Landau gauge Yang-Mills correlation functions

Anton Konrad Cyrol

Ruprecht-Karls-Universität Heidelberg

based on

- AKC, Fister, Mitter, Pawlowski, Strodthoff, PRD, arXiv:1605.01856 [hep-ph]
- AKC, Mitter, Strodthoff, FormTracer, arXiv:1610.09331 [hep-ph]
- AKC, Mitter, Pawlowski, Strodthoff, $N_f = 2$ Vacuum QCD, in preparation
- AKC, Mitter, Pawlowski, Strodthoff, T > 0 Yang-Mills, in preparation

November 9, 2016

AKC (U Heidelberg)

fQCD-collaboration:

J. Braun, L. Corell, <u>AKC</u>, L. Fister, W. J. Fu, M. Leonhardt, <u>M. Mitter</u>, <u>J. M. Pawlowski</u>, M. Pospiech, F. Rennecke, <u>N. Strodthoff</u>, N. Wink, ...

This talk:

- Vacuum Yang-Mills theory
- Preliminary T > 0 results

Aim:

- Qualitative understanding
- Quantitative precision

Motivation:

- No sign problem
- Understanding of confinementation

Prerequisite for QCD



Schaefer and Wagner, Prog.Part.Nucl.Phys. 62 (2009) 381

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QCD from the functional renormalization group

- Only perturbative QCD input
 - $\alpha_{S}(\mu = \mathcal{O}(10) \text{ GeV})$
 - $m_a(\mu = \mathcal{O}(10) \text{ GeV})$
- Wetterich equation with initial condition $S[\Phi] = \Gamma_{\Lambda}[\Phi]$

- Effective action Γ[Φ] = lim_{k→0} Γ_k[Φ]
- Exact equation
- ∂_t : integration of momentum shells controlled by regulator
- Full field-dependent equation with $(\Gamma^{(2)}[\Phi])^{-1}$ on rhs

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Vertex expansion

• Approximation necessary – vertex expansion:

$$\Gamma[\Phi] = \sum_{n} \int_{\rho_1,\ldots,\rho_{n-1}} \Gamma^{(n)}_{\Phi_1\cdots\Phi_n}(\rho_1,\ldots,\rho_{n-1}) \Phi^1(\rho_1)\cdots\Phi^n(-\rho_1-\cdots-\rho_{n-1})$$

- Wanted: "apparent convergence" of $\Gamma[\Phi] = \lim_{k \to 0} \Gamma_k[\Phi]$
- Current state-of-the-start truncation:

• Functional derivatives of $\Gamma_k[\Phi]$ with respect to fields yield equations

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Truncation - closed set of equations



Truncation – closed set of equations



Truncation - closed set of equations



FormTracer – Mathematica tracing package using FORM

- Mathematica: very powerful, flexible and convenient
- FORM: very fast and efficient

FormTracer uses FORM while it keeps the usability of Mathematica:

- Lorentz/Dirac traces in arbitrary dimensions
- Arbitrary number of group product spaces
- Intuitive, easy-to-use and highly customizable Mathematica frontend
- Support for finite temperature/density applications
- Support for FORM's optimization algorithm
- Convenient installation and update procedure within Mathematica:

Preprint: AKC, Mitter, Strodthoff; arXiv:1610.09331 [hep-ph] Open source: https://github.com/FormTracer/FormTracer

FormTracer – installation and usage

```
FormTracer.nb - Wolfram Mathematica 11.0
                                                                                                                        ×
File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
   Installing
        Import["https://raw.githubusercontent.com/FormTracer/FormTracer/master/src/FormTracerInstaller.m"]
   Tracing
      Space-Time
        Define syntax for space-time
        DefineLorentzTensors[δ[μ, ν] (*Kronecker delta*), vec[p, μ] (*vector*), p.q(*inner product*)];
        Take traces:
        FormTrace[vec[p + 2 r, \mu] \delta[\mu, \nu] vec[s, \nu]]
        FormTrace[\delta[\alpha, \nu] (\delta[\nu, \rho] + \delta[\nu, \rho] \delta[\sigma, \sigma]) \delta[\rho, \alpha]]
        FormTrace [\delta[1, v] \text{ vec}[s, v]]
        s.(p + 2r)
        20
        vec[s, 1]
```

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Truncation - closed set of equations



Regulator breaks BRST symmetry

- \bullet Breaking BRST symmetry \rightarrow modified STIs
- mSTIs reduce to STIs at k = 0
- \implies solve mSTIs to get initial action at $k = \Lambda$
- More practical solution: choose $\Gamma_{\Lambda} \approx S$ such that STIs are fulfilled k = 0

stls mSTls
$$k \rightarrow 0$$

$$\alpha_{A\bar{c}c}(p) = \frac{\alpha(\mu)}{4\pi} \frac{Z_{A\bar{c}c}^2(p)}{Z_A(p) Z_c^2(p)}$$
$$\alpha_{A^3}(p) = \frac{\alpha(\mu)}{4\pi} \frac{Z_{A^3}^2(p)}{Z_A^3(p)}$$
$$\alpha_{A^4}(p) = \frac{\alpha(\mu)}{4\pi} \frac{Z_{A^4}(p)}{Z_A^2(p)}$$

Select

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$$Z_{A^3}^{k=\Lambda}(p) = \text{const.}$$
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such that

$$\alpha_{A\bar{c}c}(\mu) = \alpha_{A^3}(\mu) = \alpha_{A^4}(\mu)$$

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Running couplings (scaling solution)



Gluon mass gap

Scaling solution $\lim_{p \to 0} Z_c(p^2) \propto (p^2)^{\kappa}$ $\lim_{p \to 0} Z_A(p^2) \propto (p^2)^{-2\kappa}$ Decoupling solution $\lim_{p \to 0} Z_c(p^2) \propto 1$ $\lim_{p \to 0} Z_A(p^2) \propto (p^2)^{-1}$

• Landau Gauge gluon STI requires longitudinally mass term to vanish:

$$p_{\mu}\left([\Gamma^{(2)}_{AA,\mathrm{L}}]^{ab}_{\mu
u}(p)-[S^{(2)}_{AA,\mathrm{L}}]^{ab}_{\mu
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ight)=0$$

- Splitting between longitudinal and transverse mass term necessary
- Splitting occures "naturally" for scaling solution
- Decoupling solution requires irregular vertices, e.g. a pole in the longitudinal sector
- Unphysical mass parameter at cutoff: $[\Gamma^{(2)}_{AA}]^{ab}_{\mu\nu}(k=\Lambda, p) = p^2 + m_{\Lambda}^2$,
- $m_{\Lambda}^2 \propto c \cdot \Lambda^2$ with $c \ll 1$; parameter can be uniquely determined

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Gluon Mass Gap

Dynamical mass generation



Propagators

Truncation dependence of the gluon propagator



Truncation dependence of the gluon propagator



Truncation dependence of the gluon propagator



Propagators

Truncation dependence of the gluon propagator



AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016; Lattice: Sternbeck et al. 2006

Gluon propagator dressing



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Propagators

Gluon propagator



AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016; Lattice: Sternbeck et al. 2006

Gluon propagator maximum over UV mass parameter



AKC (U Heidelberg)

Ghost propagator dressing



AKC (U Heidelberg)

Three-gluon vertex dressing (symmetric point)



AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016

AKC (U Heidelberg)

Finite temperature

Going to finite temperature:

• Introduce Matsubara frequencies:

$$\int \frac{\mathrm{d}^4 p}{(2\pi)^4} \to T \sum_{\omega_n} \int \frac{\mathrm{d}^3 p}{(2\pi)^3}$$

- Thermal Debye mass
- Same parameter-free truncation as in vacuum YM
- Upcoming: full splitting of magnetic and electric components Splitting of propagators only: Fister, Pawlowski, 2011

$$P_{\mu\nu}^{\mathsf{T}}(p) = (1 - \delta_{0\mu})(1 - \delta_{0\nu}) \left(\delta_{\mu\nu} - \frac{p_{\mu}p_{\nu}}{\bar{p}^2}\right) \qquad \qquad P_{\mu\nu}^{\mathsf{L}}(p) = \left(\delta_{\mu\nu} - \frac{p_{\mu}p_{\nu}}{\bar{p}^2}\right) - P_{\mu\nu}^{\mathsf{T}}(p)$$

• Also upcoming: nonzero Matsubara modes

Following results are preliminary and based on

• AKC, Mitter, Pawlowski, Strodthoff, T > 0 Yang-Mills, in preparation

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Temperature dependence of the gluon propagator

Magnetic component compared to averaged components from FRG:



Temperature dependence of vertices



A glimpse at unquenched $N_f = 2$ QCD



AKC, Mitter, Pawlowski, Strodthoff, in preparation

Preliminary unquenched gluon propagator



FRG: AKC, Mitter, Pawlowski, Strodthoff, in preparation

Lattice: A. Sternbeck, K. Maltman, M. Müller-Preussker, L. von Smekal, PoS LATTICE2012 (2012) 243

Preliminary unquenched quark propagator



FRG: AKC, Mitter, Pawlowski, Strodthoff, in preparation

Lattice: Oliveira, Kızılersu, Silva, Skullerud, Sternbeck, Williams, arXiv:1605.09632 [hep-lat]

Conclusion

- FRG first principal approach to QCD, complementary to lattice QCD
- $\bullet~{\sf Big}$ numerical effort $\rightarrow~{\sf tools}$ like FormTracer necessary
- BRST symmetry is broken by regulator, proper care needs to be taken
- STI consistent solution computed
- Evidence for dynamical mass generation
- Very good agreement with lattice results

Outlook

- Unquenched $N_f = 2$ QCD, in preparation
- T > 0 YM with splitting of el. and mag. components, in preparation
- Bound states (Bethe-Salpeter eq.), decay widths, ...
- Nonzero Matsubara modes, gluon spectral function, ...

Thank you for your attention!

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Running couplings in comparison with DSE results



Running of the gluon mass parameter



AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016

Gluon propagator



AKC (U Heidelberg)

Ghost propagator dressing



Momentum dependence of the ghost-gluon vertex



AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016

Ghost-gluon vertex

Ghost-gluon vertex at the symmetric point



Ghost-gluon vertex with vanishing gluon momentum



ANC, I ISLEI, MILLEI, FAMIOWSKI, SLIDULI

Ghost-gluon vertex with orthogonal momenta



Momentum dependence of the three-gluon vertex



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Three-gluon vertex at the symmetric point



AKC (U Heidelberg)

Three-gluon vertex with vanishing gluon momentum



Three-gluon vertex with orthogonal momenta



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Momentum dependence of the four-gluon vertex



AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016

Four-gluon vertex at the symmetric point



AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016

Regulator dressing

