Landau Gauge Quark Propagator with External Magnetic Fields

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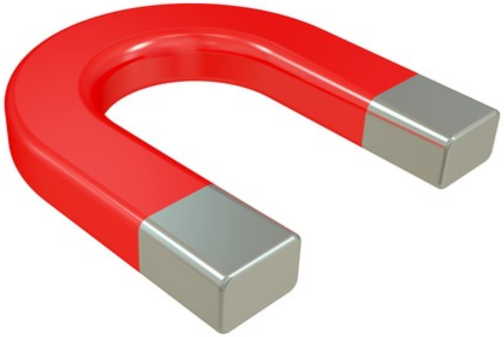


1. Motivation

Why are magnetic fields so interesting to study in quantum systems?

The **phenomenolgical** side:
magnetic fields are realized in a variety of systems!

- **non central HI collisions**
  (some debate here, relevant at thermal equilibrium? → see *“Where to go from here”*)
- expansion of the **early universe**
- dense neutron stars
- **Chiral Magnetic Effect**
  (strong CP problem)
- …

The **“mathematical / theoretical”** side:
magnetic fields are a nice tool to play with!

- **(inverse)** magnetic catalysis as a tuning knob of **chiral symmetry breaking**
- **confinement** ? (probing EM charged, confined objects – transition from charged quarks to neutral mesons)
- **QCD phase diagram**

direct test of the **topology of YM theories** (largely unexplored!)

Plus: a lot of intuition from quantum mechanics

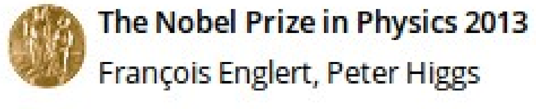
strongest magnetic field produced in a lab:	10^5 G
magnetars	10^{13} G
heavy ion collisions	10^{17} G

1.2 GW LANL 100T
pulsed magnet

2. Chiral Symmetry and Magnetic Fields

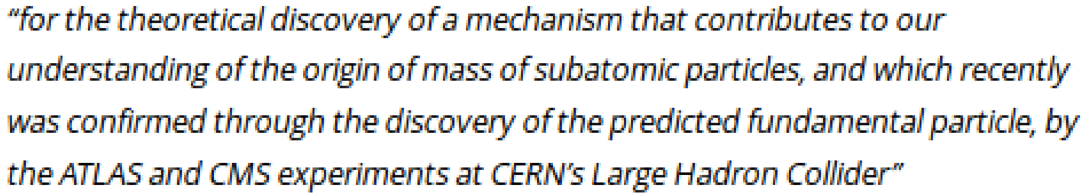
prelude: dynamical chiral symmetry breaking

This years Nobel Price:



The Nobel Prize in Physics 2013

François Englert, Peter Higgs

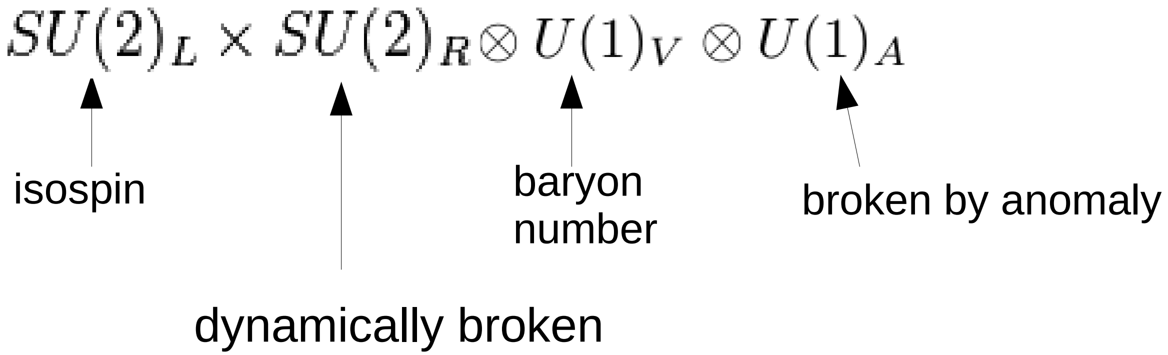


"for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider"

but as we look at a proton we realize:
only about 1% of its mass is explained by the Higgs mechanism!

→ *Dynamical chiral symmetry breaking is a fundamental feature of the strong interaction*

assume two light flavor QCD:



$$SU(2)_L \times SU(2)_R \otimes U(1)_V \otimes U(1)_A$$

isospin

baryon number

broken by anomaly

dynamically broken

signaled by non vanishing **fermion condensate ("chiral condensate")**: $\langle \bar{\Psi}_R \Psi_L \rangle$

→ Non-perturbative feature – a "true" quantum effect
→ light pions – pseudo Nambu-Goldstone bosons
(three of them since there are three broken symmetry generators)

First glimpse: the famous **NJL model** (Phys. Rev 122 (1961) 345 & Phys. Rev 124 (1961) 246)

2. Chiral Symmetry and Magnetic Fields

Specific issue: **Inverse magnetic catalysis VS. magnetic catalysis**

Magnetic Catalysis

a little bit of history: **Schwinger** 1951

→ (bare) fermion propagator in external field (to all orders) **“proper time method”**

Ritus 1972 → an equivalent approach (later more!)

Modern times: Gusynin, Miransky, Shovkoy 1995:

- **effective dimensional reduction**
- **catalysis of dynamical symmetry breaking by a magnetic field**

… in NJL & (ladder) QED:

$$\boxed{m^2_{\mathrm{dyn}} = \frac{|eB|}{\pi}\exp\left(-\frac{4\pi^2(1-g)}{|eB|N_cG}\right)} \quad g \equiv N_cG\Lambda^2/(4\pi^2) \qquad \boxed{m_{\mathrm{dyn}} = C\sqrt{|eB|}\exp\left[-\frac{\pi}{2}\left(\frac{\pi}{2\alpha}\right)^{1/2}\right]}$$

… and a very intuitive picture!

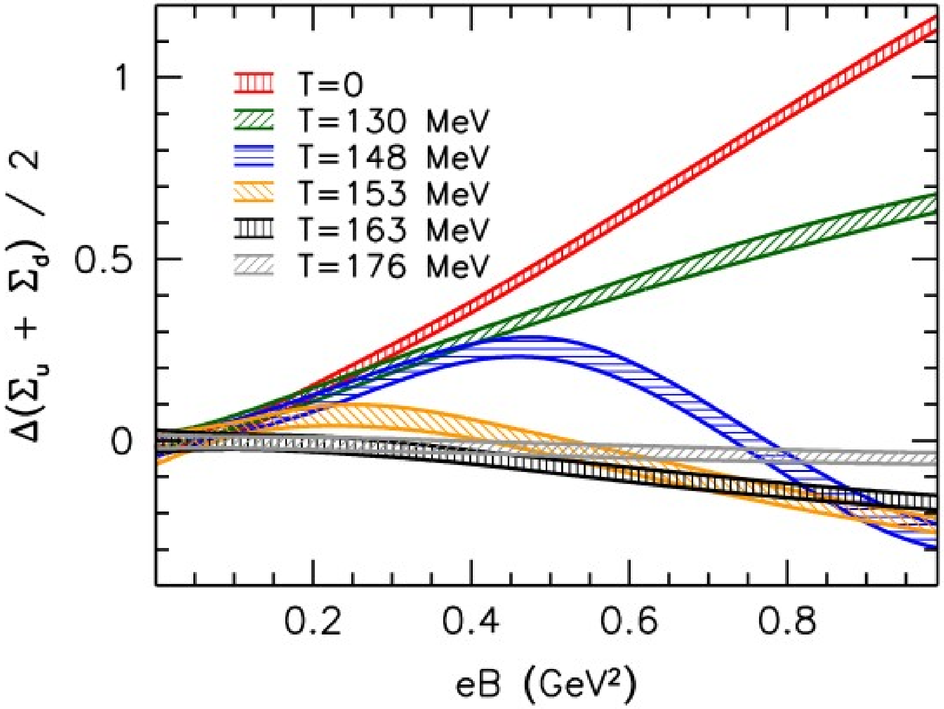
superconductors: Cooper pairs are charged (MWC theorem!), magnetic moments are antiparallel
→ magnetic field tends to break up this condensate

QCD: chiral condensate electrically neutral, magnetic moments of the quarks are aligned
→ enhancement of the condensate

thus even at weakest interaction (at T=0): **chiral symmetry breaking!**

2. Chiral Symmetry and Magnetic Fields

Specific issue: **Inverse magnetic catalysis VS. magnetic catalysis**



Bruckmann, Bali, Endrodi et al. (2012) and other similar studies

Inverse Magnetic Catalysis

but along came the lattice …

- around T_c non monotonic behavior shows up
  → **symmetry restoration**

- reason unclear, but perhaps two possible contributors:

1. Hadronic dofs:

- **mesonic fluctuations** tend to restore chiral symmetry
- Transition from neutral Nambu-Goldstone bosons to their charged constituents
  → **Mermin-Wagner-Coleman** theorem (Fukushima & Hidaka 2013)

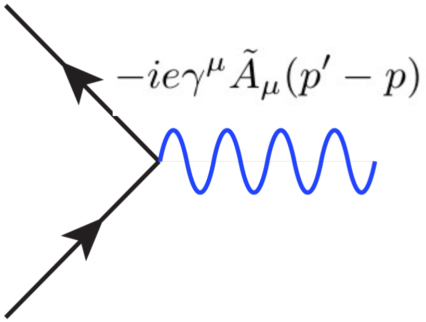
Not enough! (Kamikado & Kanazawa 2013)

2. Backcoupling from the gluonic sector

- medium modifications, charged sea of virtual quark-antiquark pairs

3. Magnetic Fields in quantum systems

we want to study **Abelian background fields!** but how to include such thing??



- ***perturbation theory:*** introduce sources that particles couple too ...

BUT: - a magnetic field **violates Poincare invariance**

→standard Fourier expansion impossible

actually that is not completely true …

→ *Schwinger's proper time method (can comment on that later if you wish)*

- will hardly find non interacting asymptotic states

<u>In reality magnetic fields are often large and have to be included to every order!</u>

→ can treat them **quasi classical!**

How about we don't do a Fourier expansion of our n-point functions but instead do an expansion in the eigenfunctions of the dirac operator including the magnetic field?

→ **Ritus' method,** *comes with a challenge*:

electrically charged and neutral particles will live in different eigensystems

→ there will be **no "momentum conservation"** at vertices

(which we normally would get from orthogonality and completeness of the eigensystem)

3. Magnetic Fields in quantum systems

Ritus method:

warning: *- some technique here, but we can learn something!*
- this will be a rather sloppy derivation!

begin with **dirac equation**:

$$(i\gamma \cdot \Pi + m)\Psi(x) = 0 \qquad \Pi_\mu = \partial_\mu + ieA_\mu(x) \qquad A_\mu(x) = (0, 0, Hx, 0)$$

(constant, along z)

2-point Greens function can only depend on:

$$\gamma\Pi, \quad \sigma F, \quad (F\Pi)^2, \quad \gamma^5 FF^*$$

all those commute with $(\gamma\Pi)^2 = \Pi^2 - \frac{1}{2}e\sigma F$ thus have same eigenfunctions:

$$(\gamma\Pi)^2 E_p = p^2 E_p$$

other operators commuting with $(\gamma\Pi)$ are $i\partial_0$, $i\partial_3$ and $i\partial_2$ corresponding to the eigenvalues $p_\parallel = (p_0, p_3)$ and p_2. → still plane waves in 0,2,3 direction

There is one other operator commuting with all those:

$$\mathcal{H}E_p = kE_p \qquad \mathcal{H} = -(\gamma\Pi)^2 + \Pi_0^2 = \Pi_1^2 + \Pi_2^2 - eH\Sigma^3$$

Nucl. Phys. B **747** (2006) 266,
Ann. Phys. **69** (1972) 555

3. Magnetic Fields in quantum systems

Ansatz: $\boxed{E_p = E_{p,\sigma}\Delta(\sigma),}$ $\qquad E_{p,\sigma} = N_\sigma e^{i(p_0x_0-p_2x_2-p_3x_3)}F_{k,p_2,\sigma}\,.$

Klein-Gordon like
scalar function

spin projector along z

Essentially we are solving for eigenfunctions of a plane wave operator in three dimensions with a harmonic oscillator in the other dimension

The solution is:

$$\boxed{E_{p,\sigma}(x) = N(n)e^{i(p_0x_0-p_2x_2-p_3x_3)}D_n(\rho)\,, \qquad \rho = \sqrt{2|eH|}\left(x_1 - \frac{p_2}{eH}\right)\,, \qquad N(n) = \frac{(4\pi\,|eH|)^{\frac{1}{4}}}{\sqrt{n!}}}$$

spin

$$n = l + \tfrac{\sigma}{2}sgn(eH) - \tfrac{1}{2}$$

total angular momentum
(Landau level)

orbital angular momentum

$$p^2 = p_0^2 - p_3^2 - k\,, \qquad k = |eH|(2n+1) + \sigma|eH|$$

$= 2|eH|l$

note: no dependence on p2!
→ **dimensional reduction**
effectively a 3-dimensional system!

3. Magnetic Fields in quantum systems

Our new found eigensystem is orthogonal and complete:

$$\int d^4x\, \bar{E}_p(x) E_{p'}(x) = (2\pi)^4 \delta^{(4)}(p-p')\, \Pi(l)$$

$$\sum\!\!\!\!\!\!\!\int \frac{d^4p}{(2\pi)^4} E_p(x)\bar{E}_p(y) = (2\pi)^4\delta^{(4)}(x-y), \qquad \text{with} \qquad \sum\!\!\!\!\!\!\!\int \frac{d^4p}{(2\pi)^4} = \sum_{l=0}^{\infty} \int \frac{d^2p_{\parallel}}{(2\pi)^4} \int\limits_{-\infty}^{\infty} dp_2$$

$$\Pi(l) = \begin{cases} \Delta(\mathrm{sgn}(eH)) & l = 0 \\ 1 & l > 0 \end{cases} .$$

Everything in complete analogy with standard textbook Feynman rules!

thus:

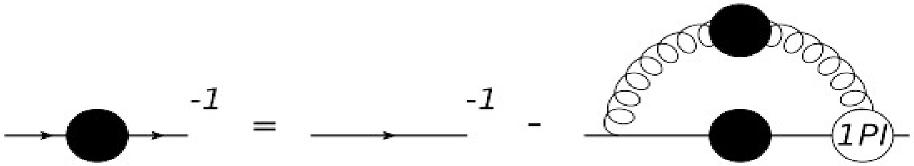
write down processes
of interest in **position space**
(where the quantum theory
/ the Lagrangian is defined)

transform into **Ritus space** →

get interaction with classical
vector potential for free
(to all orders!)

4. Quenched Quark DSE

We would like to study chiral symmetry breaking, hence want to use the **quark Dyson Schwinger Equation** to get a handle on the ***chiral condensate***



$$\boxed{S^{-1}(x,y) = S_0^{-1}(x,y) + \Sigma(x,y)}$$

$$\Sigma(x,y) = i\,g^2\,C_F\,\gamma^\mu\,S(x,y)\,\Gamma^\nu(y)\,D_{\mu\nu}(x,y)$$

now can expand this in the Ritus basis → effect of magnetic field implicitly included

$$\Sigma(p,p') = g^2 C_F \int d^4x\,d^4y\,\bar{E}_p(x)\,\gamma^\mu S(x,y)\Gamma^\nu(y)D_{\mu\nu}(x,y)E_{p'}(y)$$

$$S(x,y) = \sum\!\!\!\!\!\!\!\int \frac{d^4q}{(2\pi)^4}\,E_q(x)\frac{1}{A_{||}(q)i\gamma\cdot q_{||} + A_\perp(q)i\gamma\cdot q_\perp + B(q)}\bar{E}_q(y)$$

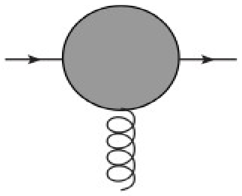
$$D_{\mu\nu}(x,y) = \int \frac{d^4k}{(2\pi)^4}\,e^{ik(x-y)}D(k^2)P_{\mu\nu}$$

quenched gluon propagator *(from lattice: Eur. Phys. J C **68** (2010) 165, ask me for details)*
→ obviously still Fourier expanded, since electrically neutral
however: will see later (unquenched): **the gluon does feel the magnetic field!**

4. Quenched Quark DSE

challenge: At qqg vertex there will be no “momentum conservation”!
→ no delta functions from orthonormality, that make life easy

instead:



$$= \int d^4x\, \bar{E}_p(x)\gamma^\mu E_q(x) e^{ikx}$$

plane wave

Ritus eigenfunctions

… modulo dressing functions and other tensor structures

$$\boxed{\begin{aligned}\int d^4x\, \bar{E}_p(x)\gamma^\mu E_q(x)e^{ikx} &= (2\pi)^4\delta^{(3)}(q+k-p)\, e^{-k_\perp^2/4|eH|} e^{ik_1(q_2+p_2)/2eH} \\ &\times \sum_{\sigma_1,\sigma_2=\pm} \frac{e^{i\mathrm{sgn}(eH)(n(\sigma_1,l)-n(\sigma_2,l_q))\phi}}{\sqrt{n(\sigma_1,l)!n(\sigma_2,l_q)!}} J_{n(\sigma_1,l)n(\sigma_2,l_q)}(k_\perp)\ \Delta(\sigma_1)\gamma^\mu\Delta(\sigma_2)\end{aligned}}$$

$$J_{n_1n_2} \equiv \sum_{m=0}^{\min(n_1,n_2)} \frac{n_1!n_2!}{m!(n_1-m)!(n_2-m)!}\left(i\mathrm{sgn}(eH)k_\perp\frac{\sqrt{2|eH|}}{2eH}\right)^{n_1+n_2-2m}$$

Sorry for this mess! But we can learn something here:

1. The Landau level (LL) is not conserved, but the total angular momentum of quark and gluon (spin 1) is conserved. → There can be transitions between LL
2. What we see here is a “modified” or smeared “momentum” conservation, something like a form factor.

4. Quenched Quark DSE

composing all the bits and pieces gives:

$$\Sigma(p,p') = (2\pi)^4\delta^{(3)}(p-p')g^2C_F\sum_{l_q}\int\frac{d^2q_\|}{(2\pi)^4}\int_{-\infty}^{\infty}dq_2\int_{-\infty}^{\infty}dk_1e^{-k_\perp^2/2|eH|}\sum_{\sigma_1,\sigma_2,\sigma_3,\sigma_4}\frac{e^{i\mathrm{sgn}(eH)(n_1-n_2+n_3-n_4)\phi}}{\sqrt{n_1!n_2!n_3!n_4!}} \qquad (24)$$

$$\times J_{n_1n_2}(k_\perp)J_{n_3n_4}(k_\perp)\Delta(\sigma_1)\gamma^\mu\Delta(\sigma_2)\frac{1}{A_\|(q)i\gamma\cdot q_\| + A_\perp(q)i\gamma\cdot q_\perp + B(q)}\Delta(\sigma_3)\gamma^\nu\Delta(\sigma_4)P^{\mu\nu}(k)\Gamma(k^2)D(k^2)\,.$$

which unfortunately is impossible to solve numerically!

However …

we note that $e^{-k_\perp^2/2|eH|}$ suppresses large values of k_T, hence for not too small eH we can keep only leading terms in $k_\perp/2\,|eH|$

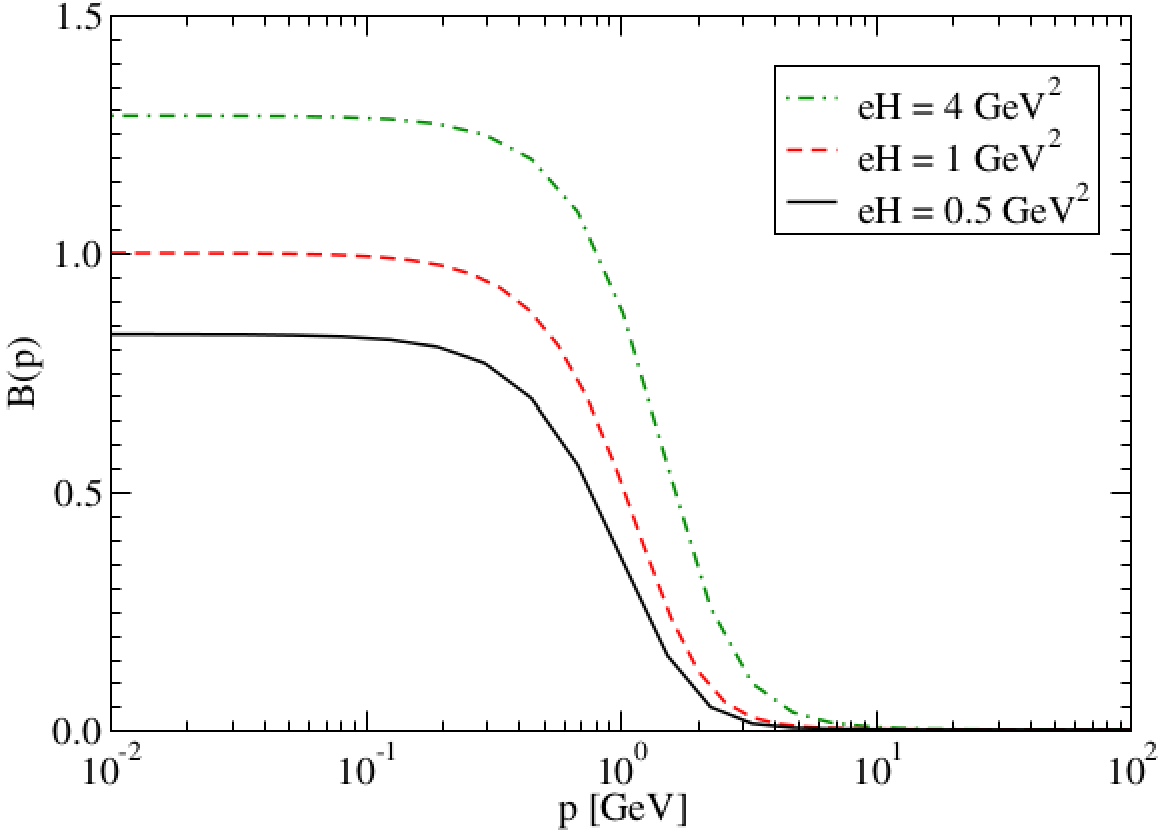
therefore $J_{nm}(k_\perp)\;\rightarrow\;\frac{[\max(n,m)]!}{|n-m|!}(ik_\perp/\sqrt{2\,|eH|})^{|n-m|}\;\rightarrow\;n!\,\delta_{nm}$

$$\boxed{\begin{aligned}\Sigma(p,p') = (2\pi)^4\delta^{(3)}(p-p')ig^2C_F\sum_{l_q=0}^{\infty}\int\frac{d^2q_\|}{(2\pi)^4}\int_{-\infty}^{\infty}dq_2\int_{-\infty}^{\infty}dk_1\;e^{-k_\perp^2/2|eH|}\sum_{\sigma_1\sigma_2\sigma_3\sigma_4}\delta_{n(\sigma_1,l)n(\sigma_2,l_q)}\delta_{n(\sigma_3,l_q)n(\sigma_4,l')}\\ \times\Delta(\sigma_1)\gamma^\mu\Delta(\sigma_2)\frac{1}{A_\|(q)\gamma\cdot q_\| + A_\perp(q)\gamma\cdot q_\perp + B(q)}\Delta(\sigma_3)\gamma^\nu\Delta(\sigma_4)D(k^2)\Gamma(k^2)P^{\mu\nu}(k)\end{aligned}}$$

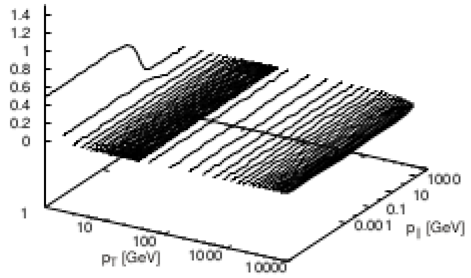
enough equations for now ...

4. Quenched Quark DSE

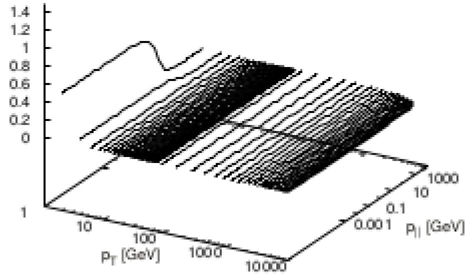
Results!



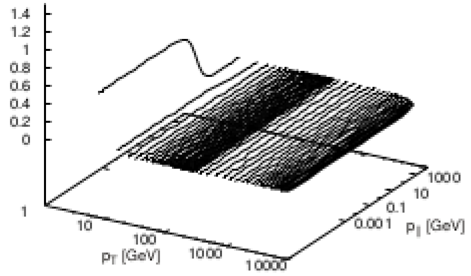
- **mass function enhanced** → **Magnetic Catalysis!**
- falling flank shifted, since there is **another scale involved**
- **flat** in the UV *(no additional divergences from “quark” self energy)*



(a) $B(p)$ [GeV] ($eH = 0.5\ \mathrm{GeV}^2$)



(d) $B(p)$ [GeV] ($eH = 1\ \mathrm{GeV}^2$)



(g) $B(p)$ [GeV] ($eH = 4\ \mathrm{GeV}^2$)

lowest landau level dominance !

(but some contribution from higher levels)

4. Quenched Quark DSE

Chiral Condensate

order parameter for chiral symmetry breaking

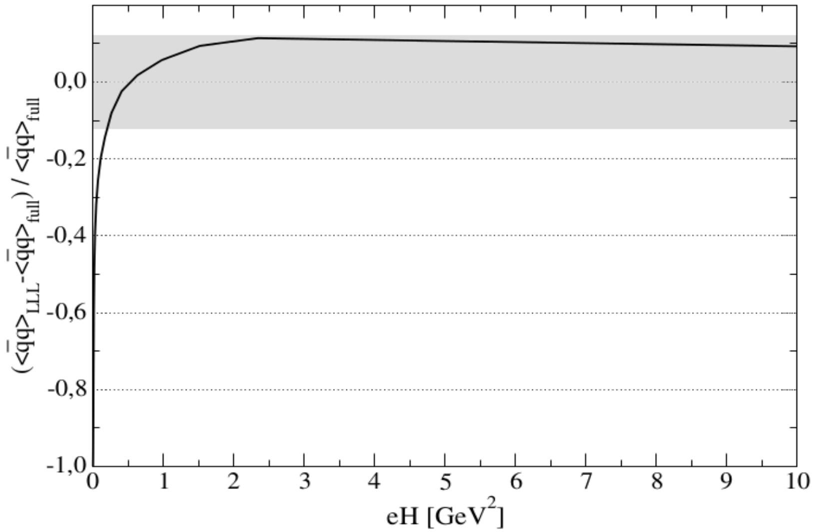
$$-\langle \bar{q}q \rangle = Z_2 \lim_{x\to 0} trS(x,0)$$

- the lowest Landau level approximation:

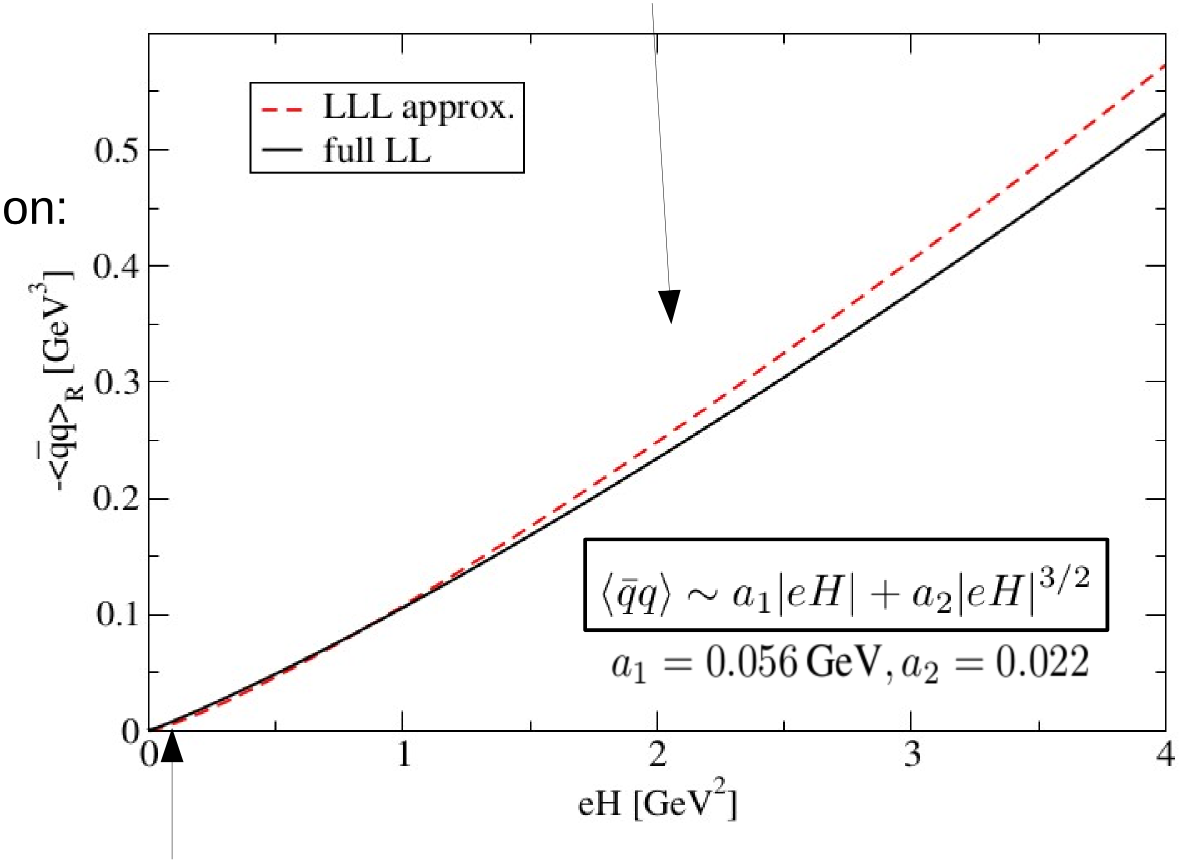
  good only at the **10% level**

  + converges only asymptotically

(I have calculated this for much much larger magnetic fields)



magnetic catalysis



$$\boxed{\langle \bar{q}q \rangle \sim a_1 |eH| + a_2 |eH|^{3/2}}$$

$$a_1 = 0.056\,\mathrm{GeV}, a_2 = 0.022$$

quadratic here
(not resolved due
to our approximation)

compare to lattice:	PRD **86** (2012) 071502
χPT:	PRD **83** (2011) 114028
dim. consid.:	PLB **402** (1997) 351

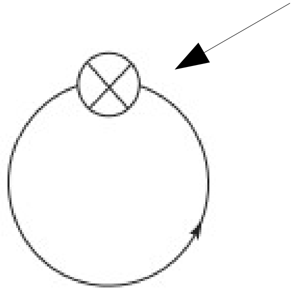
4. Quenched Quark DSE

There is more!

Spin polarization ...

insertion of σ^{12}

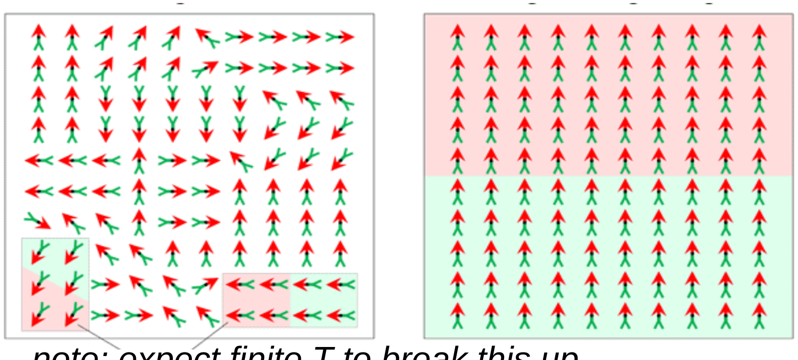
$$< \sigma^{12} >=$$



… essentially the lowest Landau level contribution to the **chiral condensate**, since all other LL are degenerate wrt. 2 spin directions and thus cancel on average.

Polarization of the QCD vacuum …

$$\mu = \frac{\langle \sigma^{12} \rangle}{\langle \bar{q} q \rangle}$$



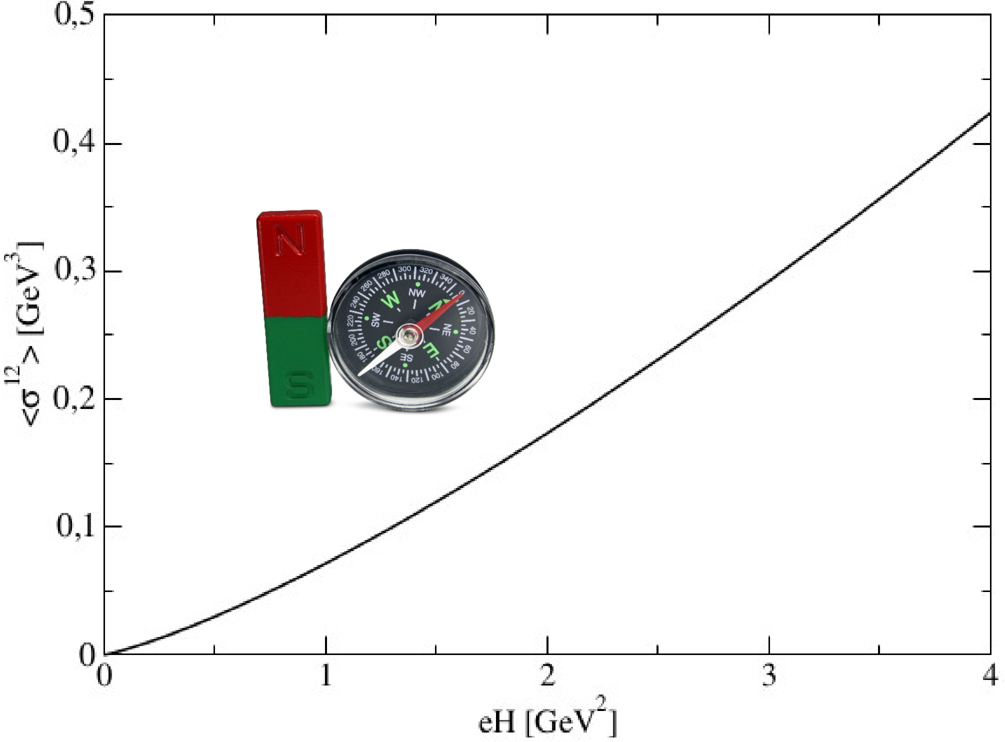
note: expect finite T to break this up
(as T tends to break up a lot of condensates ...)

tends towards its saturation μ → 1 for large magnetic fields
(this corresponds exactly to the validity of the lowest LL approximation)

$$\langle \sigma^{12} \rangle = \chi \, \langle \bar{q} q \rangle \, eH + O(eH^2)$$

sensitive to the **magnetic susceptibilty** for small eH … unfortunately not reliable within my approx. :(

4. Quenched Quark DSE



polarization of the vacuum

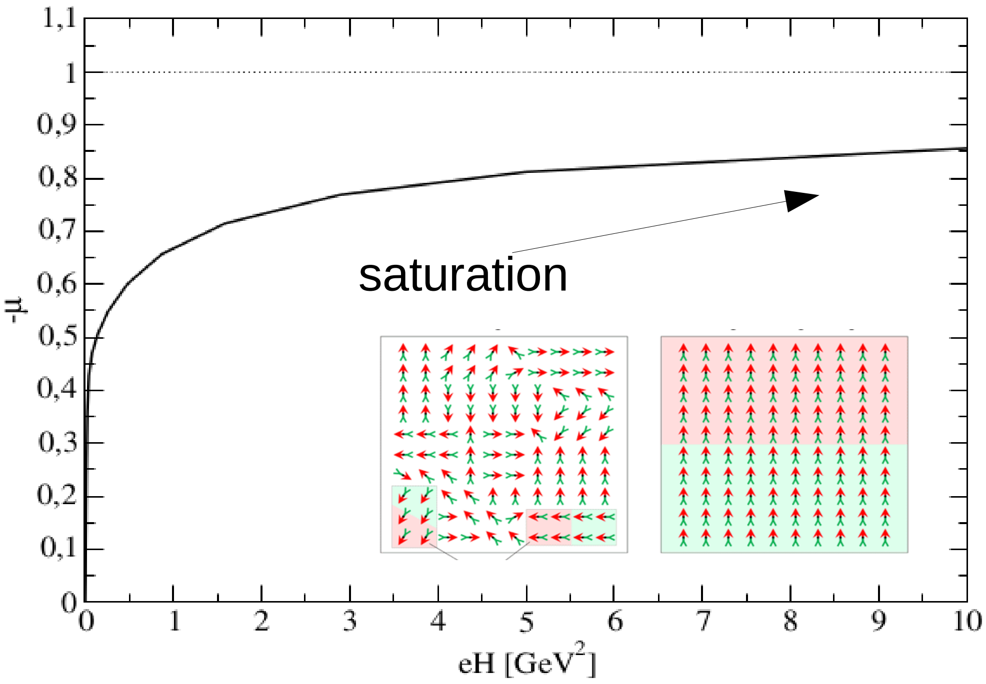
should be linear for small eH!

→ need reliable numerical method to evaluate this range

→ magnetic susceptibilty

spin expectation value

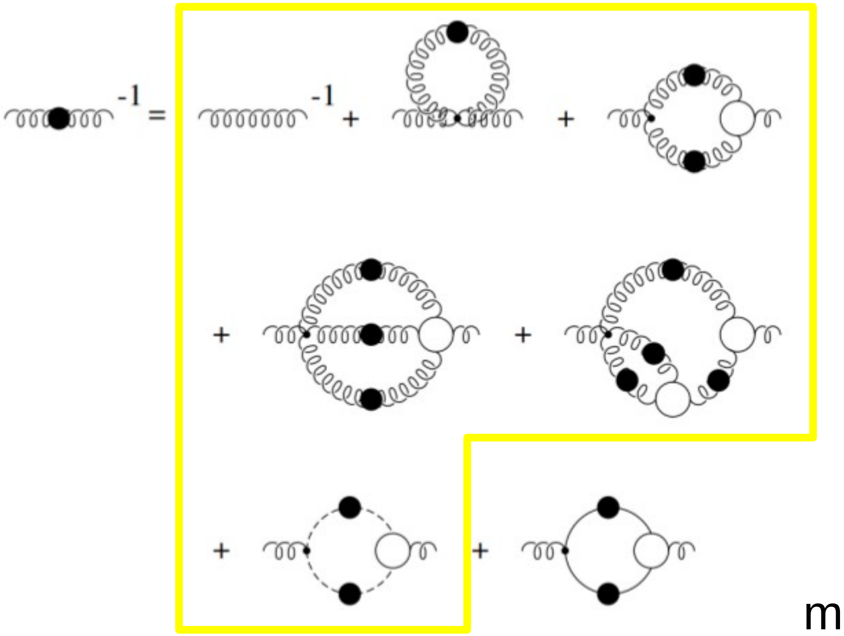
- similar behavior as the chiral condensate (obviously, since they are related)


saturation

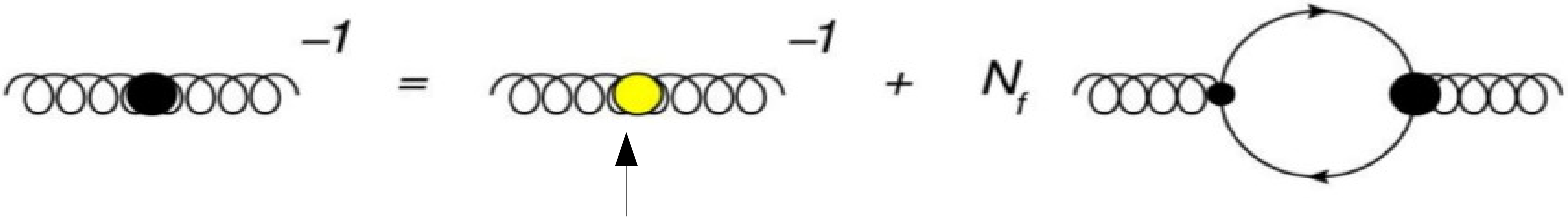
5. Unquenched Gluons

Inverse Magnetic Catalysis – back coupling from the gluons ?

gluon DSE:



magnetic field talks to those



effective “bare” propagator (quenched)

medium modification!

5. Unquenched Gluons

Polarization structure of the gluon – not that trivial anymore!

note: $\Pi^{\mu\nu}$ ...

1. … is a symmetric tensor → 10 components
2. … respects $\Pi^{\mu\nu} k_\nu = 0$
3. ... respects Furry's theorem

*this leaves **4 linear independent components**!*

write $\Pi^{\mu\nu}$ in its eigenbasis:

$$\boxed{\begin{aligned}\Pi^{\mu\nu}(k,k') &= (2\pi)^4\delta^{(4)}(k'-k)\Pi^{\mu\nu}(k)\\ \Pi^{\mu\nu}(k) &= \sum_{i=0}^{3}\kappa_i \frac{b_i^\mu b_i^\nu}{(b_i)^2}.\end{aligned}}$$

$$b_0^\mu = k^\mu \qquad \text{(eigenvalue zero)}$$

$$b_1^\mu = (F^{\mu\nu}F_{\nu\rho}k^\rho)k^2 - k^\mu(k_\nu F^{\nu\alpha}F_{\alpha\beta}k^\beta)$$

$$b_2^\mu = {}^*F^{\mu\nu}k_\nu$$

$$b_3^\mu = F^{\mu\nu}k_\nu$$

for a **constant magnetic** field this gives the projectors:

$$\left(\delta_\parallel^{\mu\nu} - \frac{k_\parallel^\mu k_\parallel^\nu}{k_\parallel^2}\right) \equiv P_\parallel^{\mu\nu} \qquad \frac{(k_\perp^2 k_\parallel^\mu - k_\parallel^2 k_\perp^\mu)(k_\perp^2 k_\parallel^\nu - k_\parallel^2 k_\perp^\nu)}{k_\parallel^2 k_\perp^2 k^2} \equiv P_0^{\mu\nu}$$

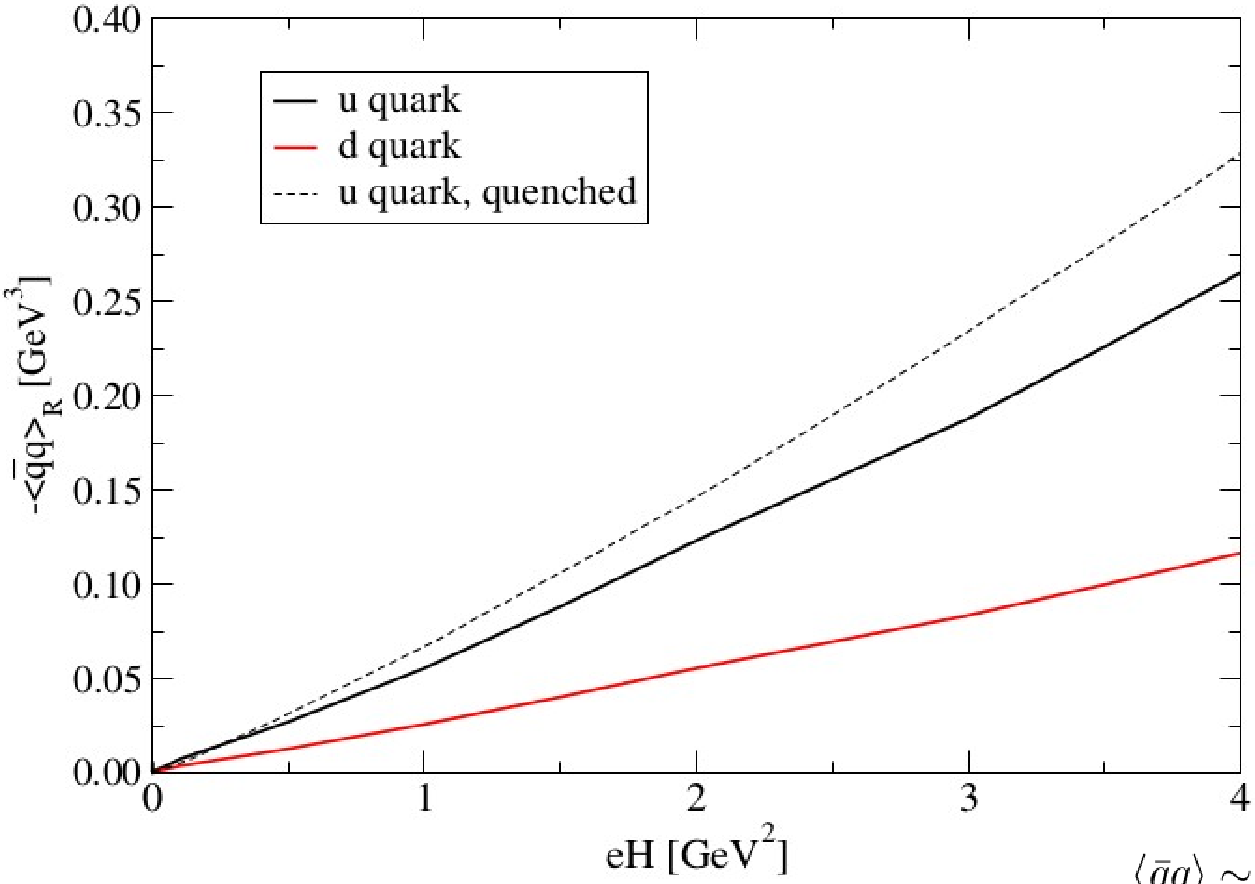
$$\left(\delta_\perp^{\mu\nu} - \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2}\right) \equiv P_\perp^{\mu\nu} \qquad \frac{k^\mu k^\nu}{k^2}$$

5. Unquenched Gluons

Unquenched QCD – gluons vs. quarks (T=0)

2 flavor, lowest Landau level unquenched only:

→ **significant reduction of chiral symmetry breaking!**



magnetic catalysis is reduced!

→ gluon propagates through magnetized medium full of virtual quark-antiquark pairs

magnetic field breaks isospin

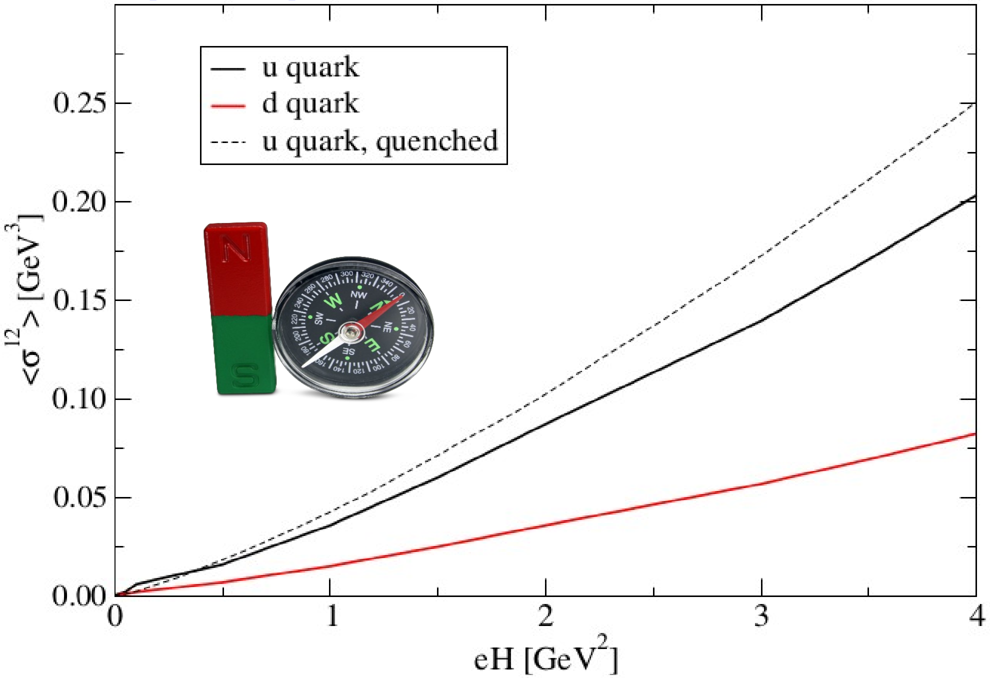
Hints towards
inverse magnetic catalysis
at finite T and μ !

$$\langle\bar{q}q\rangle \sim a_1|eH| + a_2|eH|^{3/2}$$

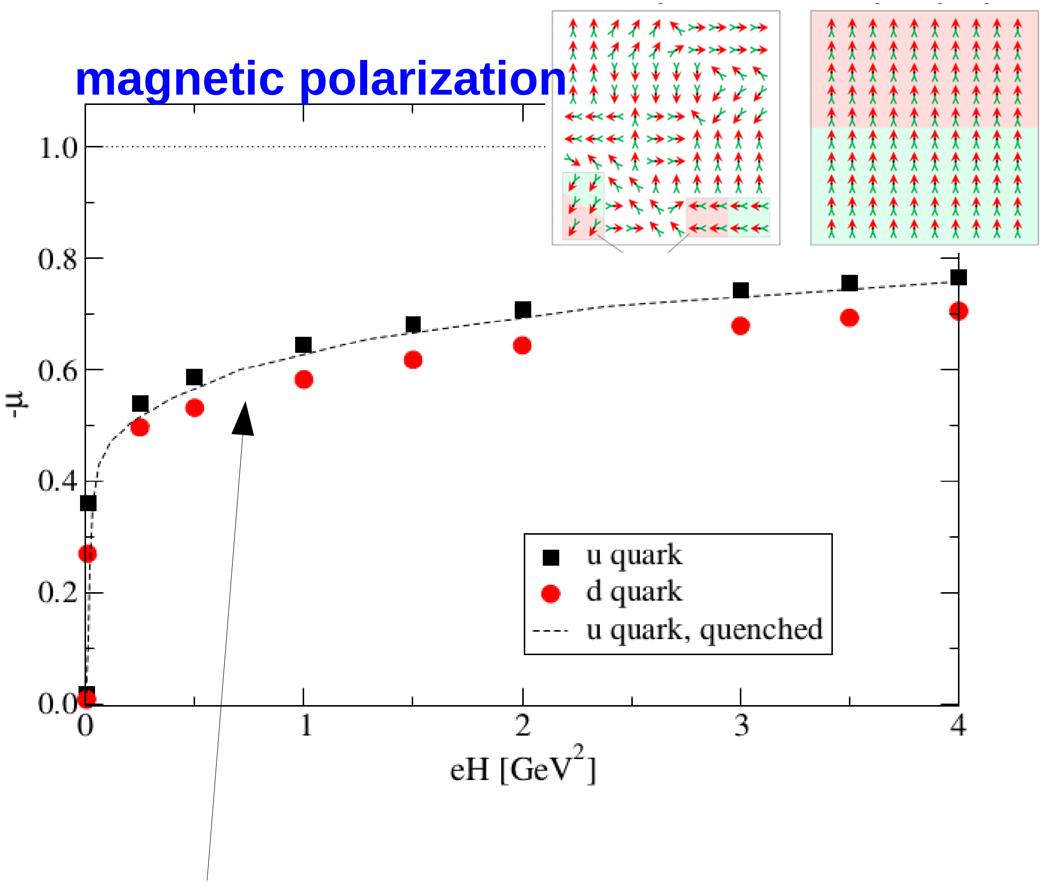
a1 = 0.047 GeV^2 a2 = 0.0095 GeV^1.5

5. Unquenched Gluons

spin expectation value



magnetic polarization



almost no effect on the polarization of the QCD vacuum

Summary

What do we learn from this?

We investigated the **influence of a strong magnetic field onto Landau gauge QCD**

- **chiral condensate and polarization structure**
- most important: **Decrease in magnetic catalysis** from back-reaction YM sector on quarks

- For chiral condensate find **linear dependence** of the magnetic field for $eH \geq \Lambda^2_{\rm QCD}$ (predicted from lattice, χPT) turning into $\sim (eH)^{3/2}$ asymptotically as predicted from dimensional considerations

- This is an **exploratory study**! Quantitative features depend on the truncations used → can be expanded easily

- Finite T and µ straightforward! → **inverse magnetic catalysis**

PS: For details: *Dynamical quark mass generation in a strong magnetic field,*
NM, J. Bonnet, C.S. Fischer, arXiv:1401.1647[hep-ph]

6. Where to go from here?

First step: Unquenched QCD at Finite T and μ + hadronic fluctuations
→ Inverse Magnetic Catalysis

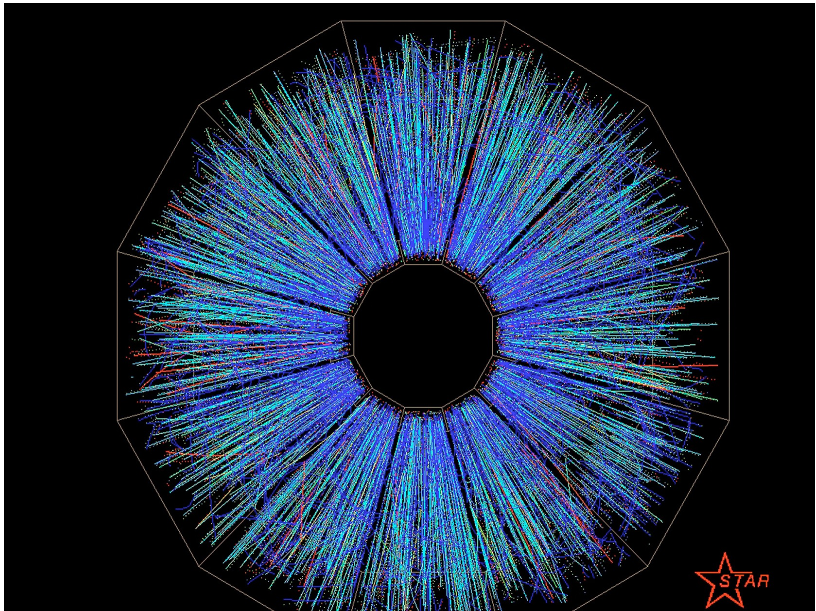
But there is more!

- Magnetic Fields of extreme strength are produced during heavy ion collisions
→ some discussions whether they are relevant when systems thermalizes

strongest magnetic field produced in a lab:	10^5 G
magnetars	10^{13} G
heavy ion collisions	10^{17} G

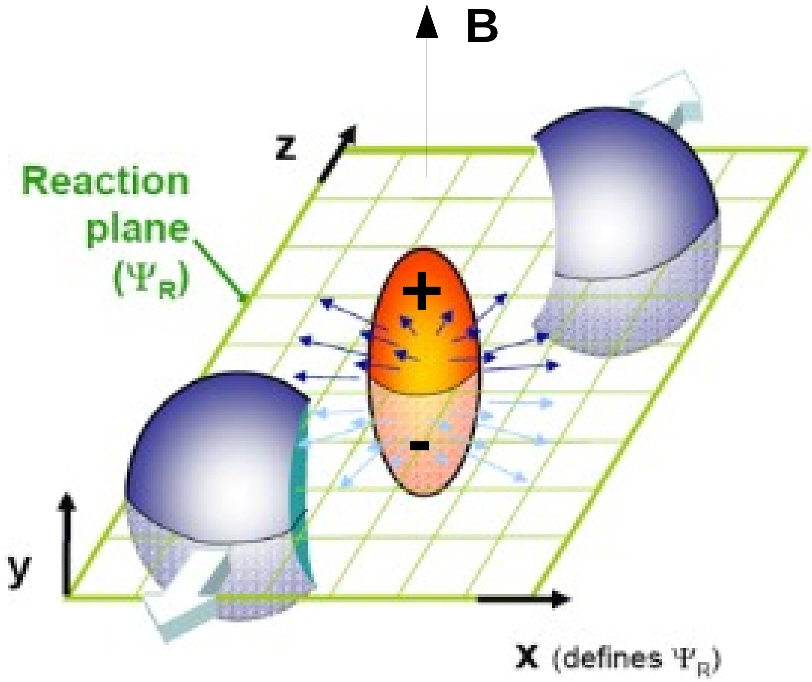
… who cares? why not **magnetic fields in QCD far from equilibrium ?**

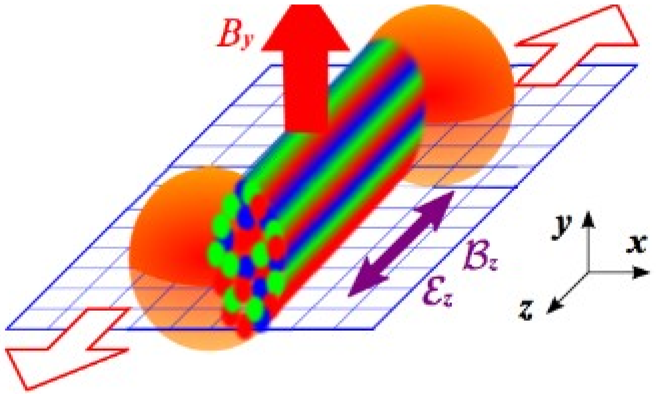
→ will effect **particle production** in early phases of HI collision
→ very relevant for **thermalization** scenarios (input for hydro!)
→ phenomenology **CP- and P-odd effects** – observables vs. theory



6. Where to go from here?

Chiral Magnetic Effect **(Kharzeev, McLerran, Warringa, Fukushima...)**





note: axial anomaly relates chirality of fermions to topological charge of YM background

intuitive picture:

- magnetic field aligns spins of positive and negative fermions at LLL in opposite directions

- charge, chirality, momentum of fermions are correlated

right-handed positive charged quarks propagate along the magnetic field

this current is usually canceled by left handed quarks …

… unless there is a **chiral imbalance**, i.e. finite axial chemical potential

Note: This is an anomaly effect! We can test a non-trivial topology of the QCD vacuum.
→ **CP violation in QCD! Non zero Chern-Simons topological charge!**

Sources

[1] I. A. Shovkovy, arXiv:1207.5081 [hep-ph].

[2] R. Gatto and M. Ruggieri, Lect. Notes Phys. **871** (2013) 87 [arXiv:1207.3190 [hep-ph]].

[3] M. D'Elia, Lect. Notes Phys. **871** (2013) 181 [arXiv:1209.0374 [hep-lat]].

[4] K. Fukushima, Lect. Notes Phys. **871** (2013) 241 [arXiv:1209.5064 [hep-ph]].

[5] M. Frasca and M. Ruggieri, Phys. Rev. D **83** (2011) 094024 [arXiv:1103.1194 [hep-ph]].

[6] T. Kojo and N. Su, Phys. Lett. B **720** (2013) 192 [arXiv:1211.7318 [hep-ph]]; T. Kojo and N. Su, Phys. Lett. B **726** (2013) 839 [arXiv:1305.4510 [hep-ph]].

[7] V. Skokov, Phys. Rev. D **85** (2012) 034026 [arXiv:1112.5137 [hep-ph]].

[8] K. Fukushima and J. M. Pawlowski, Phys. Rev. D **86** (2012) 076013 [arXiv:1203.4330 [hep-ph]];

[9] M. D'Elia and F. Negro, Phys. Rev. D **83** (2011) 114028 [arXiv:1103.2080 [hep-lat]].

[10] C. Bonati, M. D'Elia, M. Mariti, F. Negro and F. Sanfilippo, arXiv:1310.8656 [hep-lat].

[11] G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz and A. Schafer, Phys. Rev. D **86** (2012) 071502 [arXiv:1206.4205 [hep-lat]].

[12] G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz, S. Krieg, A. Schafer and K. K. Szabo, JHEP **1202** (2012) 044 [arXiv:1111.4956 [hep-lat]].

[13] G. S. Bali, F. Bruckmann, M. Constantinou, M. Costa, G. Endrodi, S. D. Katz, H. Panagopoulos and A. Schafer, Phys. Rev. D **86** (2012) 094512 [arXiv:1209.6015 [hep-lat]].

[14] G. S. Bali, F. Bruckmann, G. Endrodi, F. Gruber and A. Schaefer, JHEP **1304** (2013) 130 [arXiv:1303.1328 [hep-lat]].

[15] P. V. Buividovich, M. N. Chernodub, E. V. Luschevskaya and M. I. Polikarpov, Phys. Lett. B **682** (2010) 484 [arXiv:0812.1740 [hep-lat]].

[16] E. -M. Ilgenfritz, M. Kalinowski, M. Muller-Preussker, B. Petersson and A. Schreiber, Phys. Rev. D **85** (2012) 114504 [arXiv:1203.3360 [hep-lat]].

[17] E. -M. Ilgenfritz, M. Muller-Preussker, B. Petersson and A. Schreiber, arXiv:1310.7876 [hep-lat].

[18] V. P. Gusynin, V. A. Miransky and I. A. Shovkovy, Nucl. Phys. B **462** (1996) 249 [hep-ph/9509320].

[19] K. Fukushima and Y. Hidaka, Phys. Rev. Lett. **110** (2013) 031601 [arXiv:1209.1319 [hep-ph]].

[20] M. N. Chernodub, Phys. Rev. Lett. **106** (2011) 142003 [arXiv:1101.0117 [hep-ph]];

[21] V. V. Braguta, P. V. Buividovich, M. N. Chernodub, A. Y. .Kotov and M. I. Polikarpov, Phys. Lett. B **718** (2012) 667 [arXiv:1104.3767 [hep-lat]].

[22] Y. Hidaka and A. Yamamoto, Phys. Rev. D **87** (2013) 094502 [arXiv:1209.0007 [hep-ph]].

[23] F. Bruckmann and G. Endrodi, Phys. Rev. D **84** (2011) 074506 [arXiv:1104.5664 [hep-lat]].

[24] D. S. Lee, C. N. Leung and Y. J. Ng, Phys. Rev. D **55** (1997) 6504 [hep-th/9701172].

[25] C. N. Leung and S. -Y. Wang, Annals Phys. **322** (2007) 701 [hep-ph/0503298]; Nucl. Phys. B **747** (2006) 266 [hep-ph/0510066].

[26] A. Ayala, A. Bashir, A. Raya and E. Rojas, Phys. Rev. D **73** (2006) 105009 [hep-ph/0602209].

[27] E. Rojas, A. Ayala, A. Bashir and A. Raya, Phys. Rev. D **77** (2008) 093004 [arXiv:0803.4173 [hep-ph]].

[28] G. Murguia, A. Raya, A. Sanchez and E. Reyes, Am. J. Phys. **78** (2010) 700 [arXiv:0910.1881 [hep-th]].

[29] P. Watson and H. Reinhardt, arXiv:1310.6050 [hep-ph].

[30] V. I. Ritus, Annals Phys. **69** (1972) 555; V. I. Ritus, Sov. Phys. JETP **48** (1978) 788 [Zh. Eksp. Teor. Fiz. **75** (1978) 1560].

[31] V. I. Ritus, IN *GINZBURG, V.L. (ED.): ISSUES IN INTENSE-FIELD QUANTUM ELECTRODYNAMICS* 180-199

[32] D. B. Leinweber *et al.* [UKQCD Collaboration], Phys. Rev. D **60** (1999) 094507 [Erratum-ibid. D **61** (2000) 079901] [hep-lat/9811027].

[33] A. Cucchieri and T. Mendes, PoS LAT **2007** (2007) 297 [arXiv:0710.0412 [hep-lat]].

[36] C. S. Fischer, A. Maas and J. M. Pawlowski, Annals Phys. **324** (2009) 2408 [arXiv:0810.1987 [hep-ph]].

[37] M. Q. Huber and L. von Smekal, JHEP **1304** (2013) 149 [arXiv:1211.6092 [hep-th]].

[38] C. S. Fischer, A. Maas and J. A. Muller, Eur. Phys. J. C **68** (2010) 165 [arXiv:1003.1960 [hep-ph]].

[39] C. S. Fischer and J. Luecker, Phys. Lett. B **718** (2013) 1036 [arXiv:1206.5191 [hep-ph]].

[40] T. D. Cohen, D. A. McGady and E. S. Werbos, Phys. Rev. C **76** (2007) 055201 [arXiv:0706.3208 [hep-ph]].

[41] I. A. Shushpanov and A. V. Smilga, Phys. Lett. B **402** (1997) 351 [hep-ph/9703201].

[42] B. L. Ioffe and A. V. Smilga, Nucl. Phys. B **232** (1984) 109.

[43] V. M. Belyaev and Y. .I. Kogan, Yad. Fiz. **40** (1984) 1035.

[44] I. A. Batalin and A. E. Shabad, Zh. Eksp. Teor. Fiz. **60** (1971) 894.

[45] A. E. Shabad, Annals Phys. **90** (1975) 166.

[46] W. Dittrich and H. Gies, Springer Tracts Mod. Phys. **166** (2000) 1.

[47] W. H. Furry, Phys. Rev. **51** (1937) 125.

[48] K. Hattori and K. Itakura, Annals Phys. **330** (2013) 23 [arXiv:1209.2663 [hep-ph]].

[49] C. S. Fischer and R. Alkofer, Phys. Rev. D **67** (2003) 094020 [hep-ph/0301094].

[50] P. O. Bowman, U. M. Heller, D. B. Leinweber, M. B. Parappilly and A. G. Williams, Phys. Rev. D **70** (2004) 034509 [hep-lat/0402032].

[51] C. S. Fischer, P. Watson and W. Cassing, Phys. Rev. D **72** (2005) 094025 [hep-ph/0509213].

[52] A. C. Aguilar, D. Binosi and J. Papavassiliou, Phys. Rev. D **88** (2013) 074010 [arXiv:1304.5936 [hep-ph]].

[53] E. J. Ferrer and V. de la Incera, Phys. Rev. D **58** (1998) 065008 [hep-th/9803226].

[54] C. S. Fischer, D. Nickel and J. Wambach, Phys. Rev. D **76** (2007) 094009 [arXiv:0705.4407 [hep-ph]]; C. S. Fischer and R. Williams, Phys. Rev. D **78** (2008) 074006 [arXiv:0808.3372 [hep-ph]].

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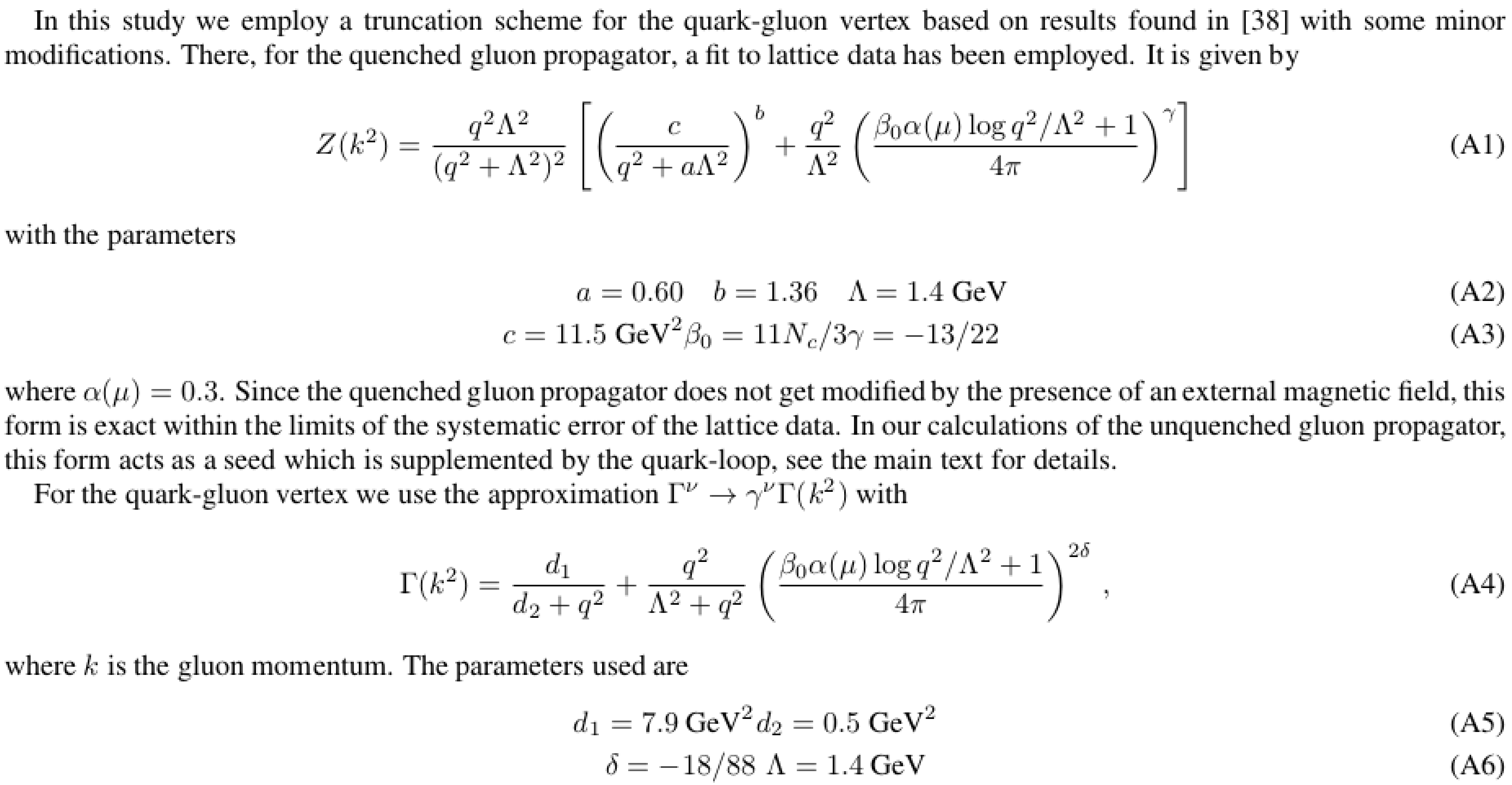


In this study we employ a truncation scheme for the quark-gluon vertex based on results found in [38] with some minor modifications. There, for the quenched gluon propagator, a fit to lattice data has been employed. It is given by

$$Z(k^2) = \frac{q^2\Lambda^2}{(q^2+\Lambda^2)^2}\left[\left(\frac{c}{q^2+a\Lambda^2}\right)^b + \frac{q^2}{\Lambda^2}\left(\frac{\beta_0\alpha(\mu)\log q^2/\Lambda^2+1}{4\pi}\right)^\gamma\right] \quad \text{(A1)}$$

with the parameters

$$a = 0.60 \quad b = 1.36 \quad \Lambda = 1.4 \text{ GeV} \quad \text{(A2)}$$

$$c = 11.5 \text{ GeV}^2 \beta_0 = 11N_c/3\gamma = -13/22 \quad \text{(A3)}$$

where $\alpha(\mu) = 0.3$. Since the quenched gluon propagator does not get modified by the presence of an external magnetic field, this form is exact within the limits of the systematic error of the lattice data. In our calculations of the unquenched gluon propagator, this form acts as a seed which is supplemented by the quark-loop, see the main text for details.

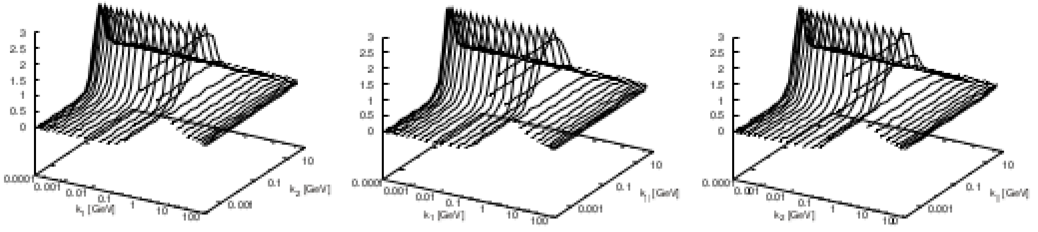
For the quark-gluon vertex we use the approximation $\Gamma^\nu \to \gamma^\nu\Gamma(k^2)$ with

$$\Gamma(k^2) = \frac{d_1}{d_2+q^2} + \frac{q^2}{\Lambda^2+q^2}\left(\frac{\beta_0\alpha(\mu)\log q^2/\Lambda^2+1}{4\pi}\right)^{2\delta}, \quad \text{(A4)}$$

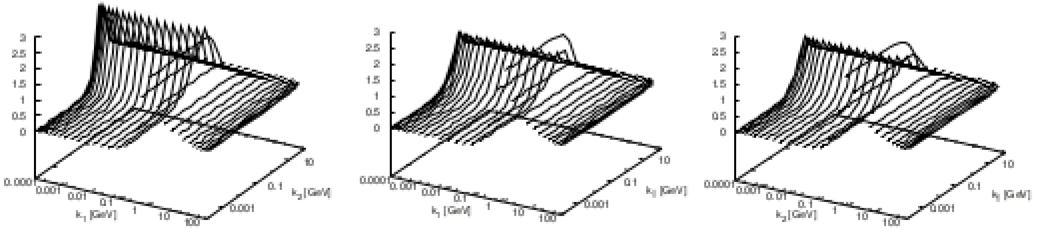
where k is the gluon momentum. The parameters used are

$$d_1 = 7.9 \text{ GeV}^2 d_2 = 0.5 \text{ GeV}^2 \quad \text{(A5)}$$

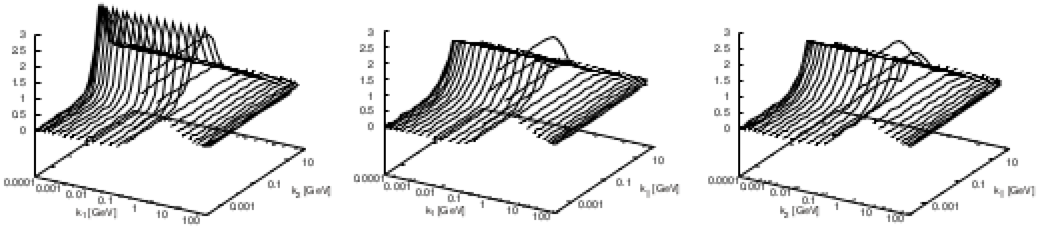
$$\delta = -18/88 \ \Lambda = 1.4 \text{ GeV} \quad \text{(A6)}$$



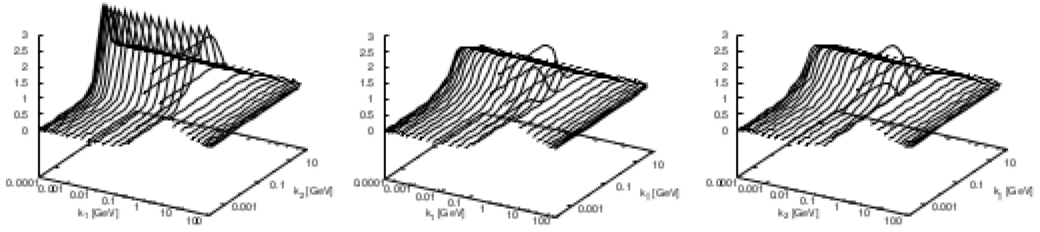
(a) $Z_{\parallel}(k1,k_2)$ $(eH = 0\,\mathrm{GeV}^2)$ (b) $Z_{\parallel}(k1,k_{\parallel})$ $(eH = 0\;\mathrm{GeV}^2)$ (c) $Z_{\parallel}(k2,k_{\parallel})$ $(eH = 0\;\mathrm{GeV}^2)$



(d) $Z_{\parallel}(k1,k2)$ [GeV] $(eH = 0.5\,\mathrm{GeV}^2)$ (e) $Z_{\parallel}(k1,k_{\parallel})$ $(eH = 0.5\;\mathrm{GeV}^2)$ (f) $Z_{\parallel}(k2,k_{\parallel})$ $(eH = 0.5\,\mathrm{GeV}^2)$



(g) $Z_{\parallel}(k1,k2)$ [GeV] $(eH = 1\,\mathrm{GeV}^2)$ (h) $Z_{\parallel}(k1,k_{\parallel})$ $(eH = 1\;\mathrm{GeV}^2)$ (i) $Z_{\parallel}(k2,k_{\parallel})$ $(eH = 1\,\mathrm{GeV}^2)$



(j) $Z_{\parallel}(k1,k2)$ [GeV] $(eH = 4\,\mathrm{GeV}^2)$ (k) $Z_{\parallel}(k1,k_{\parallel})$ $(eH = 4\;\mathrm{GeV}^2)$ (l) $Z_{\parallel}(k2,k_{\parallel})$ $(eH = 4\,\mathrm{GeV}^2)$

FIG. 6. Gluon dressing function $Z_{\parallel}(k_1,k_2,k_{\parallel})$ for $eH = 0\;\mathrm{GeV}^2$ (quenched, first line), $eH = 0.5\;\mathrm{GeV}^2$ (second line), $eH = 1\;\mathrm{GeV}^2$ (third line) and $eH = 4\;\mathrm{GeV}^2$ (fourth line) and different momentum slices, where the third momentum is set to zero respectively.

$$\begin{aligned}\delta_{n_1(l',\sigma_1)n_2(l,\sigma_2)}\delta_{n_3(l',\sigma_3)n_4(l,\sigma_4)}\delta_{\sigma_1,\sigma_2}\delta_{\sigma_3,\sigma_4}\delta_{\sigma_1,\pm\sigma_3} &\propto \delta_{l,l'}\,, \\ \delta_{n_1(l',\sigma_1)n_2(l,\sigma_2)}\delta_{n_3(l',\sigma_3)n_4(l,\sigma_4)}\delta_{\sigma_1,-\sigma_2}\delta_{\sigma_3,-\sigma_4}\delta_{\sigma_1,\mp\sigma_3} &\propto \delta_{l+\sigma_1 \mathrm{sgn}(eH),l'}\,. \end{aligned} \tag{75}$$

Thus, either the gluon splits into a quark-antiquark pair on the same Landau level, or it induces a transition from one Landau level to the next. Other cases are not compatible with the spin-one-boson nature of the gluon.

Putting everything together, the gluon DSE reads

$$\begin{aligned} k^2\left(Z_0^{-1}(k)P_0^{\mu\nu} + Z_{\|}^{-1}(k)P_{\|}^{\mu\nu} + Z_{\perp}^{-1}(k)P_{\perp}^{\mu\nu}\right) =& k^2 Z^{-1}(k)P^{\mu\nu} - 2\pi g^2 eHN_c e^{-k_\perp^2/2|eH|}\Gamma(q_{\|}^2 + (q'_{\|})^2) \\ &\times \sum_{l,l'}\int \frac{d^2 q_{\|}}{(2\pi)^4}\sum_{\{\sigma_i\}}\delta_{n_1 n_2}\delta_{n_3 n_4}\frac{T_1^{\mu\nu} + T_2^{\mu\nu} + T_3^{\mu\nu}}{\mathcal{D}(q,q')}. \end{aligned} \tag{76}$$

The equation can be decomposed into its contributions from the polarization subspaces denoted by $P_\perp^{\mu\nu}$, $P_{\|}^{\mu\nu}$ and $P_0^{\mu\nu}$. In the following, $Z(k)$ stands for the dressing function of the quenched isotropic gluon propagator. The resulting equations for the dressing functions for the full gluon propagator read in a compact notation (here we have one quark flavor, $N_f = 1$, with charge $q_f = e$ for brevity, although later on we solve for $N_f = 1 + 1$ up- and down-quarks with charges $q_f = +2/3\,e$ and $q_f = -1/3\,e$ respectively).

$$Z_{\|}^{-1}(k) = Z^{-1}(k) - \beta\sum_l\int\frac{d^2 q_{\|}}{(2\pi)^4}\,\frac{\chi(l)}{2}\,\frac{M_{\|}(q,q')}{\mathcal{D}(q,q')}\Big|_{l'=l}\Gamma(q_{\|}^2 + (q'_{\|})^2), \tag{77}$$

$$Z_{\perp}^{-1}(k) = Z^{-1}(k) - \beta\sum_l\int\frac{d^2 q_{\|}}{(2\pi)^4}\sum_{l'=l\pm1,l'\geq0}\frac{N_\perp(q,q')}{\mathcal{D}(q,q')}\,\Gamma(q_{\|}^2 + (q'_{\|})^2), \tag{78}$$

$$Z_0^{-1}(k) = Z^{-1}(k) - \beta\sum_l\int\frac{d^2 q_{\|}}{(2\pi)^4}\left\{\frac{\chi(l)}{2}\,\frac{M_0(q,q')}{\mathcal{D}(q,q')}\Big|_{l'=l} + \sum_{l'=l\pm1,l'\geq0}\frac{N_0(q,q')}{\mathcal{D}(q,q')}\right\}\Gamma(q_{\|}^2 + (q'_{\|})^2)\,. \tag{79}$$

$$\begin{aligned} M_{\|}(q,q') &= A_\perp(q)A_\perp(q')q_\perp {q'}_\perp + A_{\|}(q)A_{\|}(q')\left(q_{\|}\cdot {q'}_{\|} - 2q_{\|}^2\sin^2(\phi)\right)\,, \\ N_\perp(q,q') &= A_\perp(q)A_\perp(q')q_\perp {q'}_\perp\left(1 - 2\frac{k_2^2}{k_\perp^2}\right) + A_{\|}(q)A_{\|}(q')q_{\|}\cdot {q'}_{\|}, \\ M_0(q,q') &= A_\perp(q)A_\perp(q')q_\perp {q'}_\perp\frac{k_\perp^2}{k^2} + A_{\|}(q)A_{\|}(q')\left(q_{\|}\cdot {q'}_{\|}\frac{k_\perp^2}{k^2} - 2\frac{q_{\|}\cdot k_{\|}{q'}_{\|}\cdot k_{\|}}{k^2}\frac{k_\perp^2}{k_{\|}^2}\right)\,, \\ N_0(q,q') &= A_\perp(q)A_\perp(q')q_\perp {q'}_\perp\left(\frac{k_{\|}^2}{k^2} - 2\frac{k_1^2}{k^2}\frac{k_{\|}^2}{k_\perp^2}\right) + A_{\|}(q)A_{\|}(q')q_{\|}\cdot {q'}_{\|}\frac{k_{\|}^2}{k^2}. \end{aligned}$$

$\int_q \equiv \int \frac{d^2 q_{||}}{(2\pi)^4} \int\limits_{-\infty}^{\infty} dq_2 dk_1$ and $D_q(q) \equiv B^2(q) + A_{||}^2(q) q_{||}^2 + A_{\perp}^2(q) q_{\perp}^2$, the quark DSE then reads

$$\begin{aligned} B(p) &= m + g^2 C_F \int\limits_q \frac{B(q)}{D_q(q)} e^{-k_\perp^2/2|eH|} \Gamma(k^2) \left(\frac{Z_{||}(k)}{k^2} + \frac{k_\perp^2}{k^2} \frac{Z_0(k)}{k^2} \right) \\ &\quad + \frac{2}{\chi(l)} g^2 C_F \sum_{l_q = l \pm 1, l_q \geq 0} \int\limits_q \frac{B(q)}{D_q(q)} e^{-k_\perp^2/2|eH|} \Gamma(k^2) \left(\frac{Z_\perp(k)}{k^2} + \frac{k_{||}^2}{k^2} \frac{Z_0(k)}{k^2} \right) , \end{aligned} \tag{84}$$

$$\begin{aligned} A_{||}(p) &= 1 - g^2 C_F \int\limits_q \frac{A_{||}(q)}{D_q(q)} \frac{e^{-k_\perp^2/2|eH|}}{p_{||}^2} \Gamma(k^2) \left(\frac{Z_{||}(k)}{k^2} K_1(p,q) + \frac{Z_0(k)}{k^2} K_2(p,q) \right) \\ &\quad + \frac{2}{\chi(l)} g^2 C_F \sum_{l_q = l \pm 1, l_q \geq 0} \int\limits_q \frac{A_{||}(q)}{D_q(q)} \frac{e^{-k_\perp^2/2|eH|}}{p_{||}^2} \Gamma(k^2) \left(\frac{Z_\perp(k)}{k^2} p_{||} \cdot q_{||} + \frac{Z_0(k)}{k^2} p_{||} \cdot q_{||} \frac{k_{||}^2}{k^2} \right), \end{aligned} \tag{85}$$

with kernels

$$K_1(p,q) = 2 \frac{(p_{||} q_{||} \sin(\phi))^2}{k_{||}^2} - p_{||} \cdot q_{||} \tag{86}$$

$$K_2(p,q) = 2 \frac{k_\perp^2}{k_{||}^2} \frac{p_{||} \cdot k_{||} q_{||} \cdot k_{||}}{k^2} - p_{||} \cdot q_{||} \frac{k_\perp^2}{k^2} \tag{87}$$

Furthermore,

$$\begin{aligned} A_\perp(p) &= 1 + g^2 C_F \int\limits_q \frac{A_\perp(q)}{D_q(q)} \frac{e^{-k_\perp^2/2|eH|}}{p_\perp^2} \Gamma(k^2) \left(\frac{Z_{||}(k)}{k^2} p_\perp q_\perp + \frac{Z_0(k)}{k^2} p_\perp q_\perp \frac{k_\perp^2}{k^2} \right) \\ &\quad + \frac{2}{\chi(l)} g^2 C_F \sum_{l_q = l \pm 1, l_q \geq 0} \int\limits_q \frac{A_\perp(q)}{D_q(q)} \frac{e^{-k_\perp^2/2|eH|}}{p_\perp^2} \Gamma(k^2) \left(\frac{Z_\perp(k)}{k^2} p_\perp q_\perp \left(1 - 2 \frac{k_2^2}{k_\perp^2} \right) + \frac{Z_0(k)}{k^2} p_\perp q_\perp \left(\frac{k_{||}^2}{k^2} - 2 \frac{k_{||}^2 k_1^2}{k_\perp^2 k^2} \right) \right) . \end{aligned} \tag{88}$$