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Deutsche Forschungsgemeinschaft

Quarks and gluons in a magnetic field

Peter Watson, Hugo Reinhardt

Giessen, July 2014

P.W. & H.Reinhardt, Phys.Rev.D89 (2014) 045008

Outline of talk

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- Motivation and brief introduction (magnetic catalysis)
  - technical overview of the talk
- Landau levels (Dirac equation with a magnetic field)
- Ritus eigenfunction method (expansion in Landau levels) for fermions in a constant magnetic field
  - asymptotic nature of series (not good for QCD)
- Summation and Schwinger phase
  - tree-level
  - nonperturbatively
- Gap equation
- Results - chiral condensate, magnetic susceptibility
- Summary

Motivation

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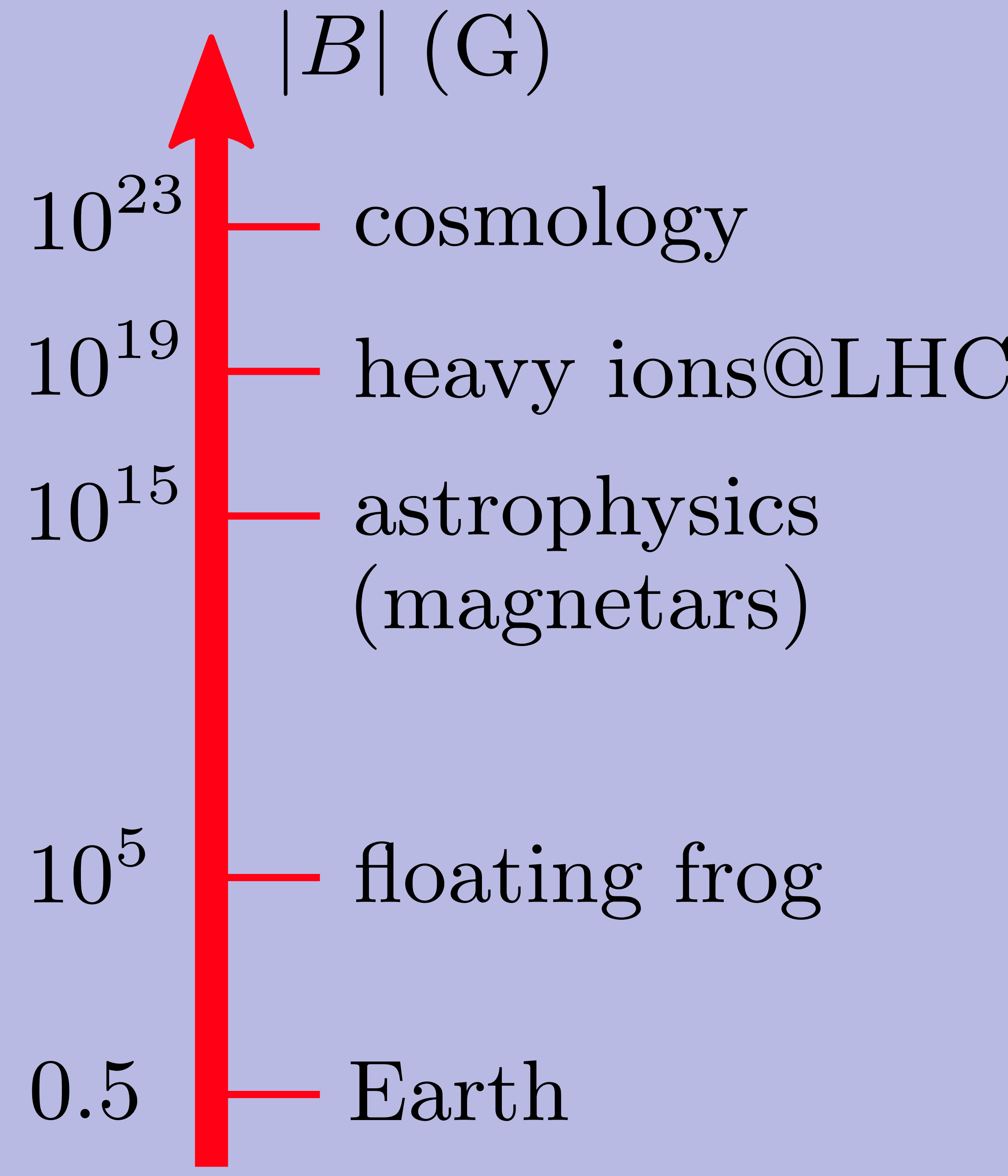
- magnetic fields happen:
  - interesting physics, but technically complicated (no translational inv.)
- my motivation:
  - heavy ion collisions
  - magnetic properties of quarks
- first questions: where from, how big, and what effect?
- early stages of noncentral heavy ion collisions@LHC:

$$|eB| \sim 15 m_\pi^2 \approx 0.3\,\mathrm{GeV}^2$$

- strong magnetic fields, or?
- effects: magnetic catalysis, inverse magnetic catalysis, chiral magnetic effect...



Skokov, Illarionov, Toneev, Int.J.Mod.Phys.A24 (2009) 5925.

Introduction

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- magnetic catalysis:

> an increase of the fermion condensate due to the presence of an external magnetic field

- known for a long time - chiral symmetry breaking takes place for strong magnetic fields in the Gross-Neveu model and QED even at small coupling (strong fields - "lowest Landau level approximation" leads to a linear rise in the condensate)
- such behavior is relevant for the QCD phase diagram e.g., in heavy ion collisions (charged particles, high velocities) there may be an effect, critical temperature might change...

Shovkovy, Lect.Notes Phys. 871 (2013) 13; Klimenko, Theor.Math.Phys. 89 (1991) 1161;
Gusynin, Miransky, Shovkovy, Phys.Lett. B349 (1995) 477.
Certainly known in Giessen! - Mueller, Bonnet, Fischer, 2014

Introduction

- magnetic catalysis is known to occur for strong fields
- but - for quarks in QCD we have a problem
  - strong magnetic fields vs. strong interaction
  (similar scales are not good for approximations)

- recall, the estimated maximum magnetic fields at the LHC are not large in the context of QCD:

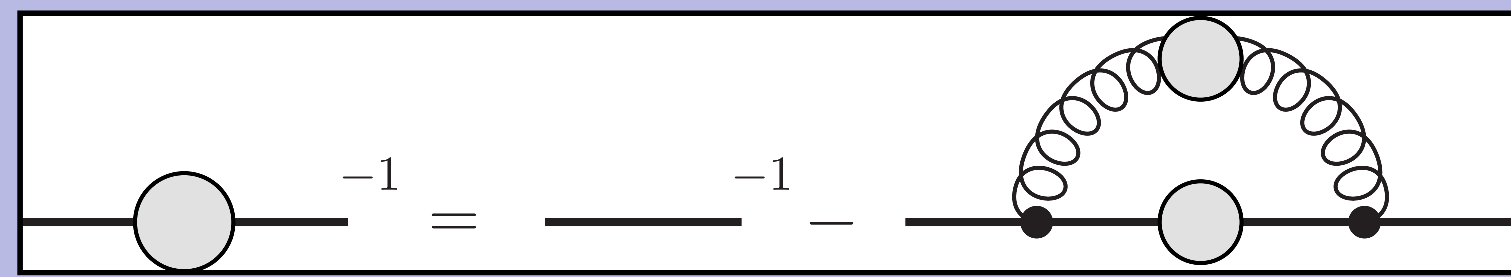
$$|eB| \sim 15 m_\pi^2 \approx 0.3\,\mathrm{GeV}^2$$

- also, we would like to know the magnetic susceptibility (calculated in the limit of vanishing field) - useful e.g., in normalizing chiral-odd transversity parton distribution functions

- we want small and moderate fields too!

Technical overview

- rainbow gap equation:



$$\Gamma(x,y) = \Gamma^{(0)}(x,y) + g^2 C_F\, \gamma^\mu S(x,y)\gamma^\kappa W_{\kappa\mu}(y,x)$$

$$\int d^4z\, \Gamma(x,z)S(z,y) = \delta(x-y)$$

- $B = 0$ - mtm space can be used: $\Gamma(p)S(p) = 1$

- $\vec{B} = B\hat{e}_z, \quad \vec{A} = Bx\hat{e}_y$
  gluon interaction is unaffected (neglecting quark loops), so the self energy is 'easy'. But, translational invariance is broken - mtm space must be replaced and the proper function is no longer simply the inverse propagator - this is where we have to work hard!

Landau levels

- consider: $\vec{B} = B\hat{e}_3,\quad \boxed{h = QB \geq 0}$
- choose a gauge: $A^0 = 0,\ \ \vec{A} = Bx_1\hat{e}_2$
- (minimal coupling) Dirac operator: $D = \imath\partial_\mu\gamma^\mu - h\gamma^2 x_1$
- for the energy levels: $(D+m)(D-m)\Psi(x) = 0$
- Fourier transform (except the x-direction), noting the spin and Hermite eigenfunctions

$$\left\{p_0^2 - p_3^2 - m^2 + h\left[\frac{\partial^2}{\partial\varepsilon^2} - \varepsilon^2 - \sigma\right]\right\} f(\varepsilon) = 0$$

$$\varepsilon = \sqrt{h}x_1 + p_2/\sqrt{h},\ \ \sigma = \pm 1,\ \ f(\varepsilon) = \psi_n(\varepsilon)$$

- Landau (energy) levels: $\boxed{E_n^2 = p_3^2 + m^2 + h(2n+1+\sigma)}$

Landau levels

- constant magnetic field introduces Landau levels
- with Hermite functions as eigenfunctions
- the Landau levels get connected to the spin $(\sigma = \pm 1)$
- translational invariance is broken!

- now we want the tree-level propagator and inverse:
- inverse propagator: $\Gamma^{(0)}(x,y) = \imath[D - m]\delta(x-y)$
- propagator: $\imath[D - m]S^{(0)}(x,y) = \delta(x-y)$

Ritus eigenfunction method

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- Ritus' solution (replace momentum modes with the Hermite function basis):

$$\Gamma^{(0)}(x,y) = \sum_{n=0}^{\infty} \int \frac{d^3\tilde{p}}{(2\pi)^3} E(x;\tilde{p},n)\Gamma^{(0)}(\bar{p},n)\overline{E}(y;\tilde{p},n)$$

$$\tilde{p}^\mu = (p_0, 0, p_2, p_3), \qquad \bar{p}^\mu = (p_0, 0, 0, p_3)$$

- Ritus matrices (orthonormal and complete) connect spin and Landau levels

$$E(x;\tilde{p},n) = h^{1/4} e^{-\imath\tilde{p}\cdot x}\left[\psi_{n-1}(\varepsilon)\Sigma^+ + \psi_n(\varepsilon)\Sigma^-\right]$$

- spin projectors: $\Sigma^\pm = \frac{1}{2}\left[1 \pm \imath\gamma^1\gamma^2\right]$

- notice that n=0 (lowest Landau level) is special!

Ritus, Sov.Phys. JETP 48 (1978) 788.

Ritus eigenfunction method

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- inverse propagator (function of eigenvalues)

$$-\imath\Gamma^{(0)}(\bar{p},n) = \bar{p}_\mu\gamma^\mu - \sqrt{2nh}\gamma^2 - m$$

- propagator

$$\imath S^{(0)}(\bar{p},n) = \frac{\bar{p}_\mu\gamma^\mu - \sqrt{2nh}\gamma^2 + m}{\bar{p}^2 - 2nh - m^2 + \imath 0_+}$$

(Landau levels appear in the denominator)

- So, by using the Ritus matrices and the associated eigenvalues instead of momentum space, we can tackle the gap equation. Usually, only the lowest Landau level (n=0) is considered (works for large fields) and gives a linearly rising condensate.

Gusynin, Miransky, Shovkovy, Phys.Lett. B349 (1995) 477.

Ritus eigenfunction method

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- BUT the general form for the chiral condensate is

$$\langle\bar{q}q\rangle = N_c\,\mathrm{Tr}_d\,S(x,x)$$
$$= N_c\frac{h}{2\pi}\int\frac{d^2\bar{p}}{(2\pi)^2}\mathrm{Tr}_d\left\{\Sigma^- S(\bar{p},n=0)+\sum_{n=1}^{\infty}S(\bar{p},n)\right\}$$

- decomposing and projecting in terms of the Ritus matrices, a pre-factor of h appears in all loop integrals, regardless of the interaction (the propagator is a function of two momentum components and the Landau level: dimensions must be maintained)!
  - we have an asymptotic expansion, not good for ‘small’ magnetic fields where we know that the quark has a nontrivial condensate!
- we have to sum up the Landau levels...

Summation and Schwinger phase

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- tree-level inverse propagator

$$\Gamma^{(0)}(x,y) = \sum_{n=0}^{\infty} \int \frac{d^3\tilde{p}}{(2\pi)^3} E(x;\tilde{p},n)\Gamma^{(0)}(\overline{p},n)\overline{E}(y;\tilde{p},n)$$

- contains the integral $I = \int dp_2\, e^{\imath p_2(x_2-y_2)} \psi_a(\varepsilon)\psi_b(\tau)$

- with $\varepsilon\,(\tau) = \sqrt{h}x_1\,(y_1) + \dfrac{p_2}{\sqrt{h}}$

- "it can be shown that" ;-) this can be written (almost) in terms of transverse momenta and Laguerre polynomials

$$I \sim e^{\imath\Phi} \int d^2p_t\, e^{\imath\vec{p}_t\cdot(\vec{x}-\vec{y})} f(\vec{p}_t) \exp\left\{-\frac{p_t^2}{h}\right\} L_n^{\alpha}\left(2\frac{p_t^2}{h}\right)$$

Gorbar, Miransky, Shovkovy, Wang, PRD88 (2013) 025025 & 025043.

Summation and Schwinger phase

- "it can be shown" ;-) that this can be written (almost) in terms of transverse momenta

$$I \sim e^{\imath\Phi} \dots$$

- where the Schwinger phase encodes the deviations from translational invariance

$$\Phi = -\frac{h}{2}(x_2 - y_2)(x_1 + y_1)$$

(vanishes for h=0)

- the sums over the Laguerre polynomials are known, to give...

Gorbar, Miransky, Shovkovy, Wang, PRD88 (2013) 025025 & 025043.

Summation and Schwinger phase

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- the tree-level inverse propagator

$$-\imath\Gamma^{(0)}(x,y) = e^{\imath\Phi}\int\frac{d^4p}{(2\pi)^4}e^{-\imath p\cdot(x-y)}\left[p_\mu\gamma^\mu - m\right]$$

- and similarly the tree-level propagator (small h)

$$\imath S^{(0)}(x,y) = e^{\imath\Phi}\int\frac{d^4p}{(2\pi)^4}e^{-\imath p\cdot(x-y)}\left\{\frac{[p_\mu\gamma^\mu + m]}{[p^2 - m^2 + \imath 0_+]} + \frac{\imath h\gamma^1\gamma^2\,[\overline{p}_\mu\gamma^\mu + m]}{[p^2 - m^2 + \imath 0_+]^2}\right\}$$

- reduction when magnetic field vanishes (unlike the Ritus decomposition),
- but no obvious relation between the two!
- up to the Schwinger phase, the momentum space expressions look promising...

Gorbar, Miransky, Shovkovy, Wang, PRD88 (2013) 025025 & 025043,
Chodos, Everding, Owen, PRD42 (1990) 2881.

Nonperturbatively

- the strategy is to take a nonperturbative ansatz for the Ritus decomposed two-point functions (where the inverse can be found), with various spin components and see if we can sum to get similar expressions...

$$-\imath\Gamma(\overline{p},n) = \Sigma^{+}(\overline{p}_{\mu}\gamma^{\mu}A - B) + \Sigma^{-}(\overline{p}_{\mu}\gamma^{\mu}C - D) - \sqrt{2nh}\gamma^{2}E$$

- (allow for different spin projections)
- sum isn't a problem for the inverse propagator:

$$-\imath\Gamma(x,y) = e^{\imath\Phi}\int\frac{d^4p}{(2\pi)^4}e^{-\imath p\cdot(x-y)} \times\left\{\Sigma^{+}(\overline{p}_{\mu}\gamma^{\mu}A - B) + \Sigma^{-}(\overline{p}_{\mu}\gamma^{\mu}C - D) - \vec{p}_t\cdot\vec{\gamma}\,E\right\}$$

- functions of two variables, $A = A(\overline{p}^2, p_t^2)$

- reduction $h \to 0: \quad (A,C,E) \to A, \quad (B,D) \to B$

Nonperturbatively

- corresponding propagator looks like...

$$\imath S(\overline{p}, n) = \Sigma^{+} \overline{p}_{\mu} \gamma^{\mu} \frac{\Delta_1 C - \Delta_2 D}{\Delta} + \ldots$$

$$\Delta_1 = \overline{p}^2 AC - BD - 2nhE^2, \;\; \Delta_2 = AD - BC, \;\; \Delta = \Delta_1^2 - \overline{p}^2 \Delta_2^2$$

- summed under approximation (suitable for small h)
    - neglect n-dependence of functions (keep explicit n factors)
    - expand denominator in (small h) Δ_2

- approximation retains the connection between spin structures
- in the end, the gap equation will determine the momentum dependence of the functions

Nonperturbatively

- approximated summed propagator

$$
\begin{aligned}
-\imath S(x,y) &= e^{\imath\Phi}\int \frac{d^4p}{(2\pi)^4} e^{-\imath p\cdot(x-y)} \\
\times \Bigg\{ & \frac{\Sigma^+(\overline{p}_\mu\gamma^\mu C + D) + \Sigma^-(\overline{p}_\mu\gamma^\mu A + B) - \vec{p}_t\cdot\vec{\gamma}\,E}{[\overline{p}^2 AC - p_t^2 E^2 - BD + \imath 0_+]} \\
+\, hE^2 & \frac{\Sigma^+(\overline{p}_\mu\gamma^\mu C + D) - \Sigma^-(\overline{p}_\mu\gamma^\mu A + B)}{[\overline{p}^2 AC - p_t^2 E^2 - BD + \imath 0_+]^2} \\
-(AD - BC) & \frac{\Sigma^+(\overline{p}_\mu\gamma^\mu D + \overline{p}^2 C) - \Sigma^-(\overline{p}_\mu\gamma^\mu B + \overline{p}^2 A)}{[\overline{p}^2 AC - p_t^2 E^2 - BD + \imath 0_+]^2} \Bigg\}
\end{aligned}
$$

- reduces to tree-level (all h), also to standard propagator in the absence of the magnetic field.

Gap equation

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- rainbow truncation (chiral quarks)

$$\Gamma(x,y) = \Gamma^{(0)}(x,y) + g^2 C_F\, \gamma^\mu S(x,y) \gamma^\kappa W_{\kappa\mu}(y,x)$$

- dressed (Landau gauge) gluon interaction

$$\imath W_{\kappa\mu}(y,x) = \int \frac{d^4q}{(2\pi)^4} e^{-\imath q\cdot(y-x)} t_{\kappa\mu}(q) \frac{G(q^2)}{q^2}$$

$$g^2 \frac{G(q^2)}{q^2} = 4\pi^2 d \exp\left\{\frac{q^2}{\omega^2}\right\} \times \left\{ \begin{array}{cc} q^2/\omega^2, & \mathrm{I} \\ -1, & \mathrm{II} \end{array} \right.$$

- Schwinger phase factorizes, so we can work in momentum space - but with functions of two variables

- consider a range of widths and fix d from the condensate: $\langle \overline{q} q \rangle_{h=0} = (-251\,\mathrm{MeV})^3$

Alkofer, Watson, Weigel, PRD65 (2002) 094026,
Aguilar, Papavassiliou, PRD83 (2011) 014013.

Results (dressing functions)

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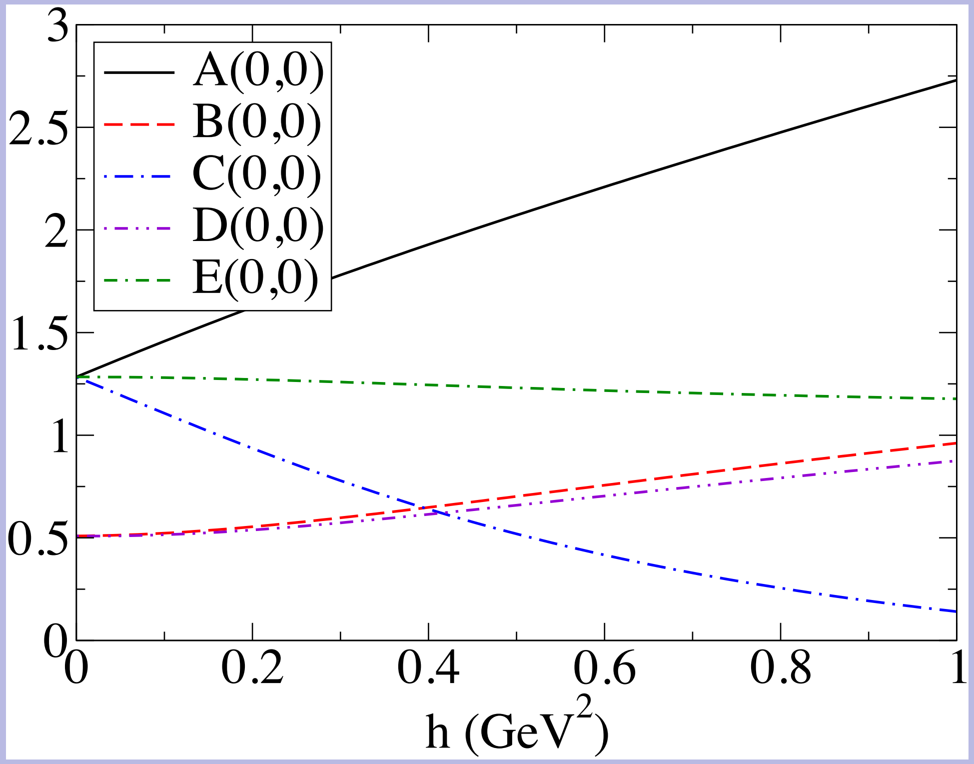
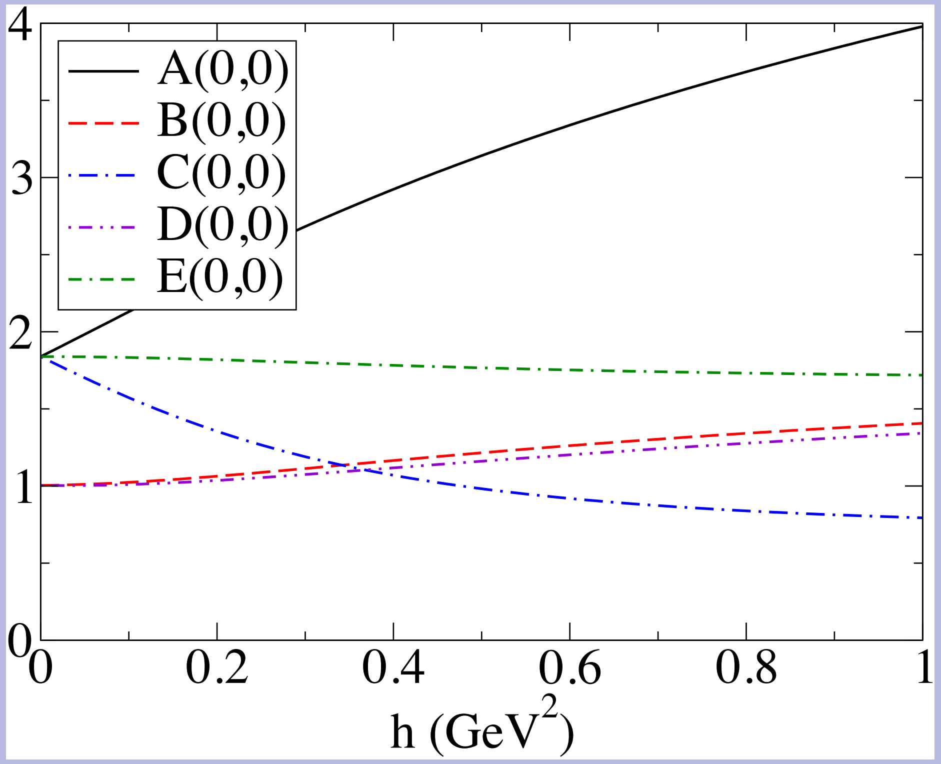
I



II



$$\mathrm{I}: \ \omega = 0.5\,\mathrm{GeV}, \ d = 16\,\mathrm{GeV}^{-2}$$
$$\mathrm{II}: \ \omega = 0.5\,\mathrm{GeV}, \ d = 41\,\mathrm{GeV}^{-2}$$

- functions at zero momentum
- reduction for vanishing magnetic field (type I explicitly matches earlier results)
- similar patterns for both interactions, since the gluon is unaffected by the magnetic field (under truncation)

Results (magnetic catalysis)

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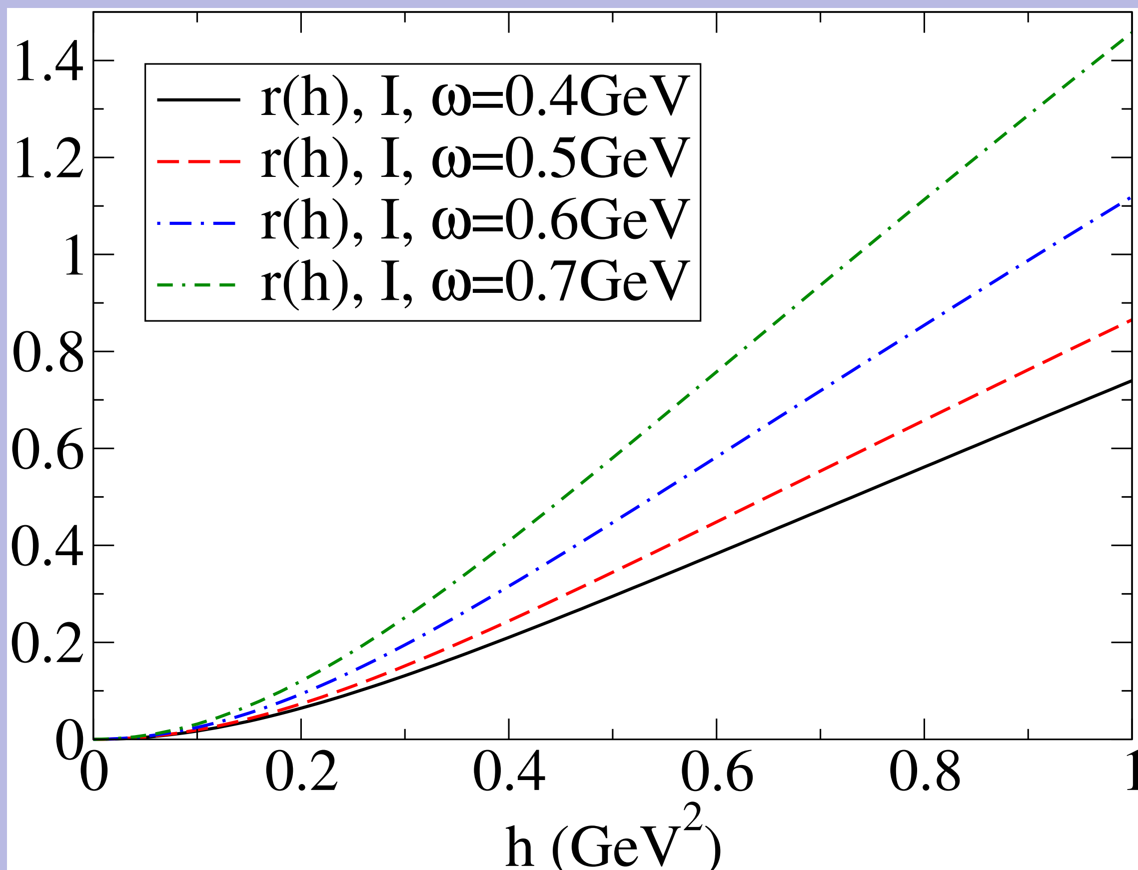

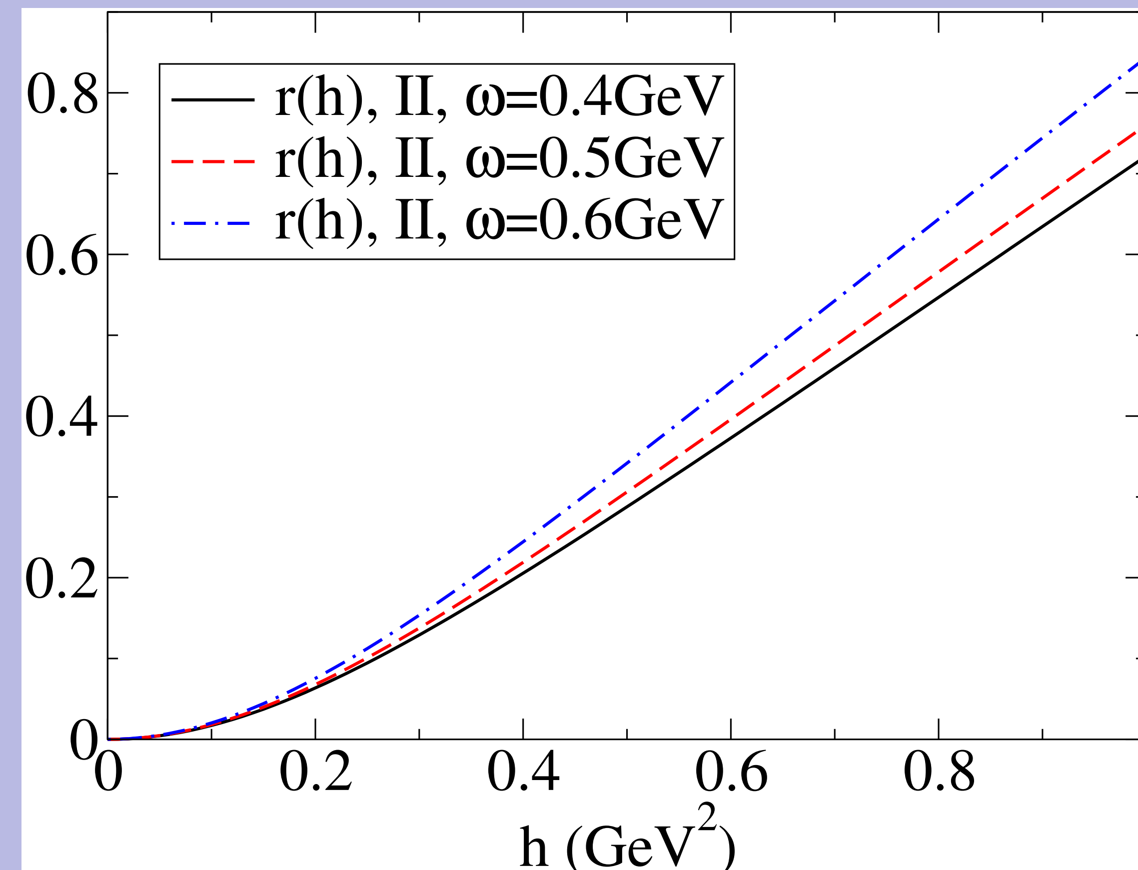

- relative increment: $r(h) = \frac{\langle \bar{q}q \rangle_h}{\langle \bar{q}q \rangle_{h=0}} - 1$

- condensate rises quadratically for small h
- and linearly for large h
- qualitative agreement with lattice is good, even for large h!

D'Elia, Negro, Phys.Rev.D83 (2011) 114028; Bali et al., PRD86 (2012) 071502; Simonov, arXiv:1212.3118.
BUT: Ilgenfritz et al., Phys.Rev.D85 (2012) 114504; Shushpanov, Smilga, Phys.Lett.B402 (1997) 351.

Results (comparison to lattice)

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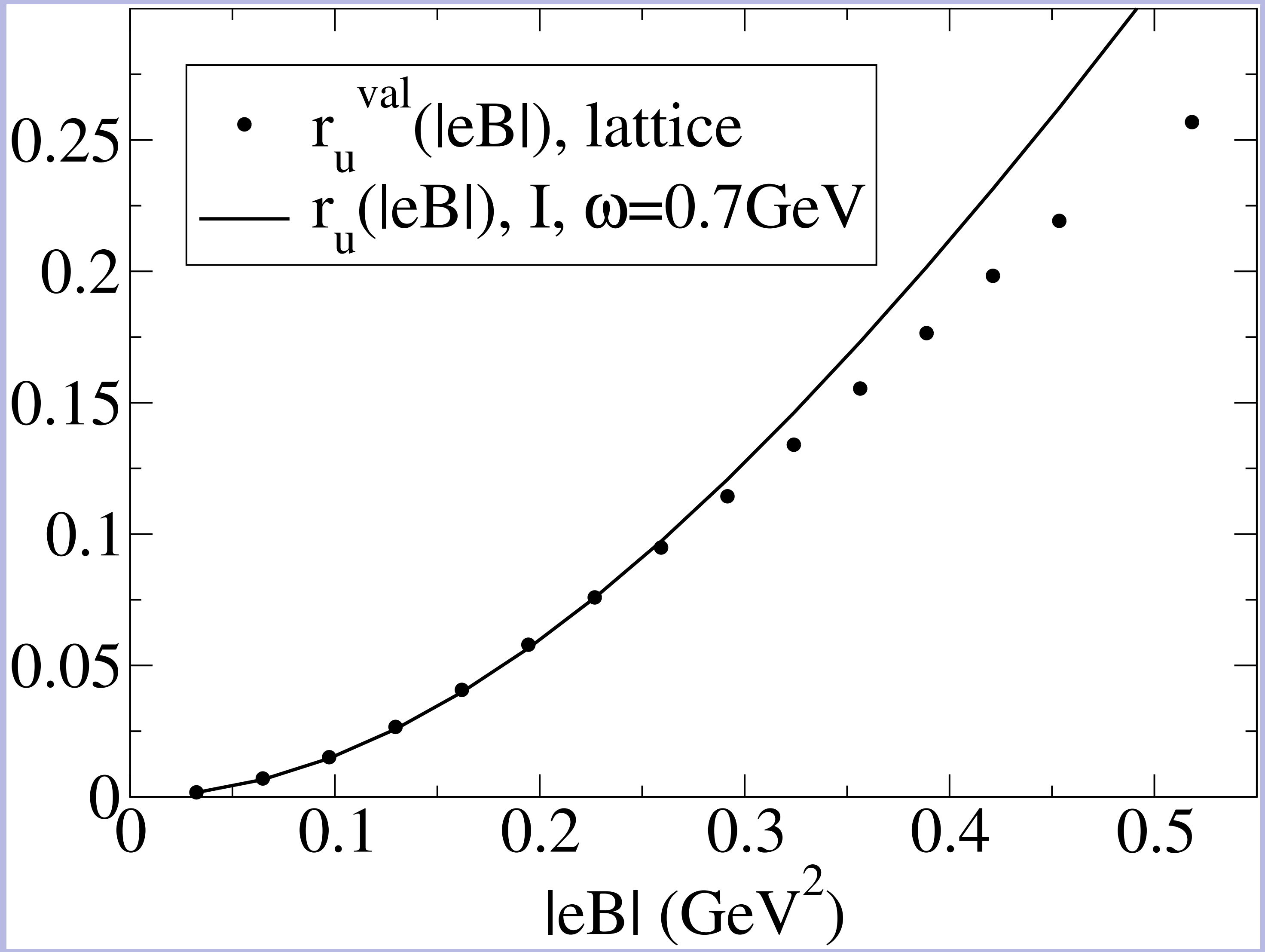

- small h quadratic behavior reproduced
- transition scale reproduced
- recall heavy ions: $|eB| \sim 15 m_\pi^2 \approx 0.3\,\mathrm{GeV}^2$
- small h matters!

D'Elia, Negro, Phys.Rev.D83 (2011) 114028

Results (magnetic moment)

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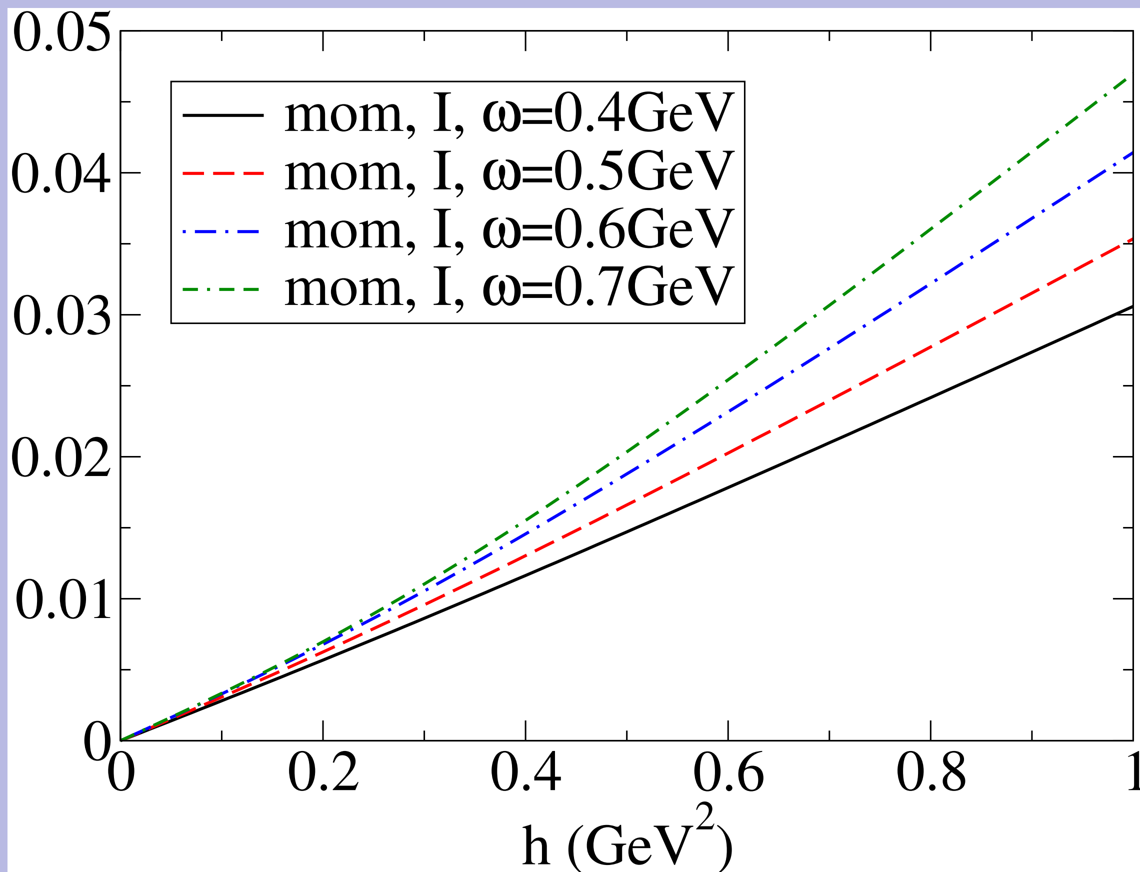

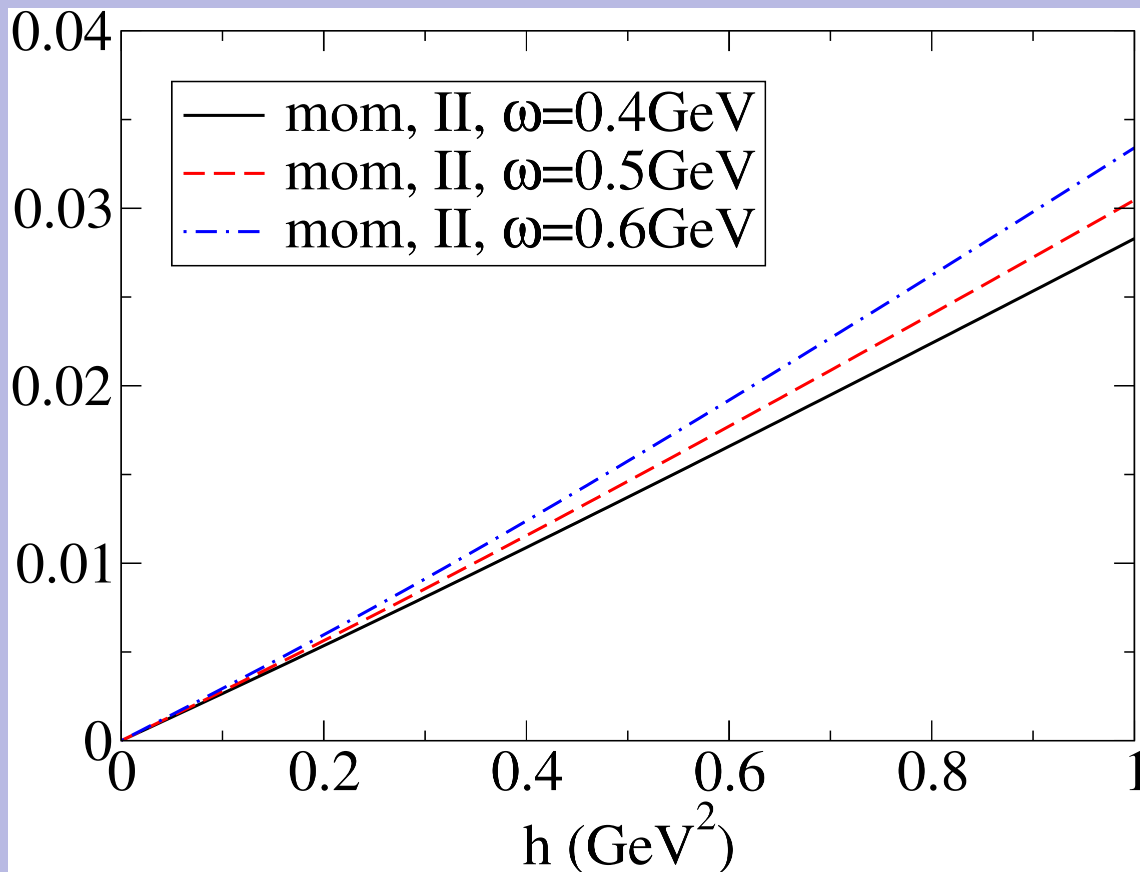

- magnetic moment: $$\langle \overline{q} \Sigma^{12} q \rangle = N_c \mathrm{Tr}_d (\Sigma^- - \Sigma^+) S(x,x)$$

(measures asymmetry between spin projected components)

- linear at small h

Results (magnetic susceptibility)

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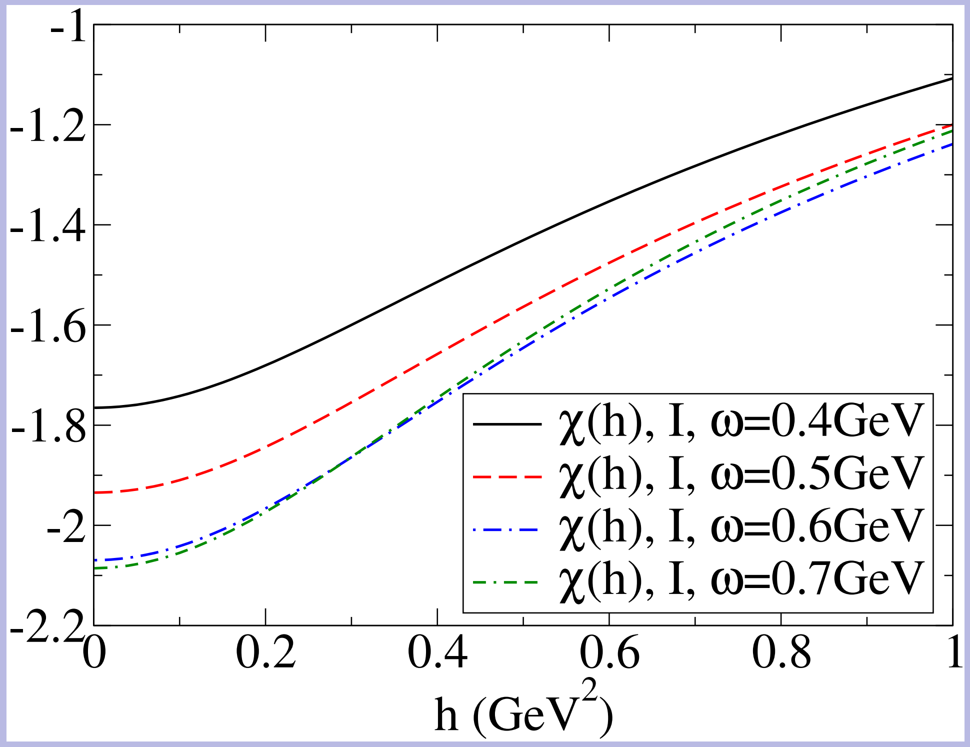

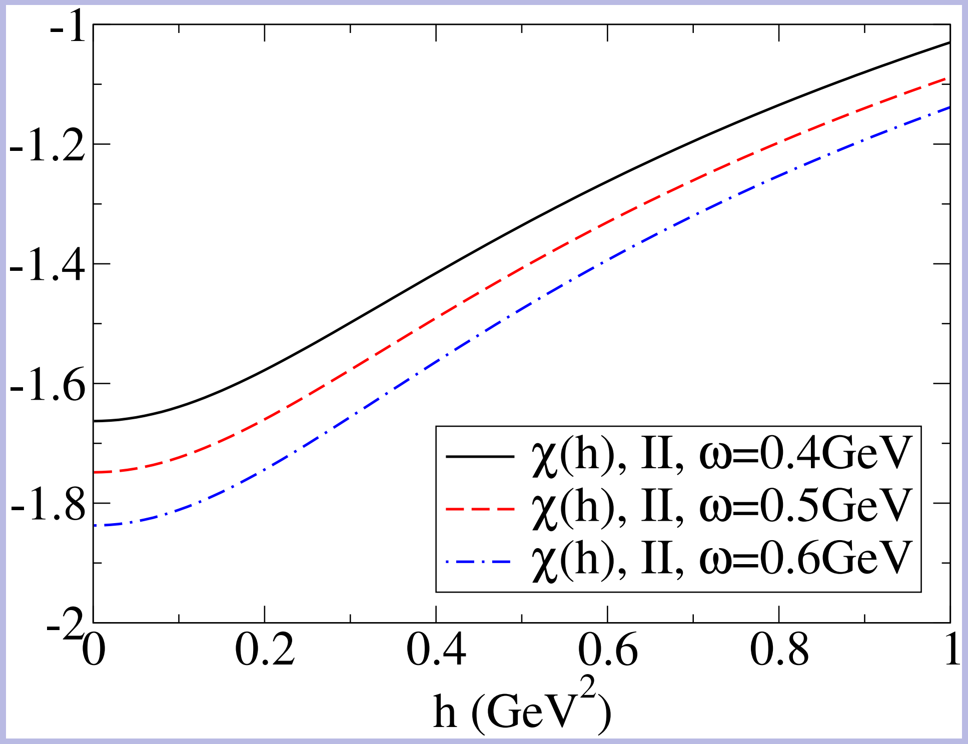

- magnetic susceptibility: $$\chi(h) = \frac{\langle \bar{q}\Sigma^{12} q\rangle}{h\langle \bar{q}q\rangle}, \qquad \chi = \lim_{h\to 0}\chi(h)$$
  - lattice (quenched, chiral): $\chi = -1.547(6)\mathrm{GeV}^{-2}$
  - lattice (unquenched, finite m): $-(2.08 \pm 0.08)\mathrm{GeV}^{-2}$
  - NJL: $-4.3\,\mathrm{GeV}^{-2}$
  - quark-meson model: $-5.25\,\mathrm{GeV}^{-2}$

Buividovich et al., Nucl.Phys.B826 (2010) 313; Bali et al., Phys.Rev.D86 (2012) 094512; Frasca, Ruggieri, Phys.Rev.D83 (2011) 094024.

Summary

- quark gap equation (rainbow truncation) with a phenomenological gluon interaction in the presence of a constant magnetic field
- strong magnetic field vs. strong interaction
  - expansion in Landau levels is not suitable in this context
- we use an approximation to sum the Landau levels, ensuring the limit when the magnetic field vanishes
- in the presence of a constant magnetic field, the chiral quark condensate increases, quadratically for small field and linearly for large field
- magnetic susceptibility
- agreement with lattice (though more to be done)
- parameter dependence - sensitive probe of the interaction?
- small h is relative to QCD!

Results (large h, n=0 dominance)

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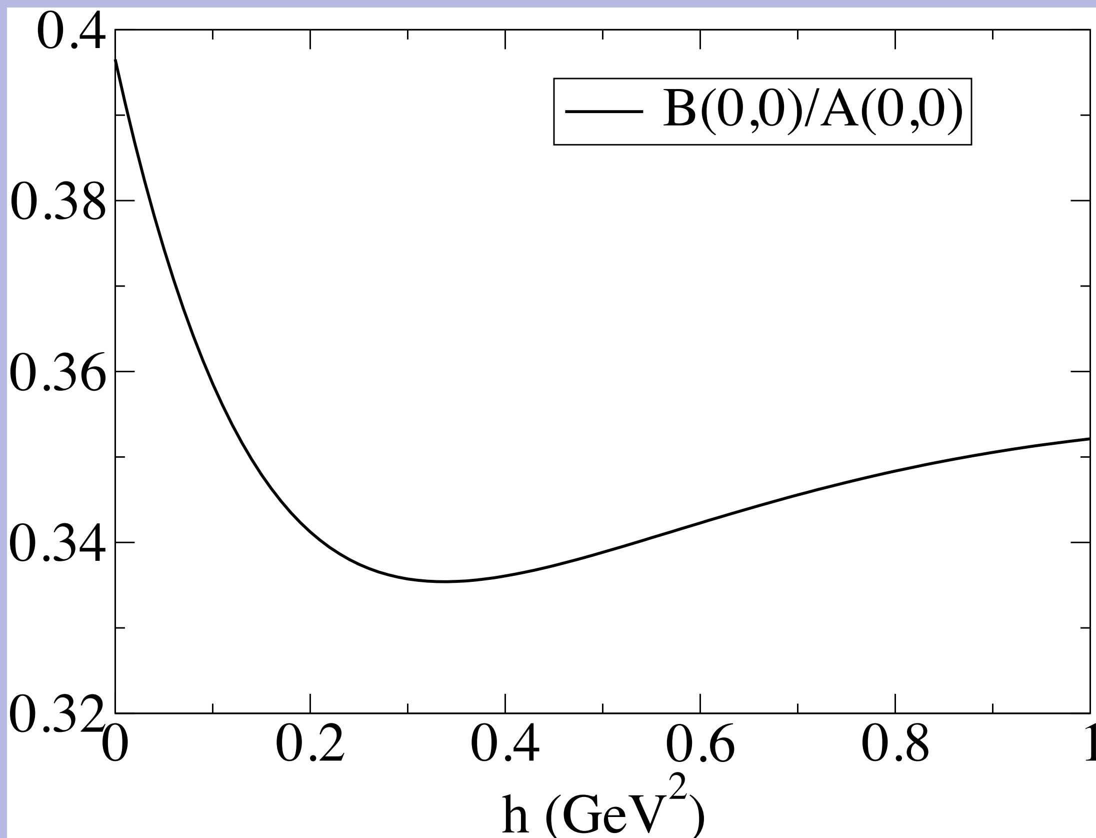

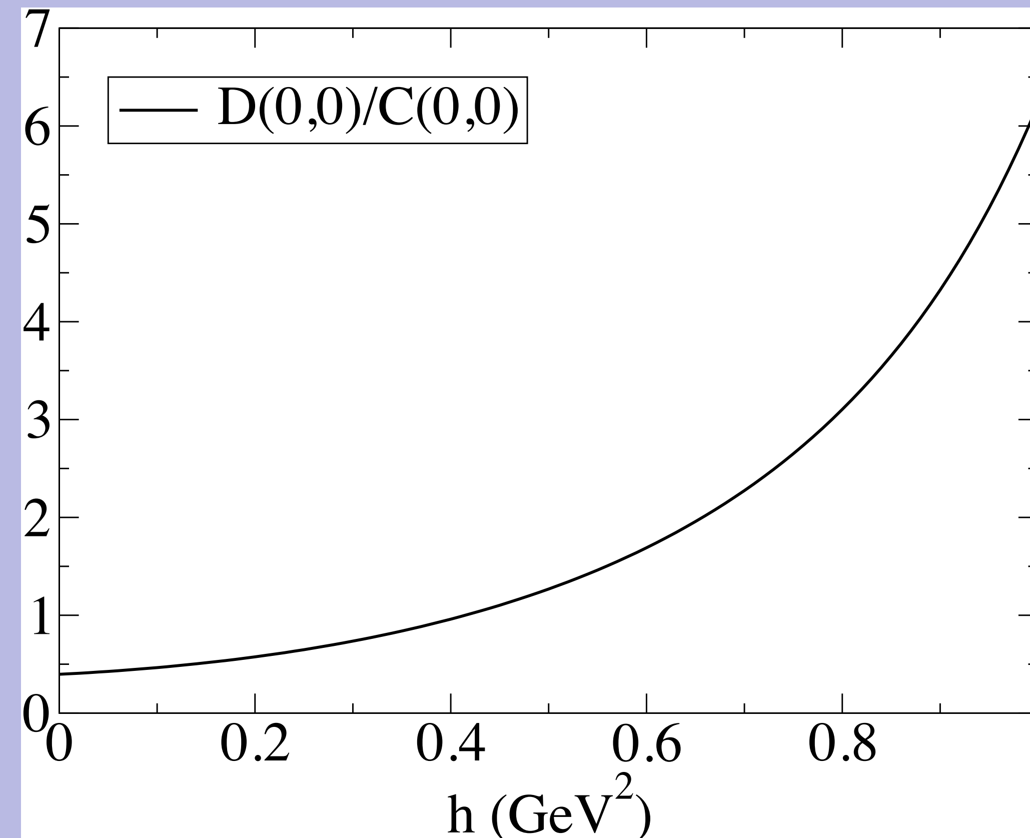

- lowest Landau level approximation:

$$[\psi_{n-1}(\varepsilon)\Sigma^{+} + \psi_n(\varepsilon)\Sigma^{-}] \rightarrow \psi_0(\varepsilon)\Sigma^{-}$$

- and the ‘mass function’ D/C dominates for large h! (because C decreases)