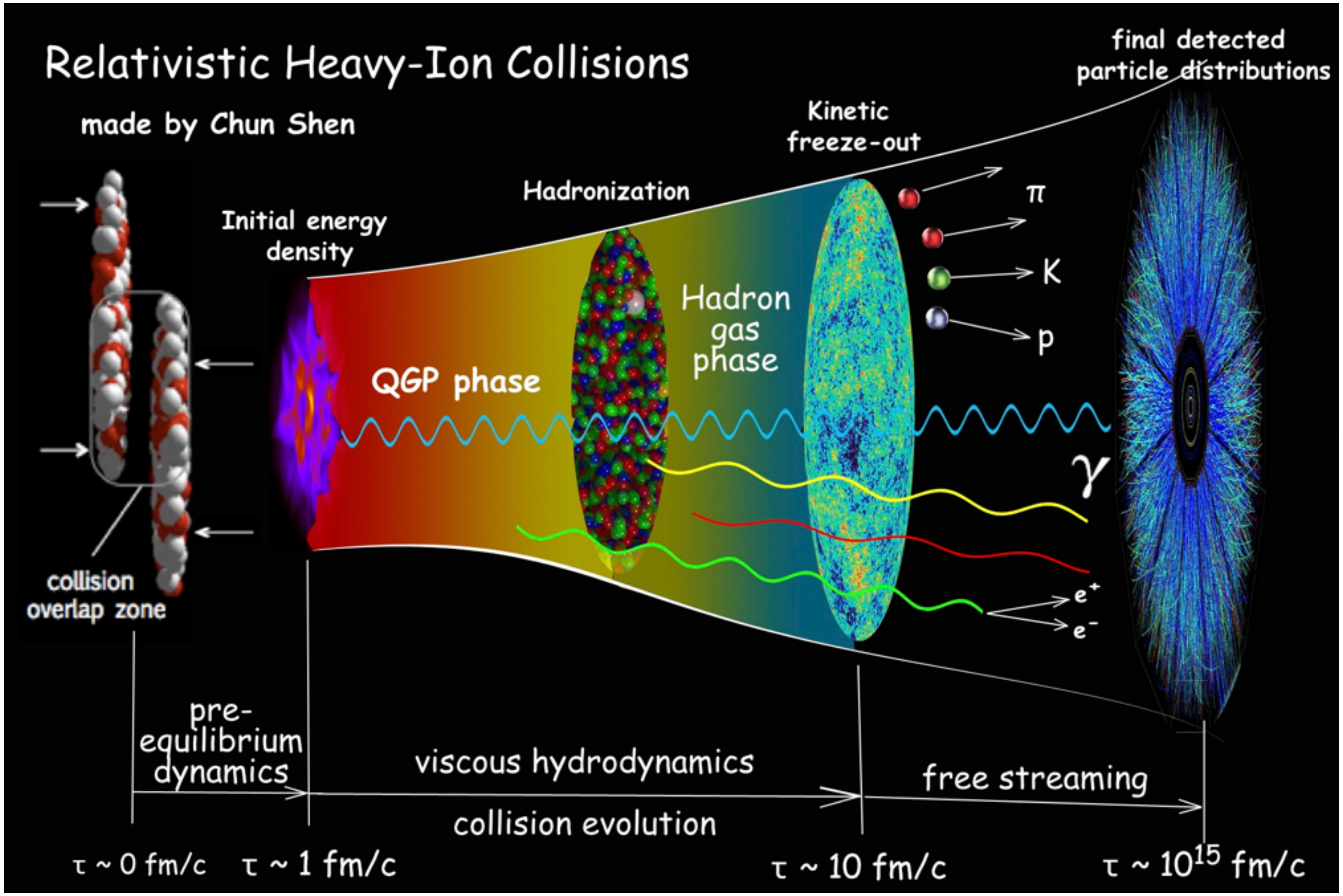
option 3:
quantum pion liquid
- only if there are Goldstones flavor symmetry breaking



this will occur in the regions where inhomogeneous phases are expected

SIGNATURES OF MOATS
IN HEAVY-ION COLLISIONS

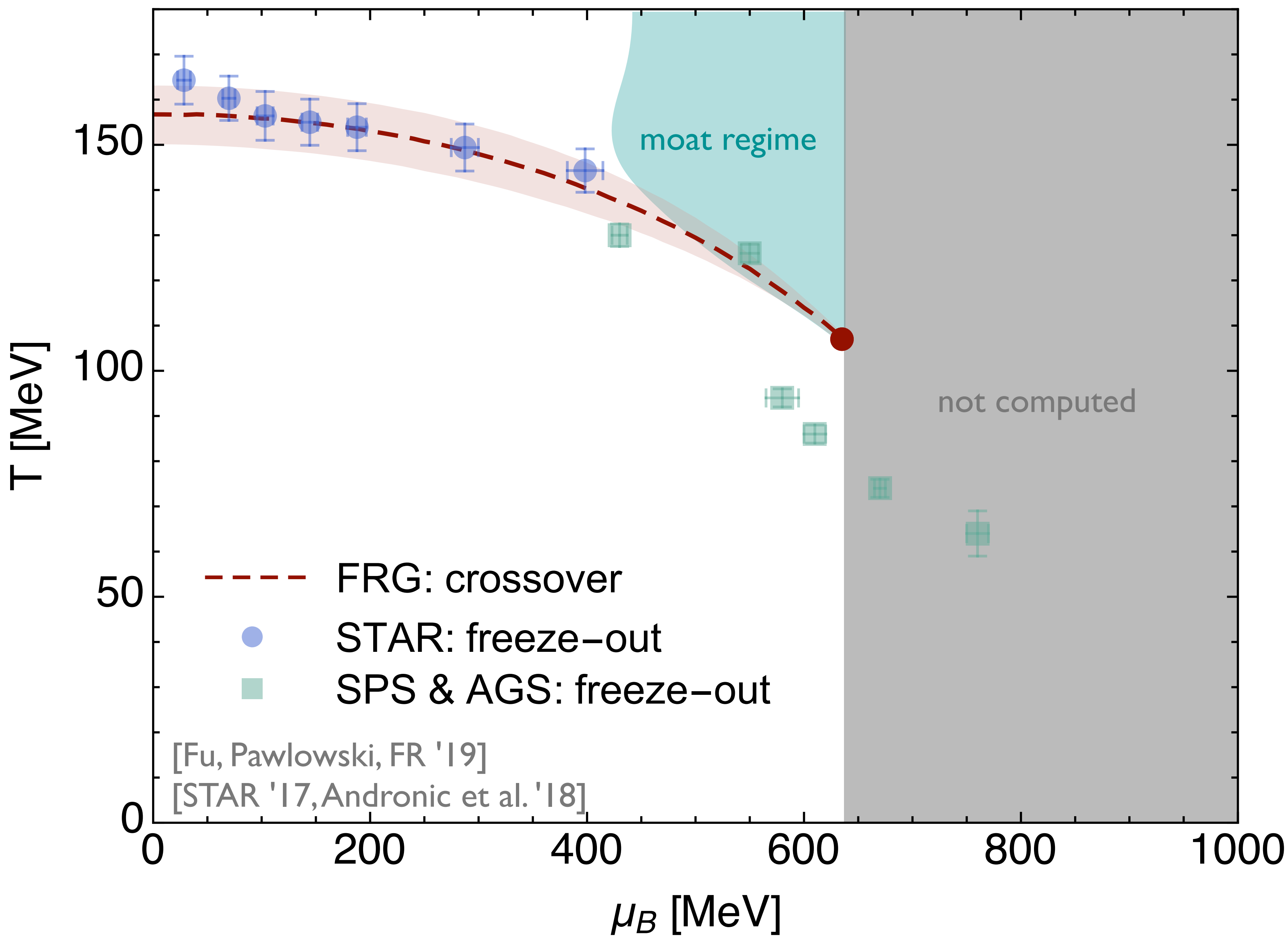
PROBING THE PHASE DIAGRAM



→ imprints of the phase structure at freeze-out?

PROBING THE PHASE DIAGRAM

Vary the beam energy to study the phase diagram different densities (smaller energy $\leftrightarrow$ lager μ)



STAR @ RHIC

$\sqrt{s} = 7.7 - 200\,\mathrm{GeV}$

$\mu_B \approx 400 - 30\,\mathrm{MeV}$

HADES @ GSI

$\sqrt{s} \approx 2.4\,\mathrm{GeV}$

$\mu_B \approx 770\,\mathrm{MeV}$

future experiments, e.g.,

CBM @ FAIR

$\sqrt{s} = 2.7 - 4.9\,\mathrm{GeV}$

$\mu_B \approx 730 - 540\,\mathrm{MeV}$

also: J-PARC, NICA, HIAF

What are the signatures of the the phase diagram in heavy-ion collisions?

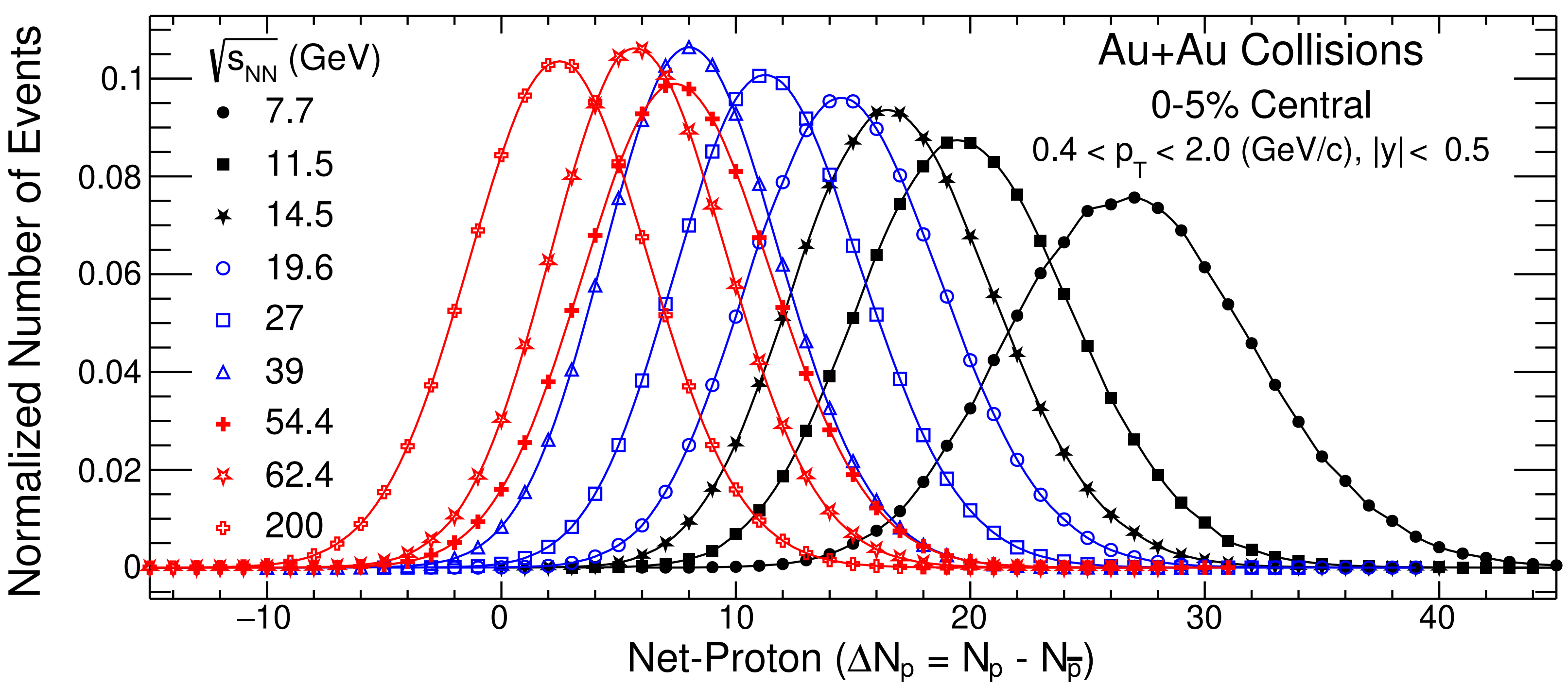
PARTICLE NUMBER DISTRIBUTION

The challenge: signatures have to be extracted from hadronic final states

A solution: consider distributions

- count particles from many collisions at fixed energy, get particle number distribution

[STAR '20]



- extract particle number correlations from the measured probability distribution $P(N_P)$:

$$\left\langle (N_P - \langle N_P \rangle)^n \right\rangle = \sum_{N_P} (N_P - \langle N_P \rangle)^n \, P(N_P)$$

→ look for signatures of the phase diagram in particle number correlations

- prominent example: CEP search

SEARCH FOR MOAT REGIMES [Pisarski, FR '21]

Moats arise in regimes with spatial modulations in the phase diagram at large μ_B

Characteristic feature: minimal energy at nonzero momentum

$\Rightarrow$ **enhanced particle production at nonzero momentum**

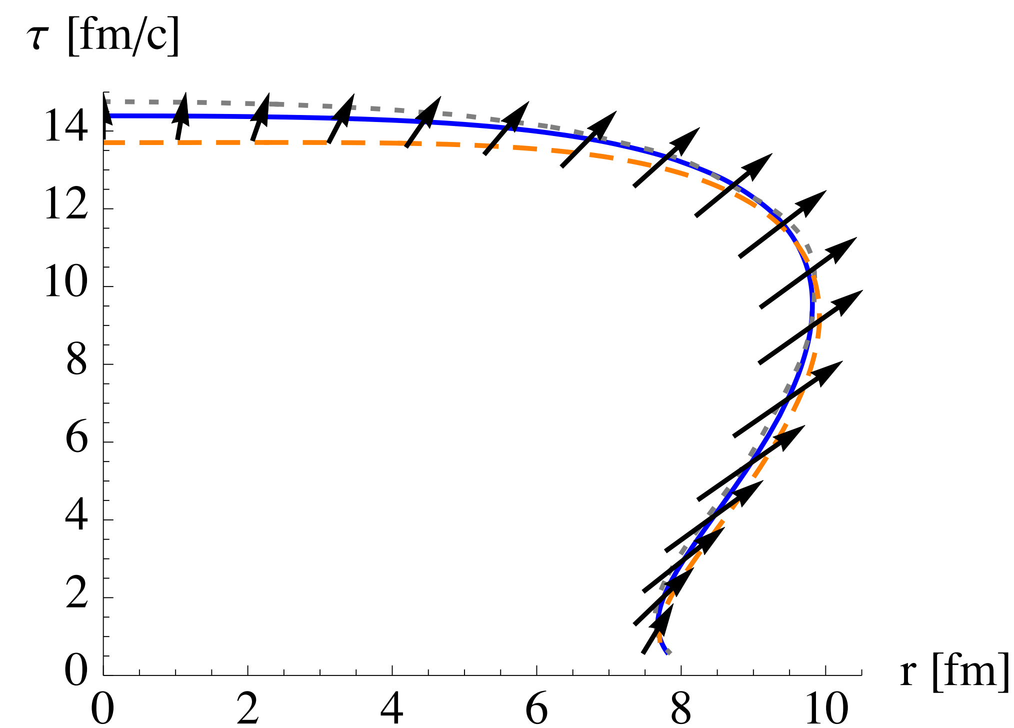
$\rightarrow$ look for signatures in the **momentum dependence** of particle numbers and correlations

- particle numbers are observables!

- particles freeze out at certain temperature T_f

$\longrightarrow$ defines 3d hypersurface: freeze-out surface Σ

How does the moat regime affect particles on Σ?



[Floerchinger, Wiedemann '13]

GENERALIZED COOPER-FRYE FORMULA

compute particle numbers on the freeze-out surface

- probability distribution of finding a particle ϕ with momentum p in thermal equilibrium: Wigner function

$$F_\phi(p) = 2\pi\,\underbrace{\rho_\phi}_{\text{spectral function}}(p_0, \mathbf{p})\, f(p_0)$$

- particles on Σ boosted with fluid velocity $u^\mu(x)$:

energy: $\breve{p}_0 = u^\mu p_\mu$

spatial momentum: $\breve{\mathbf{p}}^2 = \left(u^\mu u^\nu - g^{\mu\nu}\right) p_\mu p_\nu$

- particle spectrum from integrating particle number current over freeze-out surface:

$$\frac{d^3 N_\phi}{d\mathbf{p}^3} = \frac{2}{(2\pi)^3} \int_\Sigma d\Sigma_\mu \underbrace{\int \frac{dp_0}{2\pi}\, p^\mu\, \Theta(\breve{p}_0)\, F_\phi(\breve{p})}_{\sim \text{ particle number current density}}$$

- reduces to Cooper-Frye formula for free vacuum spectral function: $\rho_\phi(p) = \mathrm{sign}(\mathrm{p}_0)\,\delta\Big[\mathrm{p}_0^2 - \left(\mathbf{p}^2 + \mathrm{m}^2\right)\Big]$

PARTICLE SPECTRUM IN A MOAT PHASE

use simple models to show general structure

Particle in a moat regime:

- **low-energy model** of free bosons in a moat regime $(Z < 0, W > 0)$:

$$\mathscr{L}_0 = \frac{1}{2}\left(\partial_0\phi\right)^2 + \frac{Z}{2}\left(\partial_i\phi\right)^2 + \frac{W}{2}\left(\partial_i^2\phi\right)^2 + \frac{m_{\rm eff}^2}{2}\phi^2$$

- gives simple in-medium spectral function

$$\rho_\phi(p_0, \mathbf{p}^2) = \mathrm{sign}(\mathrm{p}_0)\,\delta\left[\mathrm{p}_0^2 - \mathrm{E}_\phi^2(\mathbf{p}^2)\right] \quad \text{with} \quad E_\phi(\mathbf{p}^2) = \sqrt{Z\,\mathbf{p}^2 + W\left(\mathbf{p}^2\right)^2 + m_{\rm eff}^2}$$

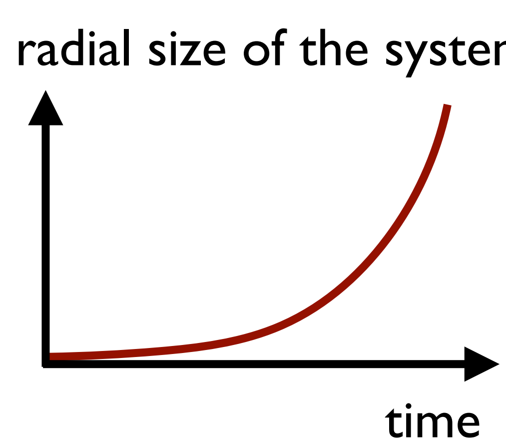
- boost symmetry broken! (but spatial rotation symmetry still intact)

Fluid velocity and freeze-out surface from hydro evolution

- boost invariant freeze-out at fixed temperature T_f and fixed proper time $\tau_f \ (= \sqrt{t^2 - z^2})$

- blast wave approximation for the fluid velocity:

$$u^r = \bar{u}\,\frac{r}{\bar{R}}\,\theta(\bar{R} - r)$$



[Schnedermann, Sollfrank, Heinz (1993)]
[Teaney (2003)]

PARTICLE SPECTRUM IN A MOAT PHASE

use simple models to show general structure

model parameters:

- pick a beam energy of $\sqrt{s} = 5\,\mathrm{GeV}$ and read off thermodynamic and blast wave parameters:

$$T_f = 115\,\mathrm{MeV}$$
$$\mu_{B,f} = 536\,\mathrm{MeV}$$

[Andronic, Braun-Munzinger, Redlich, Stachel (2018)]

$$\bar{u} = 0.3$$
$$\bar{R} = 8\,\mathrm{fm}$$
$$\tau_f = 5\,\mathrm{fm/c}$$

[Zhang, Ma, Chen, Zhong (2016)]

- thermodynamics (used later) from a hadron resonance gas [Braun-Munzinger, Redlich, Stachel (2003)]

- moat parameters: purely illustrative

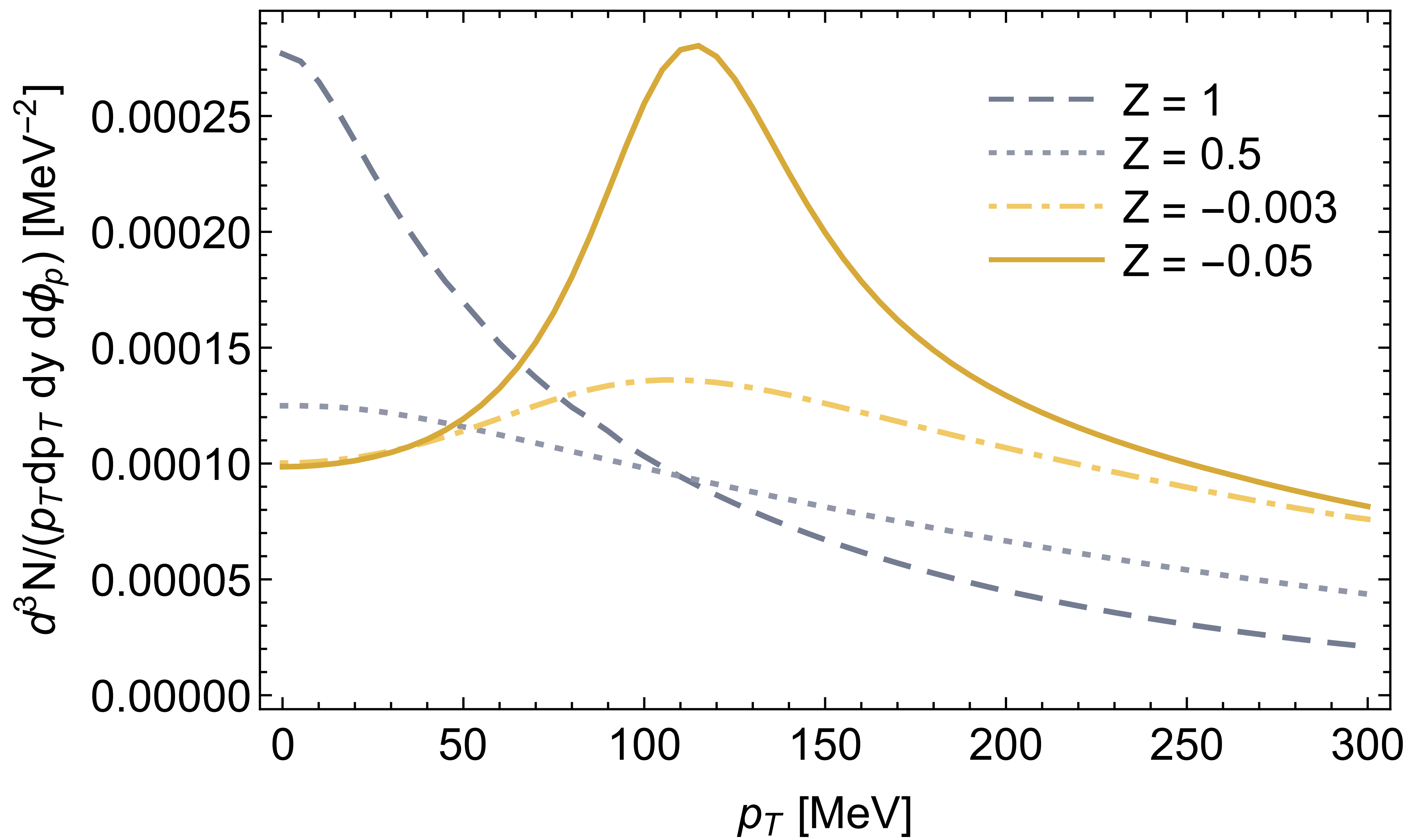
$$\text{if } Z < 0: \quad W = 2.5\,\mathrm{GeV}^{-2}$$

PARTICLE SPECTRUM IN A MOAT PHASE

transverse momentum spectrum (z: beam direction, $p_T = \sqrt{p_x^2 + p_y^2}$)

- compare normal phase (gray, $W = 0$) to moat phase (yellow, $W = 2.5\,\mathrm{GeV}^{-2}$)

$$\frac{d^3N}{p_T\,dp_T\,dy\,d\phi_p}$$



enhanced particle production at nonzero momentum!

maximum related to the wavenumber of the spatial modulation

PARTICLE NUMBER CORRELATIONS

- correlations sensitive to in-medium modifications
- difficult to compute in systems with long-range order due to multi-particle correlations
- moat regime is disordered: single particle correlations can capture relevant features

$\longrightarrow$ **correlations on Σ from (generalized) Cooper-Frye formula** [Pisarski, FR (2021)] [Floerchinger (unpublished)]

n-particle correlation:

$$\left\langle \prod_{i=1}^{n} \frac{d^3N_\phi}{d\mathbf{p}_i^3} \right\rangle = \left[\prod_{i=1}^{n} \frac{2}{(2\pi)^3} \int d\Sigma_i^\mu \int \frac{dp_i^0}{2\pi} (p_i)_\mu \, \Theta(\check{p}_i^0) \right] \left\langle \prod_{i=1}^{n} F_\phi(\check{p}_i) \right\rangle$$

thermodynamic average

- fluctuations, e.g., of thermodynamic quantities lead to fluctuations of F_ϕ
- consider small fluctuations T, μ_B, u with $\kappa_i^\mu(x) = \big(T(x), \mu_B(x), u^\mu(x)\big)_i$:

$$\left\langle F_\phi F_\phi \right\rangle_c = \frac{\partial F_\phi}{\partial \kappa_i^\mu} \frac{\partial F_\phi}{\partial \kappa_j^\nu} \bigg|_{\bar{\kappa}} \left\langle \delta\kappa_i^\mu \delta\kappa_j^\nu \right\rangle + \mathcal{O}\big(\delta\kappa^3\big)$$

connected correlator

fluctuations of T, μ_B, u

THERMODYNAMIC CORRELATIONS

- correlations $\langle \ldots \rangle$ from thermodynamic average
- weight configurations with the change in entropy due to fluctuations, Δs^μ [Landau, Lifshitz (vol. 5)]

⟶ generating functional of (connected) thermodynamic correlations

$$W[J] = \ln \int \mathscr{D}\kappa(x) \, \exp \int d\Sigma_\mu \left[\Delta s^\mu(x) + J(x)_{i\nu} \, \hat{v}^\mu \, \delta\kappa_i^\nu(x) \right]$$

normal to Σ

- connected n-particle correlations $\langle \delta\kappa^n \rangle_c$ from $\left. \dfrac{\delta^n W[J]}{\delta J^n} \right|_{J=0}$
- change of entropy in an ideal fluid $(T^{\mu\nu} = \epsilon \, u^\mu u^\nu + p\Delta^{\mu\nu})$ with Gaussian fluctuations:

$$\hat{v}_\mu \Delta s^\mu = -\frac{1}{2} \delta\kappa_{i\mu}(x) \, \mathscr{F}^{\mu\nu}_{ij}(x) \, \delta\kappa_{j\nu}(x)$$

local fluctuations!

fluctuation matrix $(\hat{u} = \hat{v}^\mu \, u_\mu)$

$$\mathscr{F}^{\mu\nu}_{ij} = \frac{1}{T} \begin{pmatrix} \hat{u} \, \dfrac{\partial s}{\partial T} & \hat{u} \, \dfrac{\partial s}{\partial \mu_B} & s\hat{v}^\nu \\ \hat{u} \, \dfrac{\partial s}{\partial \mu_B} & \hat{u} \, \dfrac{\partial n}{\partial \mu_B} & n\hat{v}^\nu \\ s\hat{v}^\mu & n\hat{v}^\mu & -\hat{u} \, (Ts + \mu_B n) g^{\mu\nu} \end{pmatrix}_{ij}$$

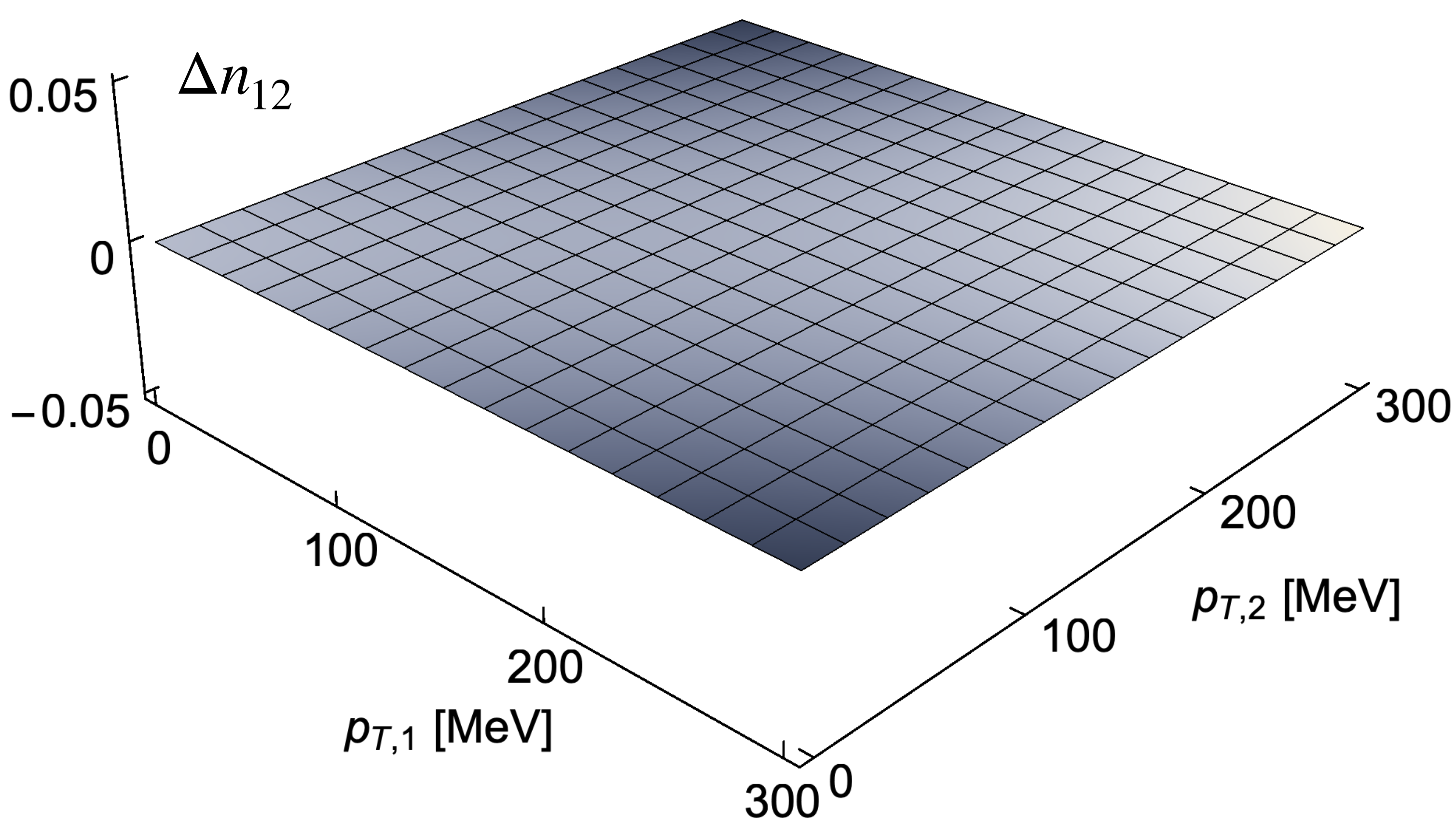
PARTICLE NUMBER CORRELATIONS

- Gaussian fluctuations: only nontrivial correlation is two-point function (all others are products thereof (Wick's theorem))

$$\left\langle \frac{d^3N_\phi}{d\mathbf{p}_1^3}\frac{d^3N_\phi}{d\mathbf{p}_2^3} \right\rangle_c = \frac{4}{(2\pi)^6}\int d\Sigma^\mu \int \frac{dp_1^0}{2\pi}\frac{dp_2^0}{2\pi}\,(p_1)_\mu\,(\hat{v}\cdot p_2)\,\Theta(\breve{p}_1^0)\,\Theta(\breve{p}_2^0)\left(\frac{\partial F_\phi(\breve{p}_1)}{\partial \kappa_i^\rho}\frac{\partial F_\phi(\breve{p}_2)}{\partial \kappa_j^\sigma}\right)\Bigg|_{\bar{\kappa}}\left(\mathscr{F}_{ij}^{\rho\sigma}(w)\right)^{-1}$$

- look at normalized two-particle correlation $\Delta n_{12} = \left\langle \left(\frac{d^3N}{d\mathbf{p}^3}\right)^2 \right\rangle_c \Big/ \left\langle \frac{d^3N}{d\mathbf{p}^3} \right\rangle^2$

normal phase



(relatively) flat two-particle p_T correlation in the normal phase

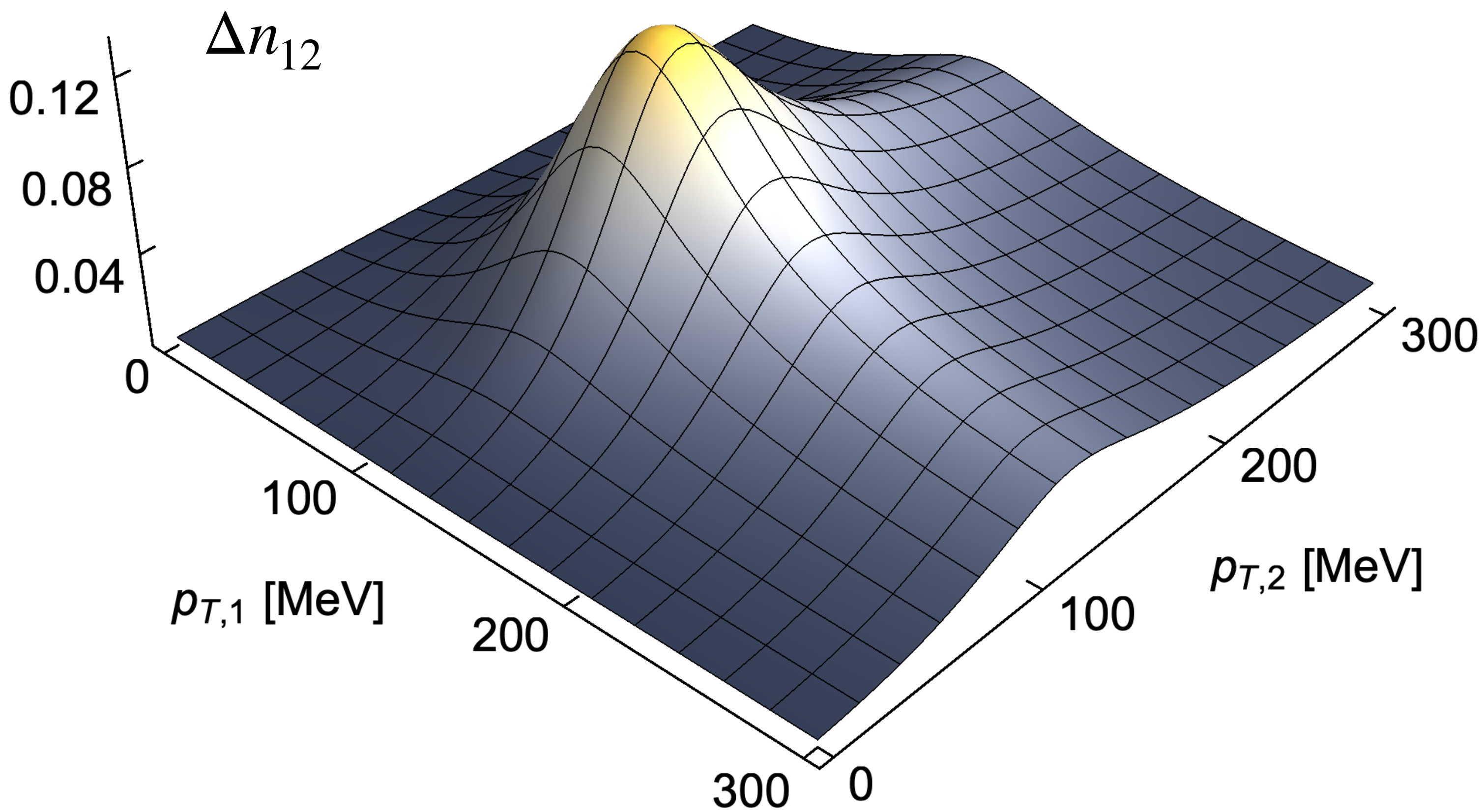
PARTICLE NUMBER CORRELATIONS

- Gaussian fluctuations: only nontrivial correlation is two-point function (all others are products thereof (Wick's theorem))

$$\left\langle \frac{d^3N_\phi}{d\mathbf{p}_1^3}\frac{d^3N_\phi}{d\mathbf{p}_2^3}\right\rangle_c = \frac{4}{(2\pi)^6}\int d\Sigma^\mu \int \frac{dp_1^0}{2\pi}\frac{dp_2^0}{2\pi}\,(p_1)_\mu\,(\hat{v}\cdot p_2)\,\Theta(\breve{p}_1^0)\,\Theta(\breve{p}_2^0)\left(\frac{\partial F_\phi(\breve{p}_1)}{\partial \kappa_i^\rho}\frac{\partial F_\phi(\breve{p}_2)}{\partial \kappa_j^\sigma}\right)\Bigg|_{\bar{\kappa}}\left(\mathcal{F}_{ij}^{\rho\sigma}(w)\right)^{-1}$$

- look at normalized two-particle correlation $\Delta n_{12} = \left\langle \left(\frac{d^3N}{d\mathbf{p}^3}\right)^2\right\rangle_c \Big/ \left\langle \frac{d^3N}{d\mathbf{p}^3}\right\rangle^2$

moat phase



pronounced peak and ridges at nonzero p_T related to wavenumber of spatial modulation!

huge enhancement:

$$\frac{\Delta n_{12}(p_{\rm peak})\Big|_{\rm moat}}{\Delta n_{12}(p_{\rm peak})\Big|_{\rm normal}} \approx 10^2$$

SUMMARY

Moats arise in regimes with spatial modulations

- expected to occur at large μ_B
- are precursors for inhomogeneous-, liquid-crystal-like or quantum pion liquid phases

Enhanced production of moat particles at nonzero momentum

- characteristic peaks (and ridges) in particle spectra and correlations at nonzero p_T

Opportunity to discover novel phases with low-energy heavy-ion collisions through differential measurements of particles and their correlations at small momenta

So far: basic description of qualitative effects
To do: quantitative description of moat regimes