τ-Decay and Hadronic Spectral Functions

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Motivation

- not all low energy resonances can be explained within the constituent quark picture
- too many observed states and some of them even have exotic quantum numbers
- broad resonances and blurred thresholds, mixing of states with same quantum numbers
- for example: a_1 meson $J^{PC} = 1^{++}$ at around 1.2 GeV
  - i) mass and decay width?
  - ii) chiral partner of the ρ meson or just a $\rho\pi$ molecule-like state? Wagner Leupold [arXiv:hep-ph/0801.0814]
- what about the other interactions that govern the phenomenology of our low energy color neutral objects?

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- what about the other interactions that govern the phenomenology of our low energy color neutral objects?
- in the constituent quark picture mesons are color neutral composite objects of quark and antiquark
- reduce complexity of QCD interaction by effective hadron hadron interaction in models with hadronic dofs and symmetries known from the QCD Lagrangian.

$$\mathscr{L} = \mathscr{L}_{\text{meson}} + \mathscr{L}_{\text{baryon}} + \mathscr{L}_{\text{dilaton}} + \mathscr{L}_{\text{weak}}$$

$$\begin{aligned}
\mathscr{L}_{\text{meson}} &= \mathrm{Tr}[(D_\mu\Phi)^\dagger(D^\mu\Phi)] - m_0^2\,\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\,\mathrm{Tr}(\Phi^\dagger\Phi)^2 \\
&+ c_1(\det\Phi - \det\Phi^\dagger)^2 + \mathrm{Tr}[H(\Phi+\Phi^\dagger)] - \frac{1}{4}\,\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) \\
&+ \mathrm{Tr}\left[\left(\frac{m_1^2}{2}+\Delta\right)(L_\mu^2 + R_\mu^2)\right] + \frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
&+ \frac{h_1}{2}\,\mathrm{Tr}(\Phi^\dagger\Phi)\,\mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2\,\mathrm{Tr}[(L_\mu\Phi)^2 + (\Phi R_\mu)^2] + 2h_3\,\mathrm{Tr}(L_\mu\Phi R^\mu\Phi^\dagger) \\
&+ \text{chirally invariant vector and axialvector four-point interaction vertices}
\end{aligned}$$

$$\begin{aligned}
\mathscr{L}_{\text{baryon}} &= \bar{\Psi}_{1L} i\gamma_\mu D_{1L}^\mu \Psi_{1L} + \bar{\Psi}_{1R} i\gamma_\mu D_{1R}^\mu \Psi_{1R} + \bar{\Psi}_{2L} i\gamma_\mu D_{2R}^\mu \Psi_{2L} + \bar{\Psi}_{2R} i\gamma_\mu D_{2L}^\mu \Psi_{2R} \\
&- \hat{g}_1(\bar{\Psi}_{1L}\Phi\Psi_{1R} + \bar{\Psi}_{1R}\Phi\Psi_{1L}) - \hat{g}_2(\bar{\Psi}_{2L}\Phi^\dagger\Psi_{2R} + \bar{\Psi}_{2R}\Phi^\dagger\Psi_{2L}) \\
&- M(\bar{\Psi}_{1L}\Psi_{2R} - \bar{\Psi}_{1R}\Psi_{2L} - \bar{\Psi}_{2L}\Psi_{1R} - \bar{\Psi}_{2R}\Psi_{L})
\end{aligned}$$

$$\mathscr{L}_{\text{dilaton}} = \frac{1}{2}(\partial^\mu G)^2 - \frac{1}{4}\frac{m_G}{\Lambda^2}\left(G^4 ln\left|\frac{G}{\Lambda}\right| - \frac{G^4}{4}\right)$$

$$\begin{aligned}
\mathscr{L}_{\text{weak}} &= \delta_w \frac{g\cos\theta_C}{2}\,\mathrm{Tr}[W_{\mu\nu}L^{\mu\nu}] + \delta_{em}\frac{e}{2}\,\mathrm{Tr}[B_{\mu\nu}R^{\mu\nu}] + \frac{1}{4}\,\mathrm{Tr}[(W^{\mu\nu})^2 + (B^{\mu\nu})^2] \\
&+ \frac{g}{2\sqrt{2}}\left(W_\mu^- \bar{u}_{\nu_\tau}\gamma_\mu(1-\gamma_5)u_\tau + \text{h.c.}\right)
\end{aligned}$$

$N_F = 2$ and $N_F = 3$ meson multiplets:

(Pseudo-)Scalars $\Phi_{ij} \simeq < q_L \bar{q}_R >_{ij} \simeq \frac{1}{\sqrt{2}}(q_i \bar{q}_j - q_i \gamma_5 \bar{q}_j)$

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_N + a_0^0)}{\sqrt{2}} + \frac{i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{\star +} + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0)}{\sqrt{2}} + \frac{i(\eta_N - \pi^0)}{\sqrt{2}} & K_0^{\star 0} + iK^0 \\ K_0^{\star -} + iK^- & \bar{K}_0^{\star 0} + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix}$$

Lefthanded $L_{ij}^\mu \simeq < q_L \bar{q}_L >_{ij} \simeq \frac{1}{\sqrt{2}}(q_i \gamma^\mu \bar{q}_j + q_i \gamma_5 \gamma^\mu \bar{q}_j)$

$$L^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} + \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ + a_1^+ & K^{\star +} + K_1^+ \\ \rho^- + a_1^- & \frac{\omega_N - \rho^0}{\sqrt{2}} + \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{\star 0} + K_1^0 \\ K^{\star -} + K_1^- & \bar{K}^{\star 0} + \bar{K}_1^0 & \omega_S + f_{1S} \end{pmatrix}^\mu$$

Righthanded $R_{ij}^\mu \simeq < q_R \bar{q}_R >_{ij} \simeq \frac{1}{\sqrt{2}}(q_i \gamma^\mu \bar{q}_j - q_i \gamma_5 \gamma^\mu \bar{q}_j)$

$$R^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} - \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ - a_1^+ & K^{\star +} - K_1^+ \\ \rho^- - a_1^- & \frac{\omega_N - \rho^0}{\sqrt{2}} - \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{\star 0} - K_1^0 \\ K^{\star -} - K_1^- & \bar{K}^{\star 0} - \bar{K}_1^0 & \omega_S - f_{1S} \end{pmatrix}^\mu$$

Mesonic Lagrangian with Global Chiral Symmetry

D. Parganlija, F. Giacosa and D. H. Rischke, Phys. Rev. D 82 (2010) 054024 arXiv:1003.4934 [hep-ph]
D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, arXiv:1208.0585 [hep-ph]

Global Chiral Symmetry:

$$\begin{aligned}\mathscr{L}_{\text{meson}} &= \text{Tr}[(D_\mu\Phi)^\dagger(D^\mu\Phi)] - m_0^2\,\text{Tr}(\Phi^\dagger\Phi) - \lambda_1[\text{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\,\text{Tr}(\Phi^\dagger\Phi)^2 \\ &+ c_1(\det\Phi - \det\Phi^\dagger)^2 + \text{Tr}[H(\Phi+\Phi^\dagger)] - \frac{1}{4}\text{Tr}({L_{\mu\nu}}^2 + {R_{\mu\nu}}^2) \\ &+ \text{Tr}\left[\left(\frac{m_1^2}{2}+\Delta\right)(L_\mu^2 + R_\mu^2)\right] + \frac{g_2}{2}(\text{Tr}\{L_{\mu\nu}[L^\mu,L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu,R^\nu]\}) \\ &+ \text{ch. inv. 4-point interactions among (pseudo-)scalars and (axial-)vectors}\end{aligned}$$

$U(N_F)_L \times U(N_F)_R$ Transformation:

$$\Phi \to U_L\Phi U_R^\dagger\,,\;\; L^\mu \to U_L L^\mu U_L^\dagger\,,\;\; R^\mu \to U_R R^\mu U_R^\dagger$$

Covariant Derivative:

$$D^\mu\Phi = \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu)$$

Field Strength Tensors:

$$L^{\mu\nu} = \partial^\mu L^\nu - \partial^\mu L^\nu\,,\;\; R^{\mu\nu} = \partial^\mu R^\nu - \partial^\nu R^\mu$$

Explicit Breaking of Chiral Symmetry

D. Parganlija, F. Giacosa and D. H. Rischke, Phys. Rev. D 82 (2010) 054024 arXiv:1003.4934 [hep-ph]
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$U(1)_A$-Anomaly

$$c_1 (\det \Phi - \det \Phi^\dagger)^2$$

non-vanishing quark masses, NO isospin breaking

$$\operatorname{Tr}[H(\Phi + \Phi^\dagger)]\,, \;\; H = h_a t^a \quad (N_f = 3\,, \Delta)$$

remaining symmetry is $U(2)_V$

Spontaneous Breaking of Chiral Symmetry

D. Parganlija, F. Giacosa and D. H. Rischke, Phys. Rev. D 82 (2010) 054024 arXiv:1003.4934 [hep-ph]

D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, arXiv:1208.0585 [hep-ph]

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Spontaneous breaking of global chiral symmetry by non-zero scalar condensate

$$\sigma \to \sigma + \phi\,,\ \ \phi = Zf_\pi$$

i) $m_\rho^2 = m_1^2 + \frac{\phi^2}{2}(h_1+h_2+h_3)\ , \quad m_{a_1}^2 = m_1^2 + (g_1\phi)^2 + \frac{\phi^2}{2}(h_1+h_2-h_3)$

ii) 3 point interaction vertices and mixing terms in $(D^\mu\Phi)^\dagger D_\mu\Phi$ that are proportional to the VEV ϕ.

$U(2)_L \times U(2)_R$ Symmetry in the Baryonic Sector

S. Gallas, F. Giacosa and D. H. Rischke, Phys. Rev. D 82 (2010) 014004 arXiv:0907.5084 [hep-ph]

S. Gallas, F. Giacosa and G. Pagliara, Nucl. Phys. A 872 (2011) 13 arXiv:1105.5003 [hep-ph]

Baryons in the mirror assignment:

$$\begin{aligned}\mathscr{L}_{\text{baryon}} &= \bar{\Psi}_{1L} i\gamma_\mu D^\mu_{1L}\Psi_{1L} + \bar{\Psi}_{1R} i\gamma_\mu D^\mu_{1R}\Psi_{1R} + \bar{\Psi}_{2L} i\gamma_\mu D^\mu_{2R}\Psi_{2L} + \bar{\Psi}_{2R} i\gamma_\mu D^\mu_{2L}\Psi_{2R} \\ &- \hat{g}_1(\bar{\Psi}_{1L}\Phi\Psi_{1R} + \bar{\Psi}_{1R}\Phi\Psi_{1L}) - \hat{g}_2(\bar{\Psi}_{2L}\Phi^\dagger\Psi_{2R} + \bar{\Psi}_{2R}\Phi^\dagger\Psi_{2L}) \\ &- M(\bar{\Psi}_{1L}\Psi_{2R} - \bar{\Psi}_{1R}\Psi_{2L} - \bar{\Psi}_{2L}\Psi_{1R} - \bar{\Psi}_{2R}\Psi_{L})\end{aligned}$$

$U(2)_L \times U(2)_R$ Transformation

$$\Psi_{1\mathrm{R}} \to U_R\Psi_{1\mathrm{R}}\,,\ \ \Psi_{1\mathrm{L}} \to U_L\Psi_{1\mathrm{L}}$$
$$\Psi_{2\mathrm{R}} \to U_L\Psi_{2\mathrm{R}}\,,\ \ \Psi_{2\mathrm{L}} \to U_R\Psi_{2\mathrm{L}}$$

Covariant Derivative

$$D^\mu_{1\mathrm{R}} = \partial^\mu - ic_1R^\mu\,,\ \ D^\mu_{1\mathrm{L}} = \partial^\mu - ic_1L^\mu$$
$$D^\mu_{2\mathrm{R}} = \partial^\mu - ic_2R^\mu\,,\ \ D^\mu_{2\mathrm{L}} = \partial^\mu - ic_2L^\mu$$

- allows for chirally invariant mass term generated by the gluon and/or tetraquark condensate
- Nucleons N, N^* are real chiral partners $N(1650)$ is favoured as chiral partner of $N(939)$
- yields correct nuclear matter saturation

Scale Invariance and the Glueball

S. Janowski, D. Parganlija, F. Giacosa and D. H. Rischke, Phys. Rev. D 84 (2011) 054007 arXiv:1103.3238 [hep-ph]

Scale invariance of the QCD Lagrangian is broken on the quantum level

$$\mathscr{L}_{\text{dilaton}} = \frac{1}{2}\left(\partial^{\mu} G\right)^{2} - \frac{1}{4}\frac{m_{\mathrm{G}}}{\Lambda^{2}}\left(G^{4} ln\left|\frac{G}{\Lambda}\right| - \frac{G^{4}}{4}\right)$$

LσM is in principle scale invariant, only mass terms and $U(1)_A$-anomaly break scale invariance

$$x^{\mu} \to \lambda^{-1} x^{\mu}\,,\ \varphi(x) \to \lambda\varphi(\lambda^{-1}x)\,,\ \Psi(x) \to \lambda^{\frac{3}{2}}\Psi(\lambda^{-1}x)$$

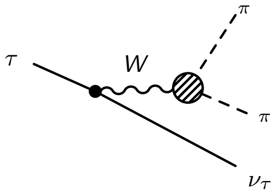
Scalar glueball is associated with fluctuations of the dilaton potential

Ground state of dilaton G_0 is related to the gluon condensate $G_0 = \Lambda = \frac{\sqrt{11}}{2m_G} C^2$

- favours $q\bar{q}$ interpretation of $f_0(1370)$ as chiral partner of the pion ($f_0(500)$ is disfavoured) and $f_0(1500)$ is 75% glueball
- scale invariance extended to $\mathscr{L}_{\text{meson}}, \mathscr{L}_{\text{baryon}}$ by parametrization of meson and baryon masses by G

τ-Decay

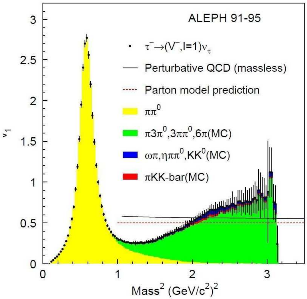
- semileptonic τ-decay involves strong and weak interactions



- describe effective electroweak interactions of hadrons in the vacuum.
- results can be further used to perform calculations at nonzero temperature and density (e.g. dilepton decay rate) and to understand more about the nature of resonances such as a_1, e.g. $\bar{q}q$ or $\rho\pi$-state?

ALEPH Spectral Functions

Vector Channel $\tau \to \nu_\tau 2\pi \nu_\tau$

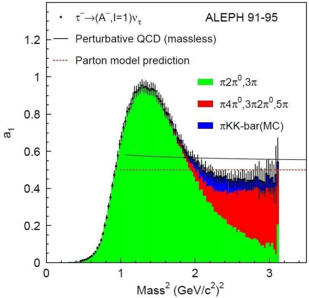


$B(\tau \to \pi^- \pi^0 \nu_\tau) = 25.47\%$

$\vec{\rho} \simeq \bar{q}\gamma_\mu \vec{\tau} q$

$J^{PC} = 1^{--}$

Axial-Vector Channel $\tau \to \nu_\tau 3\pi \nu_\tau$



$B(\tau \to \pi^- 2\pi^0 \nu_\tau + 2\pi^- \pi^+ \nu_\tau) = 18.28\%$

$\vec{a}_1 \simeq \bar{q}\gamma_\mu \vec{\tau}\gamma_5 q$

$J^{PC} = 1^{++}$

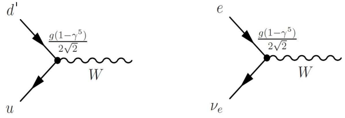
- precisely measured τ spectral functions allow for the connection between nature and phenomenological model

What do we know about weak interaction?

- $SU(2)_L \times U(1)_Y$ gauge symmetry with gauge fields W^μ and B^μ
- Weinberg mixing between $SU(2)_L \times U(1)_Y$ gauge fields B^μ, W_3^μ to physical interaction fields A^μ, Z_0^μ .
- Cabibbo mixing, flavour eigenstates are not weak eigenstates.
- Charged interaction violates symmetry under Charge and Parity transformations but preserves combined CP;
  $W_\pm^\mu$ act on left-handed particles, right-handed antiparticles only

$$P_L = \frac{1-\gamma_5}{2} \qquad P_R = \frac{1+\gamma_5}{2} \, .$$

- Charged bosons induce flavour-changing processes.



Linear Sigma Model with Weak Interaction

i) transformation of composite quarks

$\Phi_{kl} \simeq < \bar{q}_R q_L >_{kl}\,, \quad q_k \xrightarrow{U(1)_Y} e^{i y_k \Theta_Y(X)} q_k\,, \quad y_k = 2(Q_k - I_{3\,k})$

ii) $U(1)_Y$ Transformation:

Scalar and Pseudoscalar Fields:

$$\Phi \xrightarrow{U(1)_Y} \Phi U_Y^\dagger \simeq \Phi + i\Theta_Y \Phi t_3$$

Gauge Field:

$$B^\mu \xrightarrow{U(1)_Y} U_Y B^\mu U_Y^\dagger + \frac{i}{g'} U_Y \partial^\mu U_Y^\dagger$$

Righthanded Fields:

$$R^\mu \xrightarrow{U(1)_Y} U_Y R^\mu U_Y^\dagger$$

iii) Covariant Derivative:

$D_Y^\mu \Phi = \partial^\mu \Phi - i g_1 [L^\mu \Phi - \Phi(R^\mu + \frac{g'}{g_1} B_\mu t_3)]$

iv) Field Strength Tensor:

$R^{\mu\nu} = (\partial^\mu R^\nu - i g' [B^\mu, R^\nu]) - (\partial^\nu R^\mu - i g' [B^\nu, R^\mu])$

Local $SU(2)_L \times U(1)_Y$ Symmetry

$SU(2)_L \times U(1)_Y$ Transformations

$$\Phi \xrightarrow{SU(2)_L \times U(1)_Y} U_L \Phi U_Y^\dagger$$

$$L^\mu \xrightarrow{SU(2)_L} U_L L^\mu U_L^\dagger \qquad W^\mu \xrightarrow{SU(2)_L} U_L W^\mu U_L^\dagger + \frac{i}{g} U_L \partial^\mu U_L L^\mu U_L^\dagger$$

$$R^\mu \xrightarrow{U(1)_Y} U_Y R^\mu U_Y^\dagger \qquad B^\mu \xrightarrow{U(1)_Y} U_Y B^\mu U_Y^\dagger + \frac{i}{g} U_Y \partial^\mu U_Y B^\mu U_Y^\dagger$$

$SU(2)_L \times U(1)_Y$ Covariant Derivative:

$$D^\mu \Phi = \partial^\mu \phi - ig_1[(L^\mu + \frac{g}{g_1} W^\mu)\Phi - \Phi(R^\mu + \frac{g'}{g_1} B_\mu t_3)]$$

Field Strength Tensors:

$$R^{\mu\nu} = (\partial^\mu R^\nu - ig'[B^\mu, R^\nu]) - (\partial^\nu R^\mu - ig'[B^\nu, R^\mu])$$
$$L^{\mu\nu} = (\partial^\mu L^\nu - ig[W^\mu, L^\nu]) - (\partial^\nu L^\mu - ig[W^\nu, L^\mu])$$

Weinberg Mixing and Cabibbo Mixing

Weinberg Mixing:

neutral bare $SU(2)_L \times U(1)_Y$ gauge fields B^μ, W_3^μ are related to the physical fields A^μ, Z^μ by

$$\begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix}$$

and

$$e = g' \cos\theta_W = g \sin\theta_W$$

Cabibbo mixing:

strong isospin eigenstates d, s, b are related to the weak eigenstates by the CKM matrix

$N_f = 2$

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

Linear Sigma Model with Weak Interaction

$$\mathscr{L}_{\text{weak}} = \text{Tr}[(D_\mu \Phi)^\dagger (D^\mu \Phi)] + \delta_w \frac{g \cos\theta_C}{2} \text{Tr}[W_{\mu\nu} L^{\mu\nu}] + \delta_{em} \frac{e}{2} \text{Tr}[B_{\mu\nu} R^{\mu\nu}]$$
$$+ \frac{1}{4} \text{Tr}[(W^{\mu\nu})^2 + (B^{\mu\nu})^2] + \frac{g}{2\sqrt{2}} \left(W_\mu^- \bar{u}_{\nu_\tau} \gamma_\mu (1-\gamma_5) u_\tau + \text{h.c.}\right)$$

The Decay $\tau \to W\nu_\tau$ is well known from SM.

Covariant Derivative with Physical Interaction Fields:

$$D^\mu \Phi \equiv \partial^\mu \Phi - ig_1(L^\mu \Phi - \Phi R^\mu) - ie[A^\mu t_3, \Phi] - ig(W_1^\mu t_1 + W_2^\mu t_2)\Phi$$
$$- ig\cos\theta_W (Z^\mu \Phi + \tan^2\theta_W \Phi Z^\mu)$$

Field Strength Tensors:

$$L^{\mu\nu} \equiv \{\partial^\mu L^\nu - ie[A^\mu t_3, L^\nu] - ig[W_1^\mu t_1 + W_2^\mu t_2, L^\nu] - ig\cos\theta_W [Z^\mu, L^\nu]\}$$
$$- \{\partial^\nu L^\mu - ie[A^\nu t_3, L^\mu] - ig[W_1^\nu t_1 + W_2^\nu t_2, L^\mu] - ig\cos\theta_W [Z^\nu, L^\mu]\}$$
$$R^{\mu\nu} \equiv \{\partial^\mu R^\nu - ie[A^\mu t_3, R^\nu] - ig\sin\theta_W [Z^\mu, R^\nu]\}$$
$$- \{\partial^\nu R^\mu - ie[A^\nu t_3, R^\mu] - ig\sin\theta_W [Z^\nu, R^\mu]\}$$

Spectral Functions

Spectral Density is taken as the Imaginary Part of the Propagator

$$d(s) = \frac{1}{\pi}\mathrm{Im}[\Delta(s)] \ , \quad \Delta(s) = \frac{1}{s - m_0^2 + g^2\mathrm{Re}[\Sigma(s)] + g^2 i\,\mathrm{Im}[\Sigma(s)]}$$

Optical Theorem:

$$g^2\mathrm{Im}[\Sigma(s)] = \sqrt{s}\,\Gamma(s)$$

Resonance Mass:

$$m_{\mathrm{res.}}^2 = m_0^2 - \mathrm{Re}[\Sigma(m_r)]$$

Sum Rules:

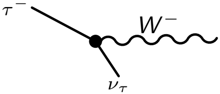
$$\int_0^\infty d_\rho(s)ds = 1 \ , \quad \int_0^\infty d_{a_1}(s)ds = 1$$

Spectral Functions

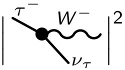
$$d_\rho(s) = \frac{1}{N_\rho}\frac{1}{\pi}\frac{\sqrt{s}\,\Gamma_{\rho\pi\pi}(s)}{(s - m_\rho^2)^2 + (\sqrt{s}\,\Gamma_\rho)^2} \ , \quad d_{a_1}(s) = \frac{1}{N_{a_1}}\frac{1}{\pi}\frac{\sqrt{s}\,\Gamma_{a_1\rho\pi}(s)}{(s - m_{a_1}^2)^2 + (\sqrt{s}\,\Gamma_{a_1})^2}$$

Vector Channel

Common to all channels is the process:

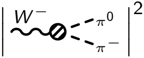


$$\Gamma_{\tau^- \to W^- 2\nu_\tau}(s) \sim \frac{|p(m_\tau^2, s, m_\nu^2)|}{m_\tau^2}$$

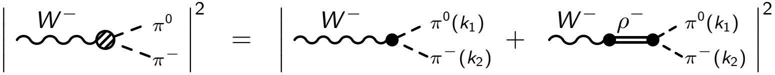


Vector Channel

$$\Gamma_{W^- \to \pi^- 2\pi^0}(s) \sim \frac{1}{s}$$



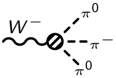
$$|p(s, m_\pi)|$$

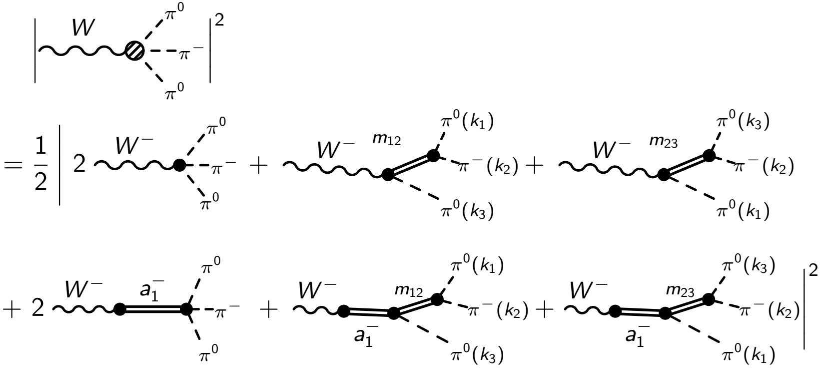


Axial-Vector Channel

Axial-Vector Channel

$$\Gamma_{W^- \to \pi^- 2\pi^0}(s) \sim \frac{1}{s}\frac{2}{2\cdot 3}\int \left| \; \right|^2 dm_{12}^2 dm_{23}^2$$

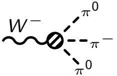


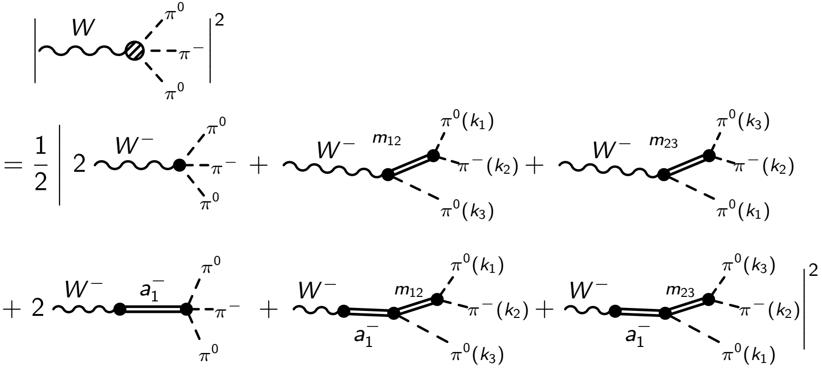


Axial-Vector Channel

Axial-Vector Channel

$$\Gamma_{W^- \to \pi^- 2\pi^0}(s) \sim \frac{1}{s}\frac{2}{2\cdot 3}\int \left| \quad \right|^2 dm_{12}^2\, dm_{23}^2$$

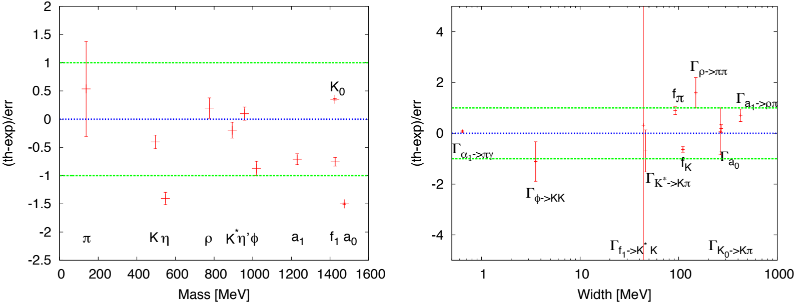




- $\Gamma(W^- \to \pi^- 2\pi^0) \simeq 1\%$ and $\Gamma(a_1^- \to \pi^- 2\pi^0) \simeq 1\%$
- in principle also contributions of σ resonance $\Gamma_{W \to \sigma 2\pi^0 \pi^-} \simeq 0$

Quest for the Parameters

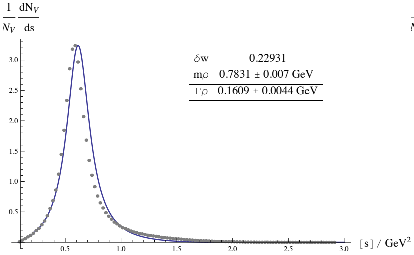
D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, D.H. Rischke
Scalar mesons in a linear sigma model with (axial-)vector mesons

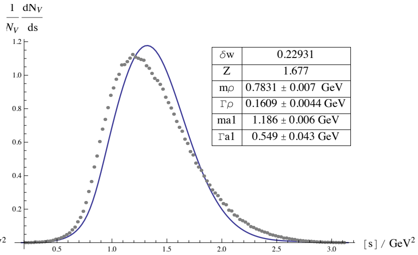


[arXiv:hep-ph/1208.0585]

- global fit of 13 parameters; test model
- 21 decay widths and masses

Results with $m_\rho, \Gamma_\rho, m_{a_1}, \Gamma_{a_1}$ and pion renormalisation constant Z as obtained from $N_f = 3$





- Only one free parameter δ_w which describes the mixing between the charged weak bosons and the (axial-)vector mesons!
- $\Gamma_{\rho^- \to \pi^- \pi^0}$, $\Gamma_{a_1^- \to \rho^- \pi^-}$
- We wanted to know if ρ and a_1 can be described as chiral partners. Yes!

  $W\rho$ mixing $\sim \delta_w s$ $\qquad$ Wa_1 mixing $\sim (\delta_w s + g_1 \phi^2)$
- the parameters have errors within range $\sim 5\%$ therefore we can still improve our results

Inclusive Spectral Functions VMA and VPA

Vector Channel not independent on values of m_{a_1} and Γ_{a_1}.

$\rightarrow$ Inclusive Spectral Functions $V - A$ and $V + A$

Inclusive Spectral Functions VMA and VPA

Vector Channel not independent on values of m_{a_1} and Γ_{a_1}.

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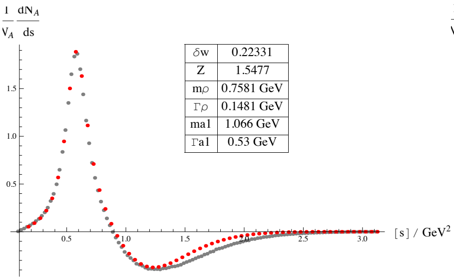
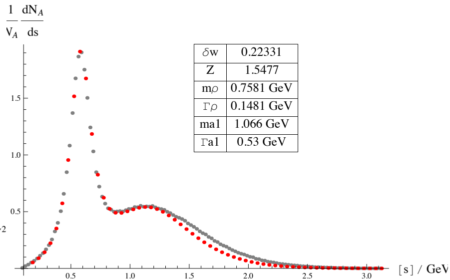


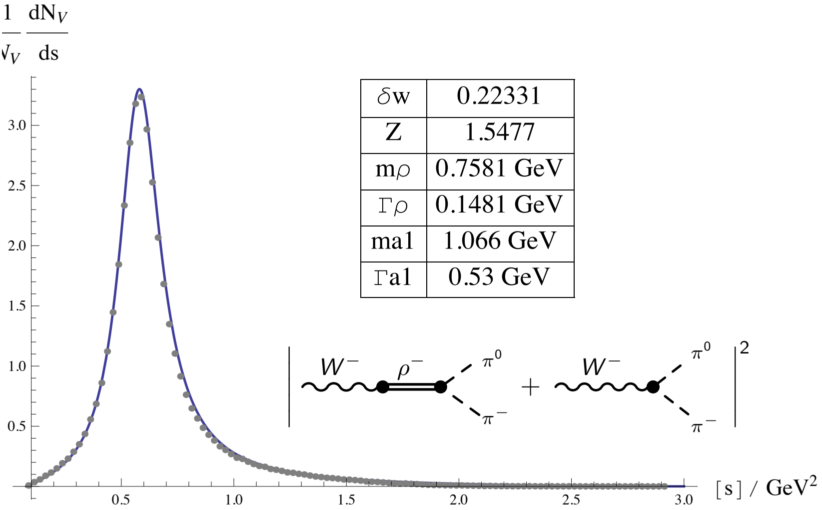
$V - A$



$V + A$

Vector Channel Spectral Function $\tau \to 2\pi\nu_\tau$

Coherent sum $|W \xrightarrow{\text{direct}} 2\pi + W \xrightarrow{\rho} 2\pi|^2$

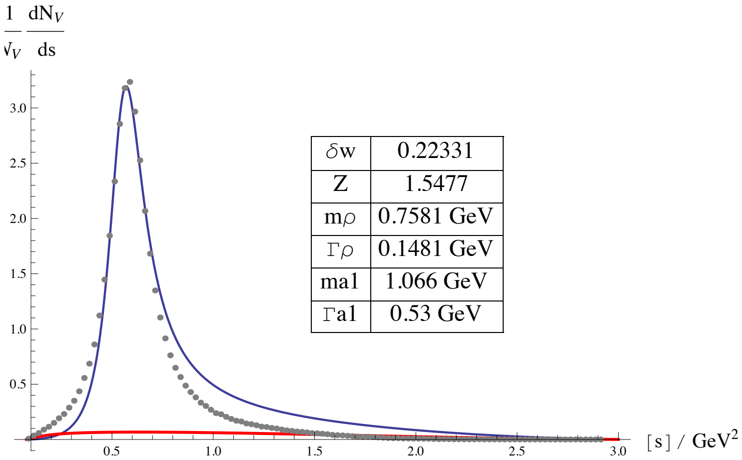


δw	0.22331
Z	1.5477
mρ	0.7581 GeV
$\Gamma\rho$	0.1481 GeV
ma1	1.066 GeV
Γa1	0.53 GeV

Vector Channel Spectral Function $\tau \to \pi^-\pi^0\nu_\tau$

Isolated contributions $W^- \to \pi^-\pi^0$ and $W^- \to \rho^- \to \pi^-\pi^0$

$$\frac{\Gamma(W^- \to \pi^-\pi^0)}{\Gamma(W^- \to \rho^- \to \pi^-\pi^0)} \simeq 0.02$$



δw	0.22331
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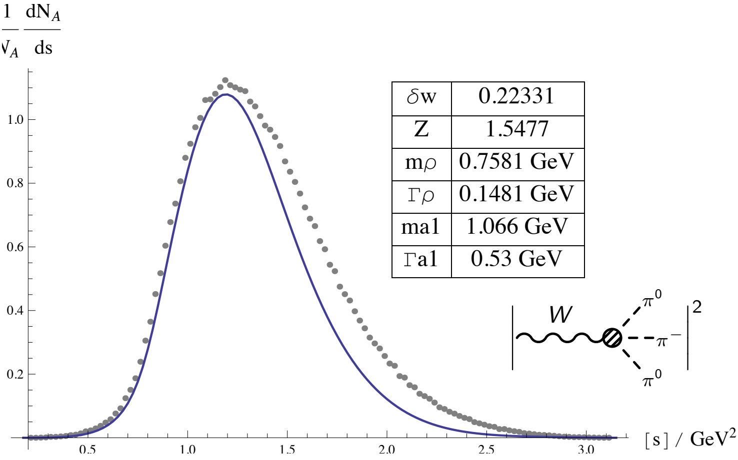
Axial Vector Channel Spectral Function $\tau^- \to \pi^- 2\pi^0 + \pi^+ 2\pi^- \nu_\tau$

Coherent Sum $|W^- \to \rho\pi \to 3\pi + W \xrightarrow{a_1} \rho\pi \to 3\pi|^2$



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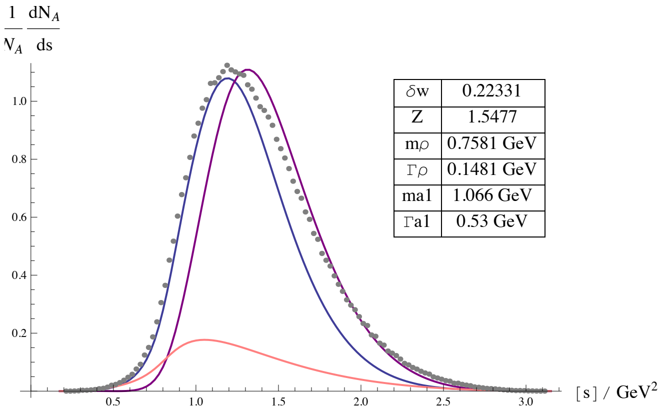
Axial Vector Channel Spectral Function $\tau^- \to \pi^- 2\pi^0 + \pi^+ 2\pi^- \nu_\tau$

Isolated contributions $W^- \to \pi^- \pi^0$ and $W^- \to \rho^- \to \pi^- \pi^0$

$$\frac{\Gamma(W^- \xrightarrow{\text{direct}} \pi^- 2\pi^0)}{\Gamma(W^- \xrightarrow{\text{coh.}} \rho^- \pi^0 \to \pi^- 2\pi^0)} \simeq 0.02$$



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Vector Decay Constant

- Vector Decay Constant f_ρ: $< 0|j^\mu\,|\rho^\mu >= m_\rho f_\rho \varepsilon^\mu$
- describes mixing of the ρ meson with the vector current,
- weak coupling with vector meson $\frac{g\cos\theta_C}{2}\delta_w s$

$\rightarrow$ $f_\rho m_\rho = \sqrt{2}\delta_w m_\rho$

- $f_\rho^{\text{exp.}} \sim 214$ MeV , $f_\rho^{\text{L}\sigma\text{M}} = 239$ MeV
- Also this result can be improved by including direct contributions in the axial-vector channel.

Sum Rules and KSFR relations

- From the Second Weinberg Sum Rule and KSFR relation

$$\frac{m_{a_1}}{m_\rho} = \sqrt{2}$$

- $m_\rho = 0.7581\,\mathrm{GeV}\,,\ m_{a_1} = 1.066\,\mathrm{GeV}$

$$\frac{m_{a_1}^{\mathrm{LsM}}}{m_\rho^{\mathrm{LSM}}} = 1.40615\ , \quad \sqrt{2} = 1.41421$$

- vector and axialvector couplings have been measured to be

$$g_\rho^{\mathrm{exp.}} = 0.116\,\mathrm{GeV}^2\,,\ g_{a_1}^{\mathrm{exp.}} = 0.145\,\mathrm{GeV}^2$$

- in our model they translate into

$$g_\rho^{\mathrm{LSM}} = \delta m_\rho^2\,,\ g_{a_1}^{\mathrm{LSM}} = \delta m_\rho^2 - g_1\phi^2$$

we obtain

$$g_\rho^{\mathrm{LSM}} = 0.12834\,\mathrm{GeV}^2\,,\ g_{a_1}^{\mathrm{LSM}} = 0.13741\,\mathrm{GeV}^2$$

Conclusion

- We described the decay of the τ lepton in an effective hadronic model
- Can we use effective chiral models to describe the phenomenology of the low energy resonances? Yes!
  When the model is comprehensive enough.
- Is a_1 a $\bar{q}q$ state? Yes!
- Are ρ and a_1 chiral partners? Yes!
  Very nice example of Vector Meson Dominance.