Investigations of the QCD phase diagram with Dyson-Schwinger equations

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arxiv 1405.4762

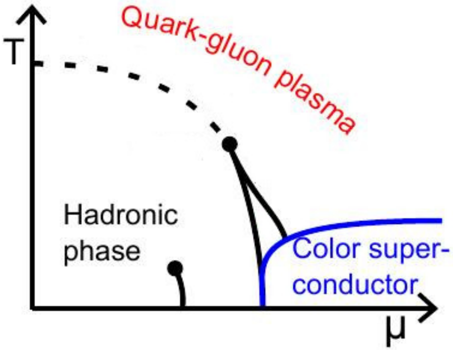
Outline

Outline

”In the beginning there was nothing, which exploded.”

(Sir Terry Pratchett)

Motivation



- Various representations
- Various interesting features (Big Bang, neutron stars, different phases and transitions)
- Various attempts to pin down (lattice QCD, Models, Functional approaches)

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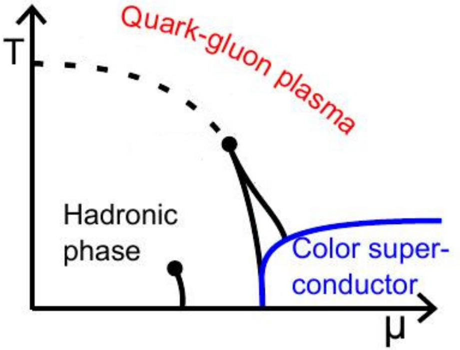

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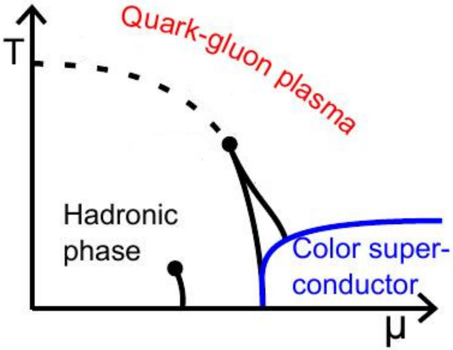

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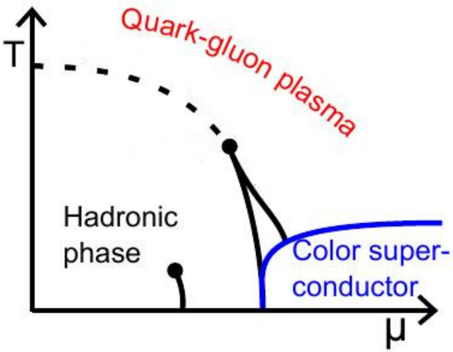

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Timeline of Dyson-Schwinger approach to phase diagram

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N_f=2

Up and down quark
HTL 2011
Full 2012

Motivation

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N_f=2+1

Strange quark
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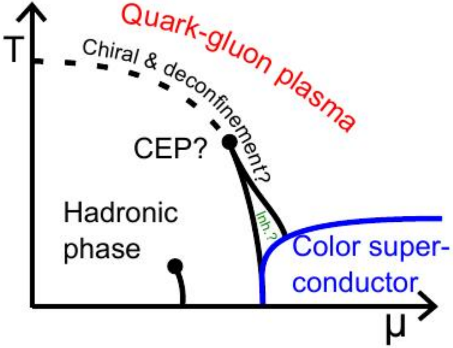
N_f=2+1+1

Charm quark
now

Motivation



- Location and existence of CEP
- Chiral and deconfinement transitions
- Evaluation of functional approach
- Impact of charm quark

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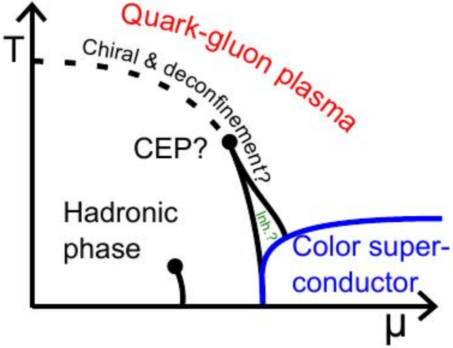

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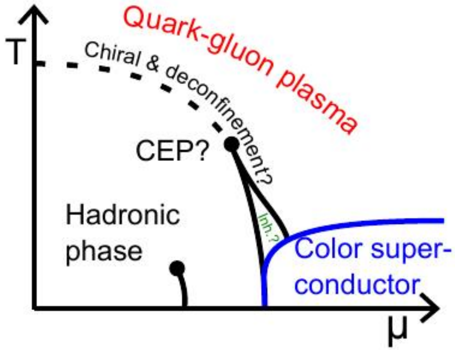

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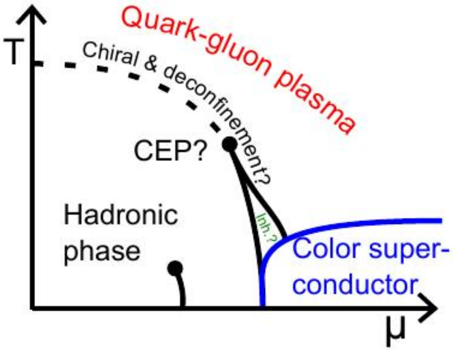

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Outline

Quark propagator in hot and dense matter

Bare quark propagator

$$S_0^{-1}(p) \;=\; ip_\mu\gamma^\mu + m_0$$

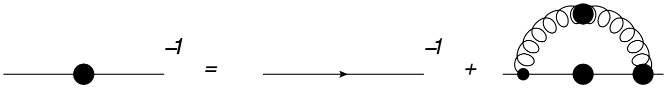
Bare quark propagator at finite T and μ

$$S_0^{-1}(p) \;=\; i\vec{p}\vec{\gamma} + i(\omega_n + i\mu)\gamma_4 + m_0$$

Dressed quark propagator at finite T and μ

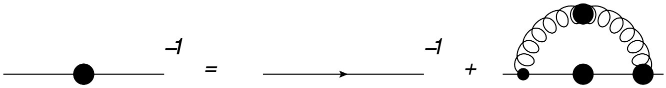
$$S^{-1}(p) \;=\; i\vec{p}\vec{\gamma}A(p) + i(\omega_n + i\mu)\gamma_4 C(p) + B(p)$$

Quark DSE in hot and dense matter



- Coupled integral equation
- Base of infinite tower of equations
- Selfconsistently calculate dressing functions A(p), B(p) and C(p)
- Depends on:
  1. full quark-gluon vertex
  2. **fully dressed gluon propagator**

Quark DSE in hot and dense matter



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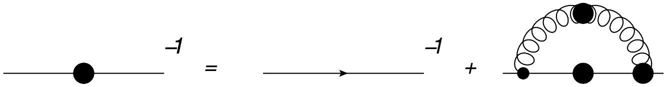

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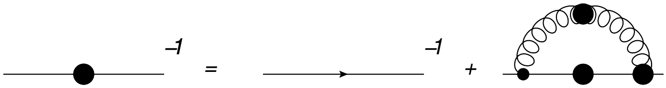

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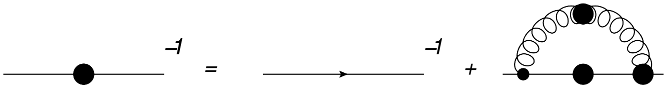

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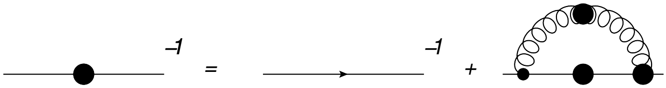

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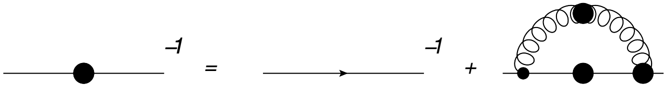

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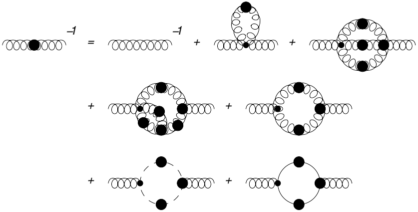
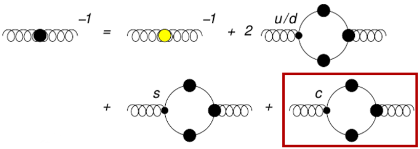
Gluon propagator and gluon DSE I

Gluon propagator at finite T (and μ)

$$D_{\mu\nu}(p) = P^L_{\mu\nu}(p)\frac{Z^L(p)}{p^2} + P^T_{\mu\nu}(p)\frac{Z^T(p)}{p^2}$$



$\rightarrow$



Gluon propagator and gluon DSE II

Gluon DSE

$$D^{-1}_{\mu\nu}(p) = \left[D^{que.}_{\mu\nu}(p)\right]^{-1} + \sum_f \Pi^f_{\mu\nu}(p)$$

- Use input from lattice QCD for quenched gluon propagator $D^{que.}_{\mu\nu}(p)$
- Calculate quark loop for each flavor
- Finite temperature gluon fully determined by dressing functions $Z^L(p)$ and $Z^T(p)$

Quark-gluon vertex

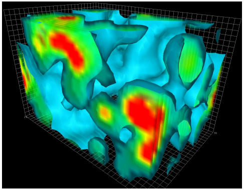
Quark-gluon vertex ansatz

$$\Gamma_\mu(l, p; q) = \gamma_\mu \cdot \Gamma_{[d_1]}(l^2, p^2, q^2) \cdot \left(\delta_{\mu,4} \frac{C(l) + C(p)}{2} + \delta_{\mu,i} \frac{A(l) + A(p)}{2} \right)$$

- Designed along symmetries and constraints
- Depends on temperature, chemical potential and quark flavor via first term of the Ball-Chiu-Vertex
- Vertex dressing function depends on parameter d_1 (interaction strength small momenta)
  $\rightarrow$ *d_1 becomes important when including the charm quark*

Outline

Chiral condensate



Quark condensate

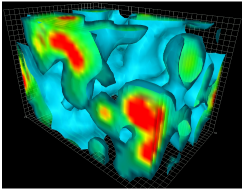
$$\langle\bar{\psi}_R\psi_L\rangle_f \propto \int \mathrm{Tr}_D\left[S^f(p)\right]$$
$$\approx const. + m_0^f\cdot\Lambda^2$$

Regularized chiral condensate

$$\Delta_{l,s} = \langle\bar{\psi}\psi\rangle_l - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_s$$

- Order paramter for chiral symmetry
- Two ways to extract T_C for crossover:
  1. Maximum of chiral susceptibility: $\frac{\partial\langle\bar{\psi}\psi\rangle}{\partial m}$
  2. Inflection point of chiral condensate: $\frac{\partial\langle\bar{\psi}\psi\rangle}{\partial T}$

Chiral condensate



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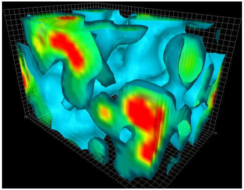

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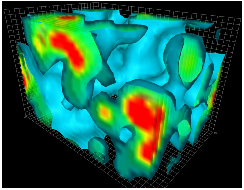

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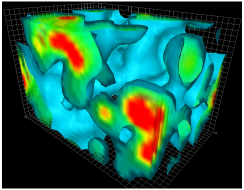

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Polyakov loop

Polyakov loop

$$L[A_0] = \frac{1}{N_C} \mathrm{Tr}\, P e^{ig\int_0^\beta d\tau A_0(\vec{x},\tau)}$$

$$\langle L[A_0] \rangle \propto e^{-F_q/T} = \begin{cases} 0 & \text{if } F_q = \infty \\ \text{non-zero} & \text{if } F_q < \infty \end{cases}$$

- Order parameter for deconfinement ($F_q = \infty \rightarrow$ no free quarks)
- Polyakov loop of minimum of background field potential upper bound for expectation value of full Polyakov loop
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Some time passes...

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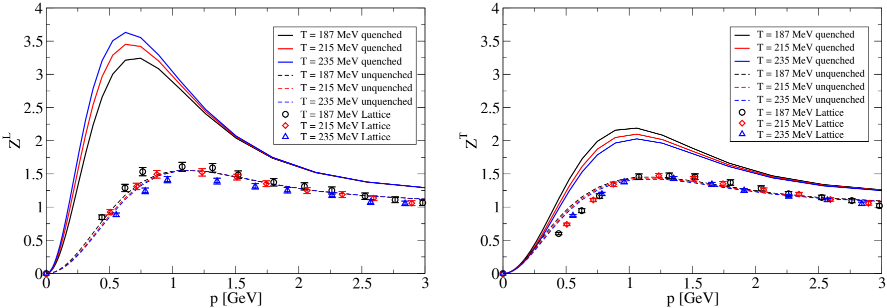
Unquenched gluon propagator N_f=2



- Gluon propagators for N_f=2 with m_π=316 MeV
- DSE results calculated before lattice results
- Lattice results from *R. Aouane et al., Phys.Rev. D87 (2013) 11, 114502*

Including the charm-quark

Remember...

Including the charm-quark

Remember...

Including the charm-quark

Remember...

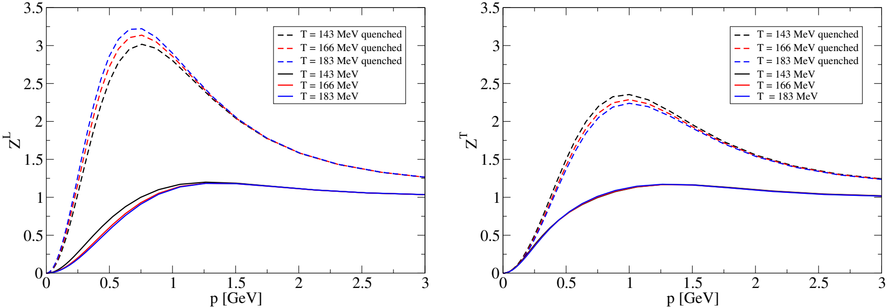
1. Fix d_1 to reproduce results from lattice QCD for N_f=2+1 at μ=0 MeV and merely add charm quark
   $\rightarrow$ Sets A

Including the charm-quark

Remember...

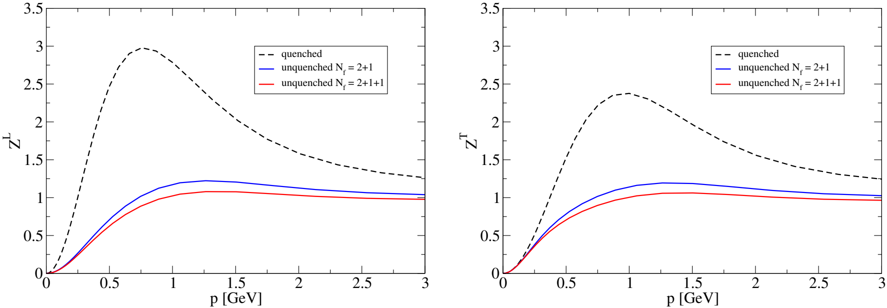
1. Fix d_1 to reproduce results from lattice QCD for N_f=2+1 at μ=0 MeV and merely add charm quark
   $\rightarrow$ Sets A
2. Fix d_1 to reproduce vacuum physics in the same truncation (N_f=2+1 and N_f=2+1+1 seperately via BSE, see *W. Heupel, T. Goecke, C.F. Fischer, Eur.Phys.J. A50 (2014) 85*)
   $\rightarrow$ Sets B

Unquenched gluon propagator N_f=2+1



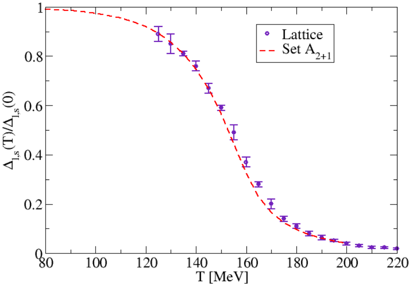
- Gluon propagators for N_f=2+1 for Set A
- Prediction for unquenched propagators on the lattice

Unquenched gluon propagator N_f=2+1 and N_f=2+1+1



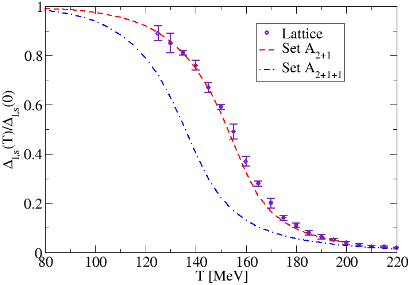
- Gluon propagators for N_f=2+1 and N_f=2+1+1 for Sets B close to the critical temperature
- *Influence of the charm quark on 10 percent level*

Results at $\mu = 0$ MeV I



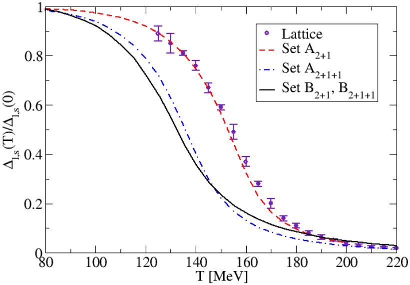
- Tuned d_1 to get good agreement with lattice data *Borsanyi et al. JHEP 1009 073*
- *Nontrivial result: perfect agreement for steepness*

Results at $\mu = 0$ MeV II



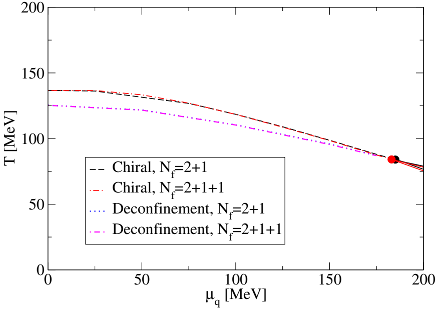
- Steepness is conserved
- *Merely adding the charm quark shifts the curve to lower temperatures*

Results at $\mu = 0$ MeV III



- Shape of the curve changed slightly
- *Chiral condensate for Sets B does not differ for N_f=2+1 and N_f=2+1+1*

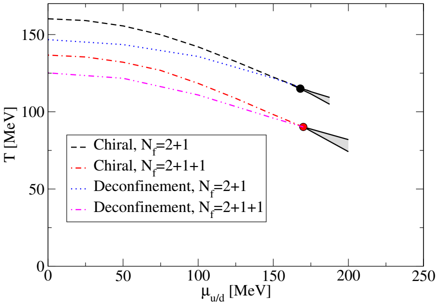
Phase diagram Sets B



Set	CEP (μ,T) [MeV]	$T_C^{\mu=0}[MeV]\,(\chi_m)$	$T_C^{\mu=0}[MeV]\,(\Delta_{l,s})$
B_{2+1}	(185,85)	136.7	132.0
B_{2+1+1}	(181,86,5)	136.8	131.6

- Difference $\Delta\, T \approx -23$ MeV compared to lattice results at $\mu = 0$ MeV *Borsanyi et al. JHEP 1009 073*
  $\rightarrow$ systematical error of truncation
- *Physics fixed in vacuum $\rightarrow$ no influence of charm quark within numerical resolution*

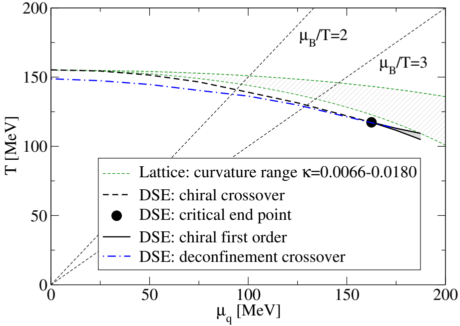
Phase diagram Sets A



Set	CEP (μ,T) [MeV]	$T_C^{\mu=0}[MeV]\,(\chi_m)$	$T_C^{\mu=0}[MeV]\,(\Delta_{l,s})$
A_{2+1}	(168,115)	160.2	155.6
A_{2+1+1}	(172,93,5)	142.6	137.2

- Disagreement of chiral and deconfinement transition: partly from definition of T_C
- *Merely adding the charm quark results in a shift of* $\Delta\, T \approx -18$ *MeV*

Phase diagram - prediction



- Chiral crossover defined via inflection point
- Curvature (μ_B):
  1. $\kappa_{DSE,chiral} = 0.0275$
  2. $\kappa_{DSE,deconf} = 0.0253$
- Curvature lattice from *G. Endrodi et al.,JHEP 1104, 001 (2011)*, *O. Kaczmarek et al.,Phys.Rev. D83, 014504 (2011)* and *P. Cea et al.,Phys.Rev. D89, 074512 (2014)*
- Baryonic and mesonic effects

Outline

Conclusion and outlook

Conclusion

- Used Dyson-Schwinger equations to calculate quark and gluon propagators in medium
- Truncation scheme gives results quantitatively comparable with lattice QCD
- Used two strategies to include charm quark
  1. Charm quark has no influence on phase structure when vacuum physics fixed
  2. Merely adding the charm quark gives shift of about -20 MeV
- Prediction for the phase diagram for N_f=2+1 holds also for N_f=2+1+1

Outlook

- Baryonic (future: mesonic) effects under investigation
- Spectral properties of the quark and gluon

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- Spectral properties of the quark and gluon

Conclusion and outlook

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Confucius, 551BC - 479

"I hear, I know.
I see, I remember.
I do, I understand."

THANK YOU FOR YOUR ATTENTION

HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

Backup - Full quark DSE I

Quark DSE at finite T and μ

$$
\begin{aligned}
\left[S^f(p)\right]^{-1} &= Z_2^f \left[S_0^f(p)\right]^{-1} + \Pi^f_{self}(p) \\
\Pi^f_{self}(p) &= C_F\, Z^f_{1F}\, g^2\, T \sum_n \int \frac{d^3 l}{(2\pi)^3} \left[\gamma_\mu\, S^f(l)\, \Gamma^f_\nu(l,p;q)\, D_{\mu\nu}(q)\right]
\end{aligned}
$$

Backup - Full gluon Propagator I

Gluon propagator at finite T (and μ)

$$
\begin{aligned}
D_{\mu\nu}(p) &= P^L_{\mu\nu}(p)\frac{Z^L(p)}{p^2} + P^T_{\mu\nu}(p)\frac{Z^T(p)}{p^2} \\
P^T_{\mu\nu} &= (1-\delta_{\mu 4})(1-\delta_{\nu 4})\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{\vec{p}^{\,2}}\right) \\
P^L_{\mu\nu} &= P_{\mu\nu} - P^T_{\mu\nu} \\
P_{\mu\nu} &= \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}
\end{aligned}
$$

Backup - Full gluon DSE I

Gluon DSE

$$D^{-1}_{\mu\nu}(p) = \left[D^{que.}_{\mu\nu}(p)\right]^{-1} + \sum_{f}^{N_f} \Pi^f_{\mu\nu}(p)$$

$$\Pi^f_{\mu\nu}(p) = -Z^f_2 \frac{g^2}{2} T \sum_n \int \frac{d^3 l}{(2\pi)^3} \,\mathrm{Tr}\left[\gamma_\mu \, S^f(l) \, \Gamma^f_\nu(l, q; p) \, S^f(q)\right]$$

Backup - Ansatz for quark-gluon vertex I

Quark-Gluon vertex ansatz

$$\Gamma^f_\mu(l,p;q) = \gamma_\mu \cdot \Gamma(l^2,p^2,q^2) \cdot \left(\delta_{\mu,4}\frac{C^f(l)+C^f(p)}{2} + \delta_{\mu,i}\frac{A^f(l)+A^f(p)}{2}\right)$$

$$\Gamma(l^2,p^2,q^2) = \frac{d_1}{d_2+x} + \frac{x}{\Lambda^2+x}\left(\frac{\beta_0\alpha(\mu)\ln[x/\Lambda^2+1]}{4\pi}\right)^{2\delta}$$

- Λ and d_2 fixed by YM sector
- Renormalized coupling $\alpha(\zeta)$=0.3
- Variable x: gluon momentum q^2 in qDSE, sum of quark momenta l^2+p^2 in quark loop (multiplicative renormalizability)
- d_1 only open parameter which sets interaction strength in IR

Backup - Parameter for different sets I

Set	m_l	m_s	d_1	m_π	m_K	f_π
A_{2+1}	0.8	21.6	8.05	107	405	107
B_{2+1}	1.32	34.1	6.8	135	497	94

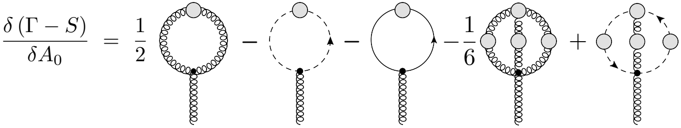
Table : Renormalized quark masses and parameter d_1 for 2+1 flavors

Set	m_l	m_s	m_c	d_1	m_π	m_K	m_{η_c}	f_π
A_{2+1+1}	0.8	21.6	300.0	8.05	109	412	2,364	95
B_{2+1+1}	1.23	31.6	440.0	7.6	135	497	2,982	94

Table : Renormalized quark masses and parameter d_1 for 2+1+1 flavors

Backup - Chiral condensate I

$$\begin{aligned}\langle\bar{\psi}\psi\rangle_f &= Z_2 Z_m N_c T \sum_n \int \frac{d^3p}{(2\pi)^3} \mathrm{Tr}_D \left[S^f(p)\right] \\ &\approx const. + m_0^f \cdot \Lambda^2\end{aligned}$$



- Gives (up to integration) background field potential
- Neglect two-loop contributions $\rightarrow$ given by QCD propagators
- Minimum of potential is $\bar{A}_0 = \langle A_0 \rangle$