Light tetraquark bound-states/Beyond Rainbow-Ladder

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JLU Giessen

(arXiv:1206.5129, arXiv:1402.5042)

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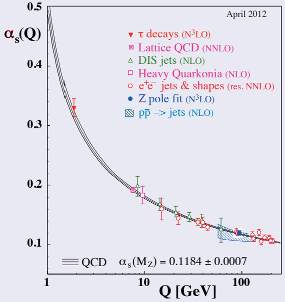
QCD boundstates

Guiding principles

- Strong coupling
  - Non-perturbative

Strong coupling α_s



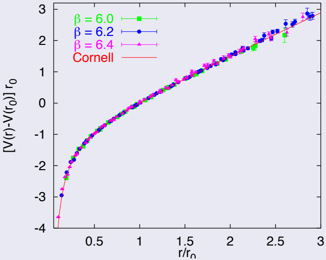
World Summary of α_s, S.Bethke

QCD boundstates

Guiding principles

- Strong coupling
  - Non-perturbative
- Confining theory
  - Single quarks cannot be observed

Linear Rising Potential



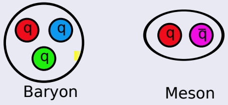
QCD forces and heavy quark bound state, G.S.Bali

QCD boundstates

Guiding principles

- Strong coupling
  - Non-perturbative
- Confining theory
  - Single quarks cannot be observed
- Colorless observables
  - “Classical” objects

“Classical“ singlet states



QCD boundstates

Guiding principles

- Strong coupling
  - Non-perturbative
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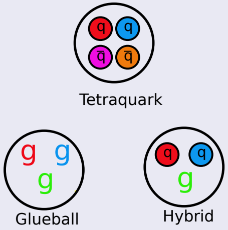
  - Single quarks cannot be observed
- Colorless observables
  - “Classical” objects
  - “Exotic” objects

"Exotic" singlet states



Tetraquarks

Reasons to investigate tetraquarks

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Tetraquarks

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Reasons to investigate light scalar tetraquarks

- Solves the “scalar puzzle“

Light scalar (σ) - 0^{++}

The bad

- Contains scattering states, glueballs, mesons, tetraquarks
- No bound state (σ) in old $\pi\,\pi$-scattering analysis
- "were exiled to the gulag of particle physics" Jaffee (2006)

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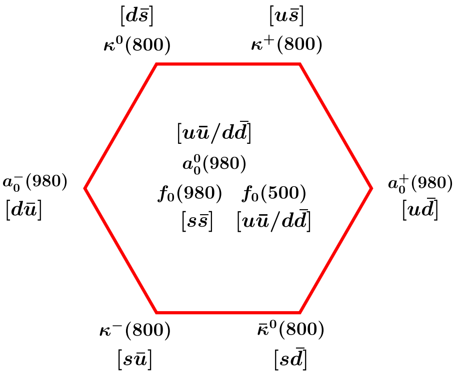
and the good

- Pole mass of $\approx 441 + i272$ MeV Leutwyler *et.al* (2006), Pelaez *et.al* (2008) deduced from experiments KLOE(2002), E791(2001), BES(2005)
- Hints on the lattice for light scalars Mathur *et.al* (2007)
- They are back in the PDG! f0(500)

Scalar Puzzle - Meson nonet vs. tetraquark nonet



Meson nonet

- Wrong mass order **in** the nonet

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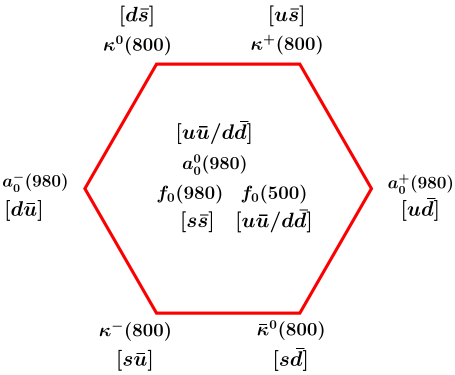

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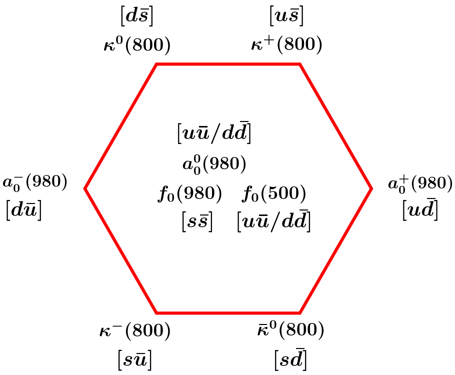

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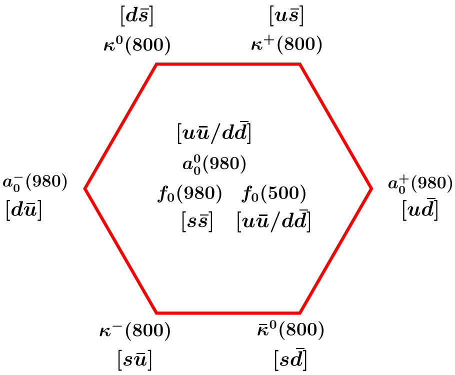

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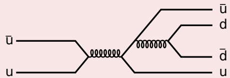
- Width of f_0 contradics OZI-rule



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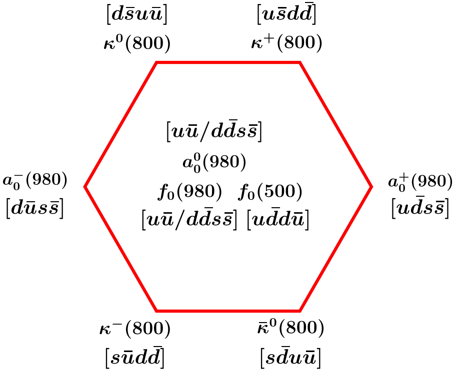

Tetraquark nonet

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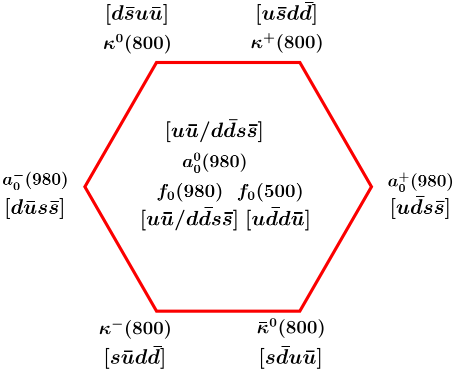

Tetraquark nonet

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- 0^{++} : L(3)=0

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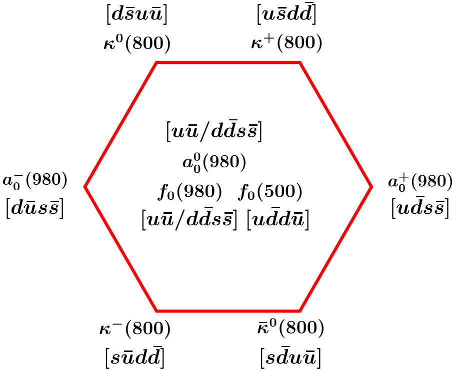

Tetraquark nonet

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- Decay channels are better understandable

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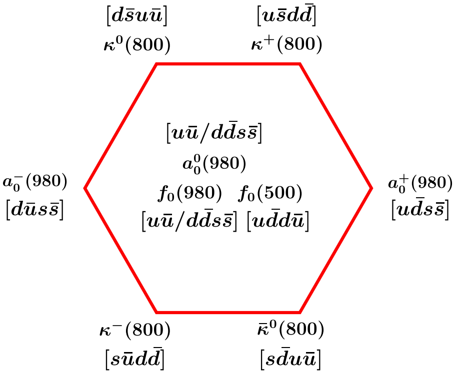
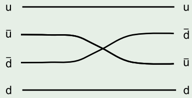


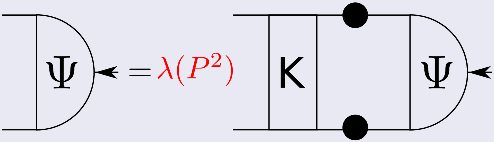
Tetraquark nonet

- Right mass ordering
- 0^{++} : L(3)=0
- Decay channels are better understandable
- Width of f_0 follows from the "gluon-less" decay



Bound-state equations in QFT

Bethe-Salpeter equation

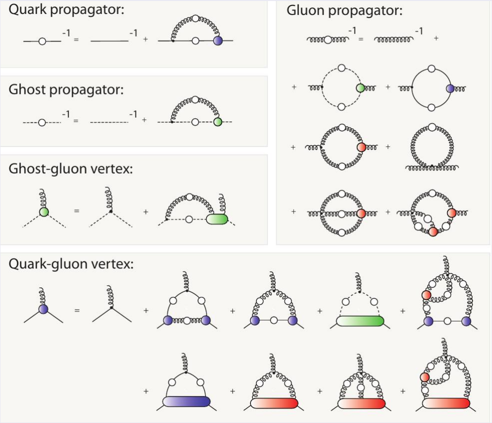


Features and ingredients

- Requires dressed propagators and suitable interaction
- Selfconsistent eigenvalue problem
- Determines mass **and** wavefunction

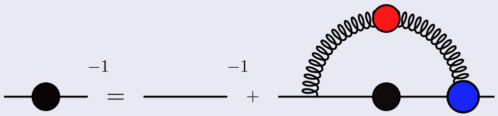
The model

DSE tower



The model

Full quark DSE



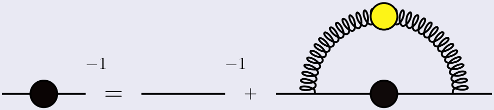
$$S(p) = \frac{-i\not{p}A(p^2) + B(p^2)}{A^2(p^2)p^2 + B^2(p^2)}$$

Kernel

$$K(p,q) = \frac{\delta\Sigma(p)}{\delta S(q)}$$

The model

Truncated DSE

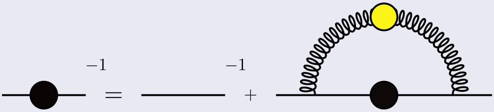


Truncation scheme

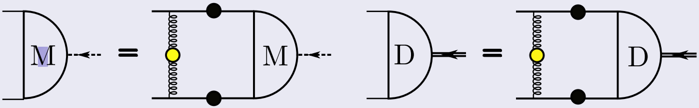
- Effective gluon Maris-Tandy
- Fixed to $f_\pi = 131$ MeV and $m_\pi = 138$ MeV
- Reproduces a variety of hadron observables

The model

Truncated DSE

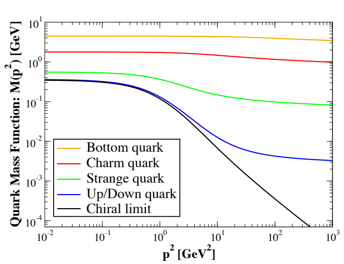


Corresponding BSEs

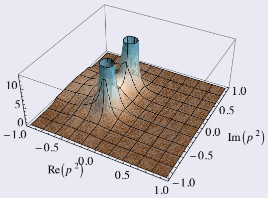


Quark propagator

Solution on the $\mathbb{R}^+$-axis:



Solution in the $\mathbb{C}$-plane:

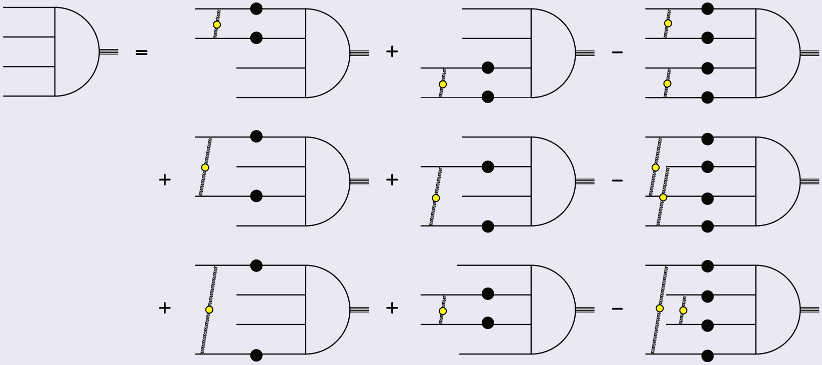


Properties

- Dynamical chiral Symmetry breaking

Tetraquark bound-state equation I

4-body problem - Quark picture



- Neglect 3- and 4-body interactions.
- Keep pair interaction. Treat overcounting properly.
- 512 wave functions, depend on 9 variables

Approximation

Ansatz for the pair interaction T-matrix:

$\mathcal{T}_1 :=$

$\mathcal{T}_2 :=$

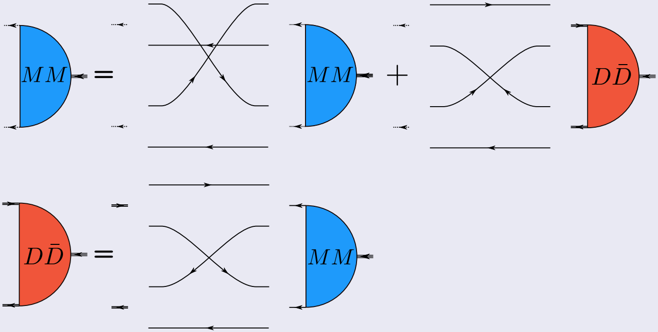
$\mathcal{T}_3 :=$

Ansatz for the 2-body T-matrix:

, = ; =

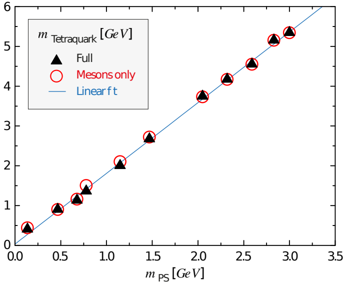
Tetraquark bound-state equation II

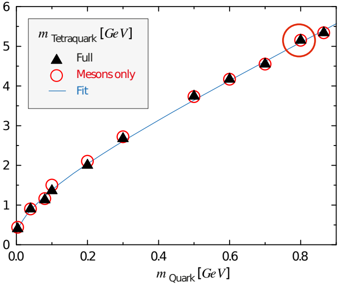
2-body problem - Meson/Meson-Diquark/Antidiquark picture



- Interaction via quark-exchange instead of a gluon-exchange.
- 2 amplitudes, depend on 2 variables.
- 2-loop structure instead of 1-loop.

Results

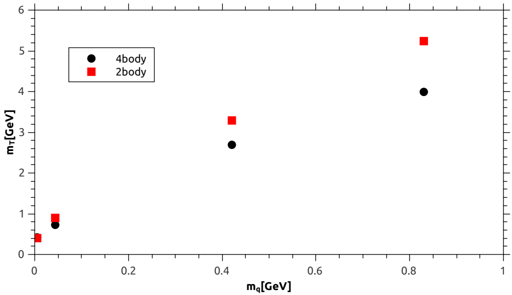




What can be learned from the 2body equation

- Bound 0^+ tetraquark state at $\approx$ 403 MeV.
- The Pion-Pion wavefunction is dominant.
- Quarkmass dependence of the tetraquark resembles the pion.
- Possible narrow $cc\bar{c}\bar{c}$ state at 5.3 GeV??

Results



What can be learned from the 4body equation - Prepreliminary

- Bound 0^+ tetraquark state at $\approx$ 425 MeV.
- Discrepancy between 2body and 4body approach.
- Low mass for $cc\bar{c}\bar{c}$ a feature or caused by neglecting 3- and 4-body terms low numerics?

Conclusion and outlook

Conclusion

- The 0^+ tetraquark boundstate equation was derived and solved in the mmdd picture and in the 4body approach.
- A mass of ≈ 403 MeV was found in both approches.
- The molecular picture of the 0^+ tetraquark is favoured by the calculation in the mmdd picture.

Conclusion and outlook

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- A mass of $\approx$ 403 MeV was found in both approches.
- The molecular picture of the 0^+ tetraquark is favoured by the calculation in the mmdd picture.

Outlook

- Solve for other quantum numbers and flavours.
- Solve the 4-body equation with better numerics and the full structure.

Beyond Rainbow-Ladder in bound state equations - Motivation

Simple Maris-Tandy model is a viable tool to investigate phenomenology:

- Meson, baryon and tetraquark masses.
- Electromagnetic form factors.
- Anamolous magnetic moment ...

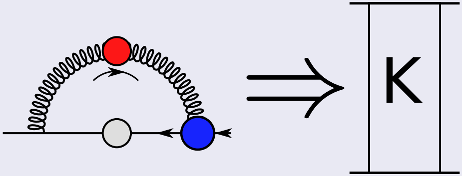
But:

- Properties beyond the ground state are of qualitative nature at best.
- The vector-axialvector splitting is incorrect.
- The scalar meson is too light. ...

Beyond Rainbow-Ladder in bound state equations - Introduction

From the DSE to the BSE

- BSE interaction kernel can be derived consistently from quark selfenergy.



- If done properly, the kernel preserves important symmetries.
- Example: If the vertex chiraly transforms as a quark, the (AxWTI) is fullfilled. Munzcek (1986)

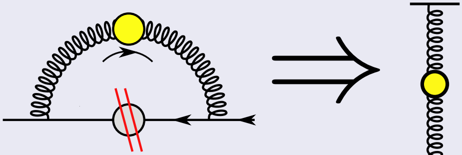
Beyond Rainbow-Ladder in bound state equations - Introduction

How to?

- Given by a functional derivative ('cutting'):

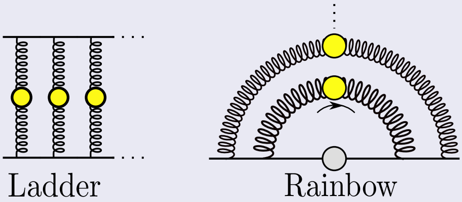
$$K_{ab}^{cd}(x, y, z, z') = \frac{\delta \Sigma^{cd}(x, y)}{\delta S^{ab}(z, z')}$$

- In the Rainbow-Ladder case:



Beyond Rainbow-Ladder in bound state equations - Introduction

By the way

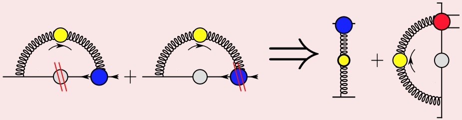


- Origin of the name Rainbow-Ladder.

Beyond Rainbow-Ladder in bound state equations - Introduction

But...

- What if the vertex itself depends on the quark?

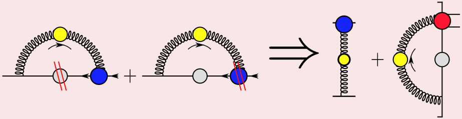


- The vertex has to be cutted aswell.
- Could be done by cutting a vertex DSE:
  - Taking functional derivatives numerically is difficult.
  - Solving for the correct 5-point function is difficult.

Beyond Rainbow-Ladder in bound state equations - Introduction

But...

- What if the vertex itself depends on the quark?



- The vertex has to be cutted aswell.
- Could be done by cutting a vertex model:
  - If the vertex depends on quarks, an explicit cutting is feasible.
  - It allows for exploratory investigations of the structure of the quark-gluon vertex on the spectrum.

First case: Ball-Chiu vertex model

Ball-Chiu construction

- From its Slavnov-Taylor identity

$$(p_1^\mu - p_2^\mu)\Gamma(p_1, p_2) = S^{-1}(p_2) - S^{-1}(p_1),$$

the longitudinal part of the vertex can be constrained.
A non-singular solution is

$$\Gamma_\mu^{BC}(p_1, p_2) = \Big[\gamma_\mu \frac{A(p_1^2) + A(p_2^2)}{2} + 2\not{k}\, k_\mu \frac{A(p_1^2) - A(p_2^2)}{p_1^2 - p_2^2} + i2p_\mu \frac{B(p_1^2) - B(p_2^2)}{p_1^2 - p_2^2}\mathbb{1}\Big]$$

Cutting the Ball-Chiu vertex

Key ideas

- The derivative has to be taken into all all 'directions' of the quark, even the unphysical ones: $\gamma_5, \gamma^\mu\gamma_5, \sigma^{\mu\nu}$.
- After taking the derivative, set all quantities back to physical point.
- Rewrite the dressing functions:

$$S^{-1}(p) = i\not{p}A(p^2) + \mathbb{1}B(p^2) + \gamma_5 C(p^2)$$

Cutting the Ball-Chiu vertex

Key ideas

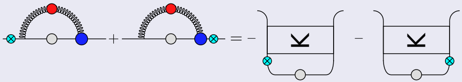
- The derivative has to be taken into all all ‘directions‘ of the quark, even the unphysical ones: $\gamma_5, \gamma^\mu\gamma_5, \sigma^{\mu\nu}$.
- After taking the derivative, set all quantities back to physical point.
- Rewrite the dressing functions:

$$A(p^2) = \frac{1}{4p^2} Tr[\not p S^{-1}]$$
$$B(p^2) = \frac{1}{4} Tr[\mathbb{1} S^{-1}]$$
$$C(p^2) = \frac{1}{4} Tr[\gamma_5 S^{-1}] \ldots$$

Cutting the Ball-Chiu vertex

The ‘problem‘

- For the pion, the additional terms stemming from cutting the vertex do **not** contribute.
- In order to fullfill the AxWTI, the kernel has to solve:



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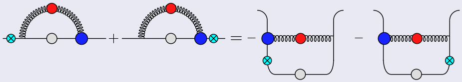


- The 3rd Ball-Chiu component breaks the AxWTI.

Cutting the Ball-Chiu vertex

The solution

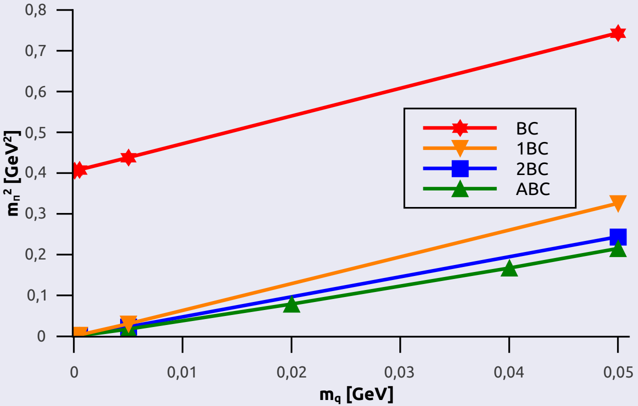
- Add a '0' to the vertex

$$\Gamma_\mu^{ABC} = \Gamma_\mu^{BC} + i2\,\gamma_5\,k_\mu \frac{C(p_1^2) - C(p_2^2)}{p_1^2 - p_2^2}.$$

- This vertex gives a contribution to the pion kernel after cutting.
- The vertex and the cutted vertex together preserve the AxWTI.

Cutting the Ball-Chiu vertex - the result

Gell-Mann-Oakes-Renner



Second case: Munczek-vertex model

Construction principle

- Has the same chiral transformation property as the quark.
  'Automatically' preserves the AxWTI.
- Given in coordinate space.
  Allows a consistent derivation of the kernel and the BSE.
- Longitudinal part restricted by the Slavnov-Taylor identity.
- Linear in the inverse quark.

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- Longitudinal part restricted by the Slavnov-Taylor identity.
- Linear in the inverse quark.

$$\Gamma^\mu(p_1, p_2) = \int_0^1 d\alpha \big[\gamma^\mu A(p^2) + \\ + 2p^\mu A'(p^2)\not{p} + i2p^\mu B'(p^2)\big]_{p=p_1+\alpha(p_1-p_2)}.$$

Second case: Munczek-vertex model

Kernel

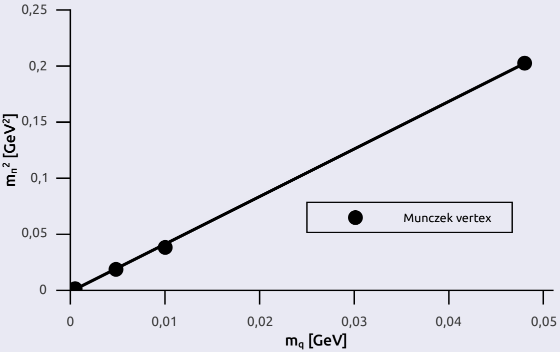
- The kernel can be stated in a closed form

$$K_{II}\left[\Gamma(P,p)\right]_{ab} = -\int_q \int_0^1 d\alpha \left[\frac{\partial}{\partial q^\nu}\left(\Gamma_{ac}(q+\alpha(q-p);P)\right)\right] \times S_{cd}(q+\frac{1}{2}P)\gamma^\mu_{db}D^{\mu\nu}(q-p). \quad (1)$$

- Because of the derivative, the kernel is different for each quantum number.

Cutting the Munczek vertex - The result

Gell-Mann-Oakes-Renner



Cutting the Munczek vertex - The result

	f_π	m_π	m_σ	m_ρ	m_{a1}
RL	0.093	0.137	0.65	0.73	0.83
MV	0.094	0.134	0.46	0.58	0.71

Spectrum

- The cutted Munczec-vertex is stable and can be used for all quantum numbers. Reason: Given in coordinate space.
- No improvement of the spectrum compared to the Rainbow-Ladder case.
- A simple Ball-Chiu like vertex seems not to be enough to increase the $\pi - \sigma$ and the $\rho - a1$ splitting.
- The cure for this are likely additional transversal parts proportional to $\sigma^{\mu\nu}$. Roberts *et.al* (2009)

Conclusion

Summary

- A procedure to cut vertices depending explicitly on the quark was presented.
- The method was applied to the Ball-Chiu vertex and the Munczek vertex.
- The Ball-Chiu vertex had to be continued into the unphysical region to preserve the AxWTI.
- The Munczek vertex worked out of the box. But the spectrum turned out to be not improved.

Outlook

- The Munczec vertex could be improved by additional transverse terms proportional to $\sigma^{\mu\nu}$.
- The functional derivative could be done numerically.

The end

Thank you for your attention!

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Helmholtz Graduate School for Hadron and Ion Research