Institut für
Theoretische Physik

JUSTUS-LIEBIG-
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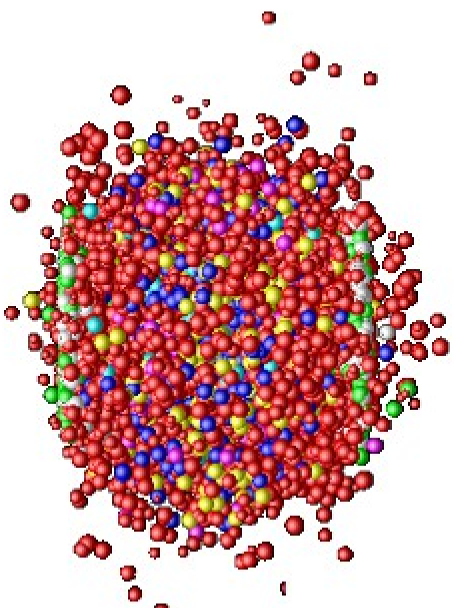
GIESSEN

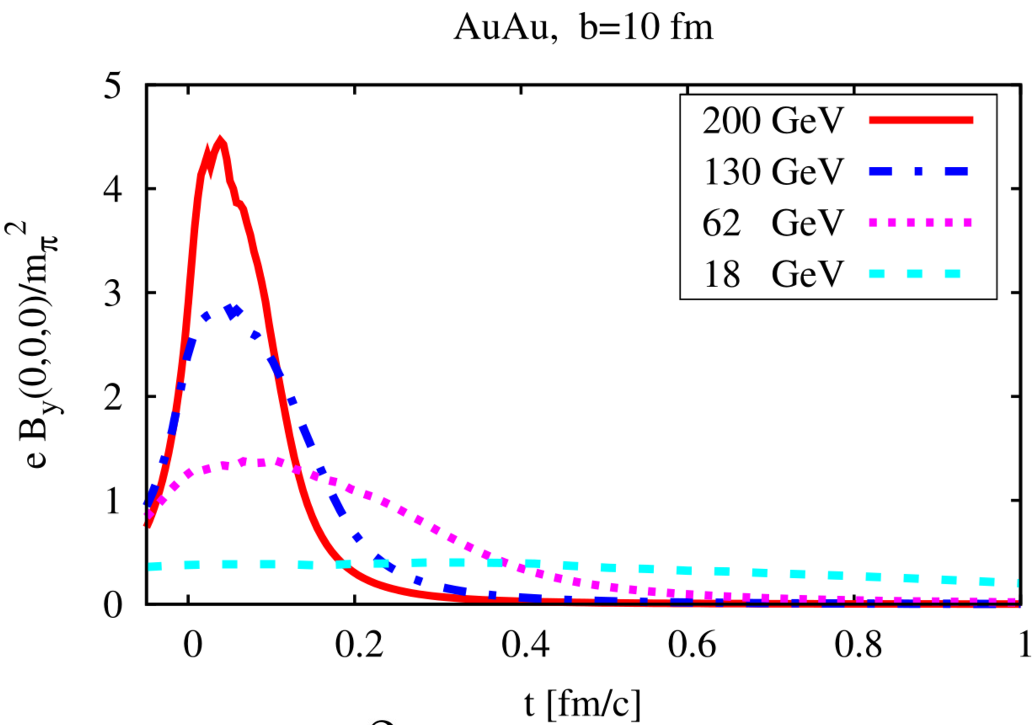
# Electric and magnetic properties of hot partonic matter



Thorsten Steinert

Gießen, 01.11.2013

# Electromagnetic fields in peripheral heavy-ion collisions




$$eE_{max} = 0.0386 \text{ GeV}^2 \qquad eB_{max} = 0.0869 \text{ GeV}^2$$

- **Huge electromagnetic fields in Heavy Ion Collisions!**
- **What is the response of the QGP to these fields?**

V. Voronyuk et al., Phys. Rev. C83 054911 (2011)

# Parton-Hadron-String-Dynamics I

- QGP consists of strongly interacting off-shell partons
- We need a transport model for off-shell particles with finite width:

**Parton-Hadron-String-Dynamics (PHSD)**

$$2p^{\mu}\partial_{\mu}^{x} i\bar{G}^{\lessgtr} - \left\{\bar{\Sigma}^{\delta} + \mathrm{Re}\bar{\Sigma}^{R}, i\bar{G}^{\lessgtr}\right\} - \left\{i\bar{\Sigma}^{\lessgtr}, \mathrm{Re}\bar{G}^{R}\right\} = i\bar{\Sigma}^{<} i\bar{G}^{>} - i\bar{\Sigma}^{>} i\bar{G}^{<}$$

$$\left\{\bar{M}, i\bar{G}^{\lessgtr}\right\} - \left\{i\bar{\Sigma}^{\lessgtr}, \mathrm{Re}\bar{G}^{R}\right\} = i\bar{\Sigma}^{<} i\bar{G}^{>} - i\bar{\Sigma}^{>} i\bar{G}^{<}$$

$$\left\{\bar{F}(p,x), \bar{G}(p,x)\right\} = \partial_{\mu}^{p}\bar{F}(p,x)\partial_{x}^{\mu}\bar{G}(p,x) - \partial_{x}^{\mu}\bar{F}(p,x)\partial_{\mu}^{p}\bar{G}(p,x)$$

W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919;
NPA831 (2009) 215;
W. Cassing, EPJ ST 168 (2009) 3

# Parton-Hadron-String-Dynamics II

- **Solve the equation with the testparticle approximation**

$$F_{XP} = iG^{<}(X,P) \sim \sum_{i=1}^{N} \delta^{(3)}(\mathbf{X}-\mathbf{X}_i(t))\,\delta^{(3)}(\mathbf{P}-\mathbf{P}_i(t))\,\delta(P_0-\epsilon_i(t))$$

$$\frac{d\mathbf{X}_i}{dt} = \frac{1}{1-C_{(i)}}\frac{1}{2\epsilon_i}\left[2\mathbf{P}_i + \nabla_{P_i}\mathrm{Re}\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2-\mathbf{P}_i^2-M_0^2-\mathrm{Re}\Sigma^{ret}_{(i)}}{\Gamma_{(i)}}\nabla_{P_i}\Gamma_{(i)}\right]$$

$$\frac{d\mathbf{P}_i}{dt} = -\frac{1}{1-C_{(i)}}\frac{1}{2\epsilon_i}\left[\nabla_{X_i}\mathrm{Re}\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2-\mathbf{P}_i^2-M_0^2-\mathrm{Re}\Sigma^{ret}_{(i)}}{\Gamma_{(i)}}\nabla_{X_i}\Gamma_{(i)}\right]$$

$$\frac{d\epsilon_i}{dt} = \frac{1}{1-C_{(i)}}\frac{1}{2\epsilon_i}\left[\frac{\partial\mathrm{Re}\Sigma^{ret}_{(i)}}{\partial t} + \frac{\epsilon_i^2-\mathbf{P}_i^2-M_0^2-\mathrm{Re}\Sigma^{ret}_{(i)}}{\Gamma_{(i)}}\frac{\partial\Gamma_{(i)}}{\partial t}\right]$$

**Effectiv selfenergies described by**

**Dynamical QuasiParticle Model (DQPM)**

W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919;
NPA831 (2009) 215;
W. Cassing, EPJ ST 168 (2009) 3

A. Peshier, W. Cassing, PRL 94 (2005) 172301;
Casssing, NPA 791 (2007) 365: NPA 793 (2007)

# Dynamical QuasiParticle Model I

**Basic idea:** **Interacting quasi-particles**

- **massive quarks and gluons (g, q, $q_{bar}$) with spectral functions :**

$$\rho_i(\omega, T) = \frac{4\,\omega\,\Gamma_i(T)}{\left(\omega^2 - \vec{p}^2 - M_i^2(T)\right)^2 + 4\,\omega^2\,\Gamma_i^2(T)} \qquad (i = q, \bar{q}, g)$$

- **running coupling (pure glue):**

$$\alpha_s(T) = \frac{g^2(T)}{4\pi} = \frac{12\pi}{(11N_c - 2N_f)\ln[\lambda^2(T/T_c - T_s/T_c)^2]}$$

- **quarks**

**mass:** $$M^2_{q(\bar{q})}(T) = \frac{N_c^2 - 1}{8N_c} g^2 \left(T^2 + \frac{\mu_q^2}{\pi^2}\right)$$

**width:** $$\Gamma_{q(\bar{q})}(T) = \frac{1}{3}\frac{N_c^2 - 1}{2N_c}\frac{g^2 T}{8\pi}\ln\left(\frac{2c}{g^2} + 1\right)$$

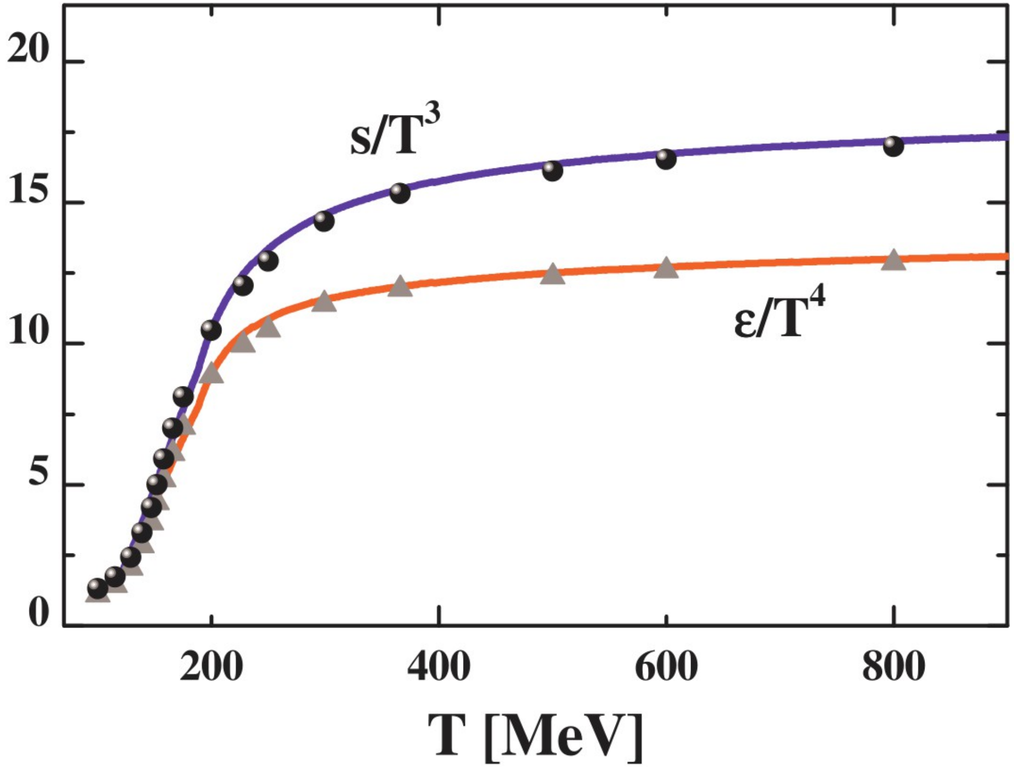
- **gluons:**

$$M_g^2(T) = \frac{g^2}{6}\left(\left(N_c + \frac{N_f}{2}\right)T^2 + \frac{N_c}{2}\sum_q \frac{\mu_q^2}{\pi^2}\right)$$

$$\Gamma_g(T) = \frac{1}{3} N_c \frac{g^2 T}{8\pi}\ln\left(\frac{2c}{g^2} + 1\right)$$

DQPM: Peshier, Cassing, PRL 94 (2005) 172301;
Cassing, NPA 791 (2007) 365: NPA 793 (2007)

# Dynamical QuasiParticle Model II




**fit to lattice (lQCD) results** (e.g. entropy density)
from Y. Aoki et al., JHEP 0906 088 (2009)

with 3 parameters: $T_s/T_c$=0.56; c=14.4; [illegible]=2.42

➔ **quasiparticle properties** (mass, width)

Bratkovskaya et al., Nucl. Phys. A 856, 162 (2011)

# Testparticle Approximation

**Propagation:**

- Use equations of motion for the testparticle propagation

**Collisions:**

- Close particles collide

$$|\vec{r}_1 - \vec{r}_2| \leq \sqrt{\frac{\sigma_{tot}(\sqrt{s})}{\pi}}$$

- Final state choosen by Monte Carlo

**Expectation values:**

- Sum up properties of interest

$$\langle \hat{A} \rangle = \sum_i A(\mathbf{X}_i, \mathbf{P}_i, \epsilon_i)$$

# Initialisation I

**Density of particle species:**

$$N_{q(g)} = d_{q(g)} \int_0^\infty \frac{d\omega}{2\pi} \int \frac{d^3p}{(2\pi)^3} 2\omega \rho_{q(g)}(\omega, \mathbf{p}) n_{F(B)}$$

**Four-momenta are choosen by Monte Carlo with respect to the distribution function I:**

$$I(\omega, p) = \frac{d_q}{2\pi^3} p^2 \ \omega \, \rho_q(\omega, p^2) \, n_F((\omega - \mu_q)/T)$$

**with $d_q$ = 18 = Spin x Flavor x Color**

# Initialisation II

- Initialize testparticles homogeneously in a finite box of $V = 9 \times 9 \times 9 \text{ fm}^3$ with periodic boundary conditions.

- For homogeneos systems the equations of motion simplify dramatically :

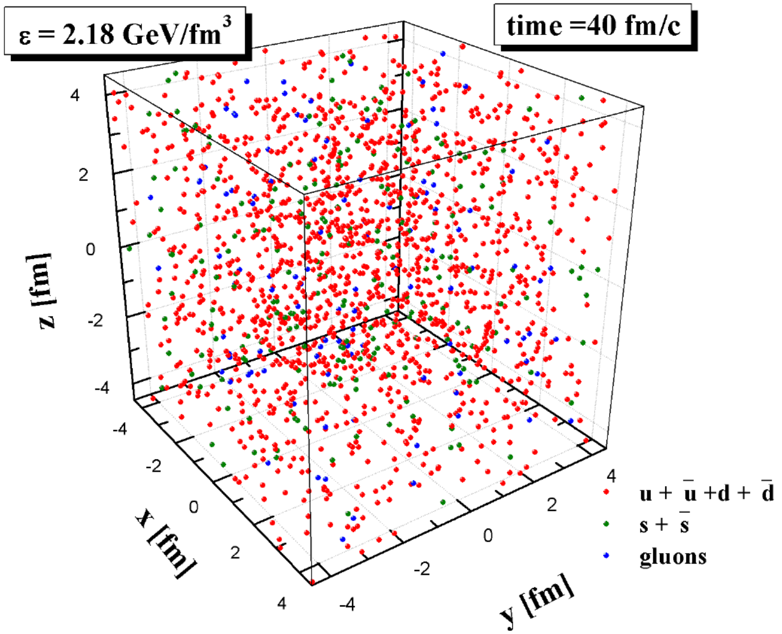
$$\mathbf{X}_i(t+dt) = \mathbf{X}_i(t) + \mathbf{P}_i(t)/\epsilon_i(t)dt$$

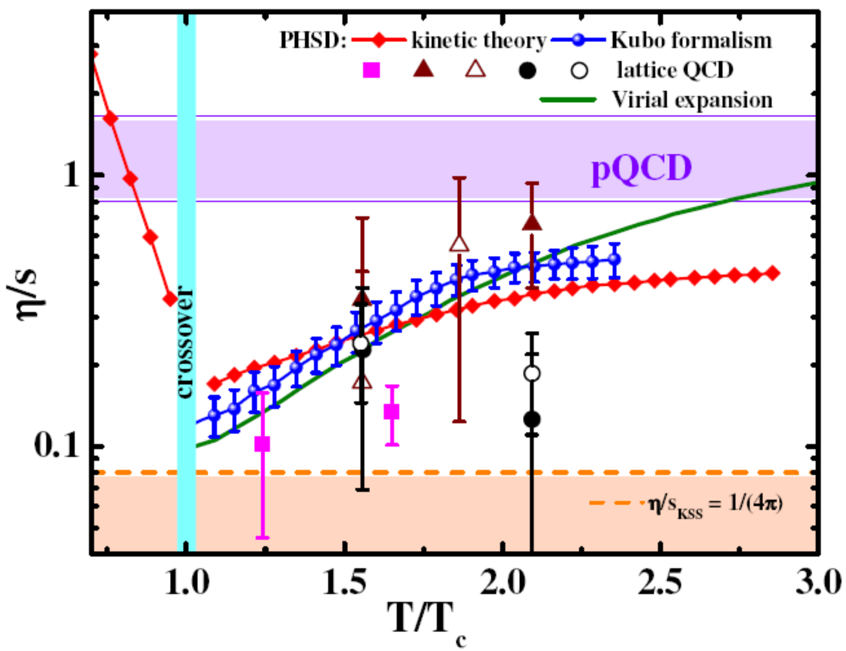
$$\mathbf{P}_i(t+dt) = \mathbf{P}_i(t)$$

$$\epsilon_i(t+dt) = \epsilon_i(t)$$

# Extracting transport coefficients

- Wait for thermodynamical equilibrium, then extract the desired properties of interest.



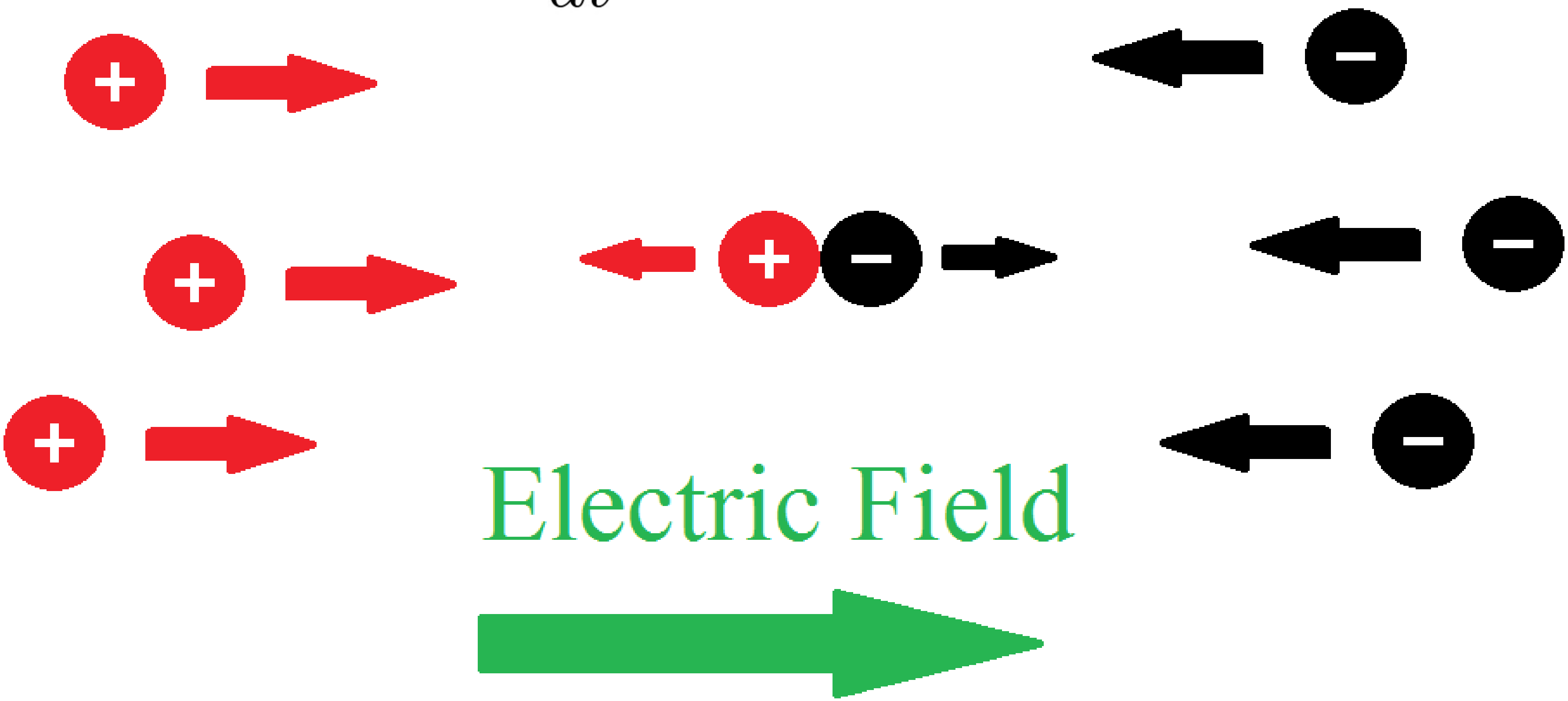

- Already done for the shear and bulk viscosity!

V. Ozvenchuk et al., PRC 87 (2013) 064903

# Electric conductivity I

**Additional force on particles of charge $q_j e$:**

$$\frac{d}{dt}p_z^j = q_j e E_z$$



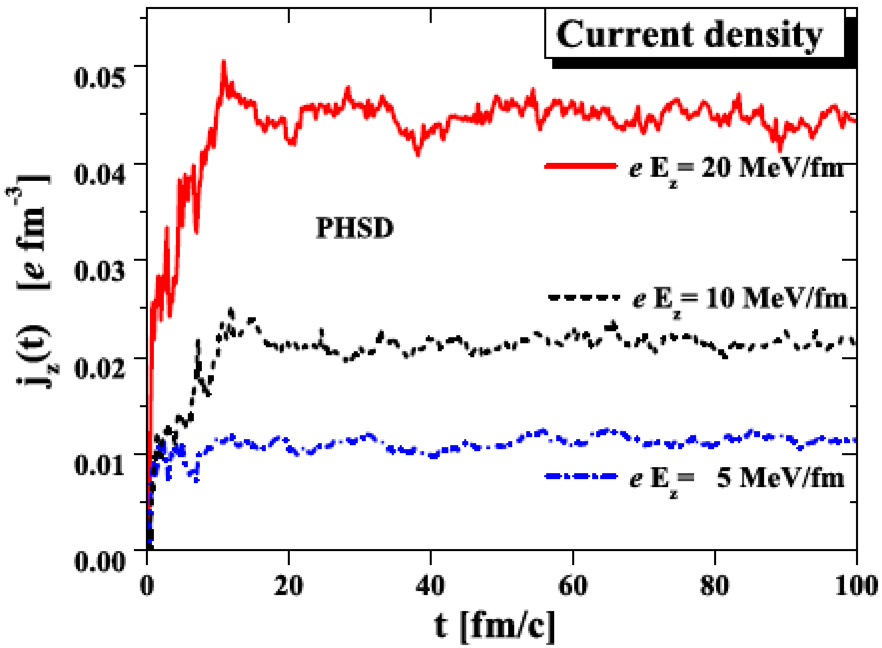
# Electric conductivity II

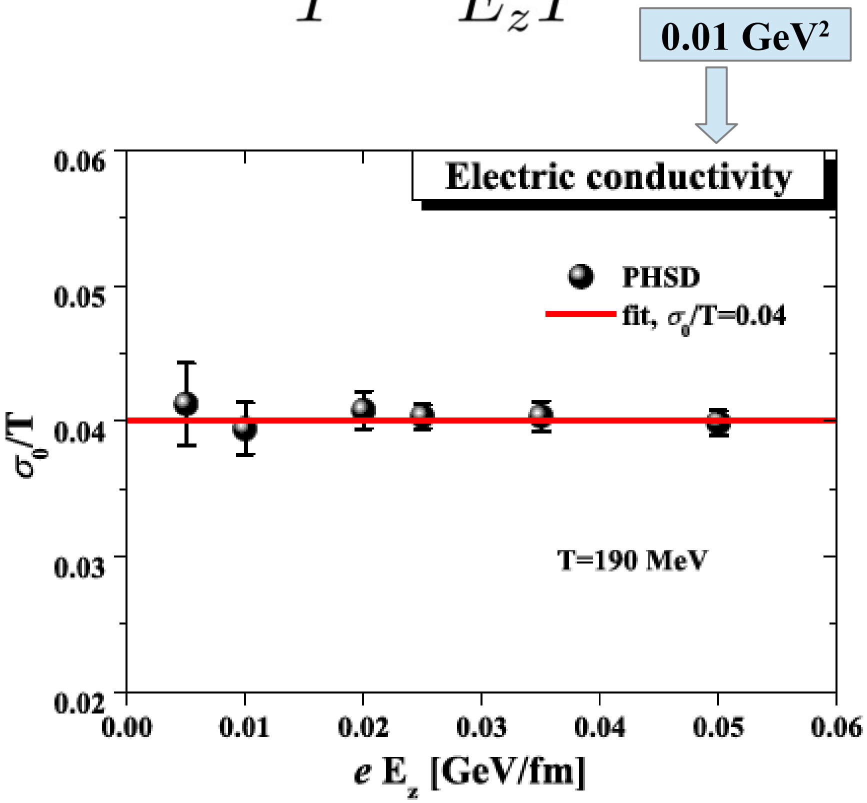
- induced current $j_z$

$$j_z(t) = \frac{1}{V} \sum_j eq_j \frac{p_z^j(t)}{M_j(t)}$$

- electric conductivity

$$\frac{\sigma_0}{T} = \frac{j_{eq}}{E_z T}$$




W. Cassing, T.S. et al., PRL 110, 182301 (2013)

# Drude model

**Charged particles are accelerated by the electric field and deaccelerated by collisions:**

$$m\dot{v} + \frac{m}{\tau} v_D = -eE$$

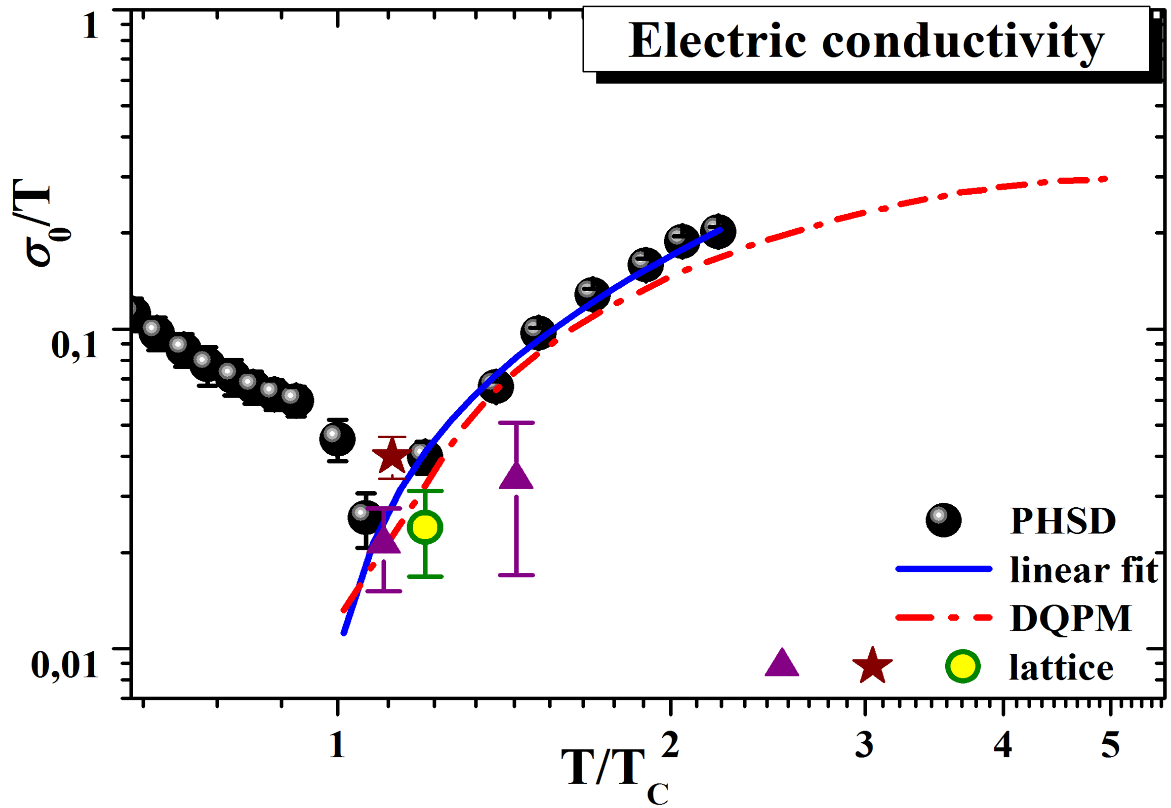
**Equilibrium for $v = v_D$:**

$$v_D = -\frac{e\tau}{m} E$$

**Drude formula:**

$$\sigma_0 = \frac{j}{E} = \frac{-env_D}{E} = \frac{e^2 \tau n}{m}$$

# Electric conductivity III




- relaxation time approach in DQPM:

  Drude formula:

$$\frac{\sigma_0(T)}{T} \approx \frac{2}{9} \frac{e^2 n_q(T)}{M_q(T)\Gamma_q(T)T}$$

W. Cassing, T.S. et al., PRL 110, 182301 (2013)

# Fermions in magentic fields I

## Start with the Dirac equation in an external field:

$$\begin{pmatrix} \mathbb{1}_2 E & 0_2 \\ 0_2 & \mathbb{1}_2 E \end{pmatrix} \psi(p) = H_{Dirac}\psi(p) = \gamma^0 \begin{pmatrix} \mathbb{1}_2 m & -\boldsymbol{\sigma}\mathbf{D} \\ \boldsymbol{\sigma}\mathbf{D} & \mathbb{1}_2 m \end{pmatrix} \psi(p)$$

$$D_\mu = p_\mu - qeA_\mu \qquad \mathbf{A} = \tfrac{1}{2}\mathbf{B} \times \mathbf{x}$$

## Klein-Gordon form:

$$2\psi_2^\dagger(p) E^2 \psi_2(p) = 2\psi_2^\dagger(p)(\mathbb{1}_2 m^2 + (\boldsymbol{\sigma}\mathbf{D})^2)\psi_2(p)$$

# Fermions in magentic fields II

## Neglect quadratic terms in B:

$$(\boldsymbol{\sigma}\mathbf{D})^2 = \mathbf{D}^2 - qe\boldsymbol{\sigma}\mathbf{B}$$

$$\mathbf{D}^2 = (\mathbf{p} - qe\mathbf{A})^2 = \mathbf{p}^2 - qe\mathbf{L}\mathbf{B}$$

## Hamiltonian with magnetic field:

$$H_{Dirac} = \sqrt{\mathbf{p}^2 + m^2 - qe(\mathbf{L} + \boldsymbol{\sigma})\mathbf{B}}$$

$$\approx E - \frac{qe}{2E}(\mathbf{L} + \boldsymbol{\sigma})\mathbf{B}$$

# Magnetic moment I

Response to magnetic field proportional to magnetic moment $\boldsymbol{\mu}$:

$$\Delta E_{mag} = -\boldsymbol{\mu} \cdot \mathbf{B}$$

- We need angular momentum L and spin S:

$$\boldsymbol{\mu} = \boldsymbol{\mu}_L + \boldsymbol{\mu}_S = \frac{qe}{2E}(\mathbf{L} + 2\mathbf{S})$$

- Angular momentum due to Lorentz force:

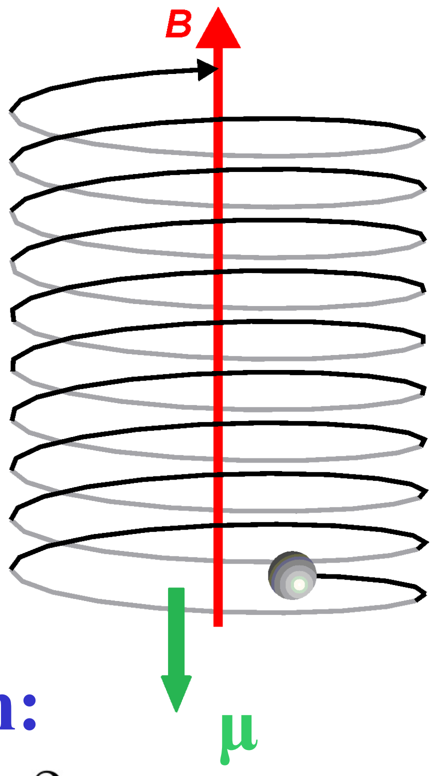
$$\mathbf{F}_L = \frac{qe}{E}(\mathbf{p} \times \mathbf{B})$$

# Angular momentum

- Angular momentum from rotation in magnetic field:

$$\mathbf{L} = (\mathbf{r} - \mathbf{r}_0) \times \mathbf{p}$$

$$L_y = \frac{-p_\perp^2}{qeB}$$




- Energy is independent from field strengh:

$$\Delta E_{mag,L} = -\mu_L B = -\frac{-p_\perp^2}{2BE} B = \frac{p_\perp^2}{2E}$$

# Spin

- Spins orient parallel to the magnetic field $\vec{S} \parallel \vec{B}$
- results in a finite magnetization of the QGP:

$$M = \frac{< \mu_S >}{V}$$

We need spinflips!

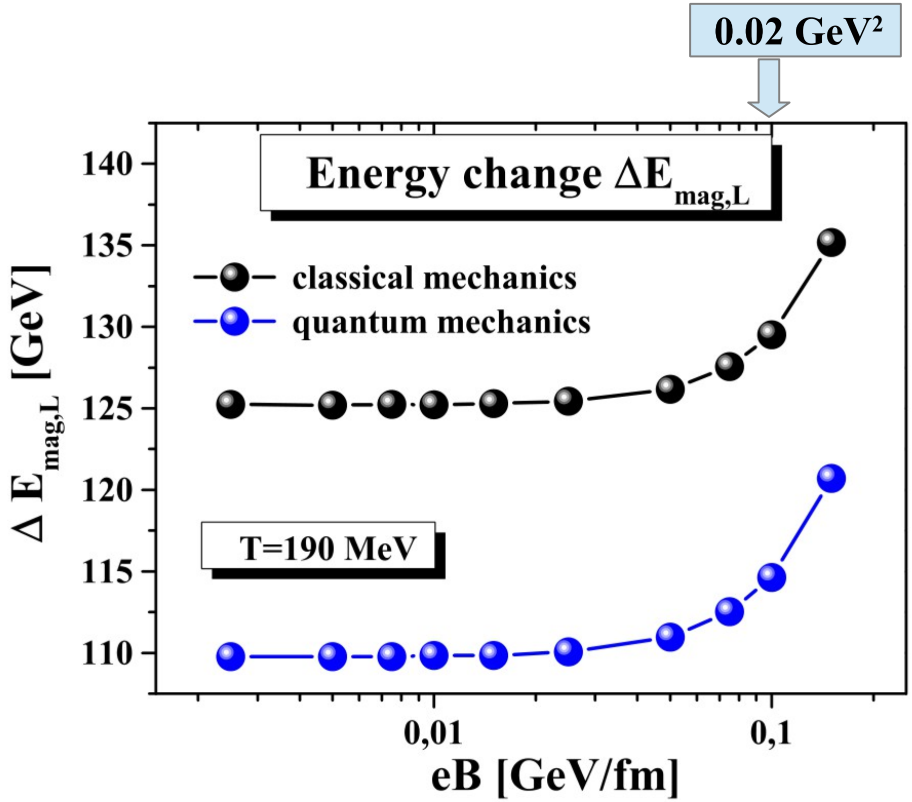
$$P_{\downarrow,\uparrow} = 1$$

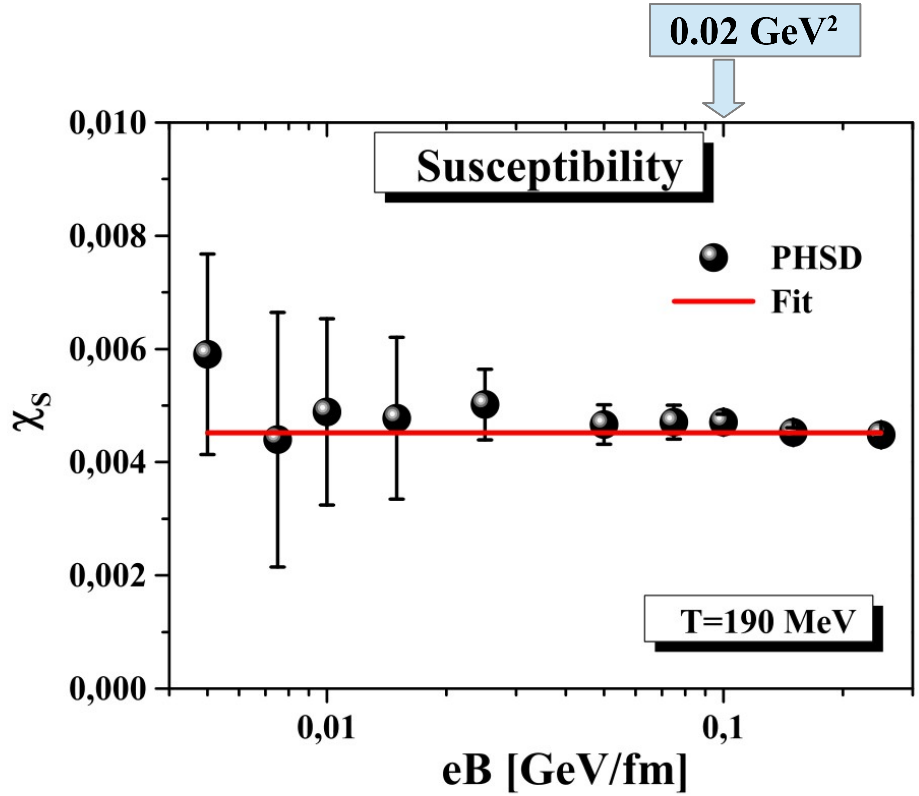
$$P_{\uparrow,\downarrow} = \exp\left(-\frac{qeB}{ET}\right)$$

Magnetization propor-
tional to magnetic field:

$$M = \chi_S B$$

# Magnetic moment II

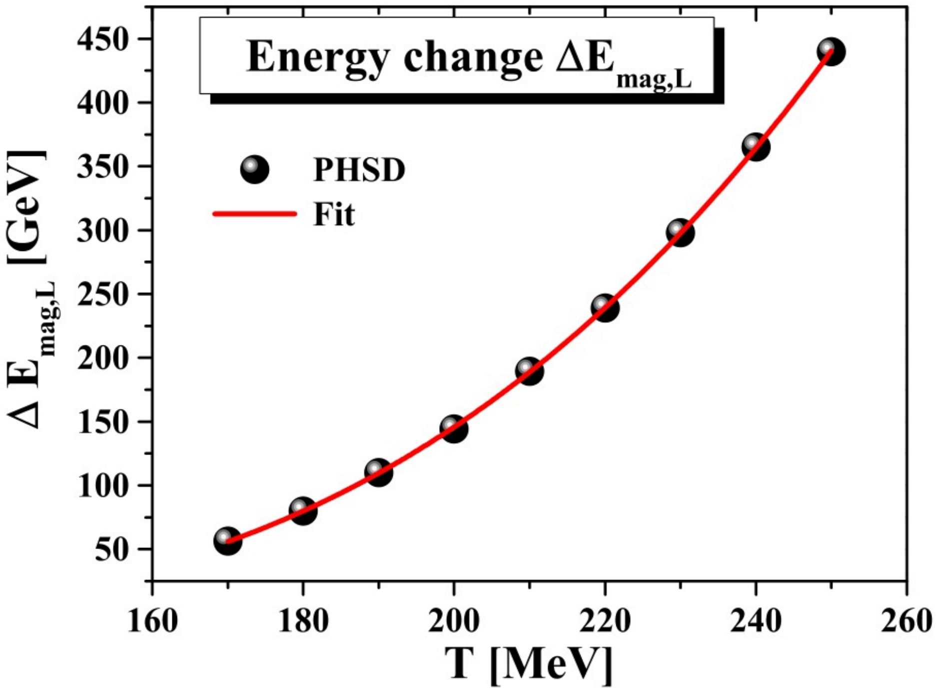


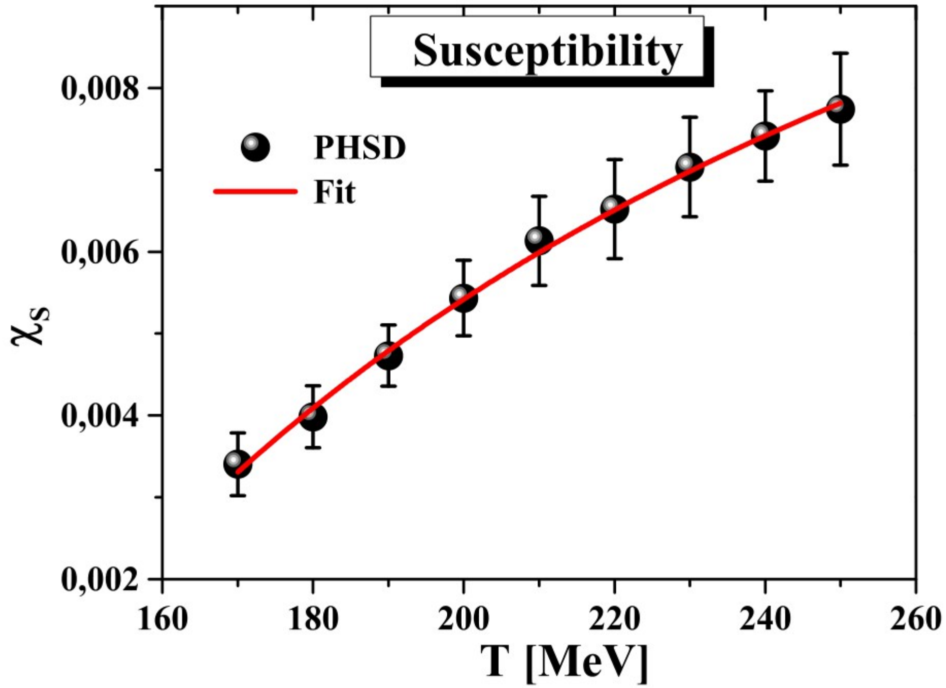




**For not too strong fields the susceptibility and the energy change are independent from the field strengh!**

**Note: Lattice QCD:** $0.1 < eB < 1.1\ GeV^2$

# Magnetic moment III




**Both rise with temperature (T>T$_c$):**

$$\Delta E_{mag,L}(T) = 0.29816 \cdot (T - 95.8592)^{2.81977} \; [MeV]$$

$$\chi_S(T) = 0.01736 - \frac{2.38671}{T}$$

# Critical magnetic field I

- The angular momentum behaves diamagnetic!
- The spin behaves paramagnetic!

$$\Delta E(T,B) = \Delta E_{mag,L}(T) - \chi_S(T) B^2$$

There is a critical field **$B_c$(T)** for which both contributions cancel out:

$$B_c(T) = \sqrt{\frac{\Delta E_{mag,L}}{\chi_S}}$$

# Critical magnetic field II

- **No thermal spin polarisation for huge magnetic fields.**
- **All spins point in the same direction:**

$$\mu_S = \frac{qe}{2E} n_{q+\bar{q}}$$

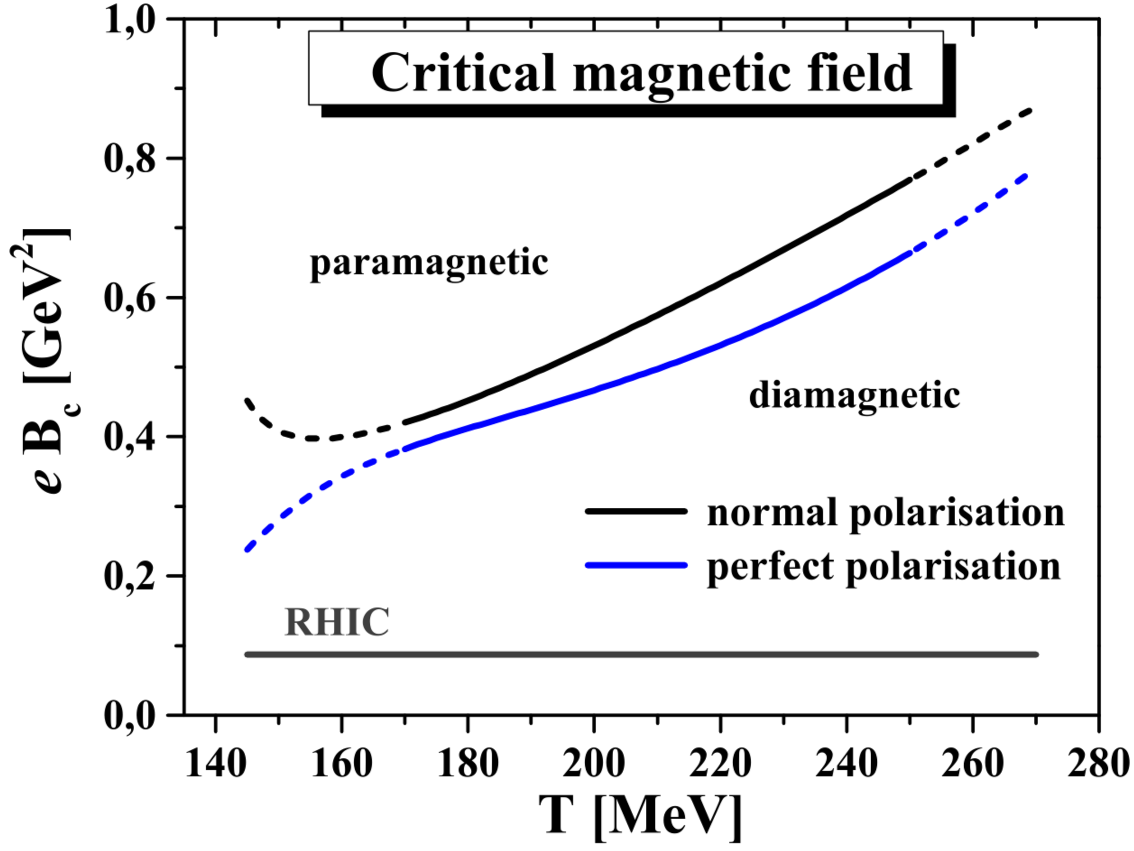
- **Energy rises linear with the field strength:**

$$\Delta E(T,B) = \Delta E_{mag,L}(T) - \mu_S(T) B$$

- **The critical field strength decreases:**

$$B_c(T) = \frac{\Delta E_{mag,L}}{\mu_S}$$

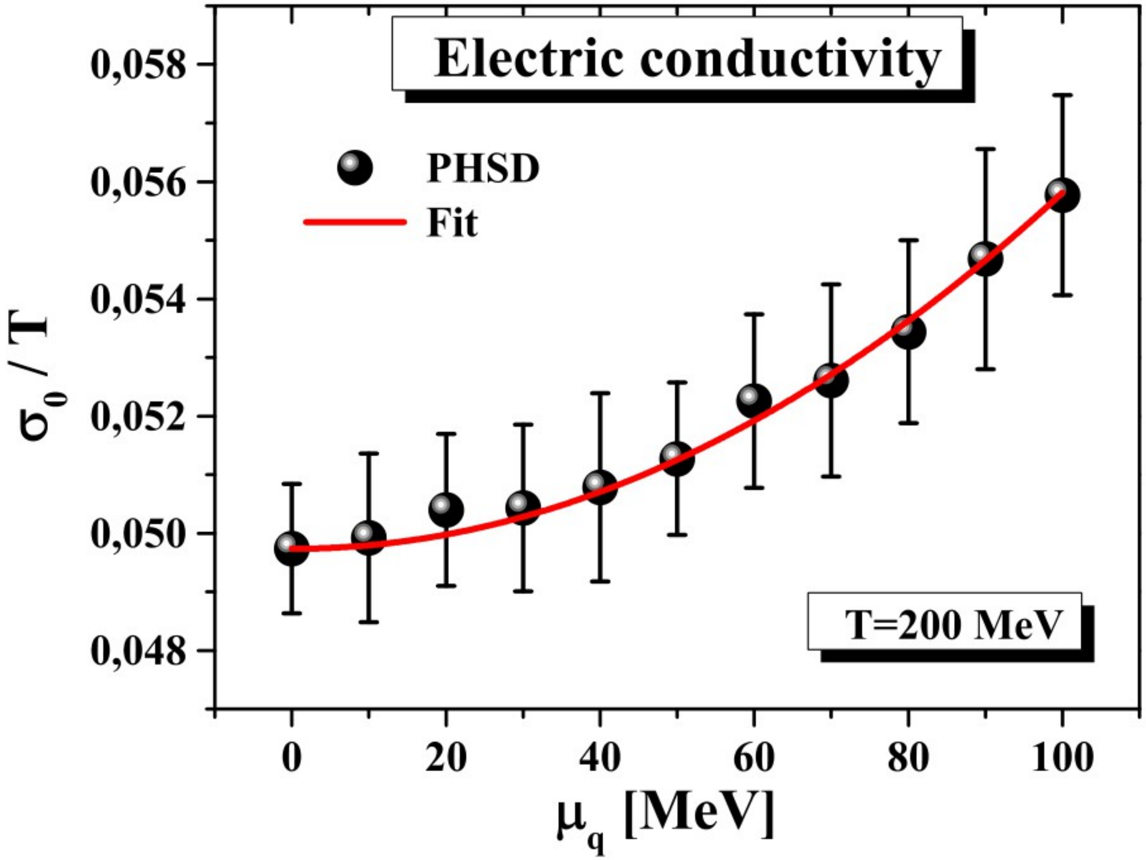
# Critical magnetic field III




- **Shows a minimum close to the critical temperature!**
- **Larger than the strongest fields at RHIC:**

$$eB_{max} = 0.0869 \text{ GeV}^2$$

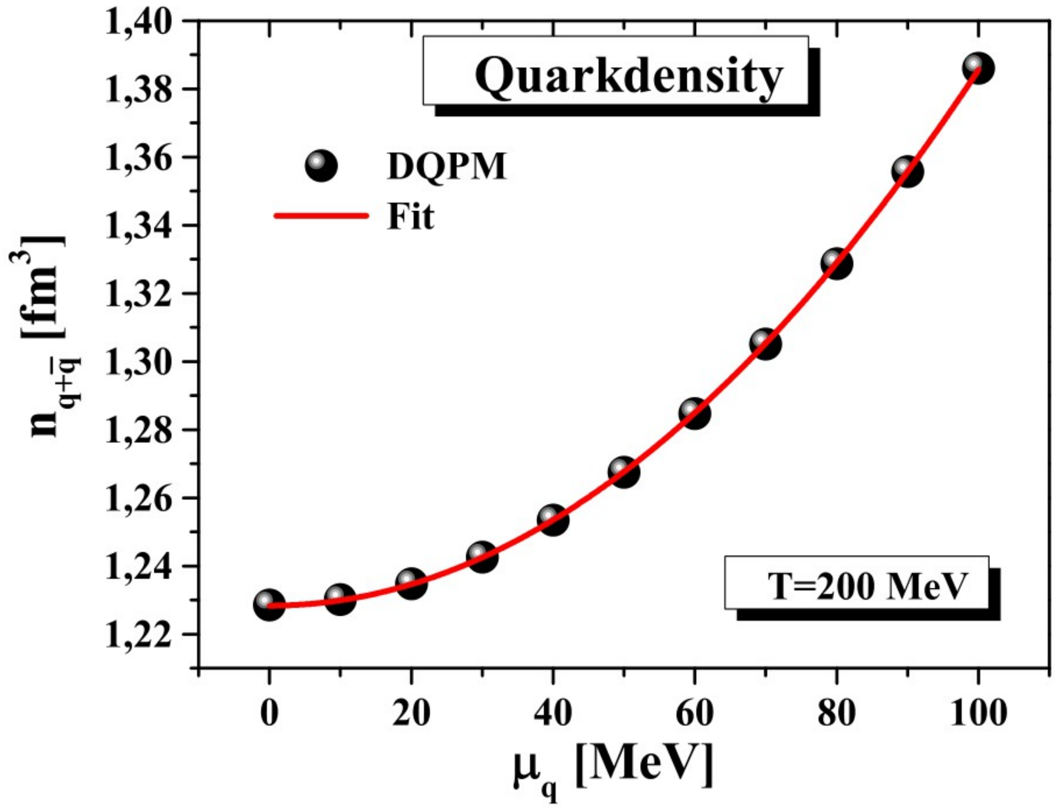
# Electric conductivity III




- **Rises like the total quarkdensity:** $$\frac{\sigma_0(T)}{T} \approx \frac{2}{9}\frac{e^2 n_q(T)}{M_q(T)\Gamma_q(T)T}$$

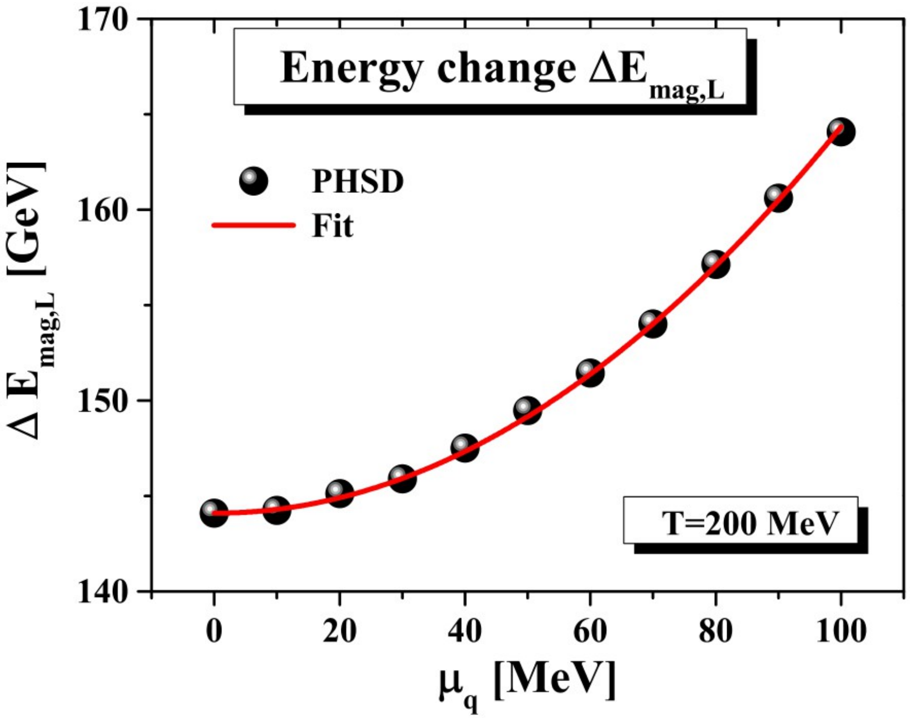
# Finite quark-chemical potential

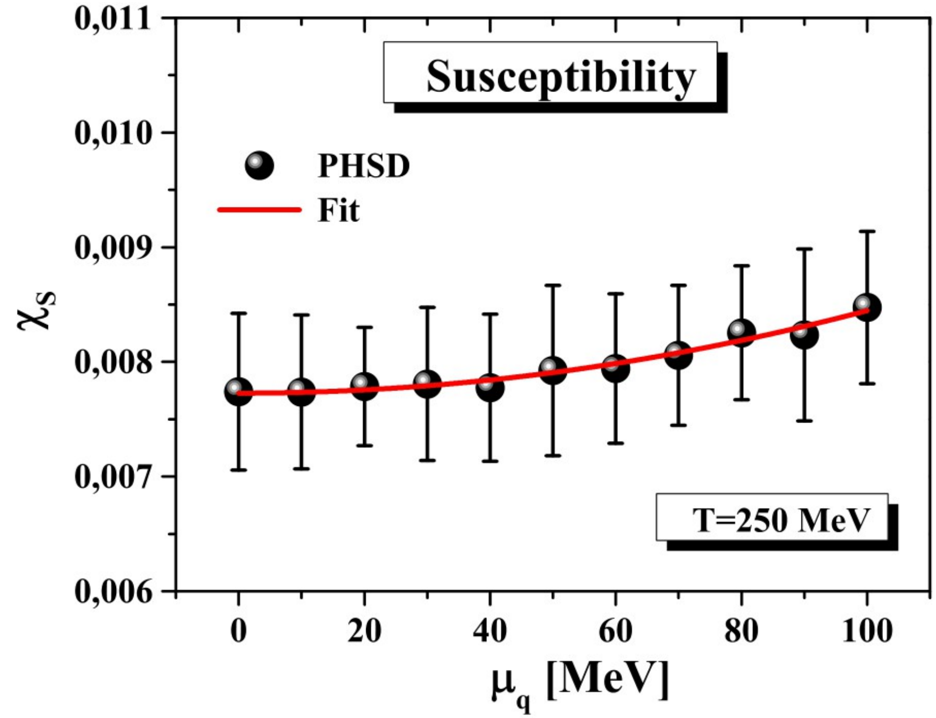
- Only small changes in the masses and the widths!
- Increase of the total quarkdensity ~ $\mu_q^2$




$$n_{q+\bar{q}}(\mu_q) = n_{q+\bar{q}}(0) \cdot (1 + a\mu_q^2)$$

# Magnetic moment IV



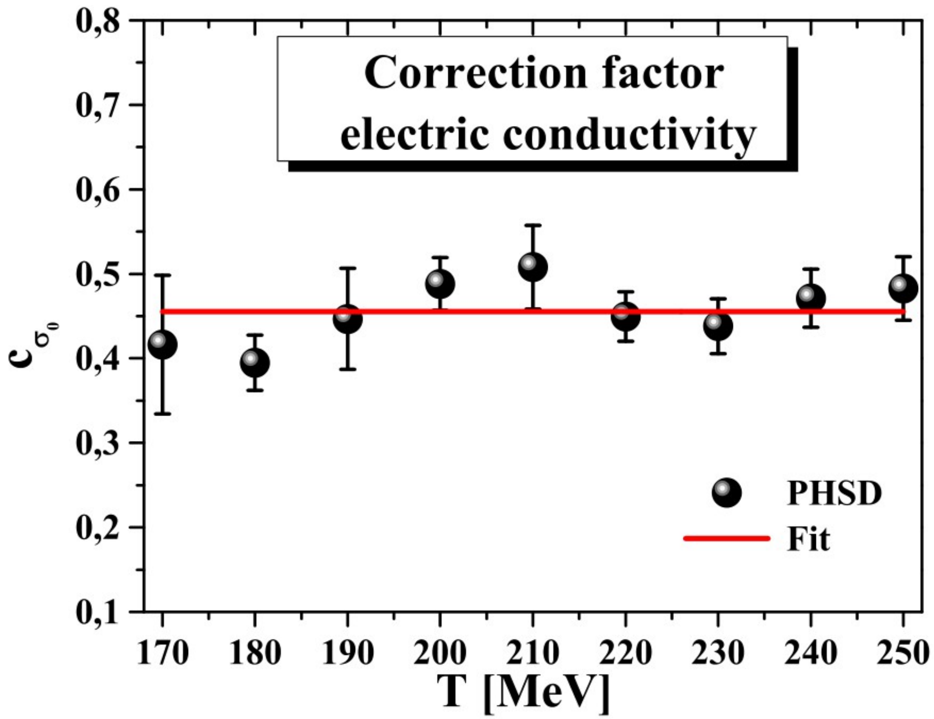

- Scales also like the total quarkdensity: $\sim \left(1 + a\mu_q^2\right)$

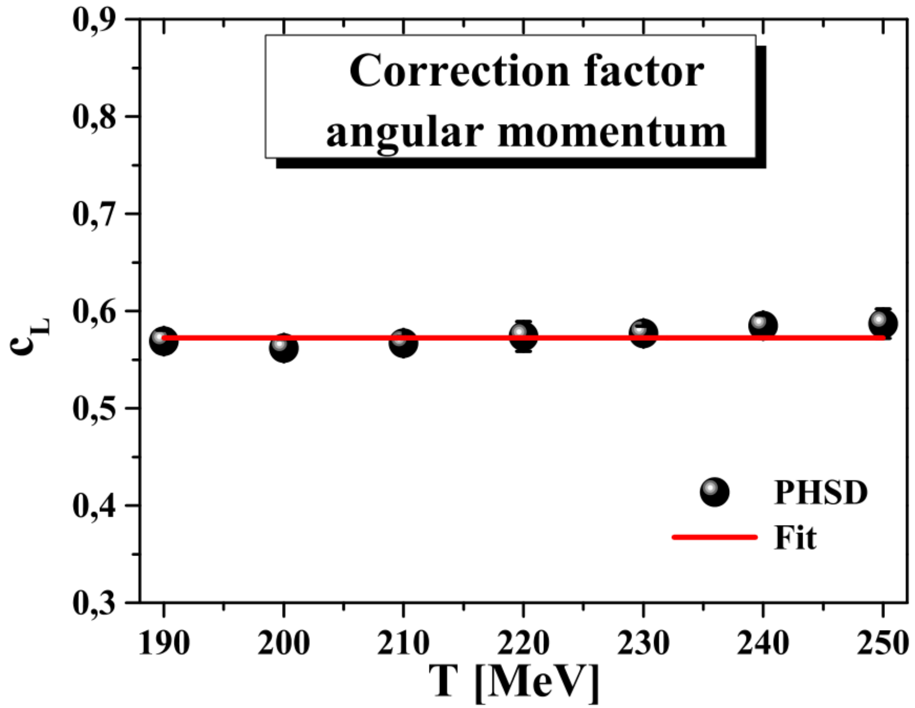
**Changes for finite quark-chemical potential seem mainly driven by increase of charged particles!**

# Correction factor

- **The correction factor is a function of temperature (T>T$_c$):**

$$a(T) = \frac{c}{T^2}$$

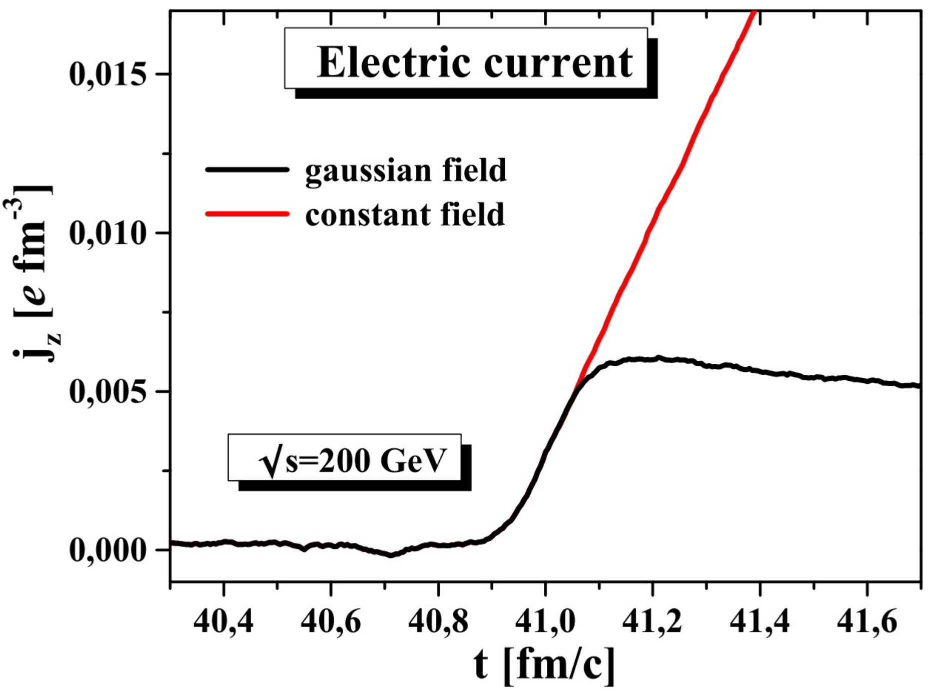


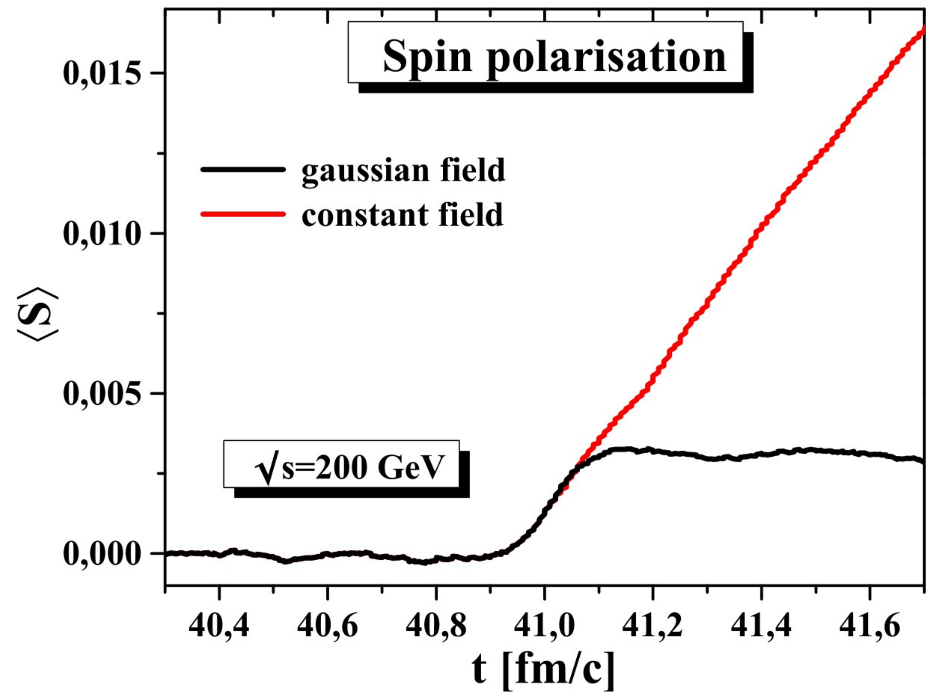




**The effects of finite quark-chemical potential decrease with increasing temperature!**

# Realistic fields I

- **Apply Gaussian shaped fields to a QGP with T=200 MeV**
- **Matches the conditions in a √s=200 GeV AuAu collision**



# Realistic fields II

- The fields last only for ≈0.4 fm/c

- Response to constant fields is 20 times larger

- An up quark needs ≈18 fm/c for a complete circle

$$\tau_{circle} = \frac{2\pi E}{qeB}$$

- The fields last to short to induce proper effects!

# Summary

- Rise with temperature T in the vicinity above $T_c$:

  $\sigma_0/T \sim T$

  $\mu_S \sim T^{-1}$

  $\mu_L \sim T^{2.8}$

- Rise with quark-chemical potential $\sim (1+c\,\mu_q^2/T^2)$
- $B_c$ is much higher than the currently accessible fields in HIC.

=> QGP response is diamagnetic!

- No huge effects in heavy ion collision.

# PHSD group

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