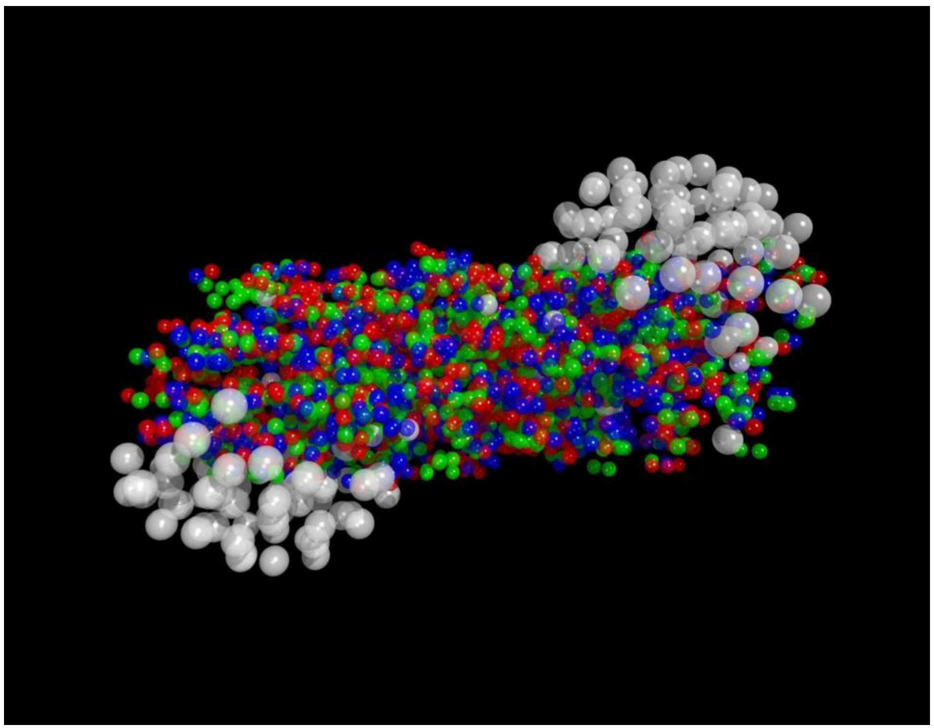
Non-equilibrium fermion production on the lattice

Daniil Gelfand | University of Heidelberg |



J. Berges, D. G., J. Pruschke, PRL 107 (2011) 061301

J. Berges, D. G., D. Sexty, hep-ph/1308.2180

F. Hebenstreit, J. Berges, D. G., PRD 87 (2013) 105006

F. Hebenstreit, J. Berges, D. G., PRL 111 (2013) 201601

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Overview

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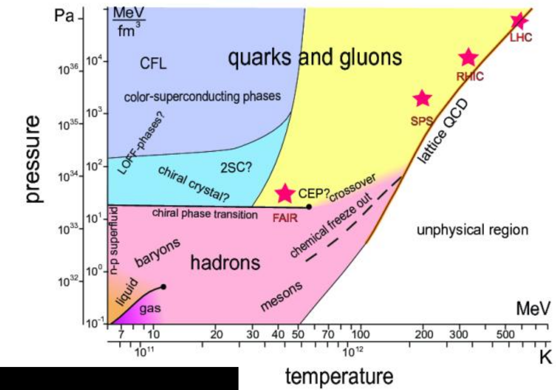
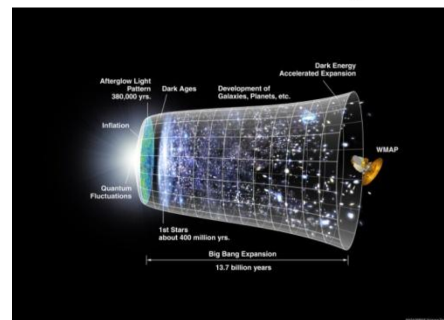
Introduction

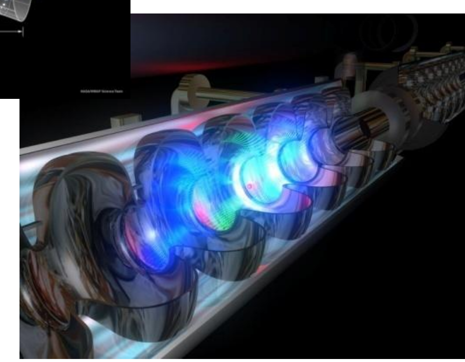
Fermion production important for:

- **Heavy-ion collisions**
  - Production of quarks from highly occupied gauge fields
- **Cosmology**
  - Production of fermionic matter from preheating after inflation
- **Intense laser beams**
  - Vacuum pair production of electron-positron pairs



Wambach, J. et al. AIP Conf.Proc. 1441 (2012) 794-796





http://scr3.golem.de/screenshots/1206/XFEL/xfel_5.jpg

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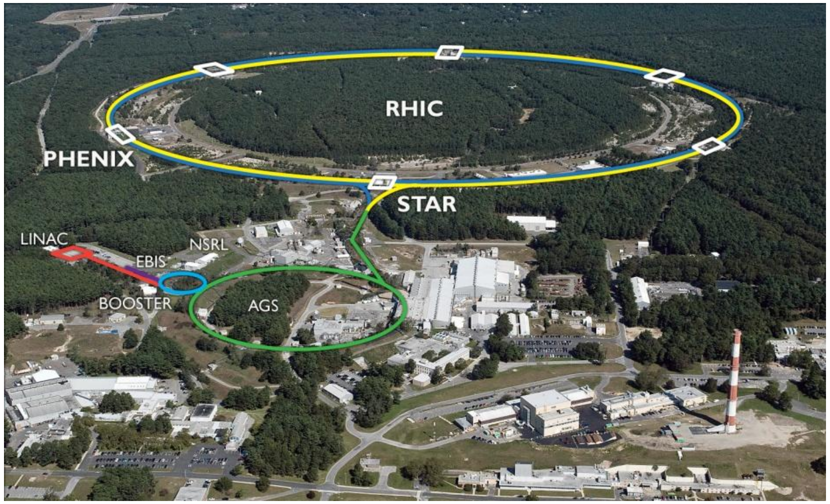
Introduction

Heavy-ion collisions:

- Unique tool to understand QCD matter
- Huge experimental and theoretical effort involved
- Explores thermal, non-thermal and vacuum properties of QCD
- Complex time evolution

Is an ab-initio description based entirely on quantum field theory possible?



http://www.rhip.utexas.edu/images/content_photos/rhic.jpg

http://startswithabang.com/wp-content/uploads/2008/05/lhc-sim.jpg

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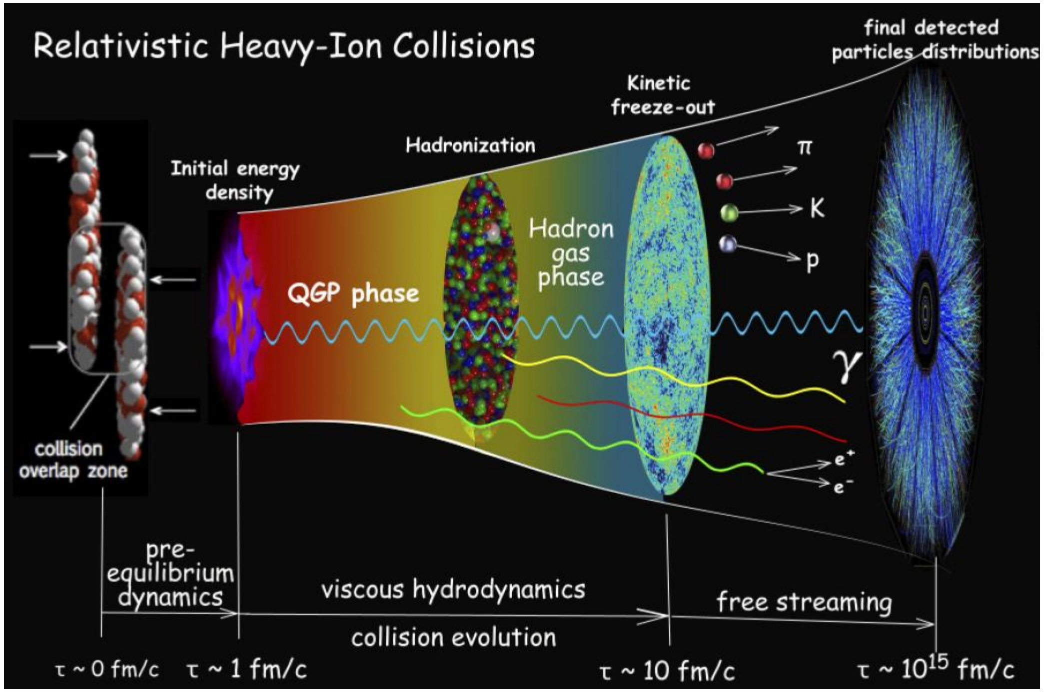
Introduction

Heavy-ion collisions:

- We focus on pre-equilibrium phase
- Goal: Derive hydrodynamics from QCD and compute transport coefficients
- Starting from a state dominated by saturated “soft” gluons



Picture by Paul Sorensen and Chun Shen

→ Initial conditions for weak coupling inspired by ***Color Glass Condensate*** picture (McLerran, Iancu, Venugopalan, Gelis, …)

Introduction

Non-equilibrium processes!

- Real-time description necessary
- Initial value problems
- So far Bjorken expansion is neglected

How to enhance bosonic fluctuations?

Usually: Initial overpopulation and/or instabilities

Quark-meson model

- Parametric resonance

QC_2D

- Initial overoccupation

Many options!

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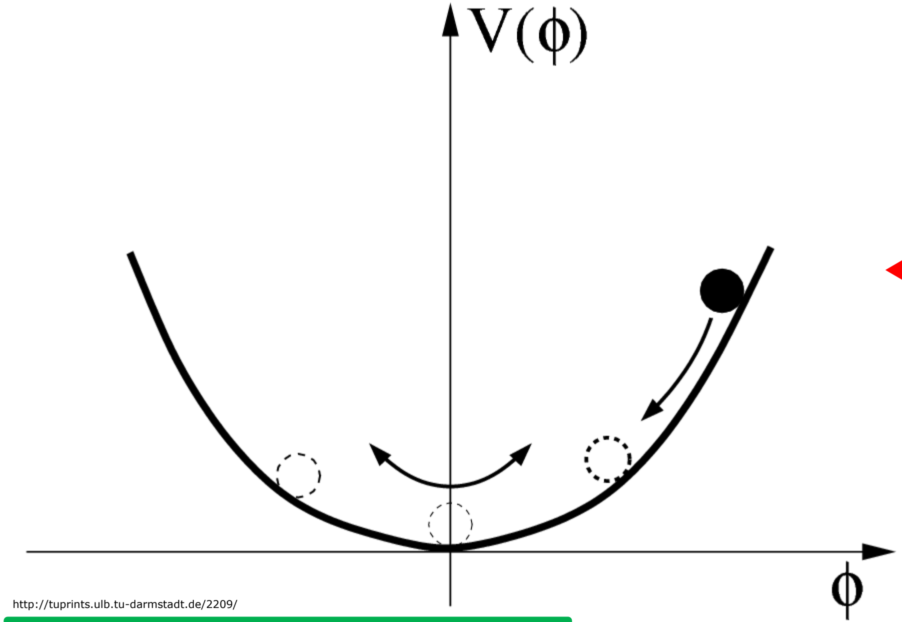
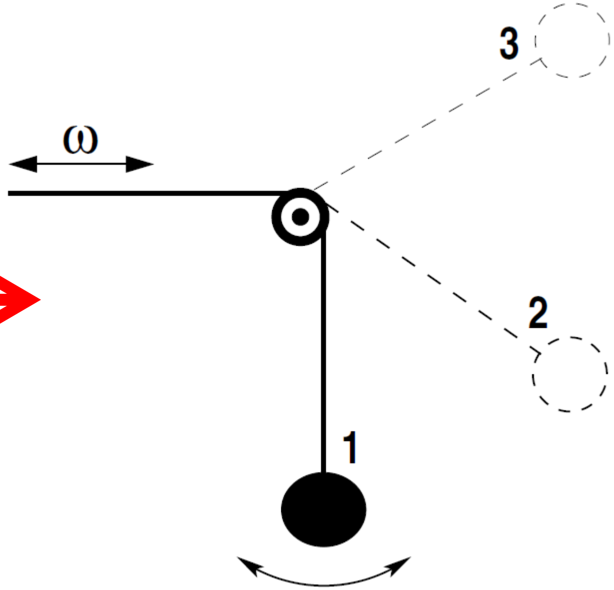
Introduction

Parametric resonance:



http://tuprints.ulb.tu-darmstadt.de/2209/

analogue



http://tuprints.ulb.tu-darmstadt.de/2209/

Relevant also for magnetic plasma instabilities:

J. Berges, S. Scheffler, S. Schlichting and D. Sexty, Phys. Rev. D 85, 034507 (2012)

$$x \leftrightarrow \langle \phi \phi \rangle (t)$$

$$\omega \leftrightarrow \langle \phi \rangle (t)$$

$$\ddot{x} + \omega^2(t) x = 0$$

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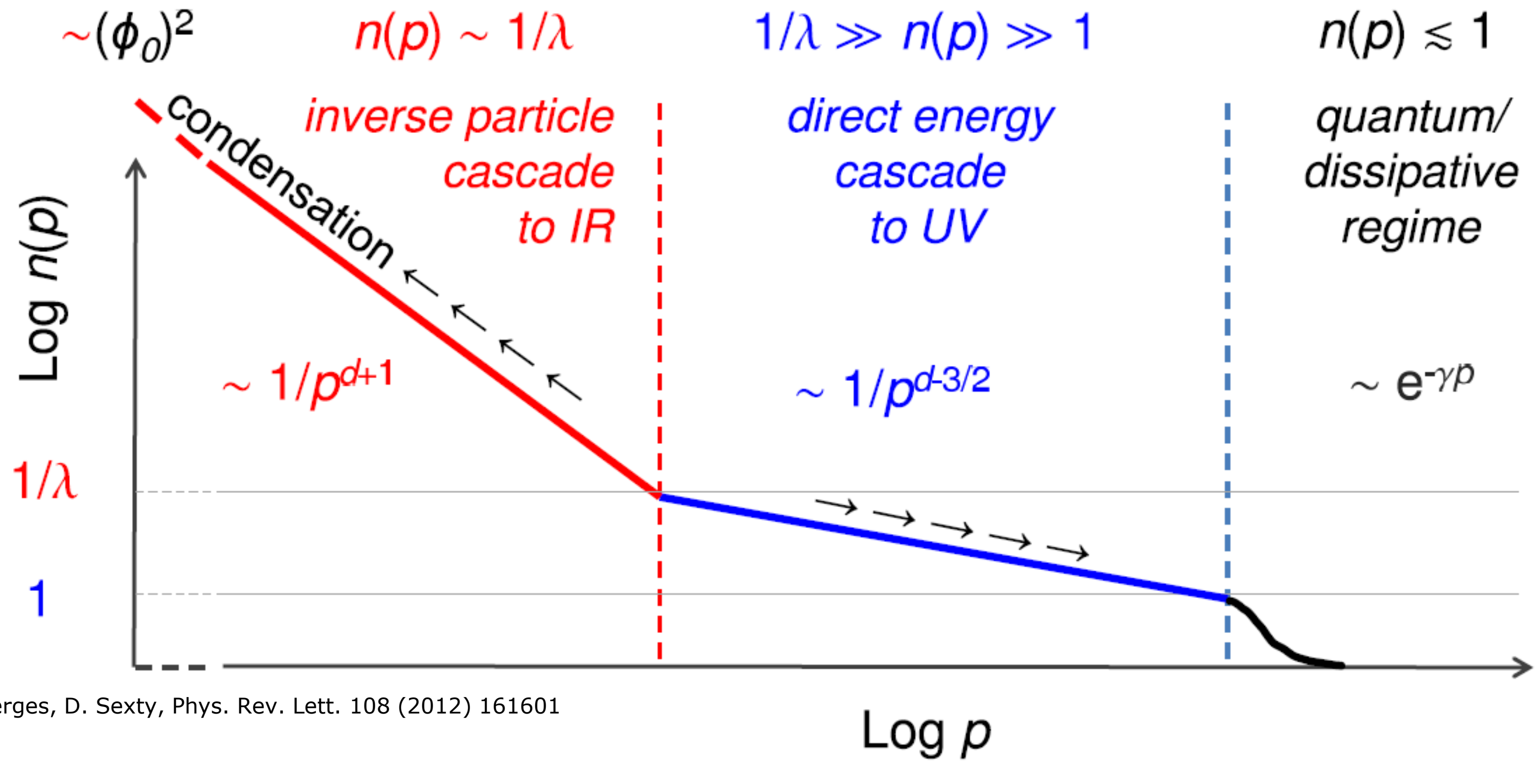
Introduction

Turbulence:

R. Micha, I. I. Tkachev, Phys.Rev. D70 (2004) 043538; J. Berges, A. Rothkopf, J. Schmidt, Phys.Rev.Lett. 101 (2008) 041603; J. Berges, D. Sexty, Phys. Rev. Lett. 108 (2012) 161601

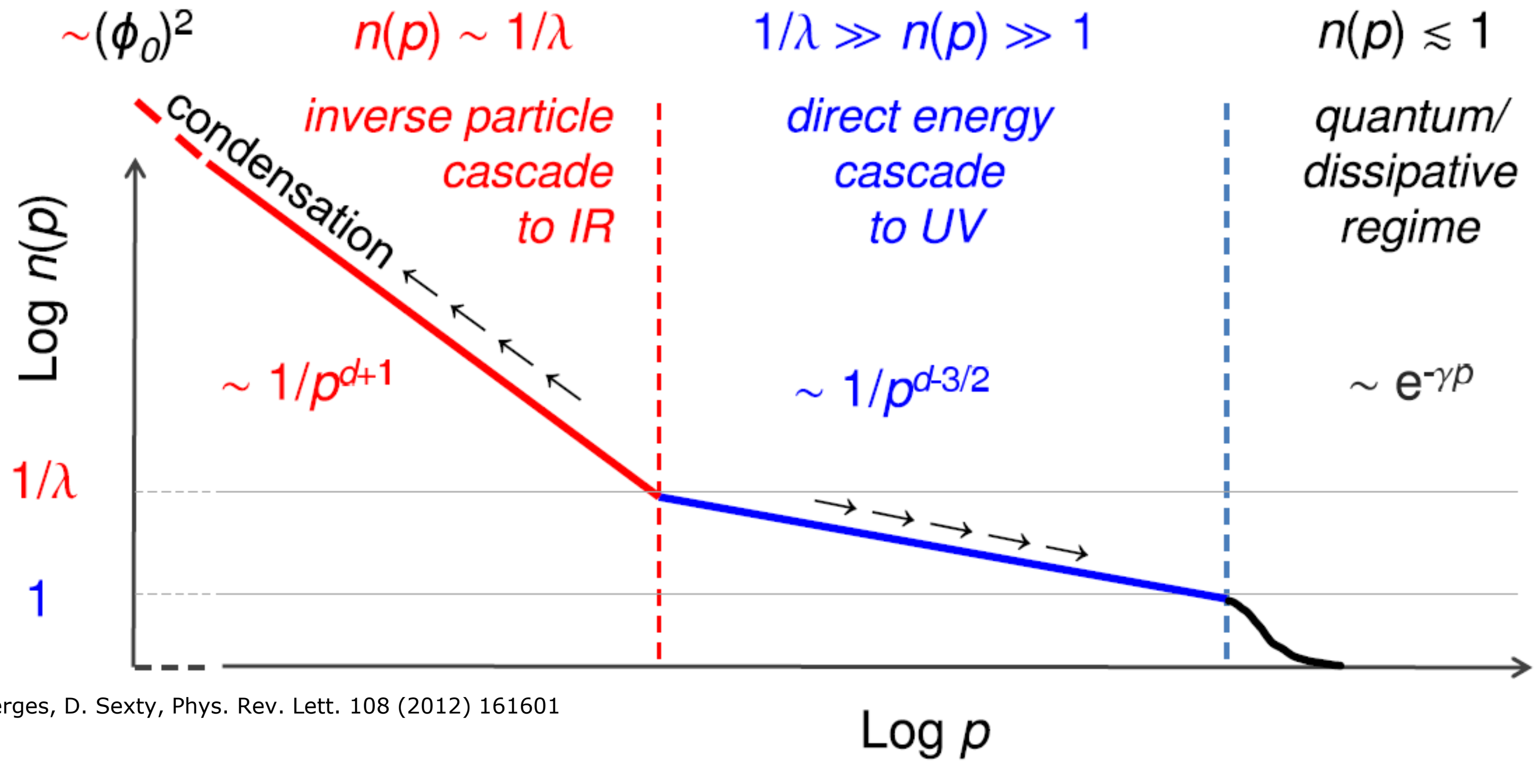
- Dual cascade



J. Berges, D. Sexty, Phys. Rev. Lett. 108 (2012) 161601

Introduction

What about quarks?



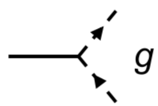
J. Berges, D. Sexty, Phys. Rev. Lett. 108 (2012) 161601

Implementation

Our model:

- 2 flavours of quarks coupled to mesons

$$\mathcal{L} = \bar{\psi}\left(i\partial_\mu\gamma^\mu\right)\psi + \frac{1}{2}\partial_\mu\phi_a\partial^\mu\phi_a - \frac{1}{2}m^2\phi_a\phi_a - \frac{\lambda}{4!\cdot N_s}(\phi_a\phi_a)^2 - \frac{g}{N_f}\bar{\psi}(\sigma + i\gamma_5\vec{\tau}\vec{\pi})\psi$$

  - Yukawa coupling  leads to dynamical mass generation:

$$m(t) = \frac{g}{N_f}\phi(t)$$

  - O(4) self-interacting  meson field $\phi = \{\sigma, \pi^1, \pi^2, \pi^3\}$
  - Macroscopic field $\phi(t) = \langle\sigma(t, \mathbf{x})\rangle$
  - 3+1 dimensions

- Start from initial conditions leading to parametric resonance!

Implementation

Lattice approach:

- Classical-statistical scalar fields: $\langle O \rangle = \int D\sigma_0 \, D\Pi_0 \; W[\sigma_0, \Pi_0] \; O_{\rm cl}[\sigma_0, \Pi_0]$

  with $O_{\rm cl}[\sigma_0, \Pi_0] \;=\; \int D\sigma_0 \; O[\sigma] \; \delta(\sigma - \sigma_{\rm cl}[\sigma_0, \Pi_0])$

  - Equations of motion including backreaction of fermions

$$\left(\Box_x + m^2\right)\sigma_{\rm cl}(x) + \frac{\lambda}{4!}\left(\sigma_{\rm cl}^2 + \vec{\pi}_{\rm cl}^2\right)\sigma_{\rm cl}(x) - \frac{g}{2}{\rm Tr}\left(F_\psi(x,x)\right) = 0$$

$$\left(\Box_x + m^2\right)\vec{\pi}_{\rm cl}(x) + \frac{\lambda}{4!}\left(\sigma_{\rm cl}^2 + \vec{\pi}_{\rm cl}^2\right)\vec{\pi}_{\rm cl}(x) - \frac{ig}{2}{\rm Tr}\left(F_\psi(x,x)\gamma_5\right) = 0$$

  - Fermion backreaction from statistical propagator

$$F(x,y;t) \equiv \frac{1}{2}\langle\left[\psi(x,t), \bar{\psi}(y,t)\right]\rangle$$

G. Aarts and J. Smit, Nucl. Phys. B 555 (1999) 35

- Quantum fermions using ♂/♀ - approach

S. Borsanyi and M. Hindmarsh, Phys. Rev. D 79 (2009) 065010

  - Statistical 'low-cost' method, scales like $\# N^d$ instead of N^{2d}

Implementation

♂/♀ - fermions:

- Set of auxiliary spinor fields $\psi_{M/F}(x)$ is introduced
- Equations of motion: $\left[i\partial_\mu \gamma^\mu - \frac{g}{2}\left(\sigma_{\rm cl}(x) + i\gamma_5 \vec{\tau}\vec{\pi}_{\rm cl}(x)\right)\right]\psi_g(x) = 0$
- Initialization:

$$\psi_{M,F}(t_0, \mathbf{x}) = \int \frac{d^3 p}{(2\pi)^3} \frac{e^{-i\mathbf{px}}}{\sqrt{2}} \sum_s \left(\xi_s(\mathbf{p}) u_s(\mathbf{p}) \pm \eta_s(\mathbf{p}) v_s(\mathbf{p})\right)$$

  - With random numbers

$$\langle \xi_s(\mathbf{p}) \xi^*_{s'}(\mathbf{q}) \rangle_{MF} = (2\pi)^3 \delta_{ss'} \delta(\mathbf{p} - \mathbf{q}) \left(1 - 2n^s_+(\mathbf{p})\right)$$
$$\langle \eta_s(\mathbf{p}) \eta^*_{s'}(\mathbf{q}) \rangle_{MF} = (2\pi)^3 \delta_{ss'} \delta(\mathbf{p} - \mathbf{q}) \left(1 - 2n^s_-(\mathbf{p})\right)$$

  - Average over all pairs of $\psi_{M/F}(x)$ for observables/backreaction

$$F_{\rm sto}(x,y;t) \equiv \left\langle \psi_M(x,t)\bar{\psi}_F(y,t)\right\rangle = \left\langle \psi_F(x,t)\bar{\psi}_M(y,t)\right\rangle$$

$$\boxed{F_{\rm sto}(x,y;t) \stackrel{!}{=} F(x,y;t)}$$

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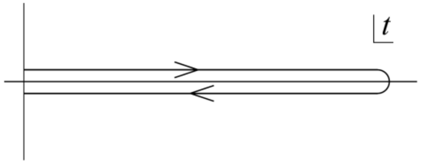
Implementation

2PI effective action: J. Berges, AIP Conf. Proc. 739, 3 (2005)

- Functional method to describe time evolution of quantum fields
  - Closed time path (in-in formalism)
  - Time evolution of two-point functions



$$\text{commutator:} \qquad \rho(x,y) = i\langle[\Phi(x),\Phi(y)]\rangle\,,$$

$$\text{anti-commutator:} \qquad F(x,y) = \frac{1}{2}\langle\{\Phi(x),\Phi(y)\}\rangle\,.$$

  - Kadanoff-Baym equations of motion

Memory integrals!

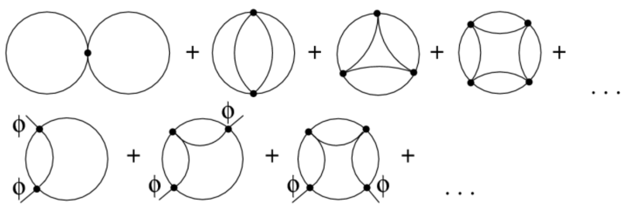
$$\left[\Box_x + M^2(x)\right]\rho(x,y) \;=\; -\int_{y^0}^{x^0} \mathrm{d}z\,\Sigma_\rho(x,z)\rho(z,y)\,,$$

$$\left[\Box_x + M^2(x)\right]F(x,y) \;=\; -\int_{0}^{x^0} \mathrm{d}z\,\Sigma_\rho(x,z)F(z,y) + \int_{0}^{y^0} \mathrm{d}z\,\Sigma_F(x,z)\rho(z,y).$$

Implementation

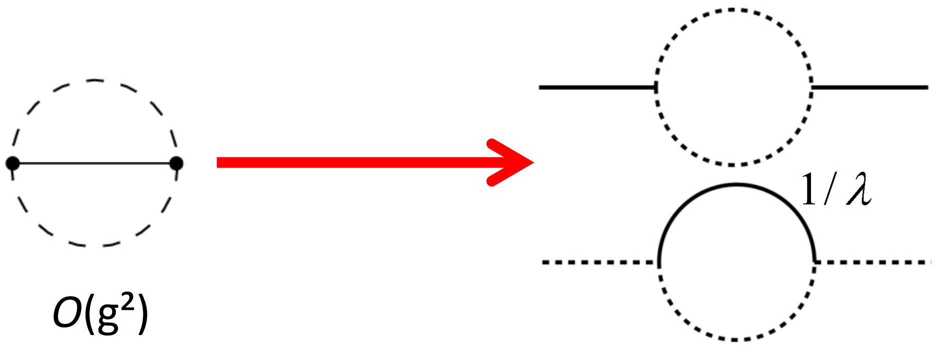
2PI effective action: J. Berges, AIP Conf. Proc. 739, 3 (2005)

- Different truncation schemes for the action
  - 1/N expansion to NLO in the number of scalar fields



J. Berges, Nucl. Phys. A 699 (2002) 847
G. Aarts et al, Phys. Rev. D 66 (2002) 045008

  - Coupling expansion to NLO in the Yukawa coupling



$O(g^2)$

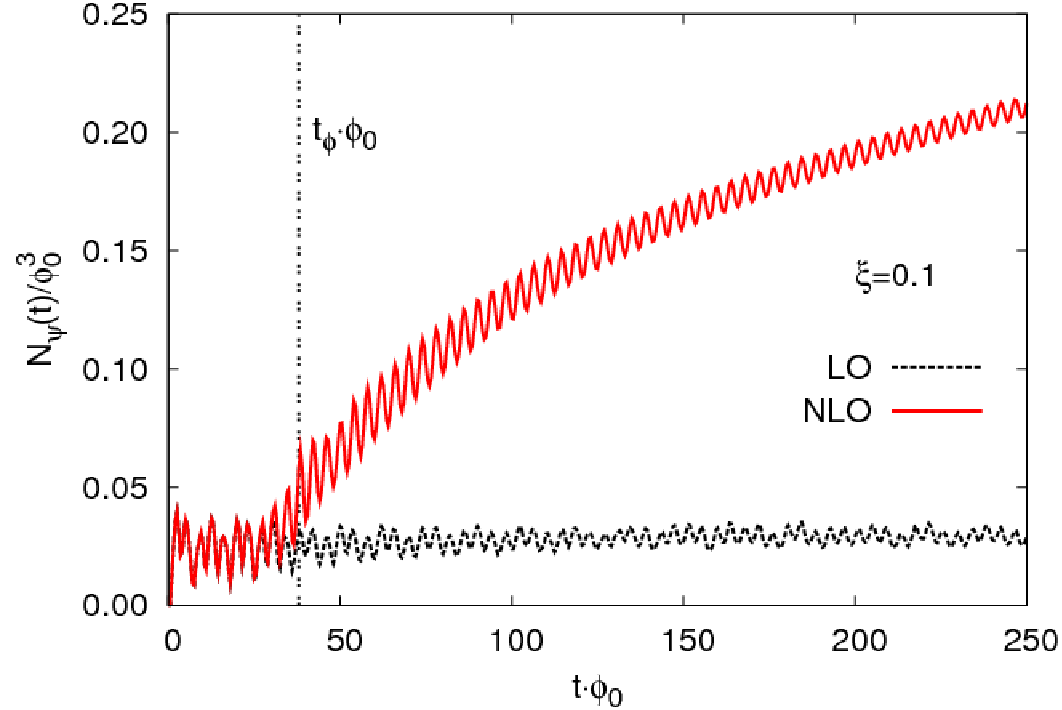
Contribution to meson self-energy

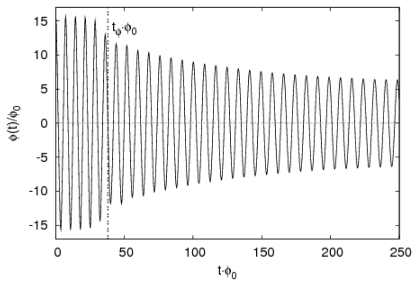
Quark self-energy

Results

Weak coupling:

- Total number of produced fermions strongly enhanced
- Quantum effects important, even at weak couplings
- Quark production rate $\sim \xi = g^2/\lambda$





LO:

$$\left[i\gamma^\mu \partial_{x,\mu} - \frac{g}{2}\phi(t)\right] F_\psi(x,y) = 0$$

No fluctuations!

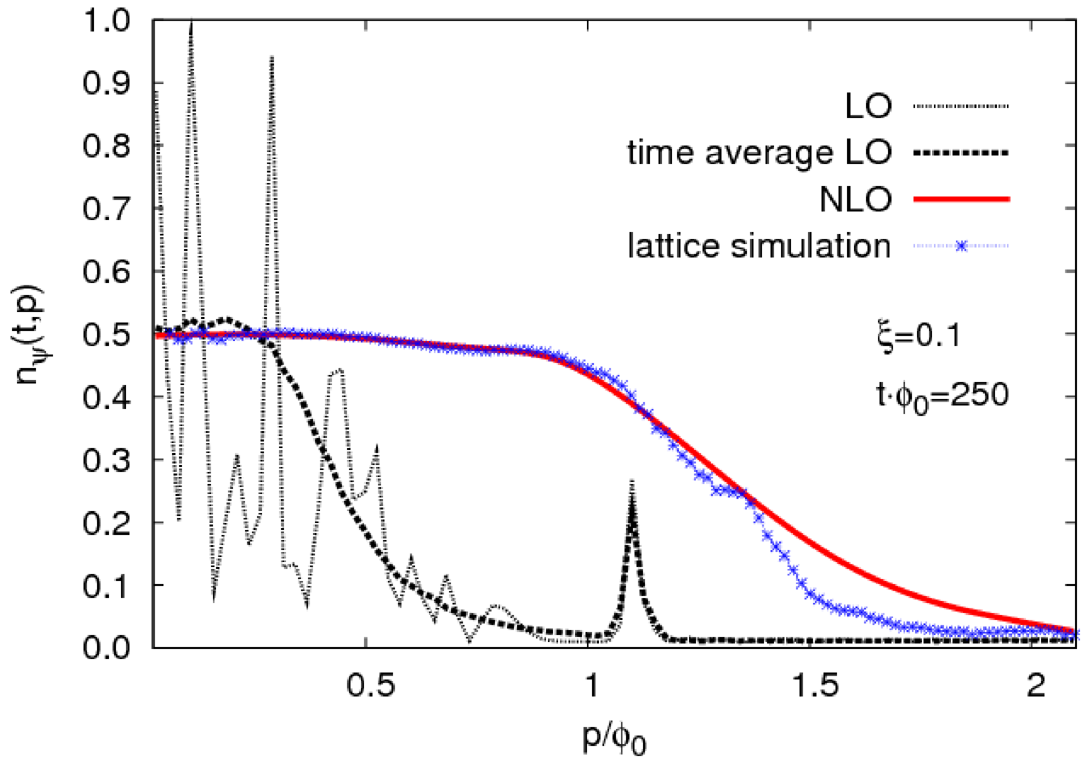
Baacke, Heitmann, Pätzold, PRD 58 (1998) 125013; Greene, Kofman, PLB 448 (1999) 6; Giudice, Peloso, Riotto, Tkachev, JHEP 9908 (1999) 014; Garcia-Bellido, Mollerach, Roulet, JHEP 0002(2000) 034; …

Results

Weak coupling:

- Qualitative and quantitative difference between LO and NLO
- Good agreement between lattice and 2PI
- Particle numbers drop at the rescaled initial field amplitude:

$$\phi_0 = \phi(t=0)/\sqrt{6N_s/\lambda}$$



LO:

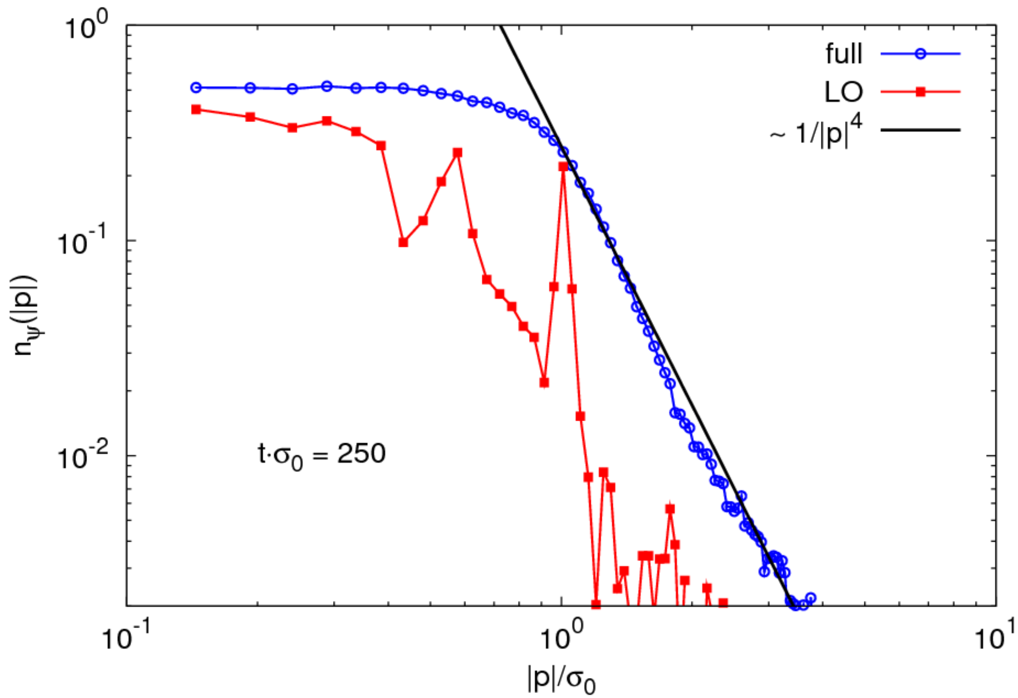
$$\left[i\gamma^\mu \partial_{x,\mu} - \frac{g}{2}\phi(t)\right] F_\psi(x,y) = 0$$

$$\xi = 0.1$$

Results

Weak coupling:

- Unexpected power-law dependence in the UV
- Exact value of exponent varies in time, stays ≈ 4
- Description in terms of perturbative decay/scattering possible?



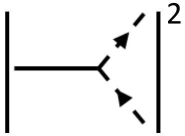
Both lattice and 2PI show similar exponents!

$\xi = 0.1$

Results

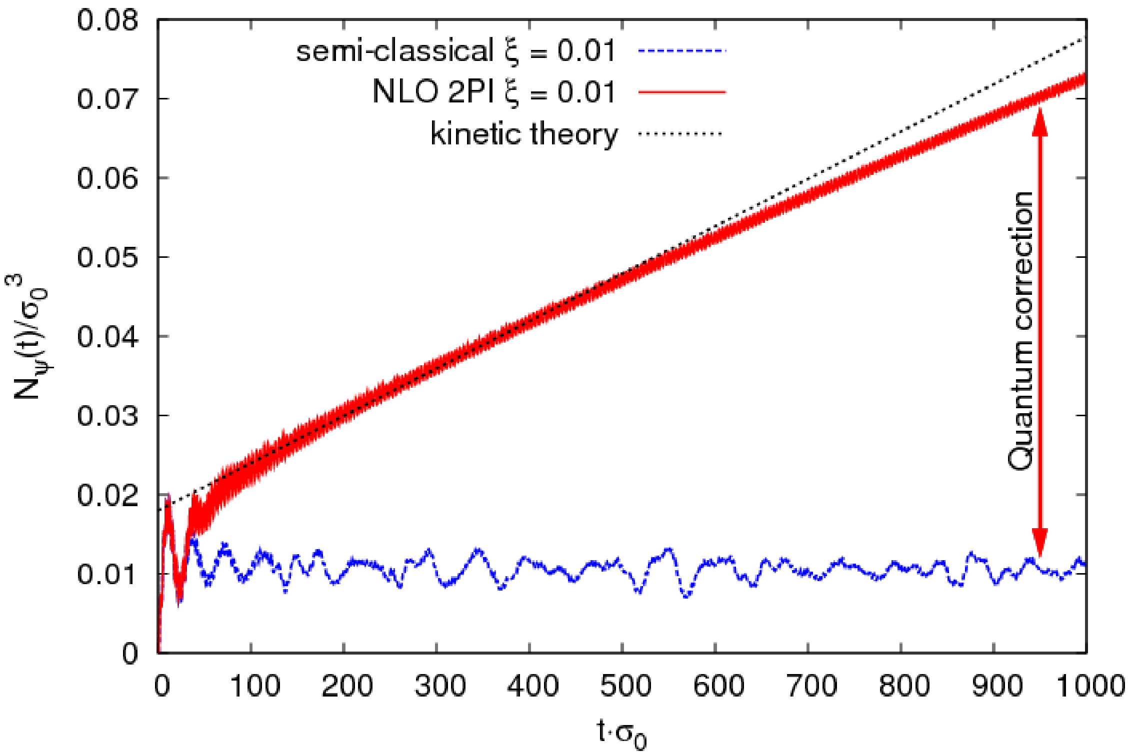
Weak coupling:

- Production rate extracted from LO perturbative decay



$O(g^2)$

- Assumptions:
  - Quasi-particles
  - Massless fermions
  - Early times ($n_\psi = 0$)
  - 'Frozen' bosonic sector

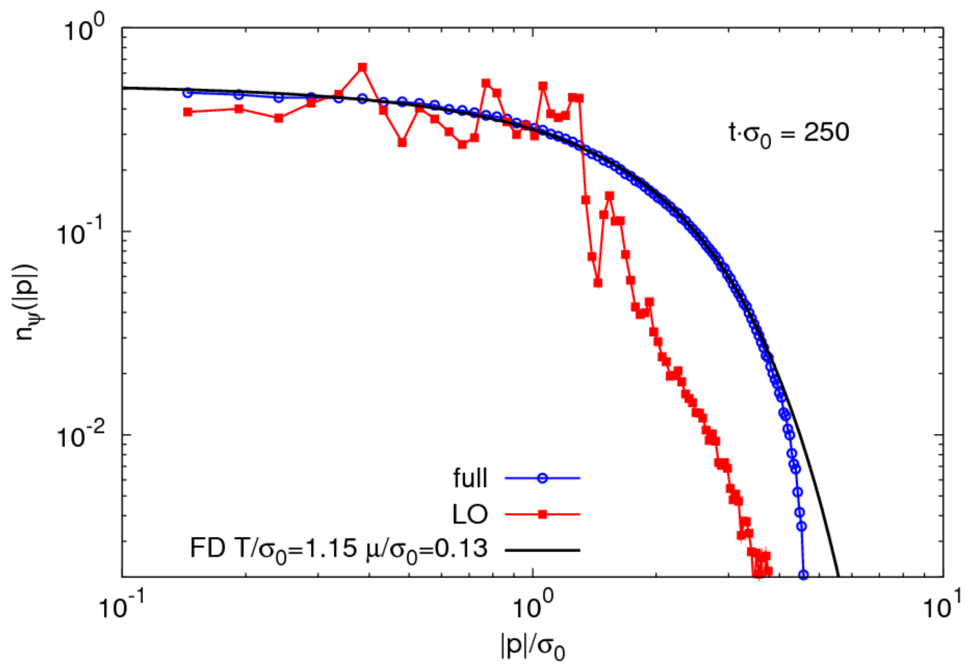


$$n_\phi(t,\mathbf{k}) \simeq \Theta(|\mathbf{k}| - \sigma_0)/\lambda \;\longrightarrow\; \partial_t N_\psi(t) \simeq \int \frac{d^3k}{(2\pi)^3} \Gamma_{\phi\to\psi\bar{\psi}}(\mathbf{k})\, n_\phi(t,\mathbf{k}) \sim \xi$$

Results

Strong coupling:

- Full dynamics shows higher occupancy in UV
- Fermions seem to take on a Fermi-Dirac distribution
- „Temperature" and „chemical potential" are fit parameters



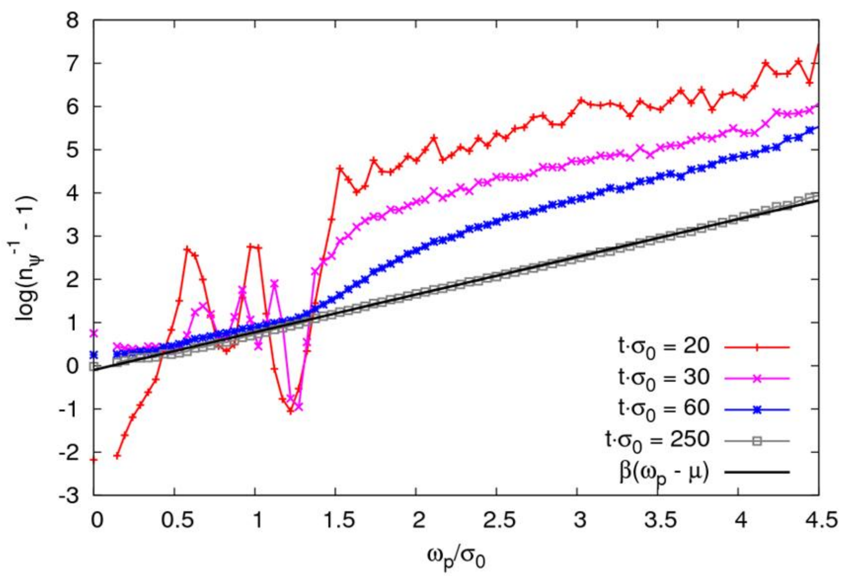
$\xi = 1.0$

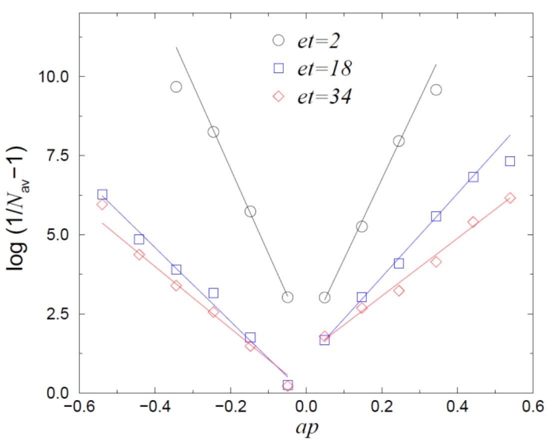
Stefan-Boltzmann limit: $T_{eq} = \sigma_0 \left(\frac{45 N_s}{\pi^2 \left(N_s + \frac{7}{2} N_f \right) \lambda} \right)^{\frac{1}{4}} \simeq 2.02 \sigma_0$

Results

Strong coupling:

- Quasi-thermalization starts in the IR and propagates to the UV
- Similar phenomenon observed in a 1+1d Abelian Higgs model





G. Aarts and J. Smit, Phys. Rev. D 61 (2000) 025002

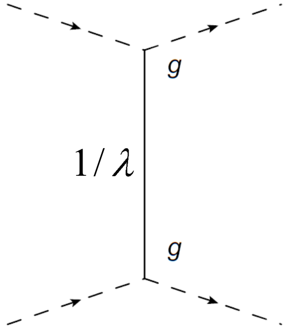
General feature of strongly interacting fermions out of equilibrium?

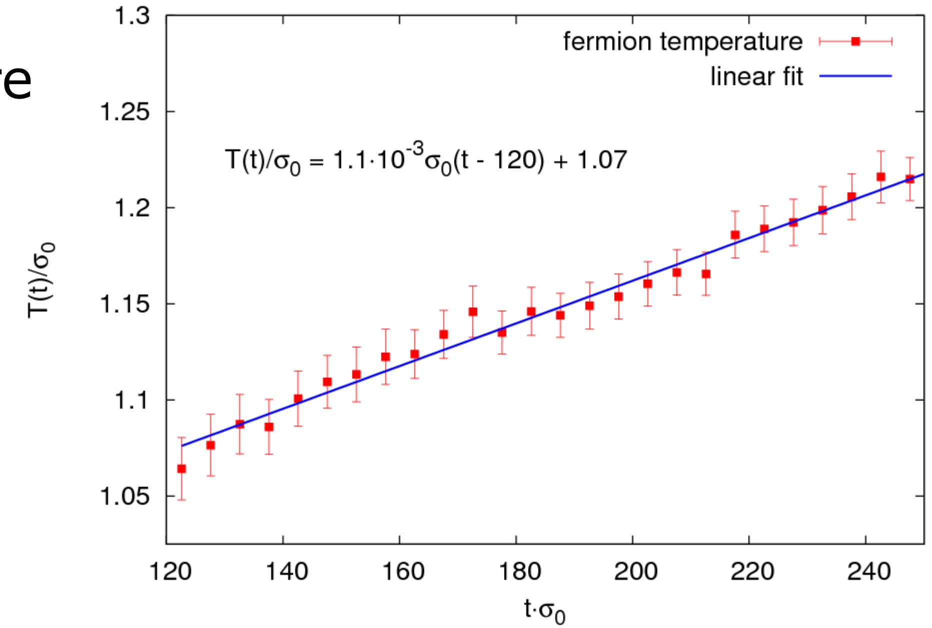
Results

Strong coupling:

- Linear rising temperature parameter
- Typical momentum of fermions grows in time
- Possible mechanism:

Efficient quark-quark scattering



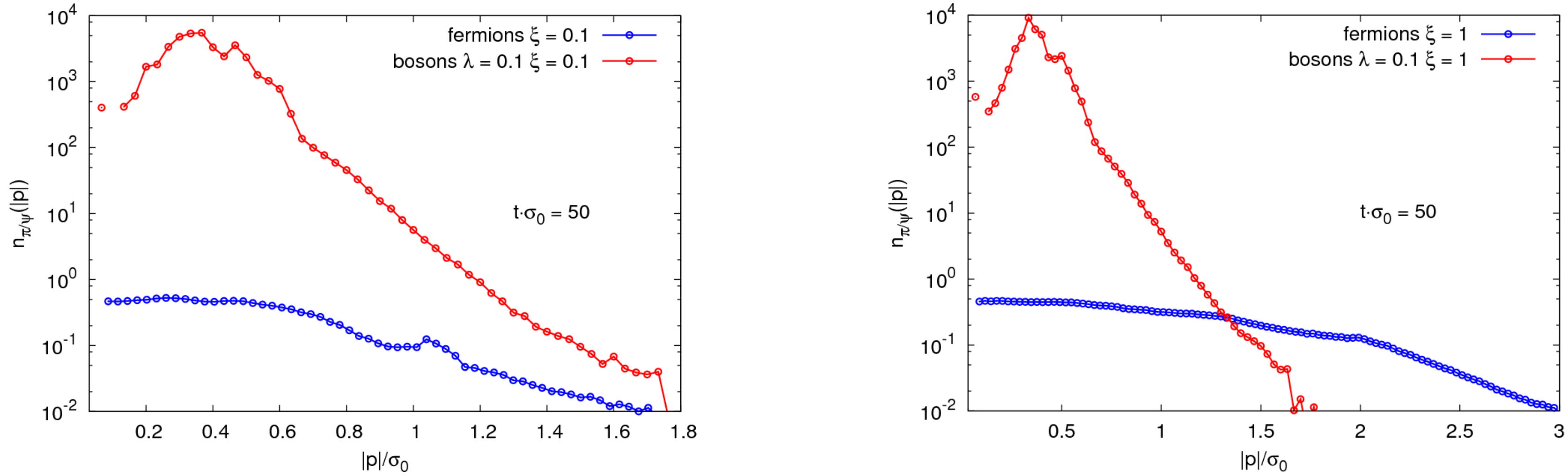


Is there a parametric separation between typical bosonic and fermionic momentum scales?

Results

From weak to strong:

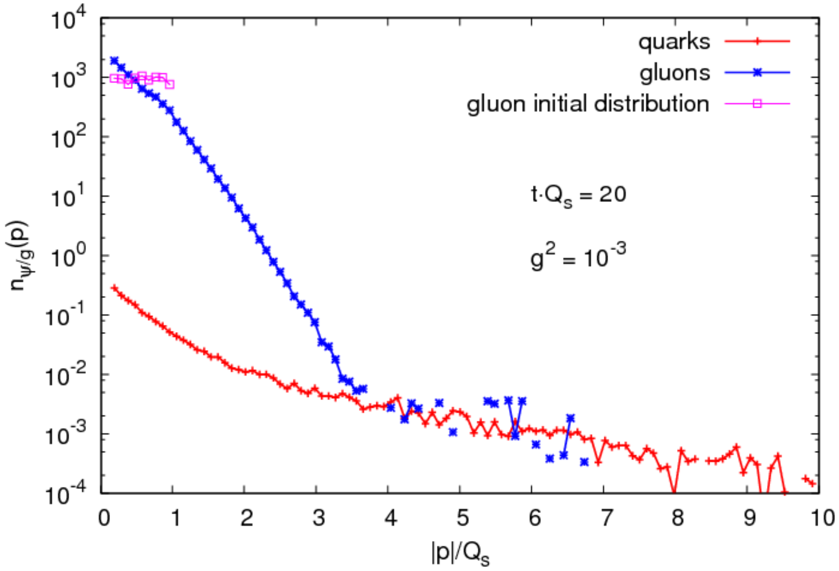
- Production of high-momentum quarks kinematically forbidden in LO perturbation theory at weak coupling
- Effective transport of energy and momentum to the UV at strong coupling

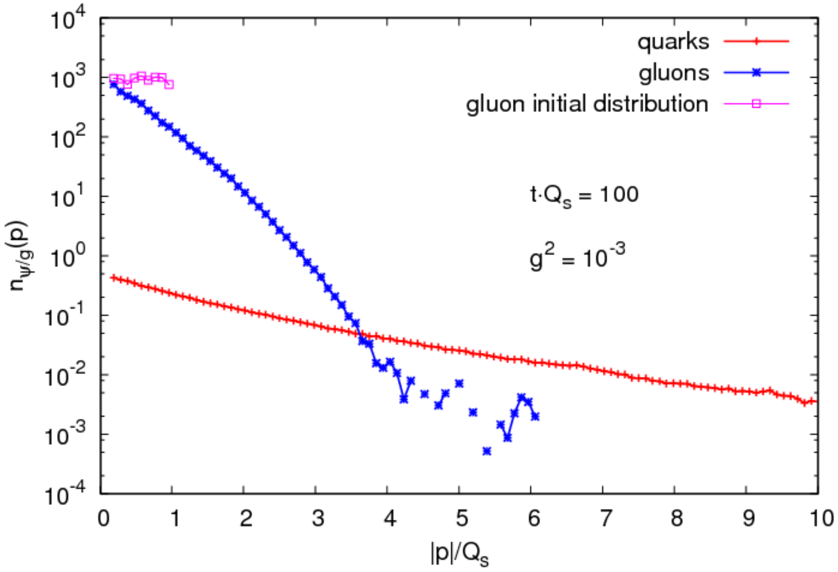


Results

QC_2D (work in progress):

- Preliminary results without backreaction
- Isotropic initial conditions with gluon occupancy of $1/g^2$ up to a scale Q_s

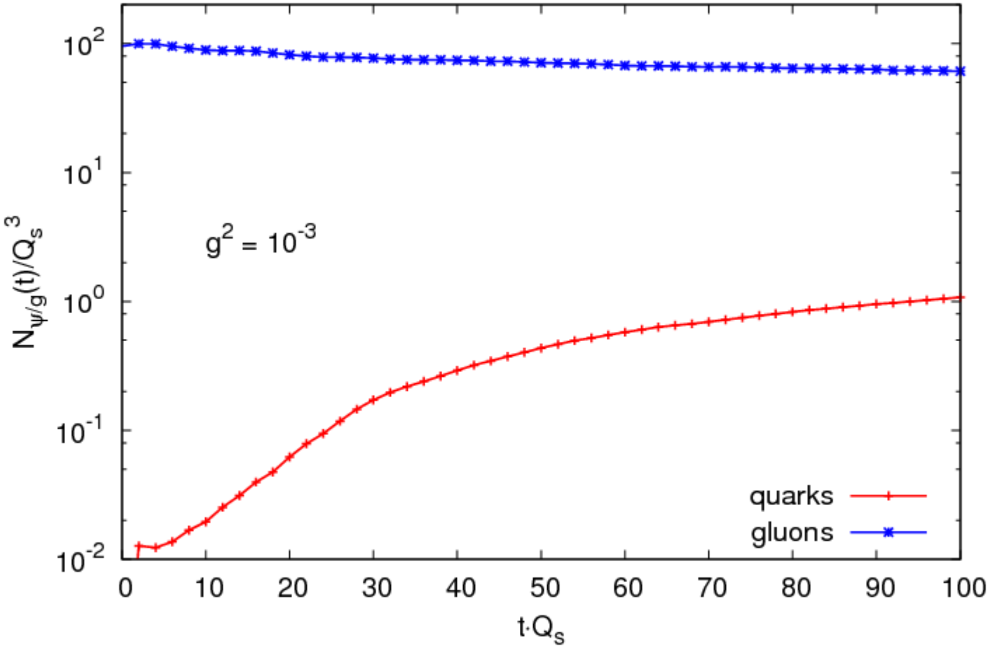




Results

QC_2D (work in progress):

- Similarities between scalar and gauge theories
- All particle numbers are defined in Coulomb-like gauge
- Many open questions wait to be answered:
  - What happens to turbulence?
  - Faster thermalization with quarks?
  - Impact of initial conditions?



Exiting challenges in both weak and strong coupling regime!

Detour: QFT of a plate capacitor

Schwinger model

- QED in 1+1 dimensions $\mathcal{S} = \int d^2x \left(\bar{\psi}[i\gamma^\mu D_\mu - m]\psi - \frac{1}{4}\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu} \right)$
  - Fermion pair production from electric fields

$$\frac{\Delta n^\pm}{LT} = \frac{eE_0}{2\pi} \exp\left(-\frac{\pi m^2}{eE_0}\right) = \frac{m^2\epsilon}{2\pi} \exp\left(-\frac{\pi}{\epsilon}\right) \qquad E_c = \frac{m^2}{e} \qquad \epsilon = \frac{E_0}{E_c}$$

  - Linear potential between charges
  - String formation/breaking similar to QCD

> - *No magnetic fields!*
> - *No propagating photons!*

- Plaquette formulation
  - Temporal axial gauge: $\mathcal{A}_0 = 0 \;\; U_0(\mathbf{x}) = 1$

$$\mathcal{S}_g[U] = \frac{1}{e^2 a_s a_t} \sum_{\mathbf{x}} \mathrm{Re}\left[1 - U_{01}(\mathbf{x})\right] \qquad \partial_\mu \mathcal{F}^{\mu\nu}(x,t) \;=\; -e\,\mathrm{Tr}\left[\gamma^\nu F(x,x;t)\right]$$

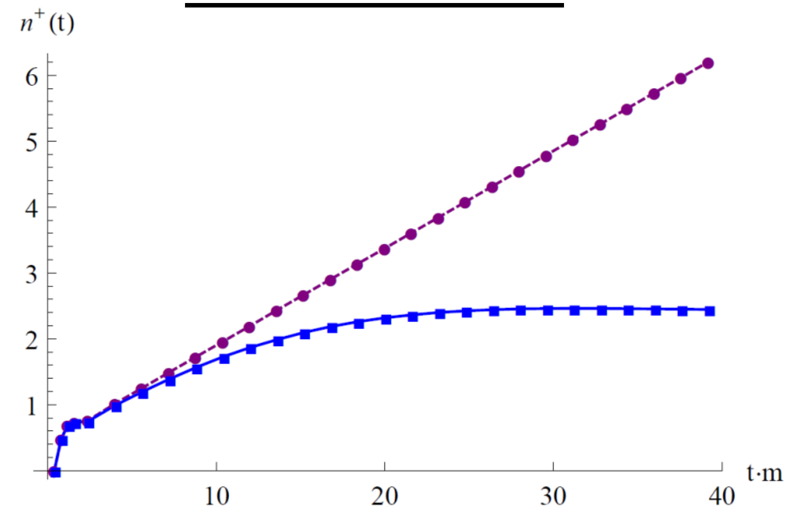
Results

Schwinger model:

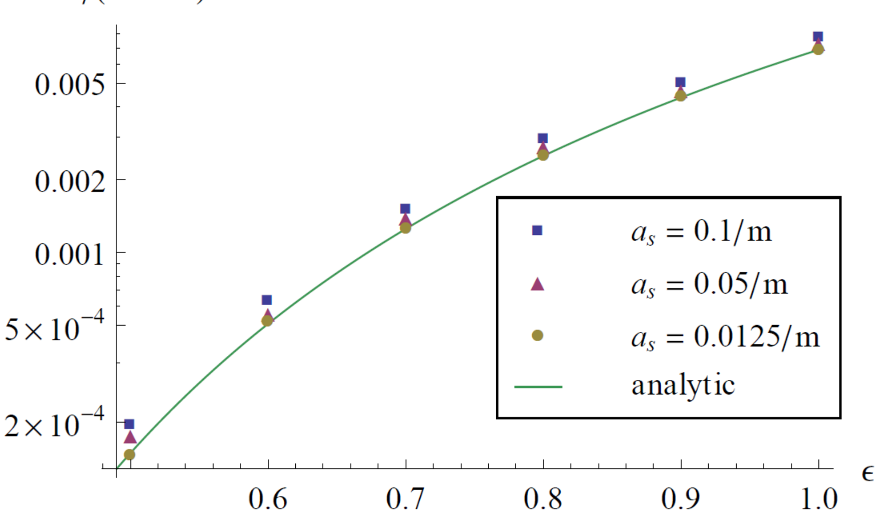
- Homogeneous field

$$E(x,t) = E_0$$

With backreaction:



No backreaction:



- *Convergence to analytic prediction in continuum limit*
- *Backreaction leads to saturation*
- *Earlier results successfully confirmed*

F. Hebenstreit, R. Alkofer and H. Gies, Phys. Rev. Lett. 107 (2011) 180403; F. Hebenstreit, PhD thesis, arXiv:1106.5965 [hep-ph]

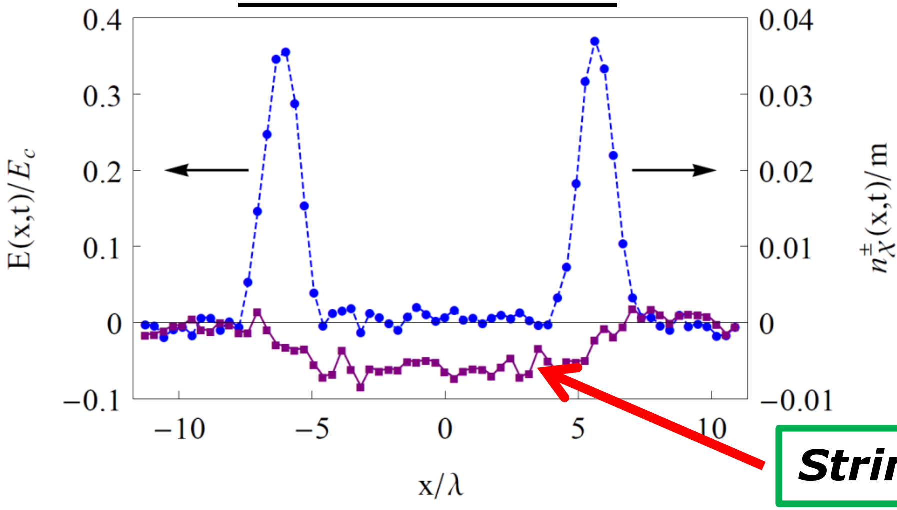
Results

Schwinger model:

- Inhomogeneous field

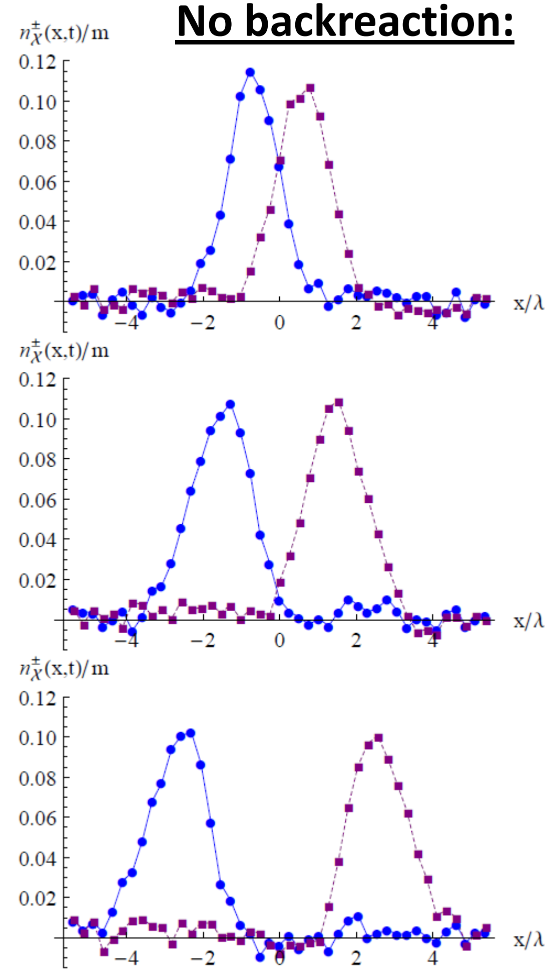
$$E(x,t) = E_0 \operatorname{sech}^2(\omega t) \exp\left(-\frac{x^2}{2\lambda^2}\right)$$

With backreaction:



String formation

No backreaction:



Results

String breaking:

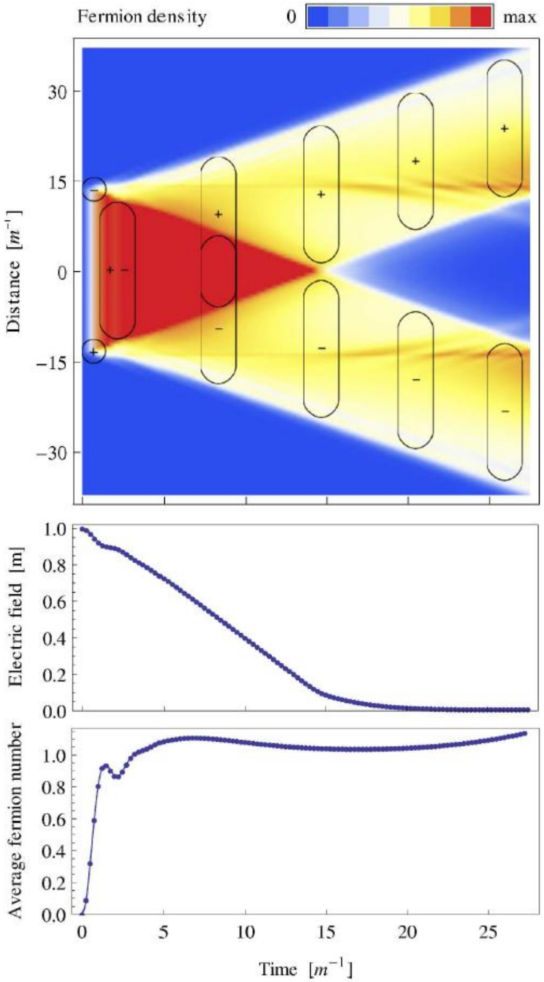
- Pair of static charges

$$\partial_x E = e\left[\delta(x + d/2) - \delta(x - d/2)\right]$$

- Dynamical two-stage process:
  - First production of overlapping oppositely charged fermion pairs
  - No screening
  - Then charges are separated by the external field
- Condition for string breaking:

$$V_{\text{str}} \gtrsim 2m + \underline{W} \longrightarrow d_{\mathbf{c}} \simeq 28.5/e$$

  - Multiple string breaking also possible!



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Summary & Outlook

Summary:

- Real-time simulations of fermions in 3+1d far from equilibrium
- Bosonic overpopulation leads to efficient quark production
- In strongly coupled systems a Fermi-Dirac distribution builds up
- Fermions transport energy to high momenta
- String breaking: Two stages, screening costs more than $2m_e$

Outlook:

- QC_2D results with backreaction
- Turbulence with quarks
- Generalization to expanding systems

+ ***Related issues in QED and cosmology***

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The End

Thank you for your attention!

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Supplement

Multiple string breaking:

- Two oppositely charged fermion bunches flying apart
- No external charges required
- Hypercritical field leads to multiple stages of matter production