Lattice QCD investigation of heavy-light four-quark systems

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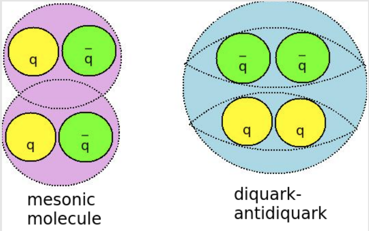
Study of four-quark states

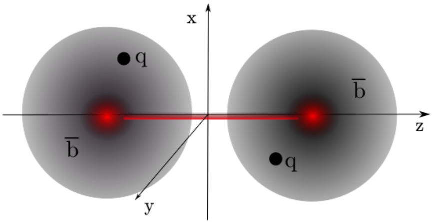
Motivation

- A number of mesons observed in particle detectors (LHCb, Belle) is not well understood.
- E.g. charged charmonium- and bottomonium-like states (${Z_c}^{\pm}$ and ${Z_b}^{\pm}$)
- They include $b\bar{b}$ or $c\bar{c}$, but are also charged: must be 4-quark states
- Different four-quark/tetraquark structures possible, two examples:



The static-light approach

- Computation of 4-quark states very difficult
- If 2 quarks are heavy and 2 quarks are light:
  Treat degrees of freedom independently (Born-Oppenheimer approximation [M. Born, R. Oppenheimer, "Zur Quantentheorie der Molekeln," Ann.Phys. 389, Nr. 20, 1927]).
  1. Lattice computation of the potential of two static quarks in the presence of two light quarks, i.e. potential can be interpreted as the potential between two B mesons



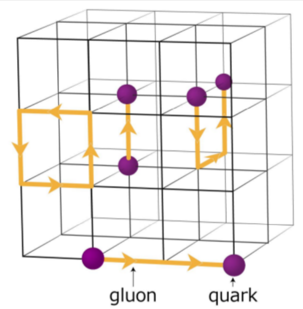
  2. Solve Schrödinger's equation to check whether potentials are sufficiently attractive to host a bound state.

Lattice QCD

- QCD: Theory to describe quarks and gluons and the forces between them
- No analytical solution for low energy observables
- No pertubative approach on the potential

Lattice QCD:

- continuum → 4-dimensional lattice
- discretize quark and gluon action (many different possibilities)
- use path integral formalism and compute vacuum expectation values numerically
- very often: necessity to use non-physically high quark masses



[http://www.jicfus.jp/en/promotion/pr/mj/guido-cossu/, April 28, 2015]

Hadron spectroscopy I

- A hadron is described by its isospin (flavor content) I, total angular momentum J, parity P and charge conjugation C.
- If two static quarks are involved, the distance r between them also characterizes the hadronic system.
- Application of a suitable operator $\mathcal{O}$ on the vacuum generates field excitations which are similar to the hadron of interest.
- Here: Use two static quarks and two quarks of finite mass
  - Static quarks: no spin, no contribution to total angular momentum and isospin
  - BB: $\bar{Q}\bar{Q}qq$ with $Q = b$ and $q \in \{u, d, s, c\}$

$$\mathcal{O}_{BB} = (\mathcal{C}\Gamma)_{AB}(\mathcal{C}\tilde{\Gamma})_{CD} \underbrace{\bar{Q}^a_C(\vec{x},t){q^{(1)}}^a_A(\vec{x},t)}_{B \text{ meson at } \vec{x}} \underbrace{\bar{Q}^b_D(\vec{y},t){q^{(2)}}^b_B(\vec{y},t)}_{B \text{ meson at } \vec{y}}$$

Hadron spectroscopy II

- Obtain the correlation function in time for each separation $r = |\vec{x} - \vec{y}|$ of the static quarks via

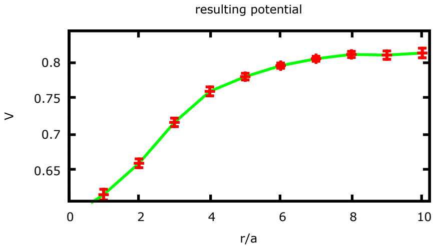
$$C_r(t) = \langle\Omega| \mathcal{O}_r^\dagger(t)\mathcal{O}_r(0) |\Omega\rangle$$

Requires $\mathcal{O}(\text{months})$ of computing time on high performance computers.

- For large t one finds the potential $V(r)$ of the hadronic state $\mathcal{O}(t)$:

$$\lim_{t\to\infty} \langle\Omega| \mathcal{O}_r^\dagger(t)\mathcal{O}_r(0) |\Omega\rangle \propto \exp(-V(r)t)$$

- Get a value of the potential for each quark separation r and obtain the full potential:



BB systems in the static-light approach

P. Bicudo, K. Cichy, A. Peters, B. Wagenbach and M. Wagner, Phys. Rev. D **92** (2015) no.1, 014507 doi:10.1103/PhysRevD.92.014507 [arXiv:1505.00613 [hep-lat]].

P. Bicudo, K. Cichy, A. Peters and M. Wagner, Phys. Rev. D **93** (2016) no.3, 034501 doi:10.1103/PhysRevD.93.034501 [arXiv:1510.03441 [hep-lat]].

related to previous papers:

C. Michael *et al.* [UKQCD Collaboration], Phys. Rev. D **60** (1999) 054012 doi:10.1103/PhysRevD.60.054012 [hep-lat/9901007].

M. S. Cook and H. R. Fiebig, hep-lat/0210054.

T. Doi, T. T. Takahashi and H. Suganuma, AIP Conf. Proc. **842** (2006) 246 doi:10.1063/1.2220239 [hep-lat/0601008].

W. Detmold, K. Orginos and M. J. Savage, Phys. Rev. D **76** (2007) 114503 doi:10.1103/PhysRevD.76.114503 [hep-lat/0703009 [HEP-LAT]].

M. Wagner [ETM and Y Collaborations], PoS LATTICE **2010** (2010) 162 [arXiv:1008.1538 [hep-lat]].

G. Bali *et al.* [QCDSF Collaboration], PoS LATTICE **2010** (2010) 142 [arXiv:1011.0571 [hep-lat]].

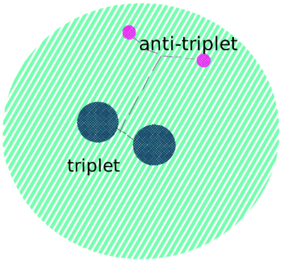
The BB system - Expectations

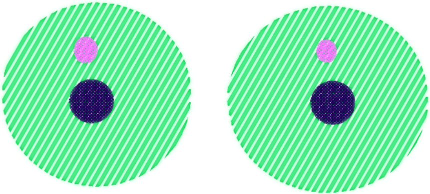
small separations of the static antiquarks:

- interaction due to 1-gluon exchange
- bound state: static $\bar{Q}\bar{Q}$ pair in a color triplet (attractive) $\longrightarrow$ antidiquark

large separations of the static antiquarks:

- screening of the antiquark-antiquark interaction due to light quarks (stronger, the more massive the light quarks)
- basically 2 static-light mesons

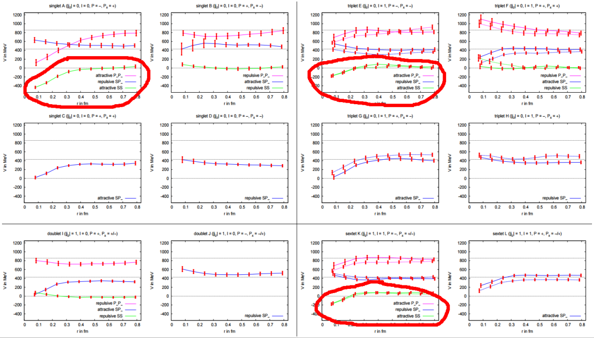




The *BB* four-quark operator

$$\mathcal{O}_{BB} = (\mathcal{C}\Gamma)_{AB}(\mathcal{C}\tilde{\Gamma})_{CD}\underbrace{\bar{Q}^a_C(\vec{x},t)q^{(1)}{}^a_A(\vec{x},t)}_{B\text{ meson at }\vec{x}}\underbrace{\bar{Q}^b_D(\vec{y},t)q^{(2)}{}^b_B(\vec{y},t)}_{B\text{ meson at }\vec{y}}$$

- different choice of Γ corresponds to different potentials (different couplings of B, B^*, B_0^* and B_1^*)
- use of gauge configurations generated by the ETMC corresponding to 3 different pion masses



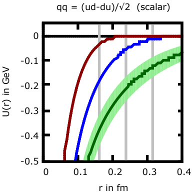
2 attractive ground state channels:

- spin scalar isosinglet with $I(J^P) = 0(1^+)$
- spin vector isotriplet $I(J^P) \in \{0(1^+), 1(1^+), 1(2^+)\}$

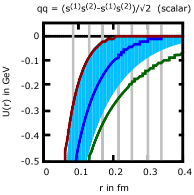
Fit an ansatz to the potentials:

$$V(r) = -\underbrace{\frac{\alpha}{r}}_{\text{Coulomb-like}} \; \underbrace{e^{-\left(\frac{r}{d}\right)^2}}_{\text{colour screening}}$$

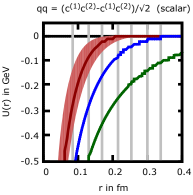
scalar isosinglet



u/d

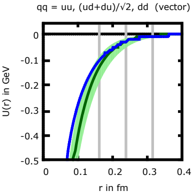


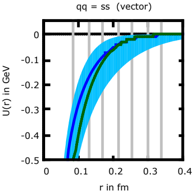
s



c

vector isotriplet





Observation: Potentials show more promise for binding the less massive the light quarks are! Strongest attraction for scalar isosinglet $qq = \frac{ud-du}{\sqrt{2}}$.

Solve Schrödinger's equation

- solve Schrödinger's equation for the radial component of the two $\bar{b}$ quarks:

$$\boxed{\left(-\frac{1}{2\mu}\frac{\mathrm{d}^2}{\mathrm{d}r^2} + V(r)\right) R(r) = E_B R(r) \quad , \qquad \psi(r) = R(r)/r}$$

- lowest eigenvalue $E_B < 0$ binding (4-quark bound state), $E_B > 0$ no binding (2 mesons)

Perform a large number of fits varying...

- the temporal separation at which the lattice potential is read off the correlation function
- the range at which the fit to the potential is performed

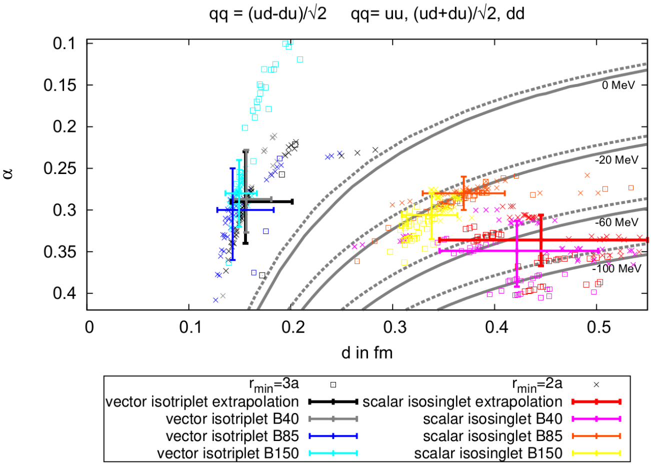
$\Longrightarrow$ Binding for light isosinglet channel only!

Most promising channel for a bound state: scalar $ud\bar{b}\bar{b}$

remember:

$$V(r) = -\frac{\alpha}{r} e^{-\left(\frac{r}{d}\right)^2}$$

Extrapolation to the physical quark mass: Binding increases



For the *BB* four-quark state with quantum numbers $I(J^P) = 0(1^+)$ we find $E_B = -90^{+43}_{-36}$ MeV at the physical point
$\longrightarrow$ binding with more than 2σ confidence level

Study of BB^* systems by means of NRQCD

A. Peters, P. Bicudo, L. Leskovec, S. Meinel and M. Wagner, "Lattice QCD study of heavy-heavy-light-light tetraquark candidates," arXiv:1609.00181 [hep-lat].

Work in progress.

The $I(J^P) = 0(1^+)$ $ud\bar{b}\bar{b}$ state with NRQCD

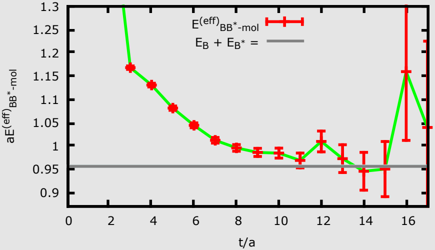
- strong evidence for a $ud\bar{b}\bar{b}$ bound state in the $I(J^P) = 0(1^+)$ channel from static-light approach
- to be confirmed with $\bar{b}$-quarks of finite mass instead of static quarks
  $\Rightarrow$ search for a bound state with nonrelativistic QCD (NRQCD)
- positions of $\bar{b}$ quarks not fixed
  $\Rightarrow$ computation of $V(r)$ not possible
  $\Rightarrow$ but direct computation of mass of lowest BB^* state in $I(J^P) = 0(1^+)$ channel possible
- $n_f = 2 + 1$ dynamical sea quarks, $m_\pi = 336$ MeV [arXiv:1409.0497]

The NRQCD correlator $\mathcal{O}_{BB^*}$:

$$\sum_{\vec{x}_1,\vec{x}_2,\vec{x}_1',\vec{x}_2'} \mathrm{e}^{i\vec{p}_1(\vec{x}_1-\vec{x}_1')}\mathrm{e}^{i\vec{p}_2(\vec{x}_2-\vec{x}_2')}\delta_{\vec{x}_1,\vec{x}_2}\delta_{\vec{x}_1',\vec{x}_2'}$$
$$\left(\bar{b}\gamma_5 d(\vec{x}_1)\bar{b}\gamma_i u(\vec{x}_2) - \bar{b}\gamma_5 u(\vec{x}_1)\bar{b}\gamma_i d(\vec{x}_2)\right)\left(\bar{d}\gamma_5 b(\vec{x}_1')\bar{u}\gamma_i b(\vec{x}_2') - \bar{u}\gamma_5 b(\vec{x}_1')\bar{d}\gamma_i b(\vec{x}_2')\right)$$

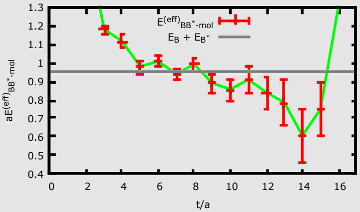
The $I(J^P) = 0(1^+)$ $ud\bar{b}\bar{b}$ state with NRQCD - preliminary results

- compute masses of B, B^* and BB^*
- if $m_{BB^*} < m_B + m_{B^*}$: bound state in $I(J^P) = 0(1^+)$ channel
- but...



Two four-quark operators

- For $I(J^P) = 0(1^+)$ there is another operator to be included to the analysis: $\mathcal{O}_{B^*B^*}$
- Build 2x2 correlation matrix using $\mathcal{O}_{BB^*}$ and $\mathcal{O}_{B^*B^*}$
- Lowest lying state lies below previous $m_B + m_{B^*}$ threshold!



$m^{(2x2)}_{BB^*} - (m_B + m_{B^*}) \approx -140$ MeV, i.e. strong indication that mass of the four-quark $ud\bar{b}\bar{b}$ is smaller than the sum of B and B^*

⟹ Qualitative confirmation of static-light result

$B\bar{B}$ systems in the static-light approach

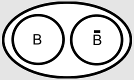
A. Peters, P. Bicudo, K. Cichy and M. Wagner, "Investigation of $B\bar{B}$ four-quark systems using lattice QCD," arXiv:1602.07621 [hep-lat].

Work in progress.

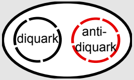
The $B\bar{B}$ system

There are several different structures possible for the experimentally relevant case of $I(J^P) = 1(1^+)$ (i.e. Z_b^+), e.g.

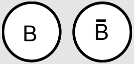
- A loosely bound $B\bar{B}$ pair, i.e. a mesonic molecule



- A bound 4-quark state made of a diquark and an antidiquark



- Two mesons: a B meson and a far separated $\bar{B}$ meson



- A bottomonium state and a far separated π^+, i.e. $Q\bar{Q} + \pi$

- **lightest state** is $Q\bar{Q} + \pi$
- **large** separations of static quarks: 2 mesons B and $\bar{B}$

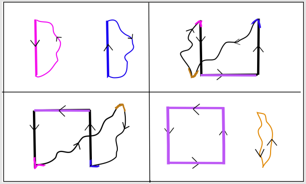
To separate these structures, we ...

1. implement a correlation matrix

$$C_{jk}(t) = \langle\Omega|\,\mathcal{O}_j^\dagger(t)\mathcal{O}_k(0)\,|\Omega\rangle \underset{\text{large } t}{\approx} A^0_{jk}\exp\left(-V_0(r)t\right) + A^1_{jk}\exp\left(-V_1(r)t\right) + \dots$$

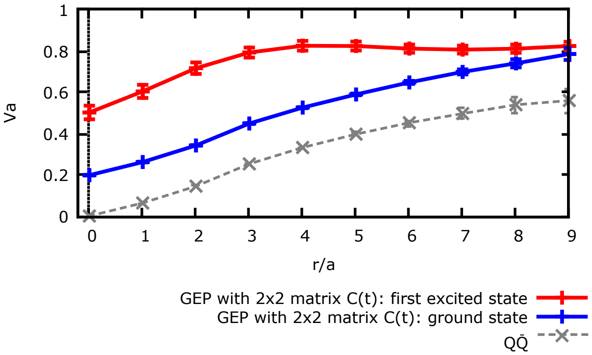
$$\mathcal{O}_0 \equiv \mathcal{O}_{B\bar{B}} = \Gamma_{AB}\tilde{\Gamma}_{CD}\,\bar{Q}^a_C(\vec{x})q^a_A(\vec{x})\bar{q}^b_B(\vec{y})Q^b_D(\vec{y})$$

$$\mathcal{O}_1 \equiv \mathcal{O}_{Q\bar{Q}+\pi} = \tilde{\Gamma}_{AB}\,\bar{Q}^a_A(\vec{x})U^{ab}(\vec{x},t;\vec{y},t)Q^b_B(\vec{y})\sum_{\vec{z}}\bar{q}^c_C(\vec{z})\left(\gamma_5\right)_{CD}q^c_D(\vec{z})$$

$C(t) =$ 

2. extract the potentials with the Generalized Eigenvalue Problem (GEP).

Preliminary results I



Potentials obtained

- $Q\bar{Q} + \pi$: ground state (blue)
- first excited state of the 2x2 matrix: free of contributions of $Q\bar{Q} + \pi$ (red)

Preliminary results II

Binding energy

- Identical analysis as in the BB case (Schrödinger's equation) yields for quantum numbers $I(J^P) = 1(1^+)$ (i.e. Z_b^+):

$$E_B = (-58 \pm 71)\mathrm{MeV}$$

- vague indication for a $\bar{u}d\bar{b}b$ bound state

Summary BB systems

- BB systems are experimentally hard to prepare, but theoretically easier to investigate
- For a BB system with light quarks $qq = ud$ and static $\bar{b}$ quarks with quantum numbers $I(J^P) = 0(1^+)$ we find $E_B = -90^{+43}_{-36}$ MeV
  $\longrightarrow$ binding with more than 2σ confidence level
- This result is supported by preliminary computations with 4 quarks of finite mass.
- next steps: repeat computations with smaller lattice spacing, investigate resonances

Summary $B\bar{B}$ systems

- $B\bar{B}$ systems are experimentally rather easy to access, but theoretically more challenging.
- Candidate for a binding $B\bar{B}$ state with $I(J^P) = 1(1^+)$ (i.e. Z_b^+) is currently investigated, we find $E_B = (-58 \pm 71)$ MeV
- next steps: analysis of possible bound state (binding energy, structure, "life time")

Backup

Meson content of a four-quark state

- One can extend the meson content of the four-quark states by application of the following light quark projectors on $\mathcal{O}$:
  - Parity projectors: $\mathcal{P}_{P=+} = \frac{1+\gamma_0}{2}$ and $\mathcal{P}_{P=-} = \frac{1-\gamma_0}{2}$
  - Spin projectors: $\mathcal{P}_{j_z=\uparrow} = \frac{1+i\gamma_0\gamma_3\gamma_5}{2}$ and $\mathcal{P}_{j_z=\downarrow} = \frac{1-i\gamma_0\gamma_3\gamma_5}{2}$
- $P = -$: S state, meson in the ground state
- $P = +$: P state, first excitation
- $\uparrow\downarrow$: light quark angular momentum
- An example for $\mathcal{O}_{B\bar{B}}$:
  - $\Gamma = \gamma_5 \quad \hat{=} \quad +S\uparrow S\uparrow + S\downarrow S\downarrow + P\uparrow P\uparrow + P\downarrow P\downarrow$
  - $\Gamma = \gamma_0\gamma_5 \quad \hat{=} \quad -S\uparrow S\uparrow - S\downarrow S\downarrow + P\uparrow P\uparrow + P\downarrow P\downarrow$
  - Therefore a B$\bar{B}$ state with $\Gamma = \gamma_5 - \gamma_0\gamma_5$ only contains S mesons.

Lattice Setups

BB and $B\bar{B}$ in the static-light approach:

Ens.	β	lattice	$a\mu$	m_π[MeV]	a [fm]	L [fm]	confs
B40.24	3.90	$24^3 \times 48$	0.0040	340	0.0790(26)	1.9	480
B85.24	3.90	$24^3 \times 48$	0.0085	480	0.0790(26)	1.9	400
B150.24	3.90	$24^3 \times 48$	0.0150	650	0.0790(26)	1.9	260

Gauge configurations generated by ETMC

BB^* with NRQCD:

Ens.	β	lattice	$am_{u,d}$	am_s	m_π[MeV]	a [fm]	L [fm]	confs
C54	2.13	$24^3 \times 64$	0.005	0.04	336	0.1119(17)	2.7	1676

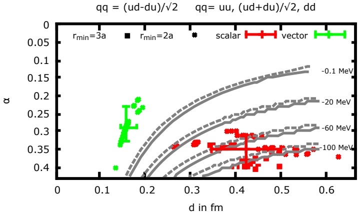
Gauge configurations generated by RBC and UKQCD collaborations

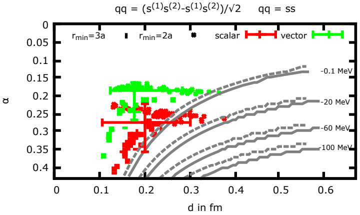
Different attractive BB channels

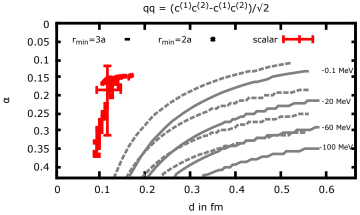
- spin scalar isosinglet:
  - qq spin $j_z = 0$
  - antisymmetric flavour $qq \in \{(ud - du)/\sqrt{2}, (s^1 s^2 - s^2 s^1)/\sqrt{2}, (c^1 c^2 - c^2 c^1)/\sqrt{2})\}$
  - $I(J^P) = 0(1^+)$
- spin vector isotriplet:
  - qq spin $j_z = 1$
  - symmetric flavour $qq \in \{uu, (ud + du)/\sqrt{2}, dd, ss, cc\}$
  - $I(J^P) \in \{1(0^+), 1(1^+), 1(2^+)\}$

Perform a large number of fits varying...

- the temporal separation at which the lattice potential is read of the correlation function
- the range at which the fit to the potential is performed



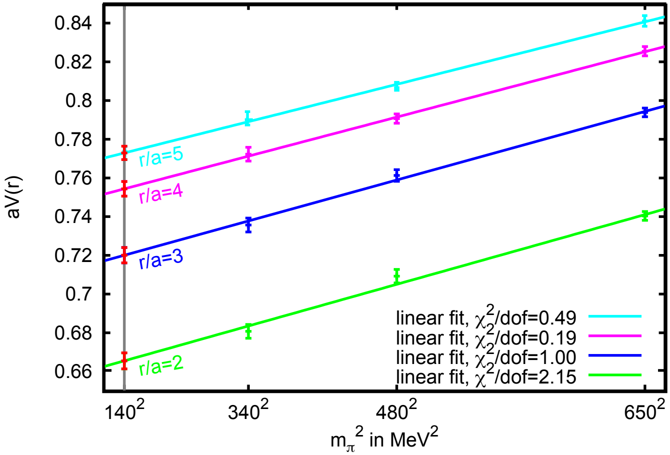




Binding for light isosinglet channel only!

Extrapolation to the physical pion mass

Example plots for a t-range $[4a \ldots 9a]$



The GEP

1. Build a matrix $C(t)$ of correlation functions $C_{ij}(t)$
2. Solve the GEP:

$$C_{jk}(t)v_k^{(n)}(t,t_0) = \lambda^{(n)}(t,t_0)C_{jk}(t_0)v_k^{(n)}(t,t_0)$$

3. And find:

$$m_{\mathrm{eff}}^{(n)}(t,t_0) = \lim_{t\to\infty}\frac{1}{a}\log\frac{\lambda^{(n)}(t,t_0)}{\lambda^{(n)}(t+a,t_0)}$$

Choice of Γ for the $B\bar{B}$ system

- The choice of the matrix Γ is constrained by the quantum numbers of the $\mathcal{O}_{Q\bar{Q}+\pi}$
- Only taking into account $\mathcal{O}_{B\bar{B}}$ we find the strongest attraction for $\Gamma = \gamma_5 - \gamma_0\gamma_5$