Quark mass gap in strong magnetic fields

– QCD under extreme conditions with nonperturbative gluons

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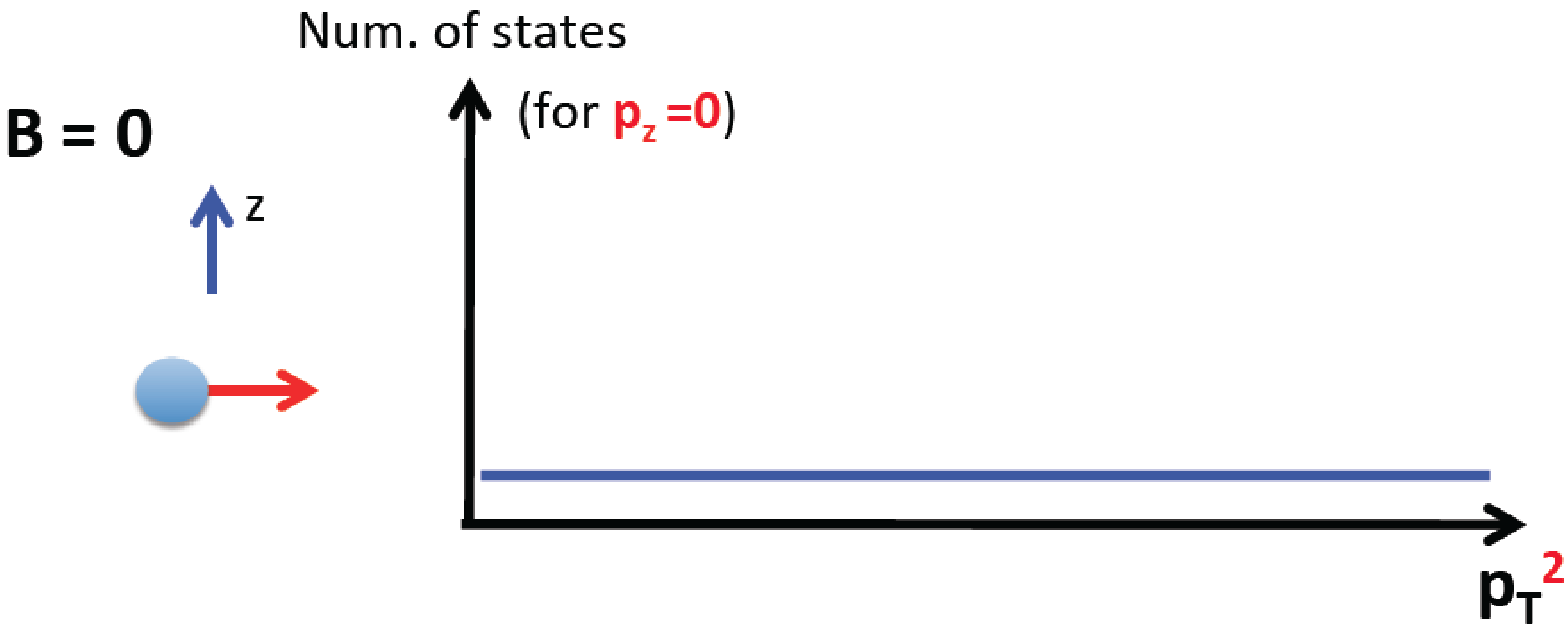
References: arXiv:1211.7318; arXiv:1305.4510

ITP, Giessen, 16/05/14

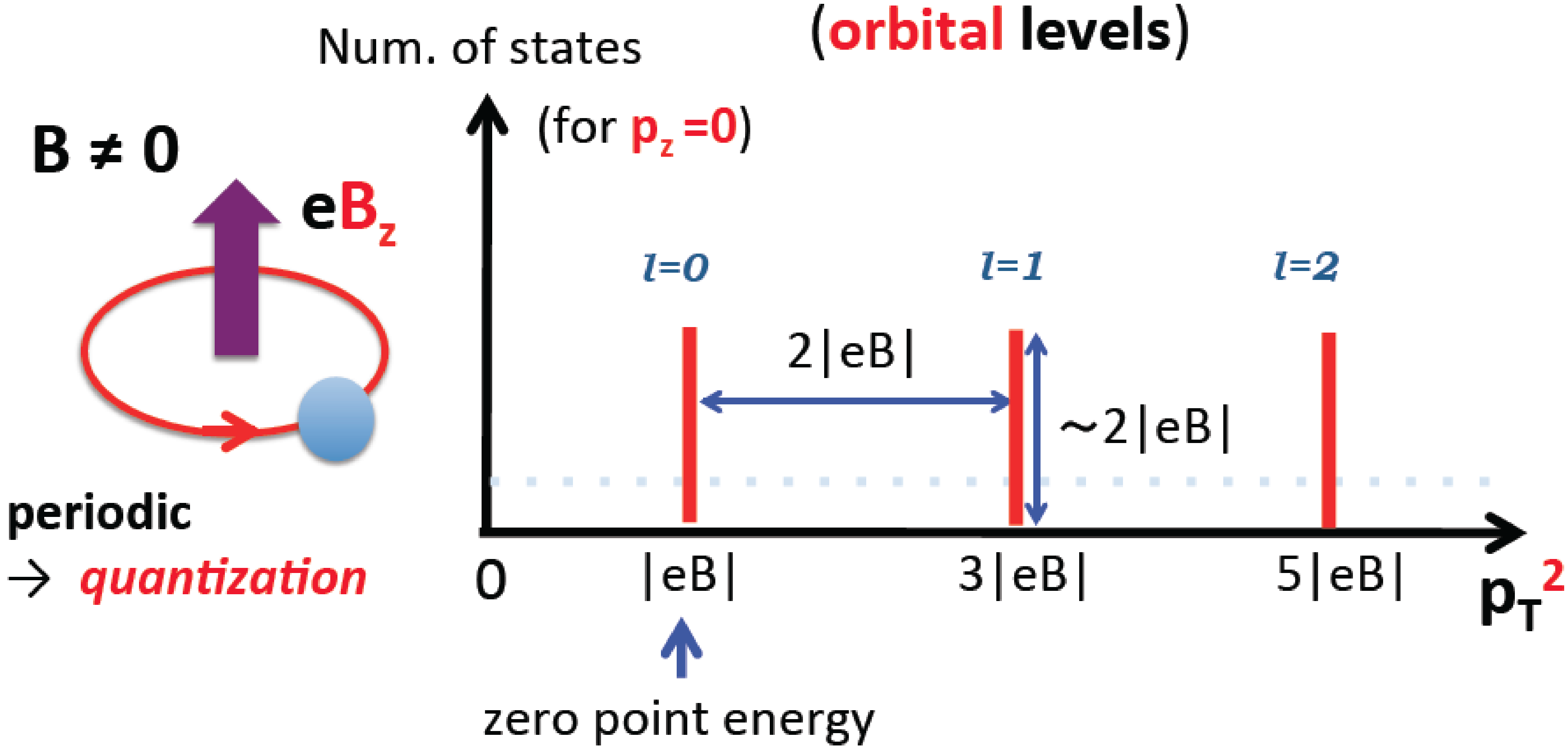
Why to study QCD in B fields?

- Theory confronts lattice (no sign problem)
- Simple qualitative problems are still available
  - Distinctions between models and QCD!
- QCD in magnetic fields (B) may be realized in Nature
  - Compact stars: $\sim 10^{-6}\mathrm{GeV}^2$
  - Heavy-ion coll.: $\mathrm{RHIC} \sim \Lambda^2_{QCD} \sim 0.01\mathrm{GeV}^2$, $\mathrm{LHC} \sim 0.3\mathrm{GeV}^2$
- Impacts of B fields to quark-gluon plasma evolution
  - Thermodynamics, EoS
  - Realtime dynamics, transport coefficients
- Impacts of confinement physics to QCD under extreme conditions
  - Role of nonperturbative gluons!

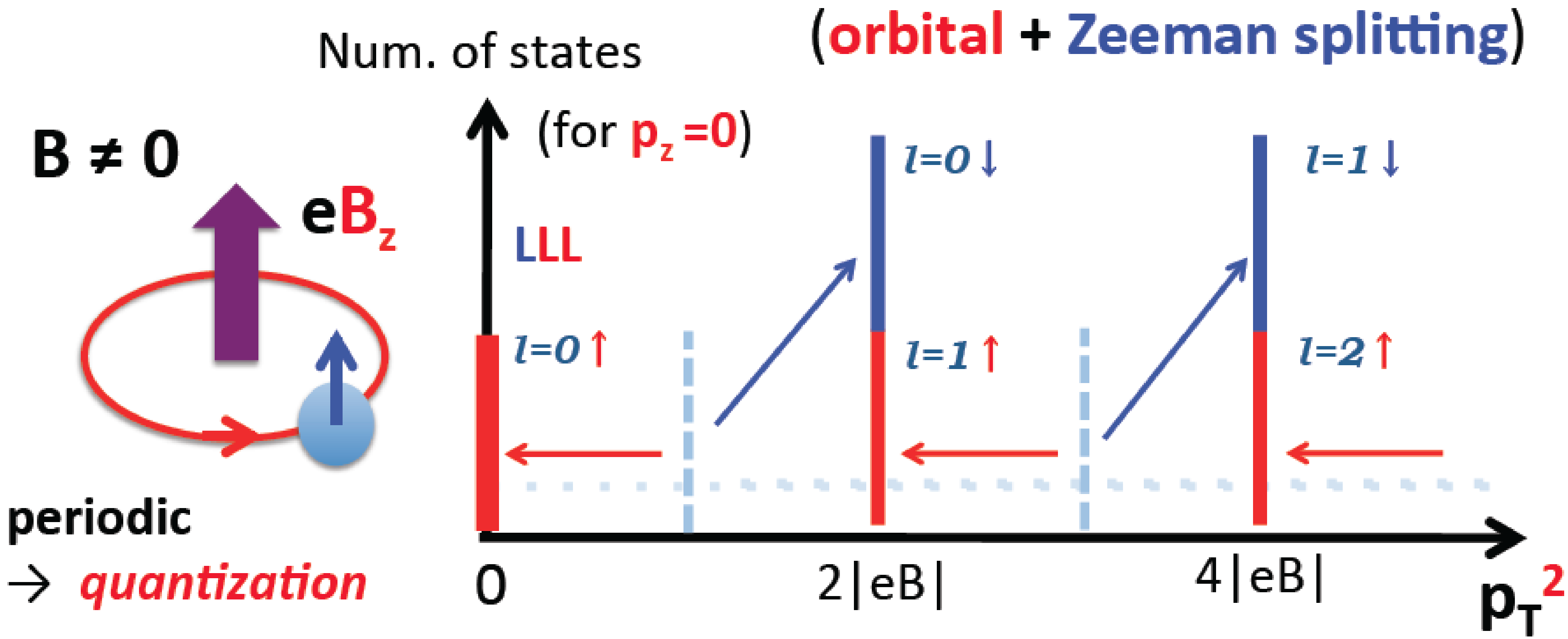
Quantum mechanics in B fields (spinless)



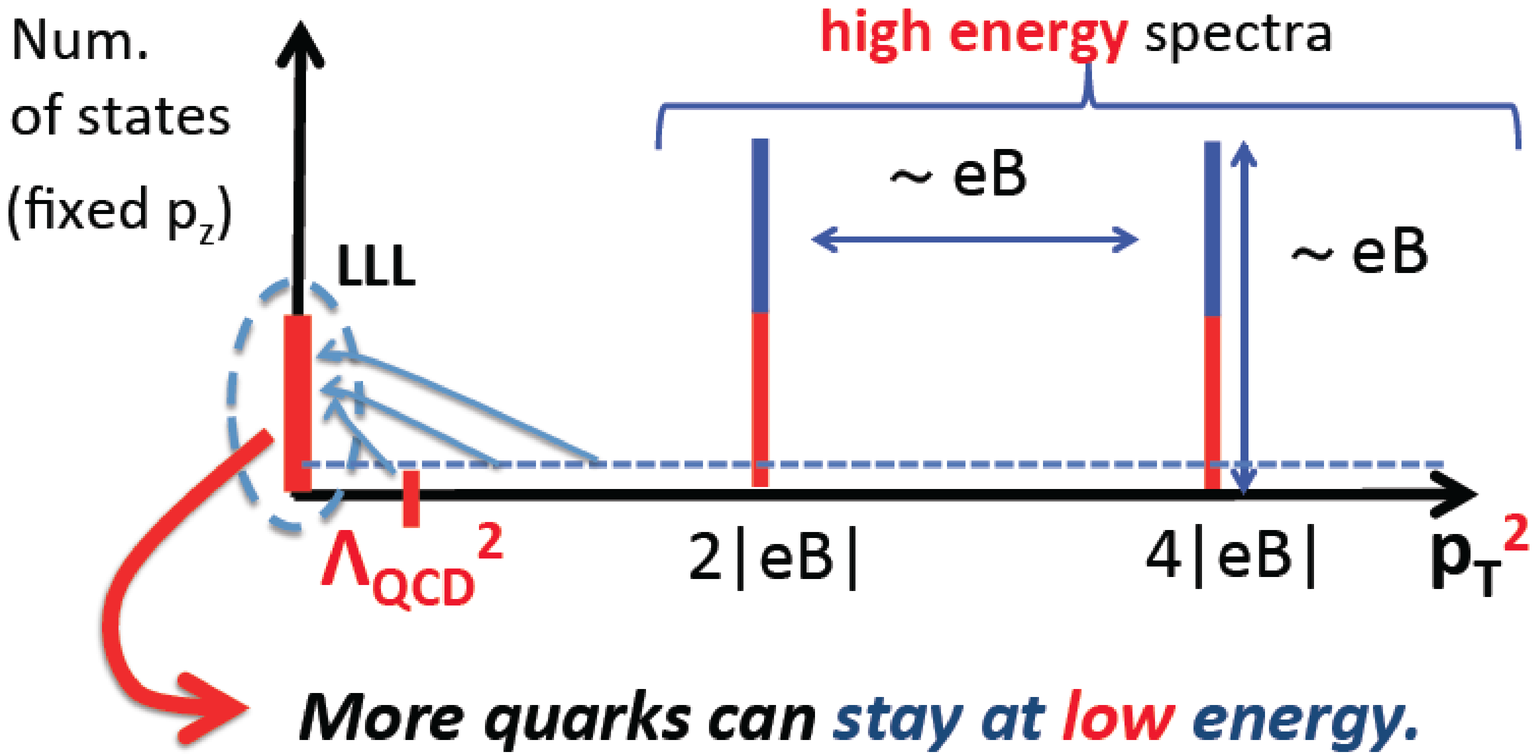
Quantum mechanics in B fields (spinless)



Quantum mechanics in B fields (spin 1/2)



Quantum mechanics in B fields



- Size of IR phase space controlled by B
- More quarks in nonperturbative dynamics
- B enhances chiral condensate: Magnetic Catalysis

Field theory in B fields primer – Ritus bases

1) Choose the gauge for EM fields : e.g.) $A_2^{\mathrm{em}} = Bx_1$

2) Apply "*spin projection*" :

$$\psi_\pm \equiv \mathcal{P}_\pm \psi \qquad \mathcal{P}_\pm = \frac{1 \pm \mathrm{i}\gamma_1\gamma_2\,\mathrm{sgn}(e_f B)}{2}$$

3) Expand by proper spatial wavefunctions :

$$\psi_\pm(x) = \sum_{l=0} \int \frac{\mathrm{d}^2 p_L \mathrm{d}p_2}{(2\pi)^3}\, \psi^\pm_{l,p_2}(p_L)\, \underline{H_l\left(x_1 - \frac{p_2}{B}\right)}\, \mathrm{e}^{-\mathrm{i}p_2 x_2}\, \mathrm{e}^{-\mathrm{i}p_L x_L}$$

$$p_L \equiv (p_0, p_z)$$

Harmonic oscillator w.f. with

$$m\omega = |eB|$$

Field theory in B fields primer – quark propagator

The action for the LLL (n=0): $\chi = \psi_+^{l=0}$

$$\mathcal{S}_{\mathrm{LLL}} = \int_{p_L, p_2} \bar{\chi}_{p_2}(p_L)\,(-\mathrm{i}\not{p}_L + m)\,\chi_{p_2}(p_L)$$

(No B-dep. !)

for the n-th hLL : $\psi_n = \psi_+^{l=n} + \psi_-^{l=n-1}$

$$\mathcal{S}_{\mathrm{nLL}} = \int_{p_L, p_2} \bar{\psi}_{n,p_2}(p_L)\left(-\mathrm{i}\not{p}_L + \underline{\mathrm{i}\,\mathrm{sgn}(eB)\sqrt{2n|eB|}\gamma_2} + m\right)\psi_{n,p_2}(p_L)$$

The propagator :

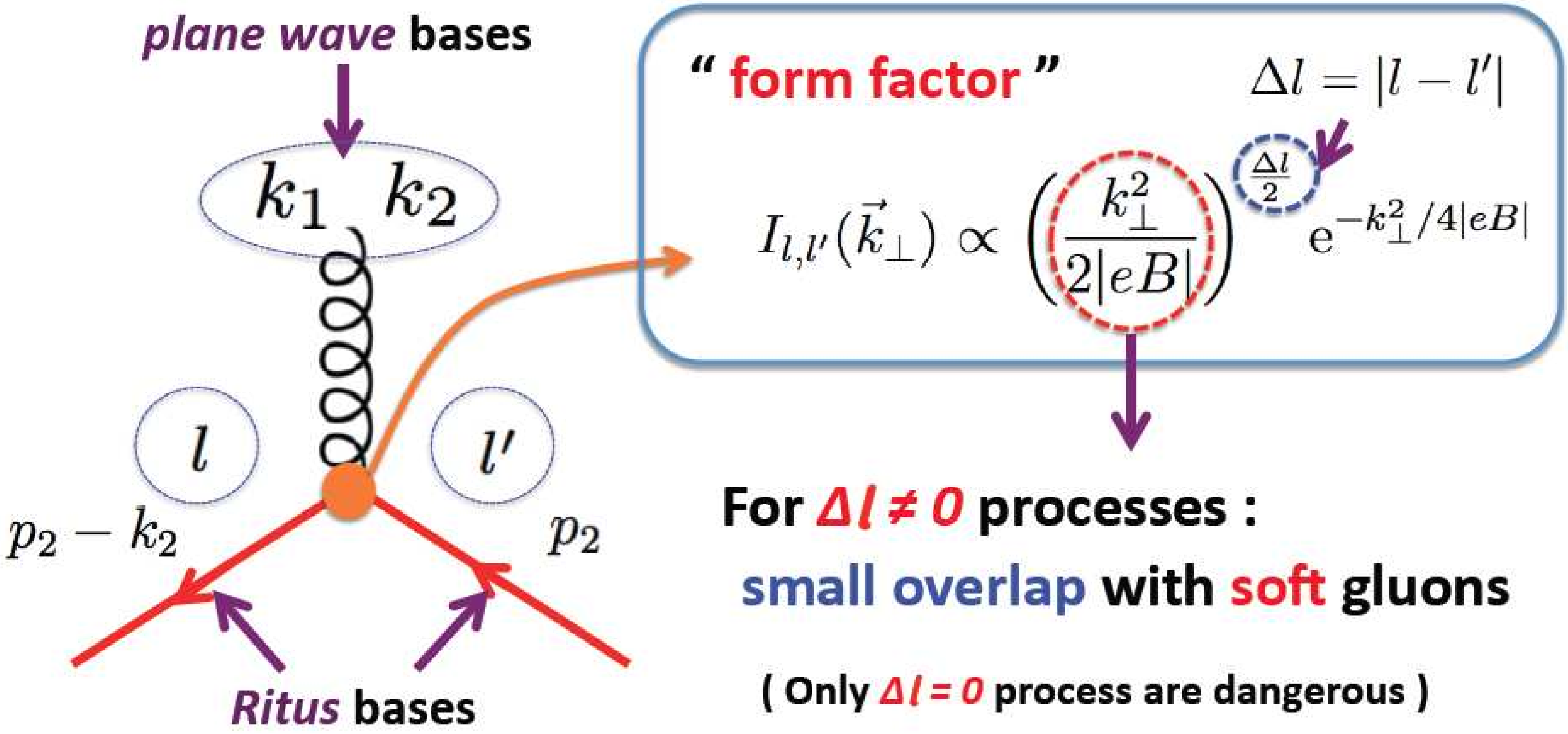
$$\langle \psi_{n,p_2}(p_L)\bar{\psi}_{n',p_2'}(p_L')\rangle = \underline{S_n^{\mathrm{2D}}(p_L)} \times \delta_{nn'}\delta(p_2 - p_2')\delta^2(p_L - p_L')$$

(1+1)-dimensional for each index "n"

(No p_2 – dep.)

Field theory in B fields primer – quark-gluon vertex

$$S_{\rm int} = \int_x \bar{\psi}(x)\gamma_\mu t_a \psi(x)\, A^a_\mu(x)$$ **4D Gluons couple to different LLs.**



(Kojo, NS, arXiv:1305.4510)

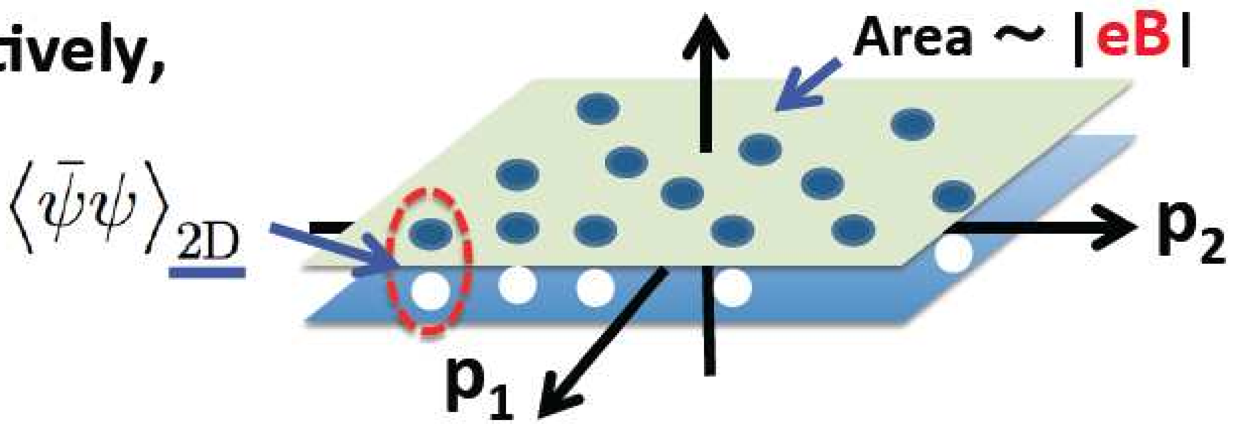
Field theory in B fields primer – chiral condensate

$$\langle\bar{\psi}(x)\psi(x)\rangle_{\underline{4\mathrm{D}}} = -\frac{|eB|}{2\pi}\int_{p_L}\mathrm{tr}\left(S_\chi^{2\mathrm{D}}(p_L) + \sum_{n=1} S_n^{2\mathrm{D}}(p_L)\right)$$

$$\langle\bar{\psi}(x)\psi(x)\rangle_{4\mathrm{D}} = \frac{|eB|}{\underline{2\pi}}\langle\bar{\psi}(x_L)\psi(x_L)\rangle_{\underline{2\mathrm{D}}}$$

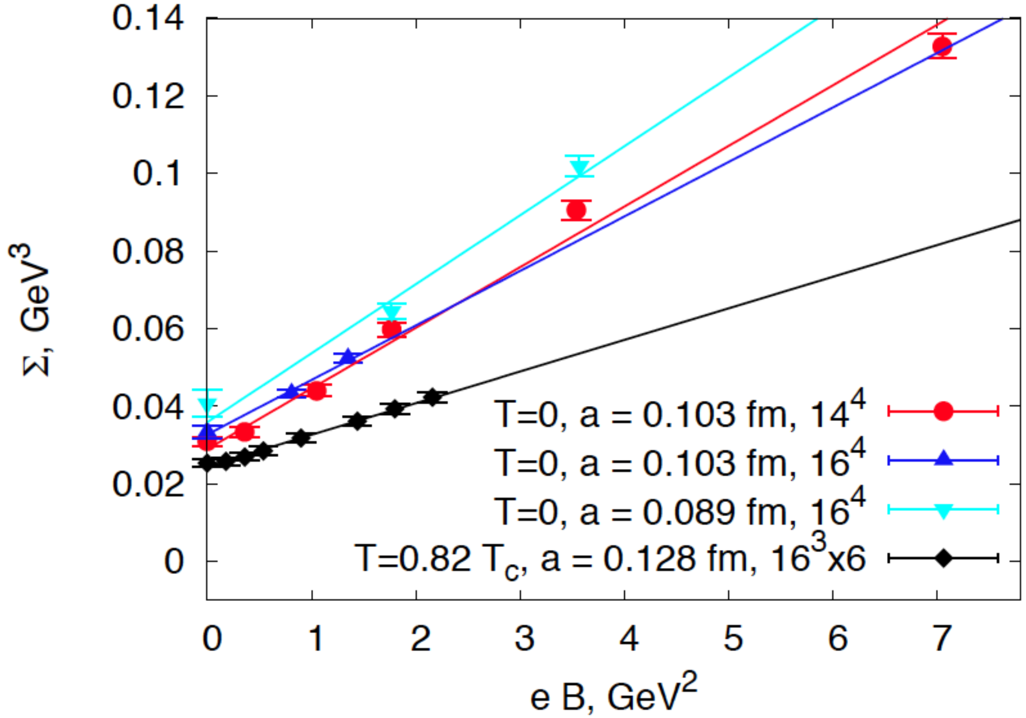
(*holds even after self-energies are included*)

Intuitively,



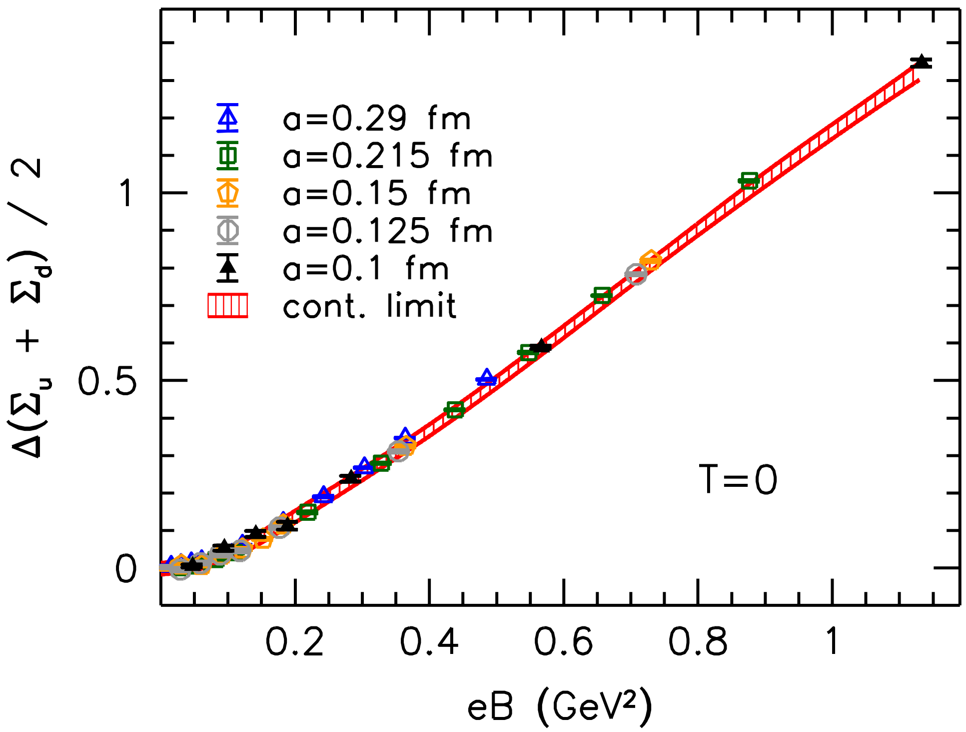
Chiral condensate on the lattice – LINEAR in B!

Quenched SU(2)



Chernodub *et al.*, arXiv:0812.1740

Full QCD

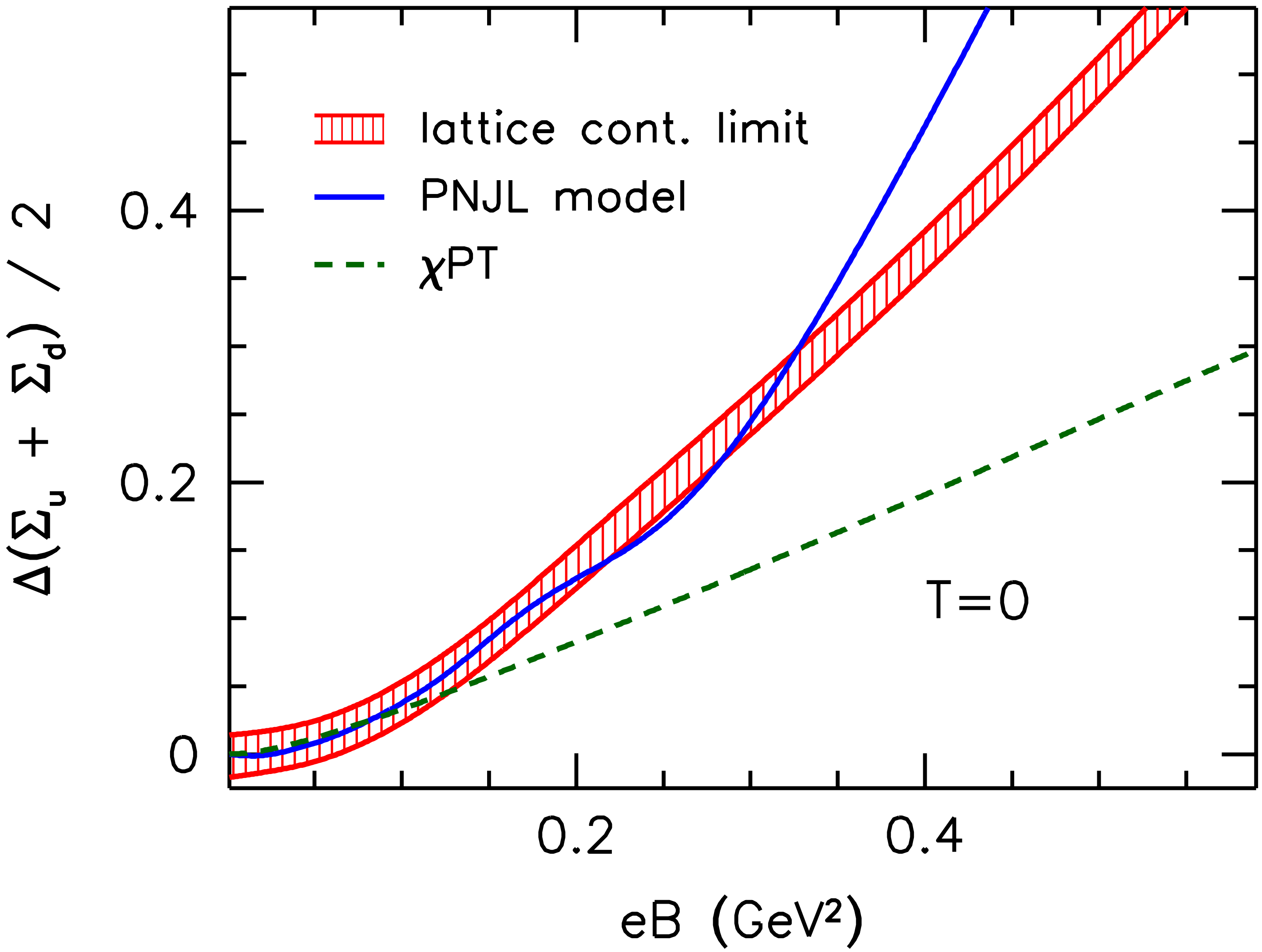


Bali *et al.*, arXiv:1206.4205

$$<\bar{\psi}|\psi>_{4D} \sim |eB| <\bar{\psi}|\psi>_{2D} \longrightarrow <\bar{\psi}|\psi>_{2D} \sim M_q^B \sim \mathrm{constant} + \ldots$$

Models vs Lattice

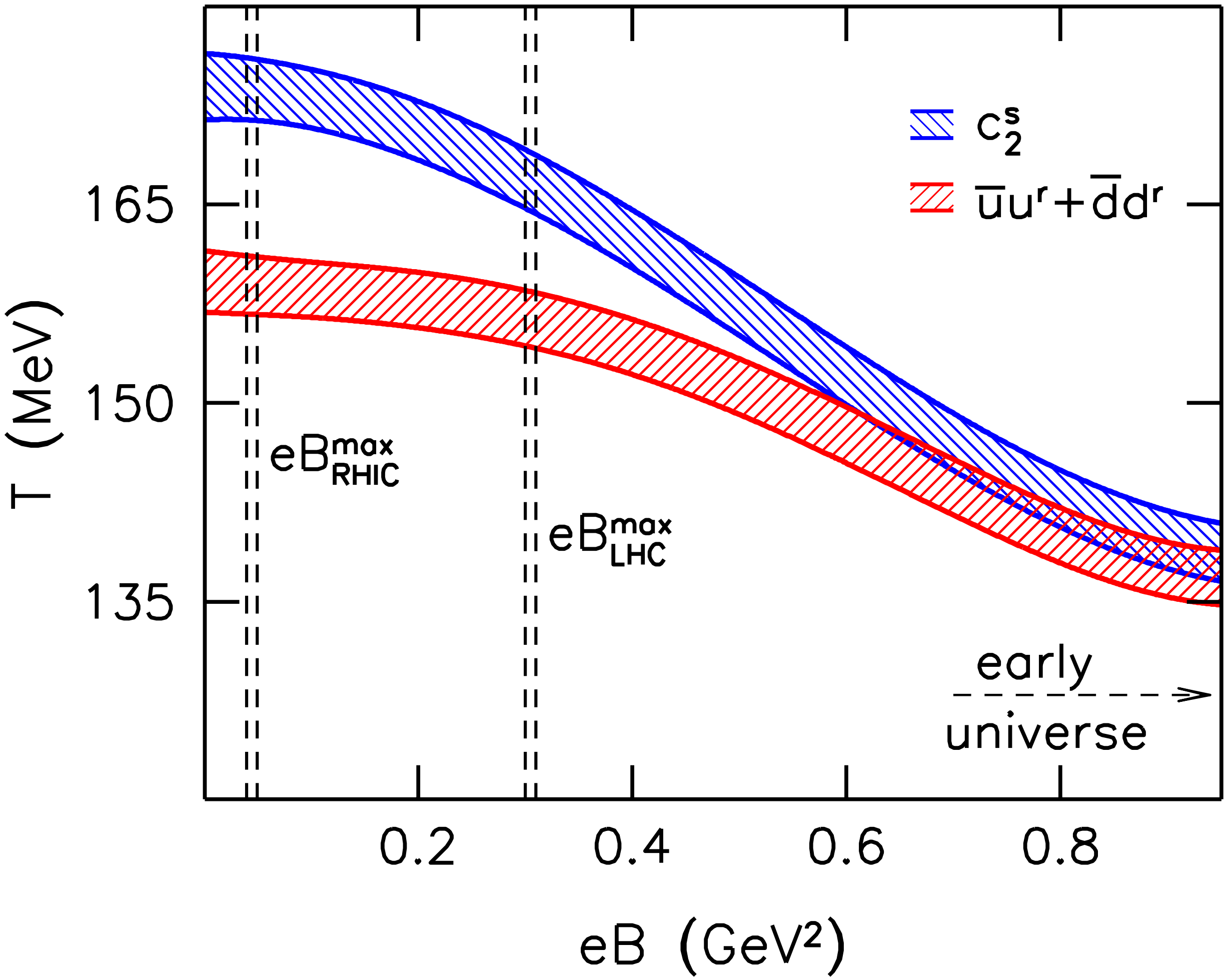
No dynamical gluons in models!



Bali *et al.*, arXiv:1206.4205

Inverse Magnetic Catalysis

T_c DECREASES with increasing B!!!



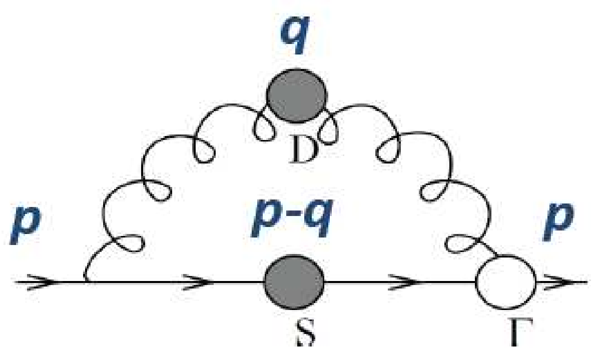
Bali *et al.*, arXiv:1111.4956

Dyson-Schwinger eq. for LLL quark mass gap

(for a more detailed study: Mueller, Bonnet, Fischer, arXiv:1401.1647)

1) No explicit B-dep. for the LLL

2) No p_T-dep. → " factorization "



$$M(p_L) \sim \int_{q_L} S^{2D}_{LLL}(p_L - q_L; M) \int_{q_\perp} e^{-\frac{q_\perp^2}{2|eB|}} D^{4D}_{NP}(q_L, q_\perp)$$

Form factor
for "ΔL = 0 process"

2D " *smeared* " force

(origin of B-dep.)

Comparison of forces 1 – NJL

$$\int_{q_\perp} e^{-\frac{q_\perp^2}{2|eB|}} D^{4D}_{NP}(q) \sim \int_0^{\sim|eB|} dq_\perp^2 \, D^{4D}_{NP}(q)$$

origin of B-dep.

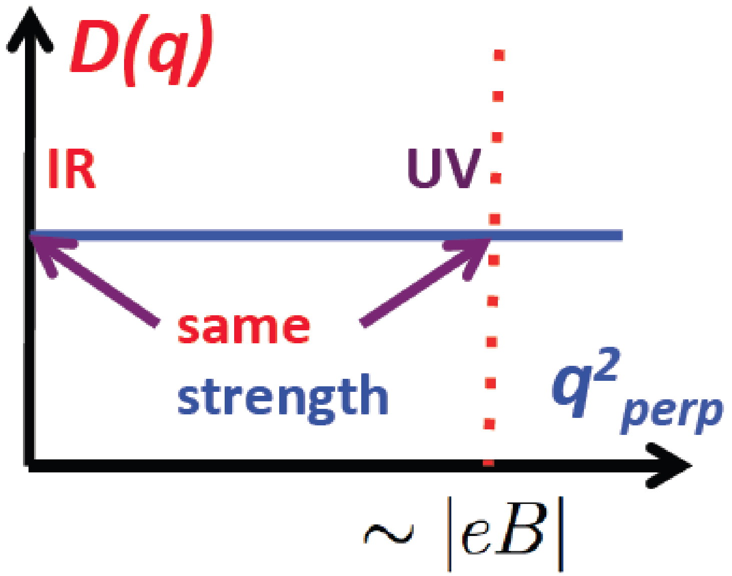
1) Contact interactions (NJL, etc.)

$$\sim \int_0^{\sim|eB|} dq_\perp^2 \text{ const.}$$

$$\Rightarrow \sim |eB| \times \text{const.}$$

2D Force is strongly B-dep.

$$M \sim |eB|^{1/2}$$



Comparison of forces 2 – QED

$$\int_{q_\perp} \mathrm{e}^{-\frac{q_\perp^2}{2|eB|}} D_{\rm NP}^{\rm 4D}(q) \sim \int_0^{\sim|eB|} \mathrm{d}q_\perp^2 \; D_{\rm NP}^{\rm 4D}(q)$$

origin of B-dep.

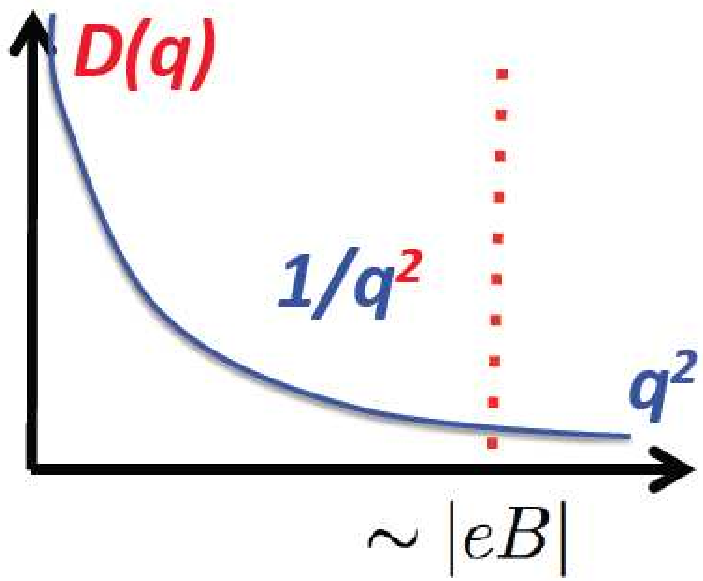
2) QED case (1/q² force : *running is negligible*)

$$\sim \int_0^{\sim|eB|} \mathrm{d}q_\perp^2 \; \frac{1}{q_\perp^2 + q_L^2}$$

$$\Rightarrow \quad \sim \ln \frac{q_L^2}{|eB|}$$

2D Force is still marginally B-dep.

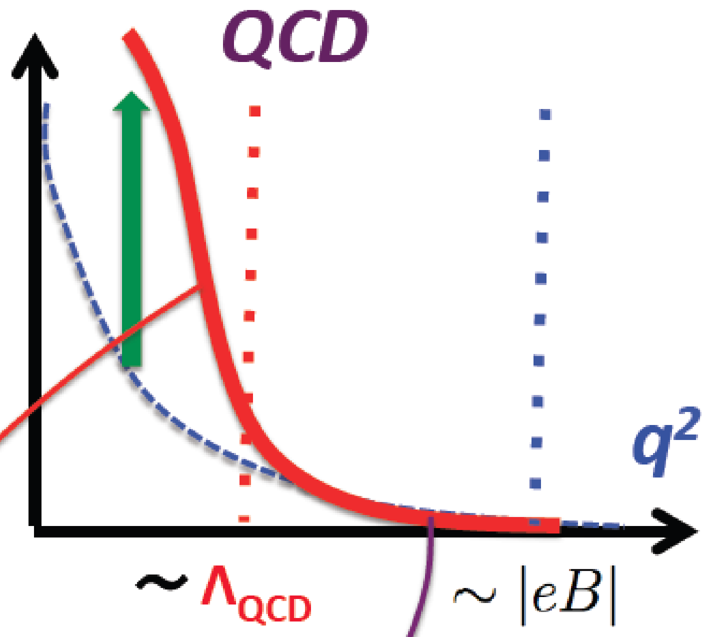
$$M \sim |eB|^{1/2}$$



Comparison of forces 3 – QCD

Now suppose: QCD force has strong "*IR enhancement*"

$$\int_{q_\perp} e^{-\frac{q_\perp^2}{2|eB|}} D^{4D}(q_L, q_\perp)$$



For small q_{perp} ~ Λ_{QCD} :

we can set : $e^{-\frac{q_\perp^2}{2|eB|}} \sim 1$

$$\sim \int_0^{\sim\Lambda_{QCD}^2} dq_\perp^2 \, D^{4D}(q_L, q_\perp)$$ + ***small B-dep. corrections***

The dominant part :
"nearly B-indep."

$$M \sim \Lambda_{QCD}$$

Comparison of forces 3 – QCD

- The more IR enhancement, the less B dependence in M_q^B!
- A toy model: Linear rising potential for color charges ($\sigma \sim \Lambda_{\mathrm{QCD}}^2$) (Kojo, NS, arXiv:1211.7318)

$$D_{\mu\nu}(q) \; = \; -C_F \times g_{\mu 0} g_{\nu 0} \times \frac{8\pi\sigma}{(\mathbf{q}^2)^2} \qquad \text{(Coulomb gauge Gribov)}$$

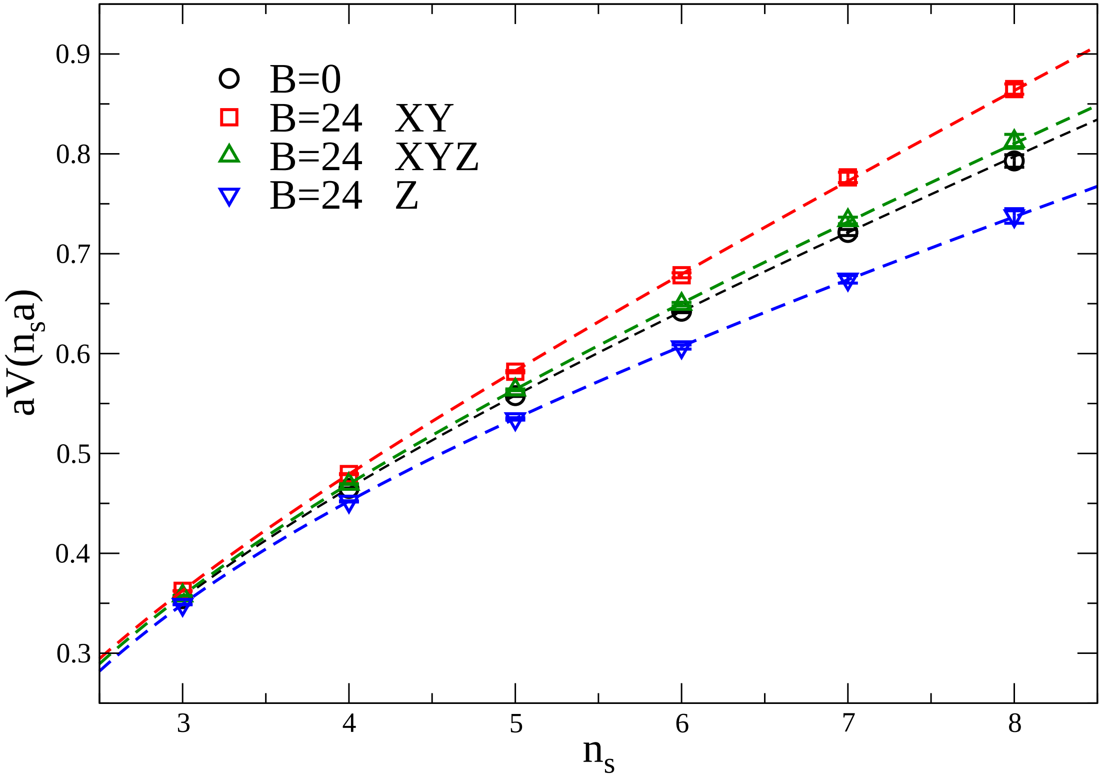
$$M \; \sim \; \sigma^{1/2} \; \sim \; \Lambda_{\mathrm{QCD}} \quad \longrightarrow \; <\bar{\psi}|\psi>_{\mathrm{4D}} \sim |eB|\Lambda_{\mathrm{QCD}} + \ldots$$

- Form factors of q-g vert. control UV fluctuations. LLL dominant at rather small $|eB| \sim (0.1 - 0.3)\,\mathrm{GeV}^2$! (Kojo, NS, arXiv:1305.4510)

$$I_{l,l'}(\mathbf{q}_\perp) \; \sim \; \left(\frac{q_\perp^2}{2|eB|} \right)^{\frac{|l-l'|}{2}} e^{-\frac{q_\perp^2}{4|eB|}}$$

- Screening effects would decrease M, therefore T_c! (IMC)

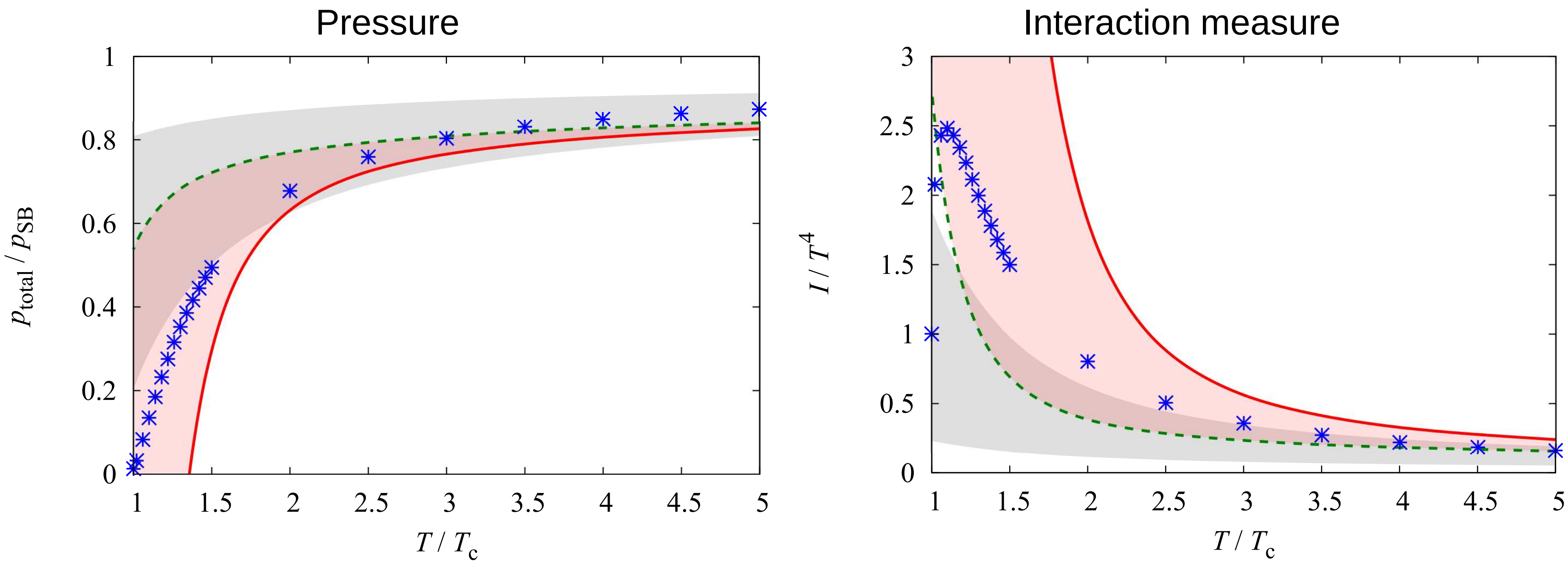
Heavy quark potential



Bonati *et al.*, arXiv:1403.6094

- Z: enhanced screening from MC which reduces HQP
- XY: reduced screening due to quantization which enhances HQP

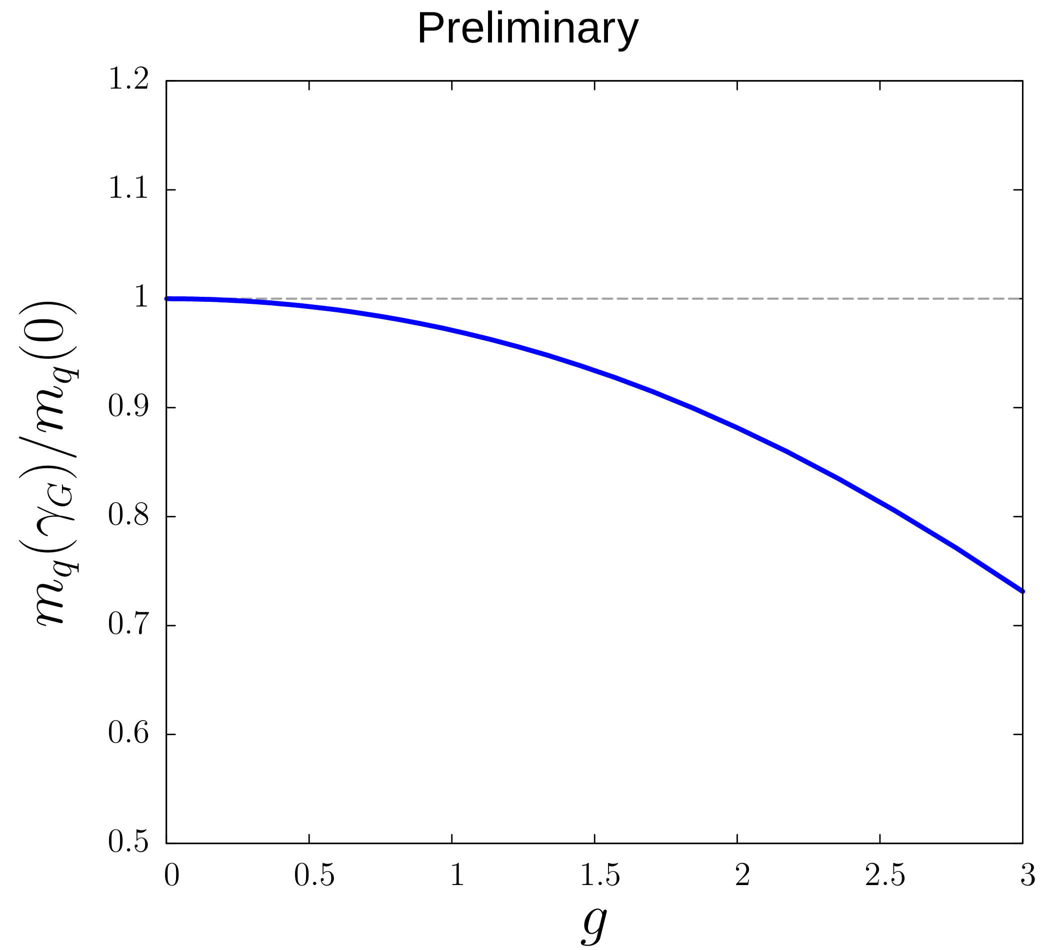
Yang-Mills EoS with Gribov gluons (Fukushima, NS, arXiv:1304.8004)



Lattice (Blue): W-B, arXiv:1204.6184; HTLpt (grey): Andersen, Strickland, NS, arXiv:0911.0676
Lattice $\alpha_s(T)$: Kaczmarek *et al.*, hep-lat/0406036

- Chromo-magnetic scale g^2T systematically included in Gribov-Zwanziger action through Gribov parameter $\gamma_G = \frac{D-1}{D}\frac{N_c}{4\sqrt{2}\pi}g^2T$.
- Sizable contributions from γ_G near T_c; Robust at higher T.

Quark thermal mass with Gribov gluons (NS, Tywoniuk, forthcoming)



- Anti-screening from g^2T: $m_q^2 = C_F \frac{(gT)^2}{8}\left[1 - 4.35972 \times 10^{-6} g^8\right]$
- Qualitatively in line with m_D from lattice (Kaczmarek *et al.*, hep-lat/0406036; Kaczmarek, Zantow, hep-lat/0503017)

Conclusions and outlook

- Nonpert. gluons are IMPORTANT for extreme QCD (B, T, μ)!
  (also from Mueller, Bonnet, Fischer, arXiv:1401.1647)
- To explain lattice data, M_q^B should be (nearly) B independent
- Solve Dyson-Schwinger eq. for quark-gluon vertex
- Impacts on QGP realtime dynamics and phenomenology:
  - Whether B fields would affect QGP evolution?!
  - Electric conductivity σ in strong B fields (Chernodub *et al.*, arXiv:1003.2180; Cassing *et al.*, arXiv:1302.0906)
    - Quark loops
    - Back reactions
  - Transport coefficients: η, ζ, $\ldots$
  - Realtime dynamics of Chiral Magnetic Effect
- New phase transition below T_c(?) (Chernodub *et al.*, arXiv:1003.2180; Cohen, Yamamoto, arXiv:1310.2234)